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Reverse transition to hydrodynamic stability through the Schwarzschild line in a supercritical fluid layer

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We consider a fluid close to its gas-liquid critical point in a bottom-heated cavity. Due to strong density stratification, both the Schwarzschild and the Rayleigh stability criteria are relevant at the same space scale. Taking advantage of the competition between these two limits of the convection-onset criterion, we numerically exhibit striking non-Boussinesq effects: the reverse transition to stability through the Schwarzschild line of a heat diffusing layer subjected to convection, and the convection onset inside a few-millimeters-thick layer according to the Schwarzschild criterion.

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When approaching the gas-liquid critical point, several physical properties of a fluid exhibit strong anomalies. Particularly, the thermal diffusivity vanishes while the isothermal compressibility and the thermal expansion coefficient tend to infinity. These specific properties lead to a puzzling heat transfer mechanism: the piston effect (PE) responsible for the fast temperature equilibrium in a supercritical fluid (SCF) [1]. In the late 1990s, the interaction between natural convection and the PE prompted many teams [2–5] to lean towards the Rayleigh-Bénard configuration. This configuration turned to be a quite interesting system to study hydrodynamic stability because, for the SCF, the convection threshold is governed by a criterion that takes into account two competitive terms, which, for a normally compressible fluid, are separately relevant at very different space scales: The term corresponding to the classical Rayleigh criterion deriving from the Boussinesq approximation in which the compressibility of a fluid is completely neglected (thus valid at small and intermediate space scales), and the term corresponding to the Schwarzschild criterion (or adiabatic temperature gradient criterion), which usually plays at large atmospheric scales where the effect of the hydrostatic pressure is prominent. Indeed, the results of the theoretical analysis, obtained in the 1970s by Gitterman and Steinberg [6], showed that, owing to the divergence of the isothermal compressibility of the SCF, the classical Rayleigh criterion should be modified to take into account the fluid stratification and thus including the Schwarzschild criterion. Experimental [2,3] and numerical [4,5] studies confirmed the relevance of the Schwarzschild contribution. Indeed, these works show, among others, the existence of a cut-off temperature difference inside the fluid layer under which no convective motion is triggered no matter how thick the layer becomes. Moreover, in a thorough numerical work [4], the authors suspected that, due to the competition between the two stability criteria, a diffusing SCF layer subjected to convection might regain stability through the Schwarzschild line without any external intervention; this long-awaited result turns now to be a fact. This paper has a twofold goal. We show first how the Schwarzschild contribution can have a stabilizing effect leading to a reverse transition to stability (RTS) of a SCF diffusing layer subjected to convection. Then, we show how convection may be triggered in a few-millimeters-thick layer according to the Schwarzschild criterion usually encountered for large air columns. This event, observed experimentally by Kogan and Meyer [2] by reducing the critical point proximity, is exhibited here in completely different conditions with the information that only a numerical solution can provide.

A SCF is enclosed in a rectangular two-dimensional (2D) cavity of height $H=15$ mm and width $L=10$ mm, and subjected to earth gravity g (Fig. 1). Periodic lateral boundaries are considered in order to simulate the infinite fluid layer used in theoretical studies [6]. The bottom and top boundaries are both rigid and no-slip walls. Initially, the fluid is at rest, in thermodynamic equilibrium at a constant temperature T_i slightly above the critical temperature T_c such that $T_i=(1+\varepsilon)T_c$, $\varepsilon \ll 1$. The fluid is stratified in pressure and density with a mean density equal to its critical value ρ_c. The numerical simulation starts by increasing the bottom plate temperature by ΔT ($\Delta T \sim$ few mK) while maintaining the top plate at its initial temperature T_i.

In addition to the classical Navier-Stokes equations, the

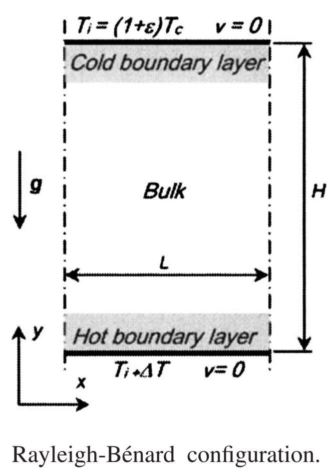

FIG. 1. The Rayleigh-Bénard configuration. The temperature field structured by the PE.

mathematical model consists of the energy and state equations written for a Newtonian van der Waals fluid [7],

$$\text{Energy equation: } \frac{\partial(\rho C_V T)}{\partial t} + \nabla(\rho C_V v T) = -(P + a\rho^2)(\nabla v) + \nabla(\lambda \nabla T) + \Phi,$$

$$\text{Equation of state: } P = \frac{\rho R T}{1 - b\rho} - a\rho^2,$$

where t is the time and v is the velocity. P, T, and ρ, are, respectively, the pressure, the temperature, and the density. Φ is the viscous dissipation function. R is the perfect gas constant, a and b are, respectively, the energy parameter and the covolume calculated from the critical coordinates: $a = 9RT_c/(8\rho_c)$ and $b = 1/(3\rho_c)$. A $(T/T_c - 1)^{-0.5}$ law is used to describe the critical divergence of the thermal conductivity λ, while the heat capacity at constant volume C_V and the dynamic viscosity μ are supposedly equal to those of a perfect gas, C_{V0} and μ_0, respectively. We consider the CO_2 critical coordinates ($T_c = 304.13$ K, $\rho_c = 467.8$ Kg/m^3), and transport properties ($C_{V0} = 472.2$ J/Kg/K, $\mu_0 = 3.44 \times 10^{-5}$ Pa s). The simulations were carried out for $T_i - T_c = 1$ K.

A fully implicit finite-volume method is used to solve the governing equations in a low Mach number approximation adapted to the SCF buoyant flows, which, by accounting for the fluid compressibility with respect to the hydrostatic pressure, is essential for a correct prediction of the convection onset induced by a weak heating in a SCF [8]. The method is second-order accurate in space and third-order in time and has been thoroughly validated [9]. The mesh size for a grid-independent solution depends on the heating applied to the bottom plate, the finest being of 100×140 nodes. The mesh is refined near the walls while it is uniform in the horizontal direction.

The bottom heating induces a thin hot boundary layer (HBL). Due to the high thermal expansion coefficient of SCF, this hot layer expands upward compressing adiabatically the rest of the fluid leading to a fast and homogeneous increase of the temperature in the bulk of the cavity by thermoacoustic effects. This sequence of events is the so-called PE. Since the top plate temperature is kept constant, a cold boundary layer settles along the top plate (Fig. 1). The thermal boundary layers grow with time at the heat diffusion speed; when the local Rayleigh number Ra, based on the HBL thickness, h, and on the temperature difference inside it, δT, exceeds the critical value $\text{Ra}_c = 1100.6$, the layer becomes unstable. The convection-onset criterion of a bottom-heated SCF layer [6] is given by

$$\text{Ra} = \frac{g h^4 \beta \rho_c C_P}{\lambda \eta}\left[\frac{\delta T}{h} - \frac{g T_i \beta}{C_P}\right] > \text{Ra}_c, \tag{1}$$

where β, η, and C_P are, respectively, the thermal expansion coefficient, the kinematic viscosity, and the heat capacity at constant pressure. The term $gT_i\beta/C_P$ is the temperature gradient obtained by moving adiabatically a fluid particle along the hydrostatic pressure gradient; it represents the contribution of the Schwarzschild criterion according to which convection arises in a fluid layer when

$$\frac{\delta T}{h} > \frac{g T_i \beta}{C_P}. \tag{2}$$

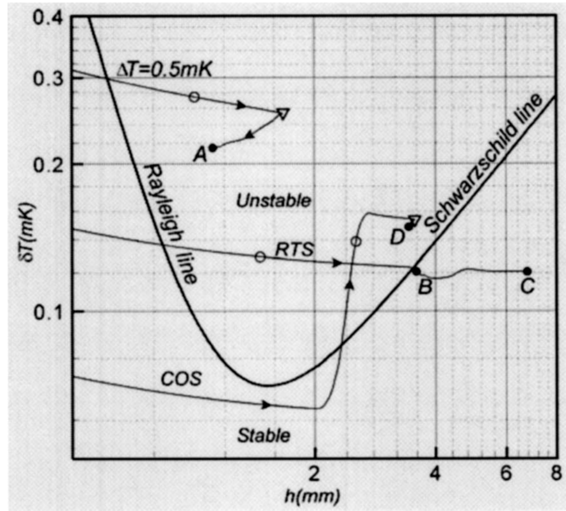

FIG. 2. Evolution of the temperature difference δT in the HBL as a function of this latter thickness h. The symbols (○) and (▽) represent, respectively, the time when the intensity of the bottom plate vortices increases exponentially, and the time when the HBL collapses. The points A, B, C, and D correspond to the snapshots of the temperature field given in Fig. 5.

In Fig. 2, the critical value of δT for the convection onset is derived from Eq. (1) and plotted vs h (the thick solid line). This neutral stability curve consists of two lines representing the limits of the convection-onset criterion [Eq. (1)] depending on h. For small values of h, the fluid compressibility can be neglected and the HBL stability is governed by the classical Rayleigh criterion, obtained from Eq. (1) by dropping the term $gT_i\beta/C_P$, while for larger values of h, viscosity and thermal diffusion are neglected, and the stability depends on the Schwarzschild criterion given by Eq. (2). As the HBL thickens with time, the evolution curve $\delta T(h)$ will cross the Rayleigh line first, which prevents the convection onset from occuring according to the Schwarzschild criterion. But Fig. 2 suggests that, after crossing the Rayleigh line, a RTS through the Schwarzschild one might occur if the HBL kept growing in the unstable zone.

However, this RTS cannot be obtained for any heating; indeed, let us consider the case of $\Delta T = 0.5$ mK. For a unique definition valid before and after the convection onset, the thickness of the HBL for this heating is evaluated as the distance from the bottom plate where the local temperature gradient reaches the global one $\Delta T/H$. In all that follows, h is the average thickness of the HBL in the horizontal direction. As the evolution curve $\delta T(h)$ (shown in Fig. 2) crosses the Rayleigh line, the HBL becomes unstable, and the infinitesimal perturbations are amplified into a number of convective cells taking place along the bottom plate. The instant corresponding to the symbol (○) on the curve is followed by an exponential increase of the vortices intensity; it is easily

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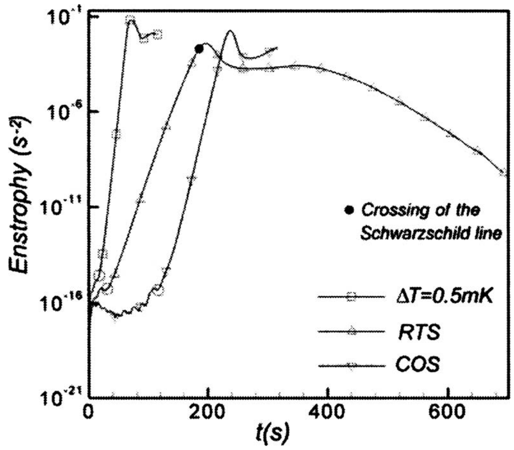

FIG. 3. Time evolution of the enstrophy field average in the lower half of the cavity. The point denoted (○) shows the beginning of the exponential increase lasting until convection deforms the isotherms.

identified on the time evolution of the enstrophy (or squared vorticity vector modulus) field average in the lower half of the cavity shown in Fig. 3. The intensity of these vortices increases until it produces an enough amount of convective transfer to deform the isotherms with several thermal plumes rising from the HBL and causing this latter to collapse. Represented by the symbol (▽) in Fig. 2 and more clearly illustrated in Fig. 4, this collapse of the HBL prevents the evolution curve $\delta T(h)$ from reaching the Schwarzschild line. Figure 5(a) shows the temperature field corresponding to point (A) in Fig. 2. The convection-dominated flow that follows the collapse of the HBL is three dimensional [10]; however, the 2D approximation remains a good approach as far as the onset of convection is sought.

In order to cross the Schwarzschild line, two scenarios are considered. (i) For a certain weak heating, the convective transfer would deform the isotherms without causing the collapse of the HBL; this latter would then keep growing to stabilize again after crossing the Schwarzschild line. (ii)

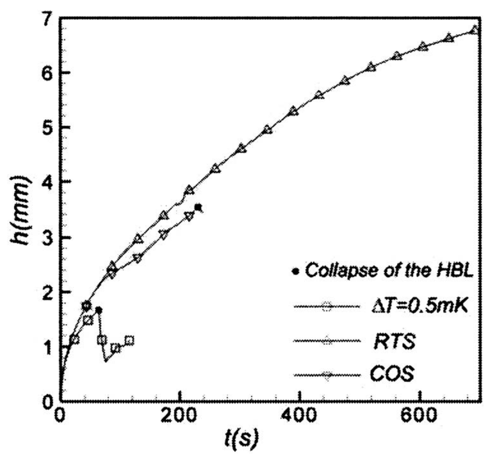

FIG. 4. Time evolution of the HBL thickness. For the RTS and the COS scenarios, the HBL thickness increases continuously whereas, for $\Delta T=0.5$ mK, it brutally decreases when thermal plumes emerge from the HBL, causing its collapse.

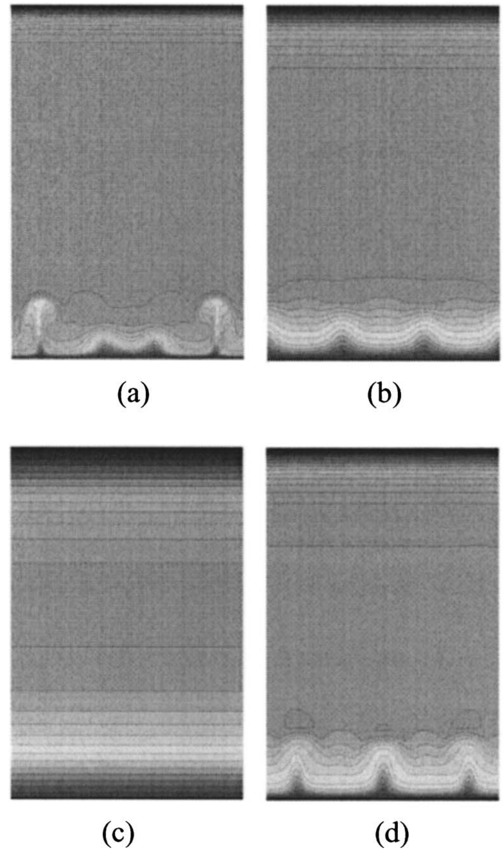

FIG. 5. Snapshots of the temperature field. (a), (b), (c), and (d) correspond, respectively, to points A, B, C, and D in Fig. 2. (a) shows the convection onset for $\Delta T=0.5$ mK ($t=69.2$ s). (b) and (c) show, respectively, for $\Delta T=0.24$ mK the destabilization ($t=196.9$ s) and the RTS ($t=692.3$ s). (d) shows the COS ($t=238.6$ s).

Starting with an even weaker heating, the curve $\delta T(h)$ would cross under the unstable zone; when its ending point gets under the Schwarzschild line, the temperature of the bottom plate might be increased and the Schwarzschild line would be crossed from below. For both of these scenarios, the height of the cavity must be large enough to allow the growth of the thermal boundary layers, and the choice $H=15$ mm is a compromise between this requirement and the mesh size. This point forward, we use the classical definition of the thermal boundary layer thickness, i.e., the distance from the bottom plate where the local temperature reaches the bulk one with a relative difference less than 1%.

A RTS through the Schwarzschild line was obtained for $\Delta T \leqslant 0.24$ mK; we consider here the case $\Delta T=0.24$ mK. Once the evolution curve $\delta T(h)$ has crossed into the unstable zone (Fig. 2), convective cells start to get organized along the bottom plate, and at point denoted by (○) in Figs. 2 and 3, the intensity of these vortices rises exponentially with time to reach a maximum and then it starts to decrease once $\delta T(h)$ has crossed the Schwarzschild line towards the stable zone (Fig. 3). When this maximum is reached, the most intense deformation of the isotherms is obtained. However, this deformation, shown in Fig. 5(b) and corresponding to point (B) in Fig. 2, is not large enough to induce the collapse of the

HBL, which keeps growing, as shown in Fig. 4, and the curve $\delta T(h)$ remains in the stable zone. Once the convective motion is sufficiently damped out, the isotherms distortion in the HBL disappears as shown in Fig. 5(c), which corresponds to point (C) in Fig. 2.

The convection onset according to the Schwarzschild criterion (COS) requires two time-phases heating process. In the first phase, a constant heating $\Delta T = 0.12$ mK is applied to the bottom plate. For this weak heating, the evolution curve $\delta T(h)$, shown in Fig. 2, remains in the stable zone. This first phase is maintained as long as necessary to bring the ending point of $\delta T(h)$ under the Schwarzschild line; it lasts about 58 s. The second heating phase consists in increasing ΔT up to 0.3 mK in 87 s according to a cosine law. This new heating induces, inside the HBL, a new thermal boundary layer that, by expanding, induces a new PE that heats up homogeneously the bulk phase. On one hand, the second heating is slow enough to prevent the convection onset inside the new thermal boundary layer according to the Rayleigh criterion, and to merge this new layer into the previously formed HBL. On the other hand, despite the diminishing factors of the temperature gradient inside the HBL (the growth of the HBL and the increase of the bulk temperature by the new PE), the bottom heating is fast enough so that the temperature gradient inside the HBL exceeds the adiabatic one [Eq. (2)]. Once the evolution curve $\delta T(h)$ has crossed the Schwarzschild line, convective cells appear in the whole thickness of the HBL (Fig. 6), and not inside the thermal boundary layer induced by the second heating phase. At point denoted (○) in Figs. 2 and 3, the intensity of these vortices increases exponentially with time, deforming the isotherms by a number of

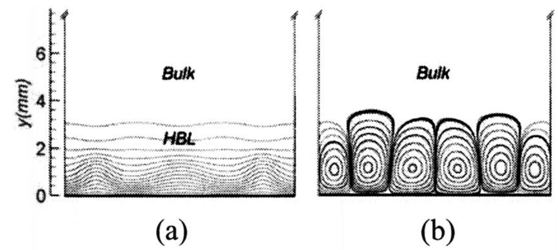

FIG. 6. Isotherms (a) and streamlines (b) during the COS scenario (t=226.4 s), showing that convection arises in the whole thickness of the HBL.

convective plumes and causing the collapse of the HBL. The temperature field after this collapse, at t=238.6 s [corresponding to point (D) in Fig. 2], is shown in Fig. 5(d).

Owing to its high compressibility, the SCF allows in a same cell the interaction between the Rayleigh and the Schwarzschild stability criteria, an interaction that is impossible in a normally compressible fluid. In this paper, we showed how the competition of these two limits of the convection-onset criterion yields to a unique hydrodynamic behaviour of a SCF: the RTS of a heat diffusion layer subjected to convection without any external intervention. We also showed how convection can be triggered in a few-millimeters-thick SCF layer according to the Schwarzschild criterion, which might be of some help in simulating atmosphericlike phenomena at small space scale.

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