Centrifugal force model for pedestrian dynamics

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In this paper, a centrifugal force model is developed for pedestrian dynamics. The effects of both the headway and the relative velocity among pedestrians are taken into account, which can be expressed by a "centrifugal force" term in dynamic equation. The jamming probability due to the arching at exits for crowd flows is provided. A quantitative analysis of the crowd flowing out of a hall shows that the average leaving time T is a function of the exit width W in negative power. The related simulation indicates that the proposed model is able to reproduce the self-organization phenomena of lane formation for sparse flows.

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I. INTRODUCTION

Pedestrian flow has been studied since the 1950s [1,2] and has attracted considerable attention in recent years. It is of importance to know the characteristics of pedestrian flow in some situations. Especially, it will be helpful to avoid the jamming state of panicking pedestrians flowing out of a hall or a room due to the arching and clogging at exits, where many tragic disasters often occur [3,4]. These pedestrian problems have been investigated with various models, including macroscopic and microscopic ones.

In macroscopic models [5,6], the interaction among individual pedestrians has not been considered in detail, which is thus not well suited for the prediction of pedestrian flow in pedestrian areas or buildings with obstacles [7].

Since microscopic models can be used to depict the detailed behavior of pedestrian flows, they have drawn more attention recently, among which are the cellular automata (CA) models [8–11], the social force model [12–14] and the magnetic force model [15].

In CA models, time, space and state variables are all discrete, which makes them ideally suitable for high-performance computer simulations [11]. Most of previous work focused on the occurrence of a jamming transition as the density of pedestrians increases. Although the CA models are based on behavioral rules prescribing walking characteristics, adapting the CA models to describing multidirectional pedestrian flow is complicated for several reasons, in particular, due to the complexity in the appropriate gridlike partition of the walking area [16].

The social force model, developed by Helbing and his co-workers, has some remarkable advantages [12–14]. The social force model is able to reproduce some observed features of pedestrian traffic flow, such as lane formation, due to the interaction of pedestrians. Nevertheless, it was argued that some of the underlying model assumptions oversimplified the process of pedestrians way-finding through the traffic flow [16]. The model consists mainly of three terms, which correspond to the acceleration towards the desired velocity of motion, the repulsive and attractive interactions with other pedestrians or obstacles and the interaction with walls and other rigid objects for a certain pedestrian moving toward the destination.

The magnetic force model was developed by Okazaki [15] and his co-workers. It was assumed that the pedestrian movement is caused by a kind of repulsive force analogous to magnetic force.

In real life, aggressive pedestrians will squeeze through those in front, if the adjustment of velocity according to the repulsive effects exerted by their neighboring walkers remain uncomfortable. This is dangerous, especially in panic situations, which nearly all pedestrians will try to squeeze through preceding pedestrians and resulted in arching and clogging at exits. However, in our study, it is shown that the jamming probability is a function of the door size and the occupancy of pedestrians. We also find that if the opening size is over 5 times of the diameter of a pedestrian, the jamming probability is close to zero whatever the occupancy is. In other words, due to the psychological response to ambience in walk, pedestrians adjust their velocities by observing the headway to others. Hence, it is natural to take the relative velocity effects into account.

In this paper, we develop a "centrifugal force" model, which takes a kind of centrifugal force as a repulsive force, with the consideration of the effects of relative velocity and the headway. The collision detection technique is used in simulation. The particular advantages of this approach include appropriate consideration of aggressive pedestrians, who squeeze through preceding pedestrians and the relative velocity effects on pedestrian behavior. Numerical simulations indicate that the nonlinear behavior in pedestrian dynamics, such as lane formation for sparse flows and arching and clogging at exits for crowd flows, can be well predicted with the presented centrifugal force model.

II. OUTLINE OF THE "CENTRIFUGAL FORCE" MODEL

In the following, the main effects that govern the motion of a pedestrian i will be examined.

A. Centrifugal force between pedestrians

The adjustment of a pedestrian's velocity is affected by neighboring pedestrians. In our model, the relative velocity effects are taken into account. We assume that those faster preceding pedestrians will not have repulsive effects on those behind them. Since the relaxation time of pedestrians is very short, a pedestrian will immediately find that the preceding

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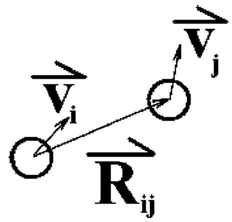

FIG. 1. $\vec{V}_i(\vec{V}_j)$ is the velocity of pedestrian $i(j)$; $\|\vec{R}_{ij}\|$ is the distance between pedestrian i and j.

pedestrians are walking faster, and the repulsive effects will not appear. If preceding pedestrians decelerate abruptly, the following one will realize it at once and decrease his/her own velocity.

Besides, the motion of a pedestrian is also influenced by the headway, as he/she wants to keep a comfortable distance from the others. A pedestrian feels increasing discomfort as he/she gets closer to a stranger [12].

Here, we assume that the repulsive effects of other pedestrian j depend not only on the relative velocity between pedestrian i and pedestrian j but also on the distance between them i.e., the headway, and hence these effects can be expressed by a force term in the following form:

$$\vec{F}_{ij} = m_i\vec{a}_{ij} = -m_i f(V_{ij},\|\vec{R}_{ij}\|)\vec{e}_{ij}, \tag{1}$$

$$\vec{R}_{ij} = \vec{R}_j - \vec{R}_i, \tag{2}$$

$$\vec{e}_{ij} = \frac{\vec{R}_{ij}}{\|\vec{R}_{ij}\|}, \tag{3}$$

$$V_{ij} = \frac{1}{2}[(\vec{V}_i - \vec{V}_j)\cdot\vec{e}_{ij} + \|(\vec{V}_i - \vec{V}_j)\cdot\vec{e}_{ij}\|], \tag{4}$$

where $\vec{a}_{ij}$ is the acceleration of pedestrian i caused by pedestrian j; $f(V_{ij},\|\vec{R}_{ij}\|)$ is a function of V_{ij}, and $\|\vec{R}_{ij}\|$ to be determined; $\vec{R}_i(\vec{R}_j)$ is the position of pedestrian $i(j)$; $\vec{V}_i(\vec{V}_j)$ is the velocity of pedestrian $i(j)$; V_{ij} denotes the projection of the relative velocity of pedestrian i and j in the direction $\vec{e}_{ij}$; $\|\vec{R}_{ij}\|$ is the distance between pedestrians i and j; m_i is the mass of pedestrian i (see Fig. 1). From Eq. (1), we obtain

$$\|\vec{a}_{ij}\| = f(V_{ij},\|\vec{R}_{ij}\|). \tag{5}$$

According to the dimension analysis [17], we propose

$$\frac{|\vec{a}_{ij}|\|\vec{R}_{ij}\|}{V_{ij}^2} = C, \tag{6}$$

where C is a constant depending on the pedestrian's character. It is necessary to determine C with empirical data. However, for simplicity, we assume $C=1$, and obtain

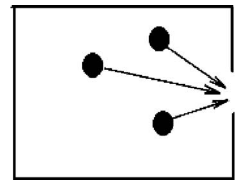

FIG. 2. Pedestrians walk towards the door, and the direction of desired speed is defined as pointed to the center of the door.

$$\vec{F}_{ij} = -m_i\frac{V_{ij}^2}{\|\vec{R}_{ij}\|}\vec{e}_{ij}, \tag{7}$$

which is similar to the form of "centrifugal force" in mechanics. This kind of repulsive force differs from the social force and the magnetic force introduced by Helbing *et al.* [12–14] and Okazaki [15], respectively, but all of them have similar characteristics. Hence, we name our model as the centrifugal force model.

If $(\vec{V}_i - \vec{V}_j)\cdot\vec{e}_{ij} > 0$, i.e., pedestrian i gets close to pedestrian j, the repulsive effects occur. If $(\vec{V}_i - \vec{V}_j)\cdot\vec{e}_{ij} < 0$, i.e., pedestrian j walks faster than pedestrian i, there are no repulsive effects. Larger V_{ij} creates greater repulsive effects in the former case. We assume that pedestrians react to those who are within their angle of view and the field of vision is 180° in front of them. Pedestrians always look forward, i.e., phenomena happening outside the visual angle has little influence on them. This can be characterized by the following coefficient:

$$K_{ij} = \frac{1}{2}\left[\frac{(\vec{V}_i\cdot\vec{e}_{ij} + \|\vec{V}_i\cdot\vec{e}_{ij}\|)}{\|\vec{V}_i\|}\right]. \tag{8}$$

Hence the "centrifugal force" should be given in the form

$$\vec{F}_{ij} = -m_i K_{ij}\frac{V_{ij}^2}{\|\vec{R}_{ij}\|}\vec{e}_{ij}. \tag{9}$$

B. Effect of borders and obstacles

A pedestrian also keeps a certain distance from borders, such as buildings, walls, various obstacles, etc. [18]. We use the similar approach presented above, and this effect is given by a force term

$$\vec{F}_{iB} = -m_i K_{iB}\frac{V_{iB}^2}{\|\vec{R}_{iB}\|}\vec{e}_{iB}, \tag{10}$$

$$\vec{R}_{iB} = \vec{R}_B - \vec{R}_i, \tag{11}$$

$$\vec{e}_{iB} = \frac{\vec{R}_{iB}}{\|\vec{R}_{iB}\|}, \tag{12}$$

$$K_{iB} = \frac{1}{2}\left[\frac{(\vec{V}_i\cdot\vec{e}_{iB} + \|\vec{V}_i\cdot\vec{e}_{iB}\|)}{\|\vec{V}_i\|}\right], \tag{13}$$

FIG. 3. (a) If a collision occurs, a pedestrian chooses one of the possible velocity, and detect whether the path of chosen direction is passable. (b) There is no way to go and the pedestrian has to stop.

$$V_{iB} = \frac{1}{2}[(\vec{V}_i - \vec{V}_B) \cdot \vec{e_{iB}} + \|(\vec{V}_i - \vec{V}_B) \cdot \vec{e_{iB}}\|], \qquad (14)$$

where $\vec{R_B}$ represents the position of the part of border that is nearest to pedestrian i, and the subscript B denotes the border.

C. Relaxation term

A pedestrian wants to reach a certain destination as comfortably as possible. If a pedestrian's motion is not disturbed, he/she will walk at the desired velocity $\vec{V_i^p}$. We use an acceleration term of the form as in [12]

$$\vec{F_i^p} = m_i \frac{\vec{V_i^p} - \vec{V_i}}{\tau_i}, \qquad (15)$$

where τ_i is a certain relaxation time.

Since all the above-mentioned effects influence a pedestrian's motion at the same moment, we can now set up the equations for a pedestrian's motion.

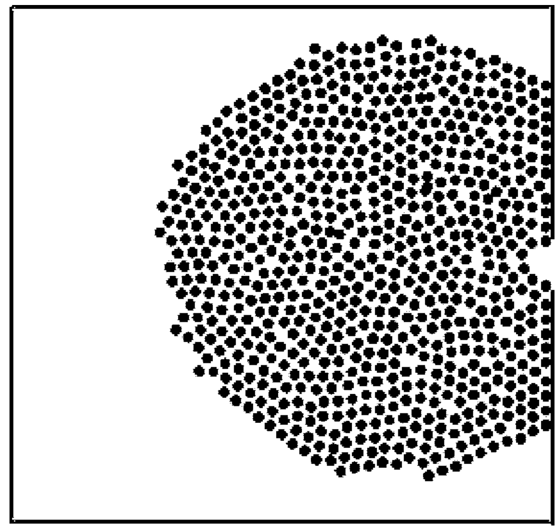

FIG. 4. At exits, arching and clogging occurs.

$$\vec{F_i} = \sum_{i \neq j} \vec{F_{ij}} + \sum_B \vec{F_{iB}} + \vec{F_i^p}, \qquad (16)$$

where $\vec{F_{ij}}$, $\vec{F_{iB}}$ and $\vec{F_i^p}$ are given respectively in Eqs. (9), (10), and (17). The centrifugal force model is now summarized as

$$m_i \frac{d\vec{V_i}}{dt} = \vec{F_i}. \qquad (17)$$

III. SIMULATION AND RESULTS

In spite of the fact that the model proposed above is simple, it can characterize various pedestrian behaviors quite

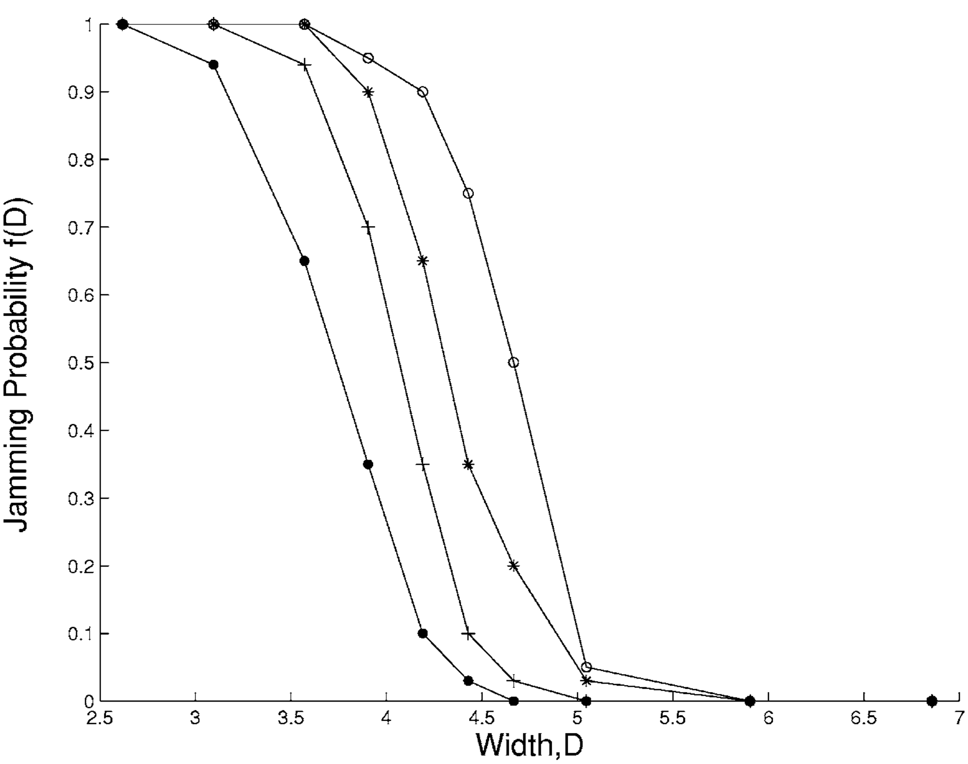

FIG. 5. Jamming probability $f(D)$ for occupancies of 0.1(·), 0.2(+), 0.4(*), and 0.8(○), while the opening varies from $2.5D$ to $7D$. The size of room is 18 m $\times$16 m.

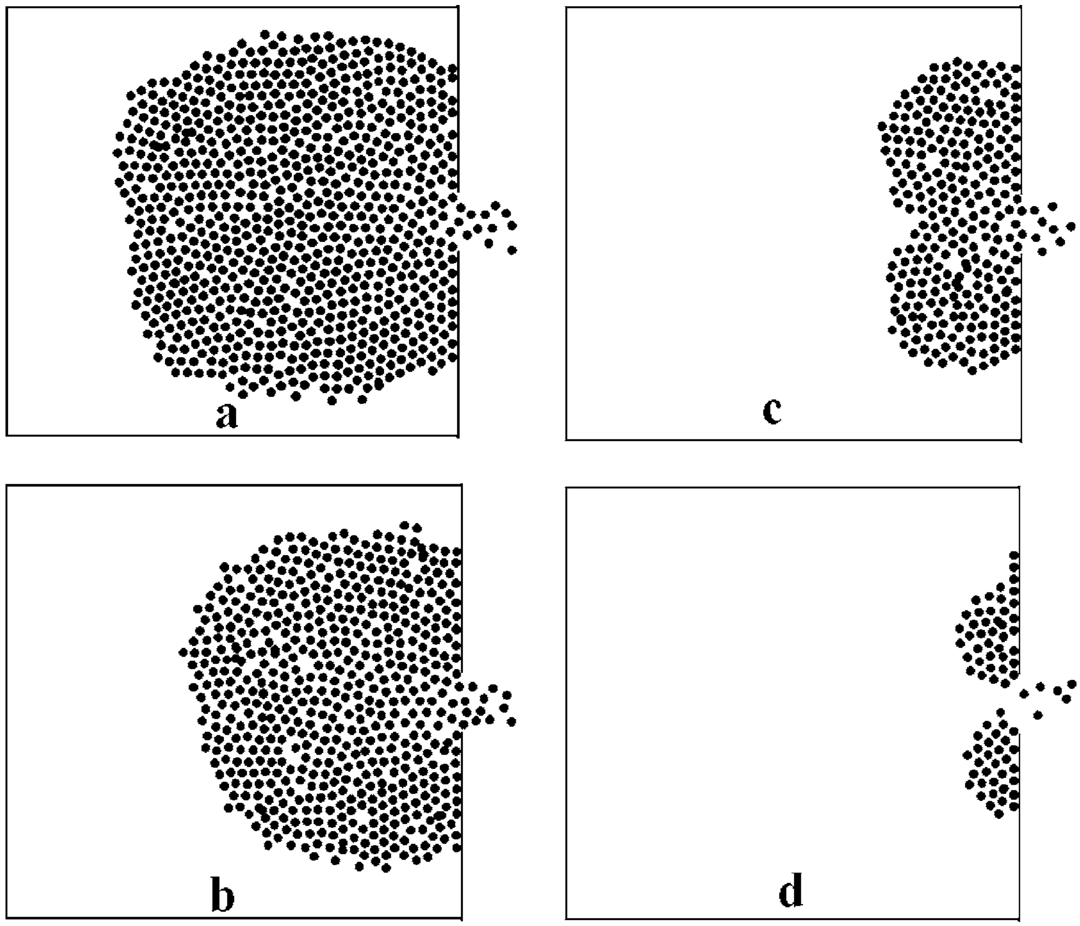

FIG. 6. Four stages of crowd flow going out of a hall.

realistically. We have reproduced some collective phenomena, and the following three cases are analyzed and depicted: (1) arching and clogging at exits; (2) crowd flowing out of a hall; (3) lane formation in sparse flow.

In simulation, the desired speeds V^p are assumed to be Gaussian distributed [19,20] with the mean value $\overline{V^p} = 1.34$ m/s and the standard deviation $\sqrt{\theta}=0.26$ m/s [21]. The relaxation time is taken as 0.5 s, and the average mass of a pedestrian is 80 kg [13]. In our first two simulations, the direction of V^p is set to be pointed to the center of the door (see Fig. 2). In the third simulation, pedestrians prefer to walk straight ahead. Since in real life especially in panic situation and the occupancy is high, pedestrians cannot look over those preceding people, we cut off the repulsive interaction between pedestrians as the distance between two pedestrians is larger than $5D$, where D is the "diameter" of a pedestrian.

A. Volume effects

In order to make the simulation more realistic, the volume of pedestrians are taken into account. In the following figures, pedestrians are represented by circles. The diameter of a circle is a measure for an actual area a pedestrian occupies. Here, we assume that each pedestrian covers the same area of $0.16\ \mathrm{m}^2 (0.4\ \mathrm{m} \times 0.4\ \mathrm{m})$. Thus the radius of each circle is 0.2 m, i.e., a pedestrian has a "hard core" of the radius 0.2 m.

B. Collision detection technique

As the model is used to simulate panic situations, collisions of pedestrian occur frequently. A key step in simulation is to use the so-called collision detection technique, which is related to the volume effects and provides a flexible route-choice method, with which aggressive pedestrians could squeeze through preceding people.

If the chosen time step in discretization is very small, there is no problem without considering volume effects, but in fact, a pedestrian cannot respond to ambience so fast and adjust the velocity in time. Besides, the simulation will be less efficient. Here, the time step we used varies from 0.05 s to 0.15 s and the results have no remarkable difference for different time steps. However, as a result, a pedestrian will be overlapped with others sometimes, because he/she keeps higher speed, which needs a longer time to be reduced. Hence, with the consideration of volume effects, the collision detection is necessary.

Since we will simulate panic situations, all pedestrians try to squeeze through others. When a collision occurs, aggressive pedestrians observe the surroundings and choose the nearest route which is temporarily expedite. The collision detection algorithm is described as follows.

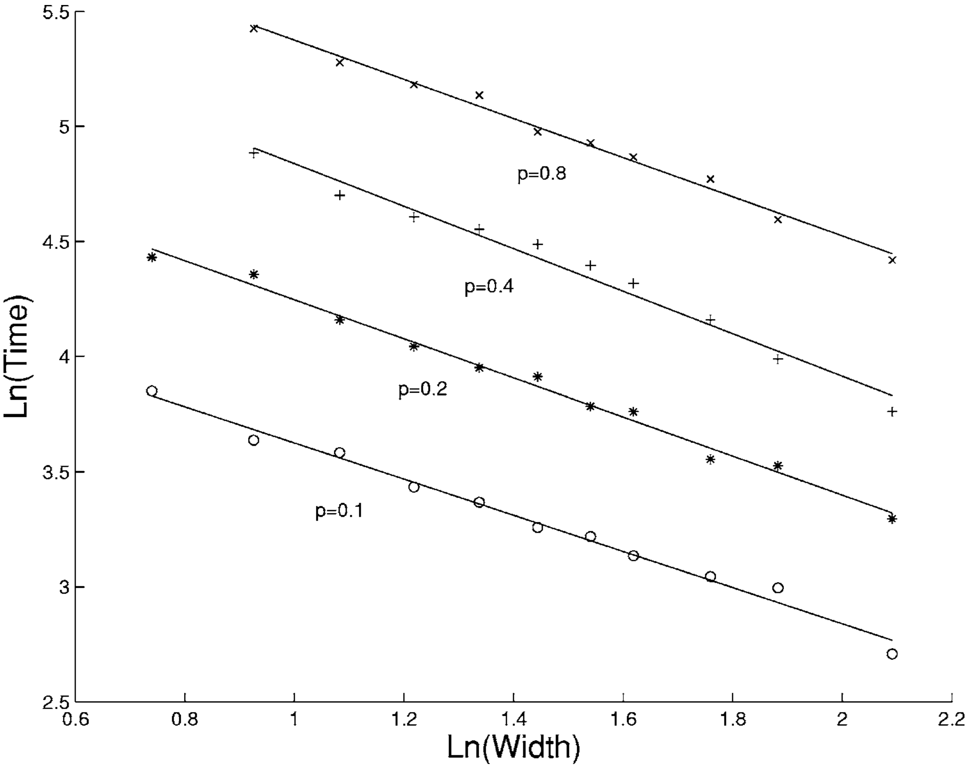

FIG. 7. The log-log plot of average leaving time T against the door size W for occupancies of 0.1(○), 0.2(*), 0.4(+), and 0.8(×). The points indicate the simulation data. The line represents the fitting results.

First, we assume that collision has not occurred and update the pedestrian's position according to Eq. (17), then detect whether a collision has occurred. If a collision has occurred, we move the pedestrian back to the original position and choose the nearest expedite route, i.e., the minimum turning angle according to the current walking direction, which is set to be discrete as the multiplication of 30° [see Fig. 3(a)]. If the minimum turning angles of both sides are equal, one chooses randomly one of them. Of course any pedestrian does not move back, so if there is no other way to go, he/she has to stop before collisions [see Fig. 3(b)].

C. Results and discussion

1. Arching and clogging at exits

In our simulation, we find that arching and clogging [3,4] occur at exits (see Fig. 4).This kind of tragic phenomenon occurs because everybody takes selfish action rather than a cooperative one. As a result, the physical interactions among people cause the arching and clogging, and jams build up. Two factors play vital roles in the arching. One is our explicit consideration of the volumes of pedestrians, the other is due to the aggressive pedestrians, who squeeze through those in front. When an individual collides with others, one chooses the temporarily passable route nearest to him/her, and squeeze through the way. However, pedestrians cannot squeeze through the gaps of preceding pedestrians, unless the gap is larger than the diameter. Thereby, an arching occurs.

From the arch configuration of the jamming event, it is obvious that the horizontal span of the arch is always greater than the opening. Furthermore, the arch is everywhere convex. This is a necessary condition for static equilibrium in the arch if friction is neglected [24].

Figure 5 shows the jamming probability against the D for different occupancies of 0.1,0.2,0.4 and 0.8, where D is the diameter of a pedestrian. The occupancy is defined as

$$\text{occupancy} = \frac{0.16 \times (\text{Number of pedestrians})}{18 \times 16}. \quad (18)$$

One can see that at higher occupancy, wider opening is needed for diminishing the jamming probability from 1 to a lower value, and a jamming is more likely to occur at higher occupancy, when the door size is commensurate. This can be explained by the fact that more pedestrians are involved in a room when the occupancy increases, which results in higher jamming probability. Although the congestion probability depends on the door size, we find that the bottleneck of the door size is 5D, when the door size is no less than 5D, the clogging probability is decreasing to zero, no matter what the occupancy is. Lots of our simulations lead to the above conclusions.

Our results are compatible to the granular flow experimental results obtained by To [24], who collected statistical data concerning the jamming probability in hoppers.

2. Crowd flow going out of a hall

We carried out a computer simulation of the crowd flow going out of a hall of size 18 m × 16 m, and focused on the patterns of walking. The initial distribution of pedestrians was randomly taken. Figure 6(a) depicts the first stage that all pedestrians walk towards the exit with the width 2.0 m. Figure 6(b) shows the second stage, the arching of pedestrians occurs, and only a few pedestrians have enough space through the exit while most pedestrians cannot go out of the exit [22]. Figure 6(c) gives the pattern of the third stage, the

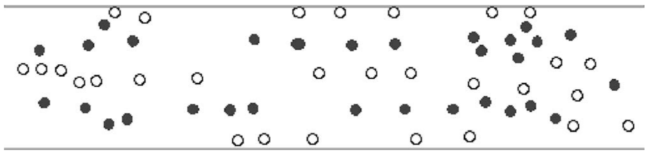

FIG. 8. The computational result shows 5 lanes in a corridor that is 5 m wide and 23 m long. The full circles represent pedestrians moving from left to right, while the empty circles are opposite. Pedestrians leave the system as soon as they reach the end of the corridor.

arching begins to decay. Figure 6(d) shows that the remaining pedestrians go out of the hall through the exit in order.

We also study the relationship between the width of the door (W), which is wider than $2m(5D)$, i.e., there is no arching and clogging at exits. and the average leaving time (T) at the occupancies of 0.1,0.2,0.4,0.8. The simulations run many times with different initial distributions to obtain the average leaving time and give the results as shown in Fig. 7.

Note that at high occupancy, much more time is needed for pedestrians to leave the room. Yet, the slope of the line is nearly the same, which shows that the average leaving time (T) is a function of the exit width (W) in negative power. Through data fitting, we find the relation

$$T \propto W^{-0.83\pm0.05}. \tag{19}$$

These results show a good agreement with the results recently obtained by Professor B.H.Wang with a CA model [26].

3. Lane formation

With open boundary conditions, we conducted simulations of pedestrians walking in opposite directions in a straight corridor. Pedestrians randomly enter the corridor from both ends. Figure 8 shows the empirically observed development of dynamically varying lanes consisting of pedestrians who intend to walk into the same direction [2]. In our model, the lane formation can be formed without assuming a preference for any side as in [23,25]. Pedestrians moving in opposite directions have strong interactions because of the high relative speed. In order to pass through each other, pedestrians move a little aside, which tends to separate oppositely walking pedestrians, and the segregation occurs as a result.

IV. SUMMARY AND OUTLOOK

We have proposed a centrifugal force model for pedestrian dynamics. The effects of relative velocity and the aggressive pedestrians have been taken into account. We have simulated some phenomena of pedestrian behavior with the consideration of volume effects explicitly. The collision detection technique is applied to determine the velocity of a pedestrian when a collision occurs. A quantitative analysis of average leaving time of the crowd flow out of a hall and the jamming probability at exits have been provided. It is shown that the proposed model is able to reproduce the self-organization phenomena of lane formation for sparse flows and arching for dense flow. Some factors have not been taken into account in this paper, such as the friction between pedestrians, which has been considered in [14], here we have neglected the effect of friction, so the presented model is probably suitable for the case of moderately high density; the injured pedestrians, who may become obstacles and affect the crow flow; the attracting force between pedestrians, for instance, friends preferring to walk together. These problems are now under consideration and the results will be presented later.

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