

Tunable sharp angular defect mode with invariant transmitted frequency range in one-dimensional photonic crystals containing negative index materials

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We propose a different photonic crystal structure with a novel defect mode. In this defect mode, the transmitted angle is sharp and the pass band is of rectangular shape. Surprisingly, there is a critical refractive index of the defect layer in the crystal. By changing the refractive index in a range higher than this critical value, the sharp transmitted angle can be tuned with transmitted frequency range maintained; when the refractive index is lower than this critical value, only the transmittance of the defect mode is adjusted, with the sharp transmitted angle and transmitted frequency kept unchanged. All these phenomena provide possible mechanisms for angular filtering, optical switching (i.e., an optical switch working in the angular domain) and setting optical limits.

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I. INTRODUCTION

Photonic crystals (PCs) have attracted extensive studies since the initial prediction of Yablonovitch [1] and John [2]. Based on one-dimensional (1D) PCs consisting of positive index materials (PIMs) with both positive permittivity and permeability, some important applications for omnidirectional filtering are proposed, such as omnidirectional reflection band (ORB) [3] and narrow frequency sharp angular defect mode [4]. Recently, negative index materials (NIMs) with simultaneous negative permittivity and permeability, which were suggested by Veselago [5], have also received a great deal of attention. Some properties of 1D PCs with NIM inclusion are revealed, for instance, omnidirectional band gap coming from the zero- \bar{n} mechanism [6] and very weak dependence of the defect mode on incident angles [7]. NIM has also been used to broaden the stop band of a 1D PC, in the case of normal propagation [8], and used as a defect layer in a defective PC with periodic structures consisting all of PIMs to obtain a flat-top transmission band at normal incidence [9]. In theory, the conditions for this flat-top band to appear should be extended to different types of defective PCs, e.g., the periodic structures are alternately consisted of NIM and PIM, or all NIMs. Moreover, for a flat-top defect mode obtained by coupling two or more defective PCs consisting all of PIMs, the defect-mode frequency varies for different incident angles. This means that light with an unwanted frequency might be transmitted through the filter, which would inevitably reduce the advantages of the filter. Hence, a flat-top pass band is needed for filtering not only in the frequency domain but also in the angular domain (i.e., it should emerge only within a sharp angular range). Such a defect mode will have further application for angular optical switching, if it can be tuned to appear at different incident angles with an invariant transmitted frequency range within an ORB, and for optical limiting, if the transmittance can be tuned with an invariant transmission angle.

In this paper, we extend the defect-mode resonant condition of the one-dimensional defective PC to several types of structures with periodic quarter-wave stacks consisted all of NIMs or alternately of NIM and PIM, and deduce the phase changes on reflection from such reflective stacks. Then the conditions for a flat-top defect mode appearing in the normal band gap of these different types of defective PCs are obtained. Based on these theories, we proposed a photonic heterostructure possessing a sharp angular and flat-top pass band, in which a flat-top pass band responses only for a sharp angular range within an ORB. The optical response of this heterostructure as a function of the refractive index of the defect layer is also analyzed, and the results provide possible mechanisms for angular optical switching (i.e., an optical switch working in the angular domain) and setting optical limits.

II. DEFECT-MODE RESONANT CONDITION FOR DEFECTIVE PCS CONTAINING NIMS

As discussed by Macleod [10], a defective PC may be completely described by the defect layer and two effective interfaces M_1 and M_2 . If we consider M_1 with the reflective stack to the left of it as system I, and M_2 with the stack to the right as system II, then the transmittance of the PC is given by

$$T(\nu) = \frac{T_1(\nu)T_2(\nu)}{[1 - \sqrt{R_1(\nu)R_2(\nu)}]^2 + 4\sqrt{R_1(\nu)R_2(\nu)}\sin^2(\frac{1}{2}\theta)}, \quad (1)$$

where T_1, T_2, R_1 , and R_2 are the transmittances and reflectances of systems I and II, respectively. For normal incidence, the transmittance $T(\nu)$ reaches a maximum when the defect mode resonant condition

$$\theta(\nu) = -\phi_1(\nu) - \phi_2(\nu) + 2O(\nu) = 2m\pi \quad (2)$$

is satisfied. Here m is an integer, $\phi_1(\nu)$ and $\phi_2(\nu)$ are the phase changes on reflection from system I and II, and $O(\nu)$ is the phase thickness of the defect layer. The precondition used

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TABLE I. The product M of individual layers of different types of quarter-wave stacks and the flat-top conditions for different types of defective PCs.

Types of quarter-wave stacks	M	Types of defective PCs	Flat-top conditons	
			Q_D	D
$Q_P > Q_N$	$(PN)^s$	$(NP)^s 2D(PN)^s$	$Q_P + Q_N$	NIM
$Q_P < Q_N$	$(NP)^s$	$(PN)^s 2D(NP)^s$		PIM
$Q_P < Q_N$	$(PN)^s$	$(NP)^s 2D(PN)^s$	$\frac{Q_P Q_N}{Q_P + Q_N}$	NIM
$Q_P > Q_N$	$(NP)^s$	$(PN)^s 2D(NP)^s$		PIM
$Q_{l_1} > Q_{l_2}$ ($l=P$ or N)	$(N_1 N_2)^s$	$(N_2 N_1)^s 2D(N_1 N_2)^s$	$Q_{N_1} - Q_{N_2}$	PIM
	$(P_1 P_2)^s$	$(P_2 P_1)^s 2D(P_1 P_2)^s$	$Q_{P_1} - Q_{P_2}$	NIM
	$(N_2 N_1)^s$	$(N_1 N_2)^s 2D(N_2 N_1)^s$	$\frac{Q_{N_1} Q_{N_2}}{Q_{N_1} - Q_{N_2}}$	PIM
	$(P_2 P_1)^s$	$(P_1 P_2)^s 2D(P_2 P_1)^s$	$\frac{Q_{P_1} Q_{P_2}}{Q_{P_1} - Q_{P_2}}$	NIM

by Macleod is that systems I and II are all consisting of PIMs, and because the phase change suffered by the wave on traversing a distance d in a PIM without absorption is $\delta(\nu) = -2\pi\nu n_{PIM}d$ [10], then he derived the amplitude of reflectance $r_{1,2}(\nu) = |r_{1,2}(\nu)|e^{i\phi_{1,2}(\nu)}$, and, consequently, minus signs are applied to the first two terms in the middle of Eq. (2).

Now we extend this defect mode resonant condition at normal incidence for defective PCs with quarter-wave reflective stacks consisting of alternate PIM and NIM layers or consisted all by NIM layers. We assume every layer is of the same quarter-wave optical thickness.

We first consider the quarter-wave stacks consisting of alternate PIM and NIM layers to be effective PIM stacks or effective NIM stacks at normal incidence. This is because quarter-wave stacks consisting of alternate PIM and NIM layers are just like series phase compensators [11,12]: whatever phase difference is developed by traversing the PIM layer, it can be canceled by traversing the NIM layer with the same absolute optical thickness as the PIM layer, since the directions of phase velocity in PIM and NIM are opposite. Hence, only the layer conjugated to the defect contributes to the phase change on reflection. According to this, the types of PCs $(NP)^s 2D(PN)^s$ and $(P_1 P_2)^s 2D(P_2 P_1)^s$ have effective PIM stacks, whereas $(PN)^s 2D(NP)^s$ and $(N_1 N_2)^s 2D(N_2 N_1)^s$ have effective NIM stacks. Here P is for PIM, N for NIM, and D for the defect layer, and the different subscripts are for different materials. For NIM, $\delta(\nu) = 2\pi\nu |n_{NIM}|d$, which has the opposite sign to that for PIM, so for effective NIM stacks, we have

$$r_{1,2}(\nu) = |r_{1,2}(\nu)|e^{-i\phi_{1,2}(\nu)}, \quad (3)$$

different from Macleod, and hence the defect mode resonant condition becomes

$$\theta(\nu) = \phi_1(\nu) + \phi_2(\nu) + 2O(\nu) = 2m\pi. \quad (4)$$

Combining Eqs. (2) and (4), we have the defect mode resonant condition for defective PCs with effective PIM or NIM stacks as follows:

$$\theta(\nu) = \pm \phi_1(\nu) \pm \phi_2(\nu) + 2O(\nu) = 2m\pi, \quad (5)$$

where the $-$ is for defective PCs with effective PIM stacks and the $+$ is for those with effective NIM stacks. For a symmetrical defective PC, $\phi_1(\nu) = \phi_2(\nu) \equiv \phi(\nu)$.

III. PHASE CHANGE ON REFLECTION

We now deduce the phase change on reflection ϕ from quarter-wave multilayers. The matrix of a quarter-wave layer without absorption can be simplified [13] to

$$\begin{bmatrix} A & \pm j/Q_l \\ \pm jQ_l & A \end{bmatrix} \quad (6)$$

at normal incidence for frequencies ν close to ν_0 (corresponding to the character wavelength λ_0 of the quarter-wave layer), where $A = \frac{1}{2}\sin[\pi(\nu/\nu_0)]$, and $+$ is for $l=P$ and the $-$ is for $l=N$, respectively. Q_l is the reciprocal of wave impedance defined as $Q_l = n_l/\mu_l$, where n_l is the refractive index and μ_l is the relative permeability of the layer.

The product

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad (7)$$

of the impedance matrices of the individual layers can be evaluated using the simplified form in Eq. (6) and eliminating higher-order terms in $\sin[\pi(\nu/\nu_0)]$ for eight types of quarter-wave stacks listed in the first two columns of Table I. Results of M are listed in the third column, where it is assumed that $Q = (Q_l^H/Q_l^L) > 1.5$ is used, and the layer with Q_l^H is of higher reciprocal of wave impedance than that with Q_l^L . Seeley [13] has shown that in the limit $Q > 1.5$, $\tan \phi$ attains limiting values

$$\tan \phi_{\text{lim}} = k \sin\left(\pi \frac{\nu}{\nu_0}\right), \quad (8)$$

where k is a parameter. For situation (i) that light from the defect layer incident to Q_l^H ,

$$\tan \phi_{\text{im}} = 2Q_D m_{12}/m_{22}, \quad (9)$$

and for situation (ii) that incident to Q_i^L ,

$$\tan \phi_{\text{im}} = 2m_{21}/m_{11}Q_D. \quad (10)$$

Then combining product M in Table I and Eqs. (8)–(10), we get the expressions of parameter k as follows:

$$k_A = \frac{Q_D}{Q_i^H \pm Q_i^L}, \quad \text{for situation (i),} \quad (11)$$

and

$$k_B = \frac{Q_i^H Q_i^L}{Q_D(Q_i^H \pm Q_i^L)}, \quad \text{for situation (ii),} \quad (12)$$

where $+$ for stacks consisting alternately of PIM and NIM layers, and the $-$ for all those consisting of PIM or NIM layers. Then, the phase change on reflection as a function of ν for frequencies close to ν_0 at normal incidence is [14]

$$\phi_j(\nu) = -k_j \pi \left(\frac{\nu - \nu_0}{\nu_0} \right), \quad j = A, B. \quad (13)$$

IV. TUNABLE SHARP ANGULAR AND FLAT-TOP DEFECT MODE

From Eqs. (5) and (13), one finds that the two terms on the phase change on reflection are monotonously increasing or decreasing when ν changes around ν_0 . If the phase thickness of the defect layer can compensate such difference of phase change on reflection, the transmittance of the spectrum will be a flat-top pass band centering at ν_0 . This can be done by inserting a PIM or NIM defect with proper indices, respectively, according to the reflective stacks: PIM defect for effective NIM stacks and NIM defect for effective PIM stacks. We call this “the flat-top condition,” as shown in columns 5 and 6 of Table I.

Take $(PN)^5 2D(NP)^5$ for example to illustrate this idea. Let light from the defect layer incident to a layer with $Q_i^H = Q_N > Q_P$, then $\phi_j(\nu) = \phi_A(\nu) = -k_A \pi[(\nu - \nu_0)/\nu_0]$, and $k_A = Q_D/(Q_P + Q_N)$. Since it has effective NIM stacks at normal incidence, one should use a PIM defect for phase-difference compensation. Let $m=1$, then Eq. (4) can be solved under the flat-top condition with a result that $Q_D = Q_P + Q_N$, and the transmission spectrum is shown in Fig. 1(a), in which we let $\mu_P = \mu_D = -\mu_N = 1$, $n_P = 1.45$, $n_N = -2.9$, and accordingly, we get $n_D = 4.35$ for the flat-top condition. For another example $(P_1 P_2)^5 2D(P_2 P_1)^5$, light from the defect layer incident to a layer with $Q_i^L = Q_{P_2} < Q_{P_1}$, then $\phi_j(\nu) = \phi_B(\nu) = -k_B \pi[(\nu - \nu_0)/\nu_0]$, and $k_B = Q_{P_1} Q_{P_2} / [Q_D(Q_{P_1} - Q_{P_2})]$. Since it has effective PIM stacks, one should use a NIM defect for phase-difference compensation. Let $m=1$, then Eq. (2) can be solved with a result that $Q_D = Q_{P_1} Q_{P_2} / (Q_{P_1} - Q_{P_2})$, and the transmission spectrum is shown in Fig. 1(b), in which we let $\mu_{P_1} = \mu_{P_2} = -\mu_D = 1$, $n_{P_1} = 2n_{P_2} = 2.9$, and, accordingly, we get $n_D = -2.9$. The transmission versus incident angles and normalized frequencies is calculated by a matrix method [15].

Figure 1 shows that, at normal incidence, a flat-top pass band emerges in the normal band gap symmetrically to ν_0 .

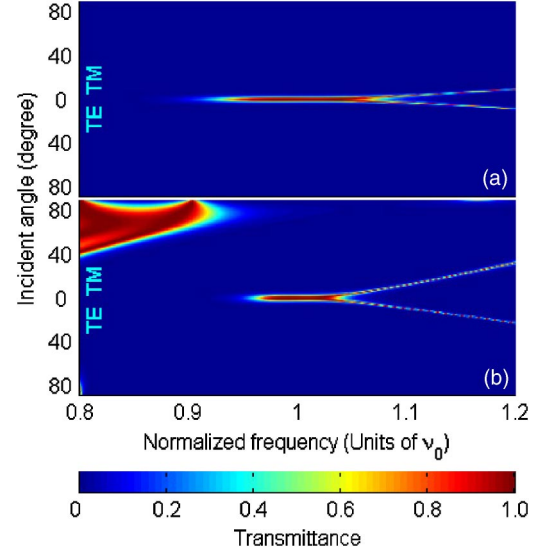


FIG. 1. (Color online) Transmission vs incident angles and normalized frequencies for structures: (a) $(PN)^5 2D(NP)^5$, with $\mu_P = \mu_D = -\mu_N = 1$, $n_P = 1.45$, $n_N = -2.9$, and $n_D = 4.35$; (b) $(P_1 P_2)^5 2D(P_2 P_1)^5$, with $\mu_{P_1} = \mu_{P_2} = -\mu_D = 1$, $n_{P_1} = 2n_{P_2} = 2.9$, and $n_D = -2.9$. Parameters in both (a) and (b) satisfy the flat-top condition, hence, a flat-top pass band emerges in the normal band gap.

As incident angle increases, the transmittance of this pass band decreases dramatically, while a resonant peak appears at the high-frequency end of this pass band and shifts to higher frequency. Transmission spectrum such as these implies that a sharp angular defect mode transmitting lights only within a narrow angular range can be achieved, as long as the propagation bands of oblique incidences are forbidden and the normal pass band is reserved simultaneously. To fulfill such requirements, we suggest defective structures discussed above (denoted by A) be coupled with a short wave pass filter (L) or a long wave pass filter (R) or both, according to the transmission spectrum of A . This is because the transmission spectrum as functions both of incident angles and frequencies varies for PCs consisting of different materials, then one must investigate the transmission spectrum of A to determine what kinds of filter it should be coupled with.

As an example, we use the defective PC in Fig. 1(b) as A to construct the heterostructure possessing sharp angular pass band, and we still use the PIMs in this structure to constitute optimized structures of L and R as follows:

$$L = 1.26[1.065[(P_2/2)P_1(P_2/2)]^2[(P_2/2)P_1(P_2/2)]^8 \times 1.065[(P_2/2)P_1(P_2/2)]^2],$$

and

$$R = 0.747[0.98[(P_1/2)P_2(P_1/2)]^2[(P_1/2)P_2(P_1/2)]^8].$$

Among structures A , L , and R , the forbidden bands compensate for each other while a pass-band intersection centering at ν_0 and normal exists. If we couple these three structures together to form a heterostructure, denoted by LAR , it can be expected that an ORB will form and a flat-top pass band existing only within a sharp incident angular range will emerge in this ORB. We prove this idea in Fig. 2(a). It can be seen that the range of ORB is $(0.85, 1.2)\nu_0$. Only light within the frequency range of $(0.94, 1.07)\nu_0$ and the angular range $0^\circ \pm 3^\circ$ can be transmitted.

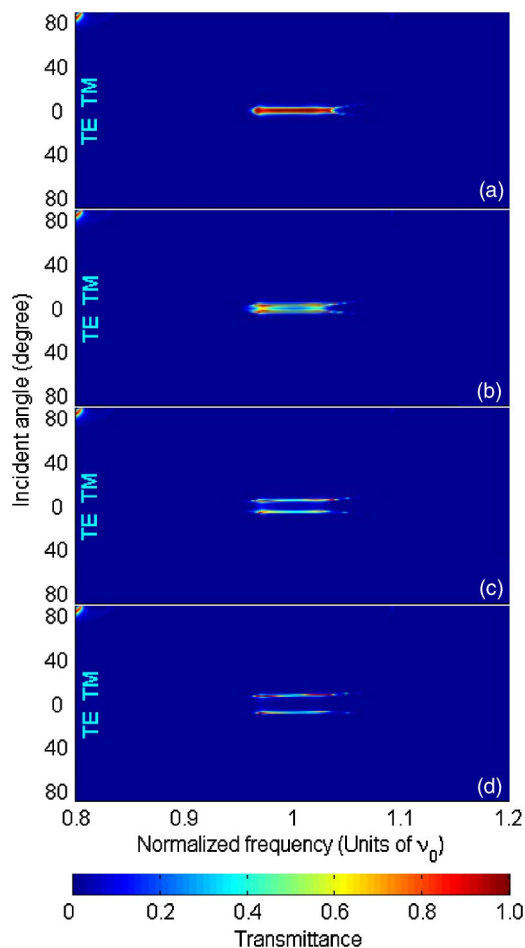


FIG. 2. (Color online) Transmittance vs incident angles and normalized frequencies of structure *LAR* with n_D equals: (a) -2.900 , (b) -2.899 , (c) -2.897 , and (d) -2.894 . Transmission of the flat-top defect mode shifts to an oblique incident angle with invariant transmitted frequency range and maintained high transmittance. This phenomenon provides a possible mechanism for angular optical switching.

From the above simulation, one will note that the pass band of *LAR* is mainly determined by the pass band of *A*. *L* and *R* are just used to forbid the propagation bands of *A* at oblique incidence. Furthermore, around the defect-mode resonant condition in Eq. (5), the transmittance T of *A* is very sensitive to the phase thickness O of its defect layer. This is because from Eqs. (1) and (5), one can get

$$\frac{\partial T}{\partial O} = - \frac{4\sqrt{R_1 R_2} \sin \theta}{\left[(1 - \sqrt{R_1 R_2})^2 + 4\sqrt{R_1 R_2} \sin^2(\frac{1}{2}\theta) \right]^2}. \quad (14)$$

On one hand, one can choose proper indices of the defect layer to compensate the difference of phase changes on reflection around the central frequency to fulfill the resonant condition and get a flat-top pass band. On the other hand, according to Eq. (14), around the resonant condition $\theta = 2m\pi$, T will be dramatically changed as a function of O . So it is interesting to investigate the variation of the transmission spectrum of *LAR* as a function of the refractive index of the defect layer in *A*. In the following, we will get results by

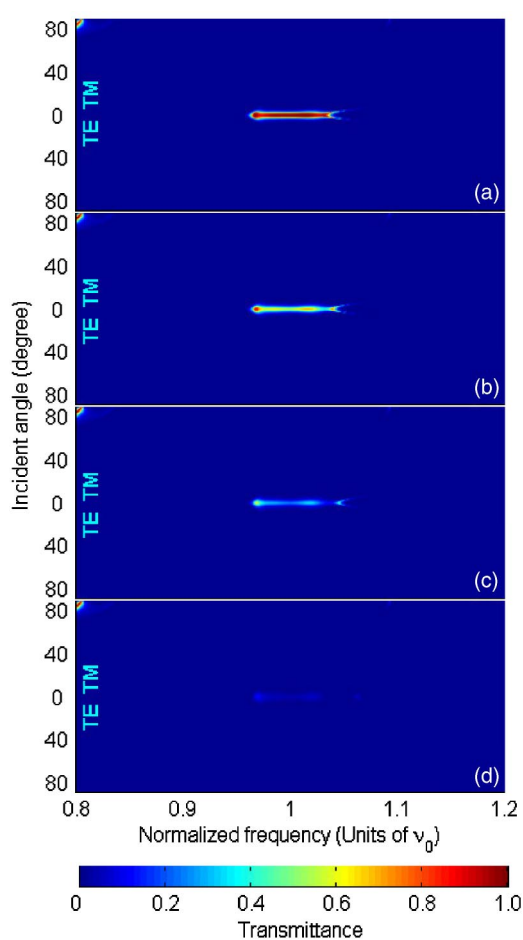


FIG. 3. (Color online) Transmittance vs incident angles and normalized frequencies of structure *LAR* with n_D equals: (a) -2.900 , (b) -2.9005 , (c) -2.901 , and (d) -2.903 . Transmittance of the flat-top defect mode decreases with invariant transmission angle. This phenomenon provides a possible mechanism for optical limiting.

analyzing the defect mode resonant condition and show examples by numerical simulation.

When the optical thickness of the defect layer is a little greater than that satisfying the flat-top condition of normal incidence, the transmission angle of the flat-top pass band shifts from the normal to an oblique one, with invariant transmitted frequency range. This is because the phase change on reflection at a small incident angle will become a little less than that at normal incidence. Increasing the optical thickness of the defect layer will compensate this difference of phase change, so that the flat-top condition will be satisfied at an oblique angle. Hence, the flat-top pass band will shift to a small incident angle, while the frequency range remains unchanged. Examples are shown in Fig. 2, where only the refractive index of the defect layer is changed and the geometry thickness of the defect layer is unchanged, i.e., $d_D \equiv 0.25\lambda_0/2.9$. Figure 2(a) shows the transmittance for *LAR* with $n_D = -2.9$. Figure 2(b) shows the transmittance for *LAR* with $n_D = -2.899$, with transmitted angular range from 1.5° to 4.3° for transmittance over 50%, and the central angle is 3.0° . When $n_D = -2.897$ and -2.894 , the central angle shifts to 5.3° and 7.5° , as shown in Figs. 2(c) and 2(d), re-

spectively. For this variation, the transmitted angular range is tunable with invariant transmitted frequency range. This phenomenon provides a possible mechanism for angular optical switching (i.e., an optical switch working in the angular domain).

When the optical thickness of the defect layer is a little less than that satisfying the flat-top condition of normal incidence, the transmittance of the whole pass band decreases, while the transmission angle remains to be 0° . This is because the defect-mode resonant condition cannot be fulfilled at any incident angle in the frequency range close to ν_0 . An example is shown in Fig. 3, where also only the refractive index of the defect layer is changed and the thickness of the defect layer is unchanged. Figure 3(a) shows the transmittance for structure *LAR* with $n_D = -2.9$ as that in Fig. 2(a). When $n_D = -2.9005$, -2.901 , and -2.903 , the average transmittance decreases to about 60, 30, and 5 %, respectively, with the invariant transmitted frequency range, as shown in Figs. 3(b)–3(d). This phenomenon provides a possible mechanism for optical limiting.

The sharp angular and flat-top defect-mode characteristics discussed above exist commonly in the defective PCs whose material indices satisfy the flat-top condition (i.e., a critical refractive index of the defect layer). By changing the refractive index in a range higher than this critical value, the *LAR* structure acts as an angular optical switch; as the refractive index is lower than this critical value, it becomes an optical limiter.

In the above we have discussed only the ideal situations in which the structure consists of lossless and nondispersive materials. In a nonideal situation, these unusual characteristics can also be expected to occur when the band structure is scalable [16], and we can scale it to a frequency range in which the material parameters have a very weak dispersion and the realization of lossless negative index material ap-

pears to be quite possible [17] in wide frequency bands with the use of active inclusions.

V. CONCLUSION

In conclusion, we extend the defect-mode resonant condition of the 1D defective PC for several types of structures with periodic quarter-wave stacks consisting alternately of NIM and PIM or by all NIMs, and deduce the phase changes on reflection from such reflective stacks. Then the conditions for a flat-top defect mode to appear in the normal band gap of such different types of defective PCs are obtained. Furthermore, we proposed a photonic heterostructure possessing a sharp angular and flat-top defect mode, in which a flat-top pass band responses only for a sharp angular range within an ORB. When the refractive index of the defect layer is a little greater than that for the flat-top defect mode appearing at normal incidence, transmission of this flat-top defect mode shifts to an oblique incident angle with an invariant transmitted frequency range and maintained high transmittance. When the refractive index of the defect layer is a little less than that for the flat-top defect mode appearing at normal incidence, the transmittance of this flat-top defect mode decreases with invariant transmission angle. All these phenomena provide possible mechanisms for sharp angular and flat-top filtering, angular optical switching, and optical limiting.

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