

Anisotropy in rupture lines of paper sheets

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We have experimentally investigated the fracture process in paper samples submitted to a uniaxial force. Five types of paper sheets (newsprint, towel, sulfite, silk, and couche papers) were fractured along two orthogonal orientations. In order to characterize the rupture lines of the paper sheets we utilized the Hurst exponent. Our results indicate a dependence of the Hurst exponent on the orientation of the paper sheets for samples of newsprint and, probably, towel and silk papers. For the other types of paper the Hurst exponent does not depend on the direction of crack propagation.

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I. INTRODUCTION

Fracture processes in disordered materials is a subject of intensive research and has attracted much scientific and industrial interest [1–3]. In any rupture experiment the presence of disorder is fundamental in the dynamics of crack propagation. It is naturally present in all materials and comes from a variety of different sources. Great theoretical and experimental efforts have been done in trying to understand the process of crack formation and propagation in disordered materials. In particular, a significant fraction of these studies have been performed from the point of view of statistical mechanics that utilize concepts such as percolation, fractals, scaling law, etc. [1,4–6].

Over the last decade fibrous materials have been attracting much attention. Several models were elaborated in describing the fracture process in these materials [7–16]. However, the small number of experimental data does not allow a reliable comparison with the theoretical results.

Experiments have shown that the crack line in disordered materials can often be described by self-affine scaling [4,17,18]. The surface irregularity is often characterized by its roughness, or width, $W(\varepsilon)$, defined as the rms value of the fluctuations of the surface height $h_i=h(x_i)$ over a length scale ε :

$$W(\varepsilon) = \frac{1}{N} \sum_{i=\varepsilon}^{N-\varepsilon} \sqrt{\frac{1}{2\varepsilon+1} \sum_{j=i-\varepsilon}^{i+\varepsilon} [h(x_j) - \langle h \rangle_i]^2} \quad (1)$$

where

$$\langle h \rangle_i = \frac{1}{2\varepsilon+1} \sum_{j=i-\varepsilon}^{i+\varepsilon} h(x_j)$$

is the average height around the position i .

For a self-affine surface, the function $h(x)$ is statistically invariant under an anisotropic scale transformation. This means that $h(x)$ has the same statistical properties as $k^{-H}h(kx)$, where H is known as the Hurst (roughness) expo-

nent, which satisfies $0 < H < 1$. For self-affine structures, the roughness W over a range ε satisfies the scaling law

$$W(\varepsilon) \sim \varepsilon^H. \quad (2)$$

The exponent H does not indicate how rough a surface is. It is a parameter that characterizes how the roughness, or the variance in the height, depends on the lateral scale over which it is measured.

Some experimental work has claimed that the Hurst exponent H does not depend on the direction of crack propagation and assumes a universal value of 0.8 [18–20]. However, this universality was first questioned by Milman *et al.* [21], who experimentally found with atomic force microscope measurements of fracture surface in crystalline metals a Hurst exponent close to 0.5, a value significantly smaller than 0.8. An exponent $H=0.5$ was also observed in materials displaying intergranular fractures, such as sandstone [22]. From the theoretical point of view, numerical models have been elaborated in order to evaluate the Hurst exponent H . Computer simulations have shown that $H \sim 0.7$ in two dimensions [23,24] and that H ranges from 0.4 to 0.5 in three dimensions [25,26]. Nowadays there is a conjecture relating the value of the Hurst exponent H to the speed of crack propagation through the sample [26]. The higher value ($H=0.8$) has been associated with a high speed of crack propagation and interpreted as a dynamic regime. In contrast, the smaller value ($H=0.5$) was related to a quasistatic regime, in which the dynamic effects of crack propagation are negligible. Experimentally, the smaller and the higher values of H are connected to the length scale at which the crack is examined [18,27,28].

Parisi and Caldarelli [29] have studied the fracture surface of three-dimensional samples using a model for quasistatic fractures known as the Born Model. They have found a Hurst exponent $H=0.5$, which does not depend on the direction of crack propagation.

An experimental procedure used to investigate the fracture process of fibrous materials is the rupture of a paper sheet into two parts [17,30,31]. Paper is a good example of a disordered material due to its structural nonuniformity, which can be confirmed even with the naked eye. When light is transmitted through it, patches of lighter and darker regions

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can be seen. Such darker and lighter parts correspond, respectively, to higher and lower local densities. Paper is made from cellulose fibers found in wood, the most frequently used being eucalyptus and pine. Different types of wood can be utilized to produce cellulose fibers with specific characteristics, therefore producing papers with different properties. Generally, the fiber network structure is highly disordered since their fibers are randomly positioned and oriented. Nevertheless, in some types of paper, the fabrication process is such that the fibers acquire some order, having a tendency to align along one direction. The fibers are held together by their ability to form hydrogen bonds with each other. The paper strength is a fundamental feature belonging to that group of physical properties associated with the paper manufacturing process. Indeed, paper strength is influenced by the kind, quality, and treatment of the constituent fibers and by the way the sheet has been formed on the paper machine.

The analysis of rupture in paper samples is very suitable because the best method to calculate the Hurst exponent demands the knowledge of the profile, i.e., of the function $h = f(x)$. In the case of a two-dimensional fracture, such a height profile is easily obtained. However, the evaluation of the Hurst exponent shows a significant dispersion, therefore demanding the analysis of a great number of samples. There are in the literature some experiments that investigated the rupture process in paper samples.

Maunuksela *et al.* [32] performed experiments on the dynamics and kinetic roughening of one-dimensional slow-combustion fronts using three types of paper. They concluded that well-controlled experiments lead to a clear asymptotic scaling that unequivocally lies in the thermal Kardar-Parizi-Zhang (KPZ) universality class.

Kertsz *et al.* [17] have done experiments using a tensile testing machine to study the morphology of rupture lines in paper. The samples were submitted to traction and the velocity of crack propagation was kept constant until the breaking process separated the sample into two pieces. They have studied five different kinds of paper and found that the Hurst exponent characterizing the rupture lines did not depend significantly on the type of paper used in the rupture process. The results obtained indicated a Hurst exponent ranging from 0.63 to 0.72, which did not depend on the direction of crack propagation.

Salminen *et al.* [33] have studied fracture in paper via acoustic emission analysis. In this case, the release of acoustic energy is related to irreversible deformation, microcracks, and, perhaps, to plasticity. It has been verified that the acoustic emission of energy and the waiting times between acoustic events followed power-law distributions. This remained true while the strain rate was varied by more than two orders of magnitude. Recently, Salminen *et al.* [31] have analyzed 6.5 m long fracture lines of paper as an example of crack propagation involving disorder. They have observed that the roughness exhibits a power-law scaling, with an exponent close to 0.60 ± 0.1 for all samples.

In this paper we have studied the dependence of the rupture line morphologies on the direction of crack propagation. Several samples of five different types of paper were submitted to a constant load until their complete rupture. Two directions, perpendicular to each other, were chosen for crack

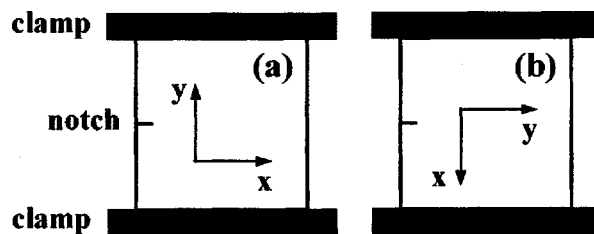


FIG. 1. Schematic representation of the experimental apparatus used in the rupture of paper. The direction of crack propagation is indicated in each figure. (a) horizontal direction and (b) vertical direction.

propagation and the fracture surfaces obtained were characterized by their Hurst exponent.

The paper is organized as follows. Section II is devoted to presenting the experimental procedure used in the rupture process. In Sec. III, the obtained results are presented and analyzed and, finally, the last section is devoted to conclusions.

II. EXPERIMENTAL PROCEDURE

We have carried out rupture experiments using five different types of commercially available papers: newsprint $[(36.8 \pm 0.2)\text{g/m}^2]$, towel $[(31.7 \pm 0.1)\text{g/m}^2]$, silk $[(51.8 \pm 0.5)\text{g/m}^2]$, couche $[(82.7 \pm 0.5)\text{g/m}^2]$, and sulfite $[(78.1 \pm 0.4)\text{g/m}^2]$. Two directions, x and y , perpendicular to each other, were assigned to the paper, along the width and the length of the sheets, respectively. We refer to x as the horizontal direction and to y as the vertical direction (Fig. 1). Square samples of 20×20 cm were cut from the paper sheets and the orientation of the directions x and y were marked on them. Each sample was fixed at both ends by two parallel clamps, either with their x or y direction parallel to the clamps. The upper clamp was maintained fixed and in the lower one a static load F was applied. The same load $F \approx 200$ N was used in all experiments. In order to initiate the propagation of the crack a notch 2 cm long was cut into the paper in the middle of one of the sides.

After the samples were fractured, they were digitized in a scanner of 450 dpi resolution with 256 gray levels and a black background. During the scanning process the beginning and the end of the rupture line were discarded. The gray level distribution near the front of the digitized figure ranges from about 170 on the side belonging to the paper to about 70 outside of the paper (on a scale that ranges from 0 to 256). The number of colors used to digitize the samples was later decreased to only two (black and white) using the function black and white from CorelPhoto-Paint 11. The threshold was adjusted until only the smaller fibers were deleted from the front. The digitized patterns were analyzed by a homemade computer program that allowed us to obtain the functions $h(x)$, which represent the patterns of the investigated rupture lines. Figure 2 shows typical functions $h(x)$ obtained for a newsprint sample along the horizontal and vertical directions.

III. EXPERIMENTAL RESULTS

From the digitized patterns we evaluated the roughness W [using Eq. (1)] of the rupture lines. A log-log plot of W

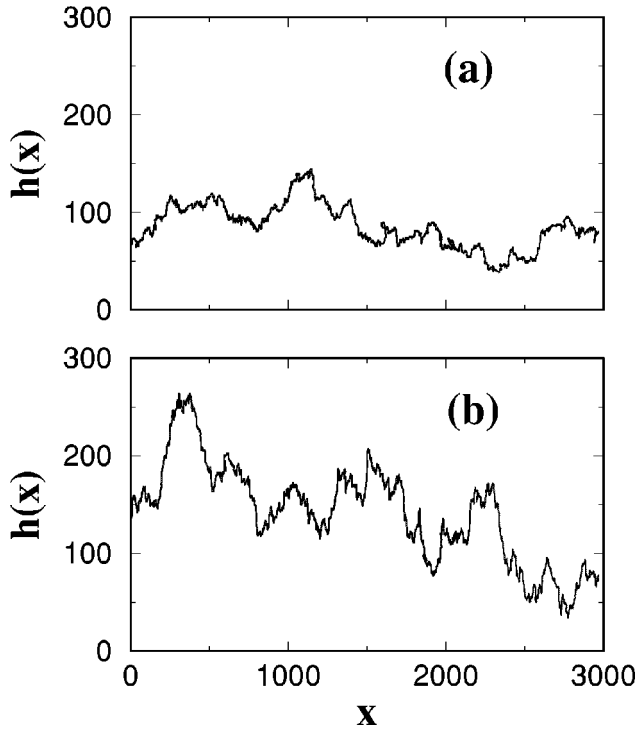


FIG. 2. Functions $h(x)$ for two samples of newsprint: (a) horizontal and (b) vertical direction. The horizontal and vertical scales are in pixels.

versus ε presents a region where the data have a linear behavior over at least two decades. The slope of this region is equal to the Hurst exponent.

Figure 3 shows typical results for samples of newsprint, silk, and towel papers along the two perpendicular directions. The slopes of the two straight lines shown are equal to 0.60 and 0.72 for the newsprint, 0.85 and 0.94 for the silk paper, and 0.71 and 0.78 for the towel paper. The lower value corresponds to the value of the Hurst exponent along the horizontal direction and the higher one to the value of the Hurst exponent along the vertical direction.

Table I presents the values obtained for the Hurst exponents calculated for each type of paper, along the horizontal and vertical directions. For all five types of paper, the data were averaged over 30 statistically independent samples.

Our results indicate a dependence of the Hurst exponent upon the direction of crack propagation for the samples of newsprint, towel, and, probably, silk papers. This dependence is related to the inhomogeneity of the paper. For these three types of paper the Hurst exponent is larger along the vertical direction than along the horizontal one. This indicates that in these papers, the propagation speed of a crack should be larger along the vertical direction.

For the two other types of paper it was observed that, within the experimental error, the Hurst exponent of the studied samples did not depend on the direction of crack propagation. Notice that in Table I the values obtained for the Hurst exponents along both directions are higher than 0.5. This result indicates that the fracture process investigated here can be identified as belonging to the dynamic regime and also corresponds to measurements at large length scales [27].

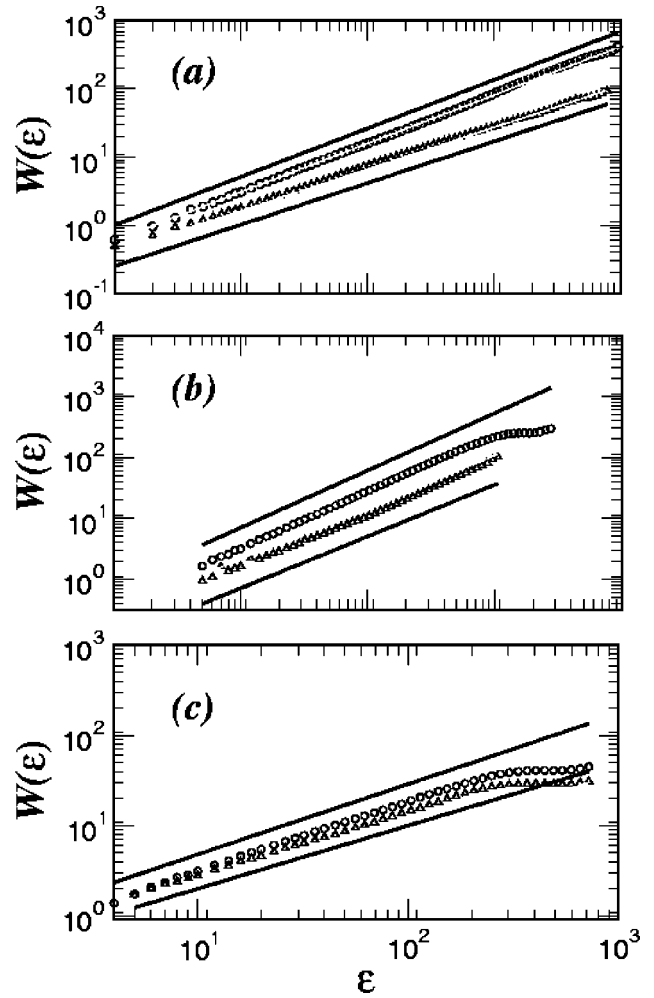


FIG. 3. Roughness of rupture lines for two samples of (a) newsprint, (b) silk, and (c) towel papers. The rupture lines are oriented along two different directions: vertical (circles) and horizontal (triangles). The slopes of the linear parts allow us to evaluate the Hurst exponents.

The results in Table I, for the newsprint, clearly reveal that the Hurst exponent depends on the direction along which it is measured. In order to confirm this result we have increased the scanner resolution to 800 dpi and used three other test methods for the crack roughness analysis: max-min difference [34], power spectrum [35], and best linear least-square fitting [36].

TABLE I. Hurst exponent for different kinds of paper sheet obtained along two directions (H_y =Hurst exponent for the vertical direction and H_x =Hurst exponent for the horizontal direction).

Paper	H_y	H_x
Newsprint	0.73 ± 0.02	0.65 ± 0.02
Couche paper	0.74 ± 0.04	0.74 ± 0.03
Towel paper	0.78 ± 0.02	0.71 ± 0.03
Sulfite paper	0.80 ± 0.04	0.77 ± 0.04
Silk paper	0.94 ± 0.02	0.90 ± 0.01

TABLE II. Hurst exponent obtained for newsprint using different methods.

Method	H_y	H_x
rms	0.69 ± 0.02	0.61 ± 0.02
max-min	0.73 ± 0.02	0.63 ± 0.02
Power spectrum	0.71 ± 0.03	0.63 ± 0.03
Best linear least-square fitting	0.68 ± 0.03	0.58 ± 0.03

The max-min method consists of the computation of $h_{max}(\epsilon)$, defined as the difference between the lowest and the highest heights h within a certain window of size ϵ , averaged over all possible origins x of the window. For a self-affine profile, h_{max} scales as a function of ϵ according to

$$h_{max} \sim \epsilon^H. \quad (3)$$

The power spectrum method measures the power spectrum of the interface, and not the height $h(x)$. In this case, the structure factor is calculated and is equal to

$$S(\vec{k}) = \langle h(\vec{k})h(-\vec{k}) \rangle, \quad (4)$$

where

$$h(\vec{k}) = \frac{1}{L^{1/2}} \sum_{\vec{x}} [h(\vec{x}) - \langle h \rangle] \exp(i\vec{k} \cdot \vec{x}), \quad (5)$$

and k is the wave factor. The structure factor scales, for a self-affine profile, as

$$s(k) \sim k^{-(2H+1)}. \quad (6)$$

In the method of the best linear least-square fitting, the roughness $W(\epsilon)$ over a length scale ϵ is given by

$$W(\epsilon) = \frac{1}{N} \sum_{i=1}^N w_i(\epsilon) \quad (7)$$

and the local roughness $w_i(\epsilon)$ is defined as

$$w_i^2(\epsilon) = \frac{1}{(2\epsilon + 1)} \sum_{j=i-\epsilon}^{i+\epsilon} [h_j - (a_i(\epsilon)x_j + b_i(\epsilon))]^2. \quad (8)$$

$a_i(\epsilon)$ and $b_i(\epsilon)$ are the linear fitting coefficients to the displacement ratio data on the interval $[i-\epsilon, i+\epsilon]$ centered at the position i . For a self-affine profile, the roughness W of the fracture surface satisfies the scaling law

$$W(\epsilon) \sim \epsilon^H. \quad (9)$$

The values of the Hurst exponents for the newsprint, estimated using the four methods are given in Table II. Notice that the three methods agree on the values of the Hurst exponents. This fact indicates that the crack surfaces are truly self-affine and confirms the conjecture that the Hurst exponent depends on the direction of fracture.

Figures 4(a) and 4(b) show optical microphotographs of newsprint and couche paper samples, respectively. For the newsprint it can be clearly seen that there are many long fibers running, preferentially, along the horizontal direction

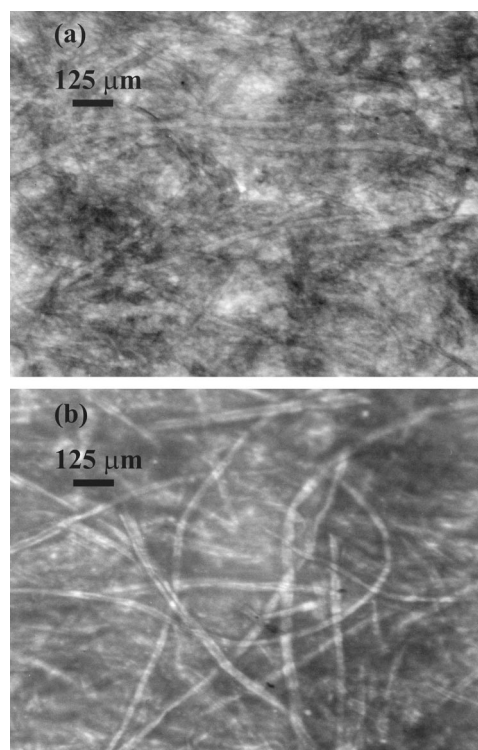


FIG. 4. Optical microphotographs for (a) newsprint and (b) couche paper. Both papers are oriented with their y direction along the length of the page.

of the figure. This orientation coincides with the horizontal direction of the paper. For the couche paper the fibers are also long but they do not appear to be oriented along any preferential direction, being more intertwined than those of the newsprint. The orientation of the fibers is produced during the paper fabrication process and seems to play an important role in the roughness of the rupture lines. The actual mechanism of the fracture is not yet completely understood at the microscopic level. For the paper to be fractured, the rupture of the hydrogen bonds between the fibers must occur in order that they can be separated from one another. However, the rupture of individual fibers seems to be not necessary. When the newsprint is fractured along the horizontal direction the crack can propagate mainly through the separation of one fiber from another by breaking the hydrogen bonds. The rupture of individual fibers, if it occurs at all, does not have to happen very often since most of the fibers are aligned parallel to the direction of crack propagation. In this case, we can suppose that the rupture process can occur due to two different possible fracture modes: tensile fracture of the hydrogen bonds between the fibers and shear fracture of the fibers. When the newsprint is fractured along the vertical direction the crack has to propagate in a direction perpendicular to the fibers. In this case, the hydrogen bonds between the fibers are mainly subjected to shear fracture mode while the fibers are mainly subjected to tensile fracture mode. Again the rupture of individual fibers is not necessary. If the hydrogen bonds between them break, they can simply slide past each other. The separation of the fibers is probably more difficult in this process as compared to that for the

horizontal direction since, as the fibers slide past each other, hydrogen bonds that were broken can be formed again. Rupture of individual fibers should be even more difficult since the force necessary to rupture one fiber must be greater than that needed to break a hydrogen bond. Consequently, the crack has to change its direction of propagation many times, trying to find a weaker point to fracture, and the rupture line formed is rougher than that formed along the horizontal direction. Microscopic examination of the rupture lines show that there are many more long fibers protruding from the paper when the paper is fractured along the vertical direction than when it is fractured along the horizontal direction. This indicates that, indeed, the fibers are just being pulled apart without being fractured. Unfortunately, comparison of the ends of the protruding fibers with the ends of the fibers away from the rupture line could not reveal if the former were broken during the fracture process. It is possible to make an analogy between paper fracture and microscopic failure mechanisms of a unidirectional composite material [1] if the hydrogen bonds are considered as being the matrix.

For the other types of paper that presented a dependence of the Hurst exponent on the orientation of the paper sheet, it was also possible to see the same kind of preferential fiber orientation, although much less pronounced than for the newsprint. This probably is the reason why the difference between H_x and H_y for the towel and the silk papers is smaller than for the newsprint. For the two types of paper that did not show dependence between the Hurst exponent and the orientation of the paper sheet, it was not possible to detect any orientation of the fibers. There seems to be, also, a relation between the size of the fibers and the value of the Hurst exponent. Papers composed of long fibers have smaller exponents than those of papers composed of short fibers.

IV. CONCLUSION

In this paper we have experimentally investigated the fracture process in five different types of paper submitted to a static load. The rupture lines of the samples were characterized by the Hurst exponent; calculated along two orthogonal orientations of the paper: vertical and horizontal. This procedure was utilized to investigate the effect of anisotropy on the Hurst exponent.

For the samples of newsprint our results clearly indicate that the Hurst exponent depends on the orientation of the paper. This dependence is associated with the orientation of the fibers within the paper, which is determined by its fabrication process. For the samples of silk and towel papers the difference between the Hurst exponents is small, almost within the experimental error, indicating a probable dependence of the Hurst exponent upon the orientation of the paper.

For all paper samples studied the rupture process is very rapid. This can be verified by the value of the Hurst exponent, higher than 0.5 for all samples tested. Therefore, the fracture process can be classified as a dynamic one.

Contrary to the results reported on the literature, our results support the idea that the value of the Hurst exponent for paper sheets is not universal. It depends on both the type of paper used and, more interestingly, it can also depend on the direction along which it is measured for the same paper sheet.

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