Stabilization analysis and modified Korteweg–de Vries equation in a cooperative driving system

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Two lattice traffic models are proposed by incorporating a cooperative driving system. The lattice versions of the hydrodynamic model of traffic flow are described by the differential-difference equation and difference-difference equation, respectively. The stability conditions for the two models are obtained using the linear stability theory. The results show that considering more than one site ahead in vehicle motion leads to the stabilization of the system. The modified Korteweg–de Vries equation (the mKdV equation, for short) near the critical point is derived by using the reductive perturbation method to show the traffic jam which is proved to be described by kink-anti-kink soliton solutions obtained from the mKdV equations.

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I. INTRODUCTION

Traffic jam is an important issue from the viewpoint of transportation efficiency and pollution. Therefore the issue has attracted much attention recently. There are various approaches to describe the characteristics of traffic flow, such as the cellular automaton models, car-following models, gas kinetic models and hydrodynamic models [1–8]. Recently, some researchers have investigated the traffic jam by use of nonlinear analysis. Kerner and Konhäuser [9] presented the nonlinear theory of the cluster effect in traffic flow, and derived the structures of a stationary moving cluster. Kurtze and Hong [10] derived the KdV equation with the hydrodynamic model and showed that the traffic soliton appears near the neutral stability line. Komatsu and Sasa [11] deduced the mKdV equation with the optimal velocity model proposed by Bando *et al.*. [12,13], while Nagatani [14,15] worked out the mKdV equation from the hydrodynamic model to describe the density waves in congestion. In real traffic flow, Kerner *et al.*. [16] and Knospe [17] made many of experimental investigations of traffic on highways, and found the wide moving traffic jams. Moreover Kerner *et al.*. [18] used an asymptotic theory of traffic jams based on the singular perturbation methods to derive formulas for the characteristic parameters of traffic flow.

For public demand, it is necessary to enhance the traffic current and avoid jams. Traffic control systems have been utilized as a part of intelligent transport system (for short, ITS). Drivers can receive information about other vehicles on roads, and then determine the velocity of their own vehicles. Thus the stability of traffic flow can be raised and the appearance of traffic jam might be suppressed. Some work has been done on the traffic behavior with the consideration of ITS control. Helbing [5] presented an improved gas-kinetic traffic model, which differs from others mainly by its nonlocal interaction term that takes into account the space requirements of vehicles and the correlations of successive vehicle velocities. The model reflects the anticipation behavior of drivers, which is responsible for a smoothing effect that acts only in the backward direction. Nagatani [19] put forward an extended optimal velocity model involving the vehicle interaction with the next car ahead (i.e., the next-nearest-neighbor interaction). Xue [3] proposed a lattice model of optimal traffic flow considering the optimal current of the next-nearest-neighbor interaction. Lenz, Wanger and Sollacher [20] discussed a model in which a driver looks at many vehicles ahead of him or her. Hasebe, Nakayama and Sugiyama [21,22] proposed an extended optimal velocity model which is applicable to a cooperative driving control system. In their model, drivers are assumed to be able to adjust their velocity by utilizing the information of an arbitrary number of vehicles that precedes or follows them. They found that there exists a certain set of parameters that makes traffic flow ”most stable” in the ”forward looking” optimal velocity model.

Nagatani proposed a simplified versions of the hydrodynamic models [14] in 1998. He used the continuum models to describe the jamming transition in traffic flow on a highway. Model I is described as

$$\partial_t \rho + \partial_x(\rho v) = 0, \tag{1}$$

$$\partial_t \rho v = a\rho_0 V(\rho(x+\delta)) - a\rho v, \tag{2}$$

where ρ_0 is the average density, and a is the sensitivity of a driver; $\rho(x+\delta)$ is the local density at position $x+\delta$ at time t; δ represents the average headway, which means $\delta = 1/\rho_0$; local density $\rho(x+\delta)$ is related with the inverse of headway $h(x,t): \rho(x+\delta) = 1/h(x,t)$. The idea is that a driver adjusts the car velocity according to the observed headway $h(x,t)$ [or density ahead $\rho(x+\delta)$].

Model II is the lattice version of model I with dimensionless space x

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0, \tag{3}$$

$$\partial_t \rho_j v_j = a\rho_0 V(\rho_{j+1}) - a\rho_j v_j. \tag{4}$$

where j denotes site j on the one-dimensional lattice, and $\rho_j(t), v_j(t)$ represent the local density and the local average velocity on site j at time t, respectively.

On the basis of lattice model of Nagatani [14,15,23], we propose the extended lattice version of the continuum models considering an arbitrary number of sites ahead on a

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single-lane highway. We obtain the stability conditions of the two models using the linear stability theory, and then derive the mKdV equations near the critical point by using nonlinear analysis. We find the traffic jam described by kink-antikink soliton solutions for the mKdV equations.

II. MODELS

The extended lattice version of the continuum models considering an arbitrary number of sites ahead on a single-lane highway is proposed. The vehicle motion is described by the following differential-difference equations, called model A:

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0, \tag{5}$$

$$\rho_j(t+\tau) v_j(t+\tau) = \rho_0 V(\rho_{j+1}(t), \rho_{j+2}(t), \ldots, \rho_{j+n}(t)). \tag{6}$$

where n denotes the number of sites ahead considered. As $n=1$, the original lattice version of the continuum model [23] on a single-lane highway is recovered. Equation (5) is the lattice version of a continuity equation, while Eq.(6) is the evolution equation. We select the optimal velocity as

$$V(\rho_{j+1}, \rho_{j+2}, \ldots, \rho_{j+n}) = \tanh\left(\frac{2}{\rho_0} - \frac{\sum_{l=1}^{n} \beta_l \rho_{j+l}(t)}{\rho_0^2} - \frac{1}{\rho_c}\right) + \tanh\left(\frac{1}{\rho_c}\right). \tag{7}$$

Here β_l is the weighting function to ρ_{j+l}, which corresponds to sensitivity a_i in the multi-anticipative car-following model [20]. The difference between them lies in that the optimal velocity in Ref. [20] is related to a certain position, while in this paper the traffic flow ahead is regarded as a whole, considering nonlocal effect. It is necessary to point out that $\beta_l (l=1,2,\ldots,n)$ have the properties as follows (1) β_l decreases monotonically with increasing value of l, which means $\beta_l/\beta_{l-1} < 1$, for we know that the influence of the sites ahead on the vehicle motion reduces gradually as the distance between the considered site and a site ahead increases. (2) $\Sigma_{l=1}^{n} \beta_l = 1$. Equation (7) has the inflection point at $\rho_j = \rho_c$ when $\rho_0 = \rho_c$. τ is introduced to denote the delay time that it takes the traffic current to reach the optimal current and τ is the inverse of the sensitivity a. We assume that a driver can obtain the information of any site density ahead. The traffic current $\rho_j(t+\tau) v_j(t+\tau)$ on the site j at time $t+\tau$ is determined by the optimal current $\rho_0 V(\rho_{j+1}(t), \rho_{j+2}(t), \ldots, \rho_{j+n}(t))$ on site $j+1, j+2, \ldots, j+n$ at time t.

We present another model, model B, which is described by following a difference-difference equation in which both space and time are discrete variables:

$$\rho_j(t+\tau) - \rho_j(t) + \tau \rho_0 [\rho_j(t) v_j(t) - \rho_{j-1}(t) v_{j-1}(t)] = 0, \tag{8}$$

$$\rho_j(t+\tau) v_j(t+\tau) = \rho_0 V(\rho_{j+1}(t), \rho_{j+2}(t), \ldots, \rho_{j+n}(t)). \tag{9}$$

By eliminating velocity in Eqs. (5) and (6) [and also in Eqs. (8) and (9)], we obtain the density equations for models A and B:

$$\partial_t \rho_j(t+\tau) + \rho_0^2 \left[V\left(\sum_{l=1}^{n} \beta_l \rho_{j+l}(t)\right) - V\left(\sum_{l=1}^{n} \beta_l \rho_{j+l-1}(t)\right)\right] = 0, \tag{10}$$

$$\rho_j(t+2\tau) - \rho_j(t+\tau) + \tau \rho_0^2 \left[V\left(\sum_{l=1}^{n} \beta_l \rho_{j+l}(t)\right) - V\left(\sum_{l=1}^{n} \beta_l \rho_{j+l-1}(t)\right)\right] = 0. \tag{11}$$

III. LINEAR STABILITY ANALYSIS

The linear stability analysis is made for the above traffic models. It is obvious that the uniform traffic flow with constant density ρ_0 and constant optimal velocity $V(\rho_0, \rho_0, \ldots, \rho_0)$ is the steady state solution for Eqs. (10) and (11), given as

$$\rho_j(t) = \rho_0 \text{ and } v_j(t) = V(\rho_0, \rho_0, \ldots, \rho_0), \tag{12}$$

Suppose $y_j(t)$ to be a small deviation from the steady state density of the jth vehicle

$$\rho_j(t) = \rho_0 + y_j. \tag{13}$$

Substituting Eq. (13) into Eqs. (10) and (11) and linearizing them yield

$$\partial_t y_j(t+\tau) + \rho_0^2 V'(\rho_0) \left[\sum_{l=1}^{n} \beta_l (y_{j+l}(t) - y_{j+l-1}(t))\right] = 0, \tag{14}$$

$$y_j(t+2\tau) - y_j(t+\tau) + \tau \rho_0^2 V'(\rho_0) \left[\sum_{l=1}^{n} \beta_l (y_{j+l}(t) - y_{j+l-1}(t))\right] = 0, \tag{15}$$

where $V'(\rho_0) = [dV(\rho_j)/d\rho_j]|_{\rho_j=\rho_0}$ because $\Sigma_{l=1}^{n} \beta_l y_{j+l}(t)$ is a small and bounded function.

Expanding y_j in the Fourier modes: $y_j(t) = \exp(ikj + zt)$, we obtain

$$z e^{z\tau} + \rho_0^2 V' \left(\sum_{l=1}^{n} \beta_l (e^{ikl} - e^{ik(l-1)})\right) = 0, \tag{16}$$

$$e^{2z\tau} - e^{z\tau} + \tau \rho_0^2 V' \left(\sum_{l=1}^{n} \beta_l (e^{ikl} - e^{ik(l-1)})\right) = 0. \tag{17}$$

For simplicity, $V'(\rho_0)$ is indicated as V' in the above equations and hereafter. Expanding $z = z_1(ik) + z_2(ik)^2 + \ldots$ and inserting it into Eqs. (16) and (17) lead to the first- and second-order terms of coefficient in the expression of z, respectively,

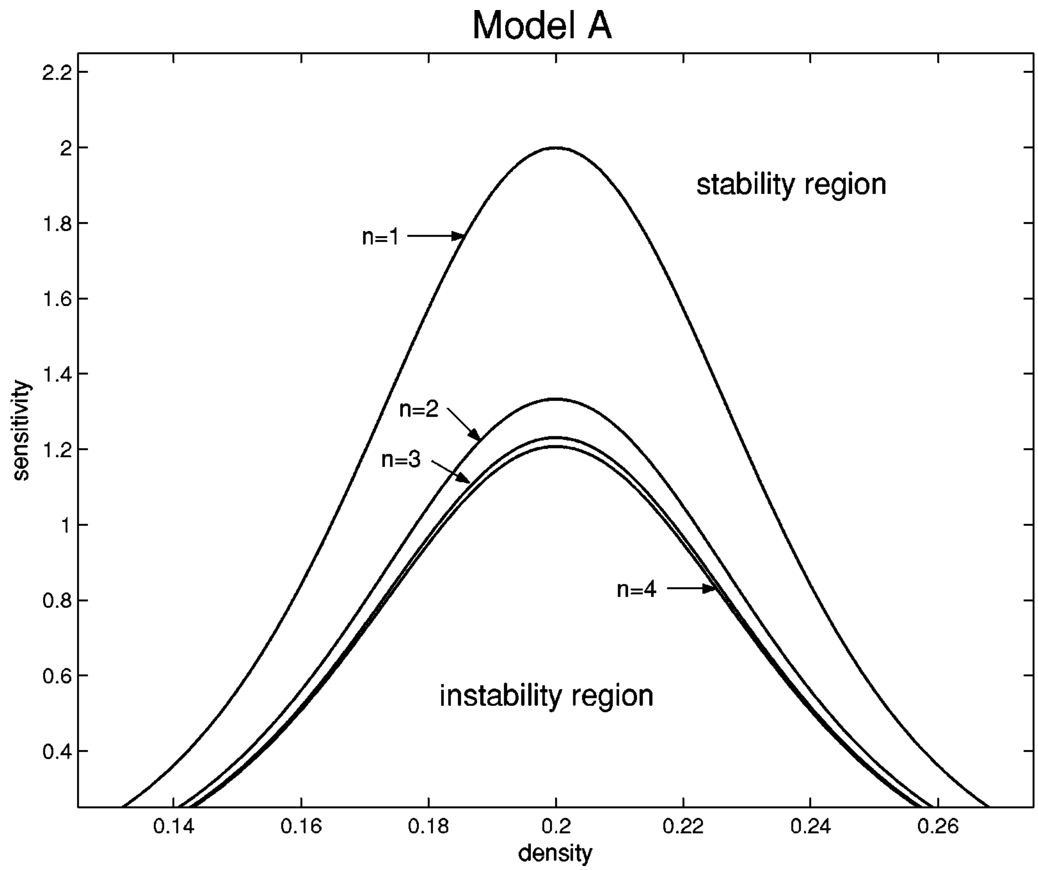

FIG. 1. The neutral stability lines considering different lattices ahead.

$$z_1 = -\rho_0^2 V' \text{ and } z_2 = -\tau(\rho_0^2 V')^2 - \rho_0^2 V' \sum_{l=1}^{n} \beta_l \left(l - \frac{1}{2}\right), \tag{18}$$

$$z_1 = -\rho_0^2 V' \text{ and } z_2 = -\frac{2}{3}\tau(\rho_0^2 V')^2 - \rho_0^2 V' \sum_{l=1}^{n} \beta_l \left(l - \frac{1}{2}\right), \tag{19}$$

The neutral stability condition is given by

$$\tau = -\frac{\sum_{l=1}^{n} \beta_l (2l-1)}{2\rho_0^2 V'}, \quad \text{for model A}, \tag{20}$$

$$\tau = -\frac{\sum_{l=1}^{n} \beta_l (2l-1)}{3\rho_0^2 V'}, \quad \text{for model B}. \tag{21}$$

For small disturbances with long wavelengths, the uniform traffic flow is unstable in the condition that

$$\tau > -\frac{\sum_{l=1}^{n} \beta_l (2l-1)}{2\rho_0^2 V'}, \quad \text{for model A}, \tag{22}$$

$$\tau > -\frac{\sum_{l=1}^{n} \beta_l (2l-1)}{3\rho_0^2 V'}, \quad \text{for model B}. \tag{23}$$

The neutral stability lines in the parameter space (ρ, a) are shown in Fig. 1 for model A. There exist critical points (ρ_c, a_c) for the neutral stability lines, such that uniform states with any density are always linearly stable for $a > a_c$, while the uniform states in a neighborhood of ρ_c are unstable for $a < a_c$. As $n=1$, the critical points and the neutral stability lines are consistent with those in a single-lane highway traffic flow [23]. From the figure it can be seen that with taking into account more sites ahead, the critical points and the neutral stability lines decrease, which means the stability of the uniform traffic flow has been strengthened. The traffic jam is thus efficiently suppressed.

IV. NONLINEAR ANALYSIS

Because of the complexity of Eqs. (10) and (11), it is difficult to extract the essential properties of solutions. Thus we use the reductive perturbation method to Eqs. (10) and (11) focusing on the system behavior near the critical point (ρ_c, a_c). With such simplification, the nature of kink-antikink solitons can be described by the mKdV equations. We introduce slow scales for space variable j and time variable t [24,25], and define the slow variables X and T

$$X = \varepsilon(j + bt) \text{and } T = \varepsilon^3 t, 0 < \varepsilon \ll 1, \tag{24}$$

where b is a constant to be determined. Let

$$\rho_j(t) = \rho_c + \varepsilon R(X,T). \tag{25}$$

Substituting Eqs. (24) and (25) into Eqs. (10), (11) and making the Taylor expansions to the fifth order of ε lead to the expression.

$$\begin{aligned}&\varepsilon^2(b+\rho_c^2 V')\partial_X R + \varepsilon^3\left[b^2\tau + \frac{\rho_c^2 V'}{2}\sum_{l=1}^{n}\beta_l(2l-1)\right]\partial_X^2 R \\ &+ \varepsilon^4\left\{\partial_T R + \left[\frac{b^3\tau^2}{2} + \frac{\rho_c^2 V'}{6}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right]\partial_X^3 R + \frac{\rho_c^2 V'''}{6}\partial_X R^3\right\} + \varepsilon^5\left\{2b\tau\partial_X\partial_T R \right.\\ &+ \left[\frac{b^4\tau^3}{6} + \frac{\rho_c^2 V'}{24}\sum_{l=1}^{n}\beta_l(4l^3-6l^2+4l-1)\right]\partial_X^4 R \\ &\left.+ \frac{\rho_c^2 V'''}{4}\sum_{l=1}^{n}\beta_l(2l-1)[R^2\partial_X^2 R + 2R(\partial_X R)^2]\right\} = 0,\end{aligned} \tag{26}$$

$$\begin{aligned}&\varepsilon^2(b+\rho_c^2 V')\partial_X R + \varepsilon^3\left[\frac{3}{2}b^2\tau + \frac{\rho_c^2 V'}{2}\sum_{l=1}^{n}\beta_l(2l-1)\right]\partial_X^2 R \\ &+ \varepsilon^4\left\{\partial_T R + \left[\frac{7b^3\tau^2}{6} + \frac{\rho_c^2 V'}{6}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right]\partial_X^3 R + \frac{\rho_c^2 V'''}{6}\partial_X R^3\right\} + \varepsilon^5\left\{3b\tau\partial_X\partial_T R \right.\\ &+ \left[\frac{5b^4\tau^3}{8} + \frac{\rho_c^2 V'}{24}\sum_{l=1}^{n}\beta_l(4l^3-6l^2+4l-1)\right]\partial_X^4 R \\ &\left.+ \frac{\rho_c^2 V'''}{4}\sum_{l=1}^{n}\beta_l(2l-1)[R^2\partial_X^2 R + 2R(\partial_X R)^2]\right\} = 0,\end{aligned} \tag{27}$$

where $V' = [dV(\rho_j)/d\rho_j]|_{\rho_j=\rho_c}$ and $V''' = [d^3V(\rho_j)/d\rho_j^3]|_{\rho_j=\rho_c}$. V' and V''' correspond to $V'(\rho_c), V'''(\rho_c)$ in the above equation and hereafter. Near the critical point $(\rho_c, a_c), \tau = (1+\varepsilon^2)\tau_c$, taking $b = -\rho_c^2 V'$ and eliminating the second- and third-order terms of ε from Eqs. (26) and (27) result in the simplified equation:

$$\begin{aligned}&\varepsilon^4\left\{\partial_T R + \left[\frac{b^3\tau_c^2}{2} + \frac{\rho_c^2 V'}{6}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right]\partial_X^3 R + \frac{\rho_c^2 V'''}{6}\partial_X R^3\right\} + \varepsilon^5\left\{b^2\tau_c\partial_X^2 R + \frac{\rho_c^2 V'''}{4}\sum_{l=1}^{n}\beta_l(2l-1)\right.\\ &\times[R^2\partial_X^2 R + 2R(\partial_X R)^2] + \left[-\frac{5b^4\tau_c^3}{6} - \frac{b\tau_c\rho_c^2 V'}{3}\sum_{l=1}^{n}\beta_l(3l^2-3l+1) + \frac{\rho_c^2 V'}{24}\sum_{l=1}^{n}\beta_l(4l^3-6l^2 \right.\\ &\left.\left. + 4l-1)\right]\partial_X^4 R\right\} = 0,\end{aligned} \tag{28}$$

$$\begin{aligned}&\varepsilon^4\left\{\partial_T R + \left[\frac{7b^3\tau_c^2}{6} + \frac{\rho_c^2 V'}{6}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right]\partial_X^3 R + \frac{\rho_c^2 V'''}{6}\partial_X R^3\right\} + \varepsilon^5\left\{\frac{3}{2}b^2\tau_c\partial_X^2 R + \frac{\rho_c^2 V'''}{4}\right.\\ &\times\left[\sum_{l=1}^{n}\beta_l(2l-1) - 6b\tau_c\right][R^2\partial_X^2 R + 2R(\partial_X R)^2] + \left[-\frac{69b^4\tau_c^3}{24} - \frac{b\tau_c\rho_c^2 V'}{2}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right.\\ &\left.\left. + \frac{\rho_c^2 V'}{24}\sum_{l=1}^{n}\beta_l(4l^3-6l^2+4l-1)\right]\partial_X^4 R\right\} = 0.\end{aligned} \tag{29}$$

In order to obtain the standard mKdV equation with higher order correction, we make the following transformations for Eqs. (28) and (29):

$$T' = -\left[\frac{b^3\tau_c^2}{2} + \frac{\rho_c^2 V'}{6}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right]T, \tag{30}$$

$$R = \left[\frac{-3b^3\tau_c^2}{\rho_c^2 V'''} - \frac{\rho_c^2 V'}{\rho_c^2 V'''}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right]^{1/2} R', \tag{31}$$

$$T' = -\left[\frac{7b^3\tau_c^2}{6} + \frac{\rho_c^2 V'}{6}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right]T, \tag{32}$$

$$R = \left[\frac{-7b^3\tau_c^2}{\rho_c^2 V'''} - \frac{\rho_c^2 V'}{\rho_c^2 V'''}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right]^{1/2} R'. \tag{33}$$

Considering Eqs. (20) and (21) we obtain the regularized equations

$$\partial_{T'} R' = \partial_X^3 R' - \partial_X R'^3 - \varepsilon M_1[R'], \tag{34}$$

$$\partial_{T'} R' = \partial_X^3 R' - \partial_X R'^3 - \varepsilon M_2[R'], \tag{35}$$

where

TABLE I. The critical sensitivity a_c and the propagation velocity c_1 in model A.

	n	1	2	3	4	5	11	12	20
F_1	a_c	2.0	1.33333	1.23077	1.20755	1.20188	1.20001	1.2	1.2
	c_1	24	24	24	24	24	24	24	24
F_2	a_c	2.0	1.2	1.05882	1.01887	1.00621	1.00001	1.0	1.0
	c_1	24	24	24	24	24	24	24	24

$$M_1[R'] = \frac{24\sum \beta_l(2l-1)}{2\sum \beta_l(1-12l^2)+24(\sum \beta_l l)^2}\partial_X^2 R' - \frac{3}{2}\sum_{l=1}^{n}\beta_l(2l-1)[R'^2\partial_X^2 R' + 2R'(\partial_X R')^2]$$
$$+\frac{-1+2\sum \beta_l(-4l^3-6l^2+l)+12(\sum \beta_l l)^2+48(\sum \beta_l l^{3/2})^2-40(\sum \beta_l l)^3}{2\sum \beta_l(1-12l^2)+24(\sum \beta_l l)^2}\partial_X^4 R', \tag{36}$$

$$M_2[R'] = \frac{24\sum \beta_l(2l-1)}{\sum \beta_l(2+l+27l^2)-28(\sum \beta_l l)^2}\partial_X^2 R' - \frac{3}{2}\sum_{l=1}^{n}\beta_l(2l-1)[R'^2\partial_X^2 R' + 2R'(\partial_X R')^2]$$
$$+\frac{2-3\sum \beta_l(-6l^3-9l^2+l)-30(\sum \beta_l l)^2-108(\sum \beta_l l^{3/2})^2+92(\sum \beta_l l)^3}{-2\sum \beta_l(2+l+27l^2)+56(\sum \beta_l l)^2}\partial_X^4 R', \tag{37}$$

where Σ denotes $\Sigma_{l=1}^{n}$. Equations (34) and (35) are the modified KdV equations with $O(\varepsilon)$ correction terms on the right-hand side. First, we ignore the $O(\varepsilon)$ terms in Eqs. (34) and (35), and get the mKdV equation with the kink solution

$$R_0'(X,T') = \sqrt{c}\tanh\sqrt{\frac{c}{2}}(X-cT'). \tag{38}$$

Next, supposing $R'(X,T')=R_0'(X,T')+\varepsilon R_1'(X,T')$, we take into account the $O(\varepsilon)$ correction. To determine the selected value of the propagation velocity c for the kink solution (38), it is necessary to consider the solvability condition [26,27]

$$(R_0', M_1[R_0']) \equiv \int_{-\infty}^{+\infty} dX R_0' M_1[R_0'] = 0,$$

$$(R_0', M_2[R_0']) \equiv \int_{-\infty}^{+\infty} dX R_0' M_2[R_0'] = 0, \tag{39}$$

where

$$M_1[R_0'] = M_1[R'] \text{ and } M_2[R_0'] = M_2[R'].$$

By performing the integration, we obtain the selected velocity c

$$c_1 = \frac{-120\sum \beta_l(2l-1)}{\sum \beta_l(1-10l+60l^2+16l^3)-60(\sum \beta_l l)^2-168(\sum \beta_l l^{3/2})^2+152(\sum \beta_l l)^3}, \tag{40}$$

$$c_2 = \frac{-270\sum \beta_l(2l-1)}{\sum \beta_l(10-15l+135l^2+36l^3)-150(\sum \beta_l l)^2-378(\sum \beta_l l^{3/2})^2+352(\sum \beta_l l)^3}. \tag{41}$$

We obtain the kink-antikink soliton solutions for models A and B

$$R(X,T) = \left[\frac{-3b^3\tau_c^2 c_1}{\rho_c^2 V'''} - \frac{\rho_c^2 V' c_1}{\rho_c^2 V'''}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right]^{1/2}\tanh\sqrt{\frac{c_1}{2}}\left[X + c_1\left(\frac{b^3\tau_c^2}{2} + \frac{\rho_c^2 V'}{6}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right)T\right], \tag{42}$$

$$R(X,T) = \left[\frac{-7b^3\tau_c^2 c_2}{\rho_c^2 V'''} - \frac{\rho_c^2 V' c_2}{\rho_c^2 V'''}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right]^{1/2}\tanh\sqrt{\frac{c_2}{2}}\left[X + c_2\left(\frac{7b^3\tau_c^2}{6} + \frac{\rho_c^2 V'}{6}\sum_{l=1}^{n}\beta_l(3l^2-3l+1)\right)T\right], \tag{43}$$

where b, τ_c, c_1, c_2 given before.

V. RESULT ANALYSIS AND DISCUSSION

On the basis of the linear stability theory and the nonlinear wave analysis, we obtain the critical points (ρ_c, a_c) and the propagation velocities c_1, c_2 of the kink-antikink solutions for models A and B. Now we select two specific optimal functions ($\beta_l=1$ for $n=1$). In this paper, we take tentatively for $n>1$

TABLE II. The critical sensitivity a_c and the propagation velocity c_2 in model B.

	n	1	2	3	4	5	6	11	20
F_1	a_c	3.0	2.0	1.84615	1.81132	1.80282	1.8007	1.8	1.8
	c_2	27	32	33.9656	34.5222	34.666	34.7022	34.7143	34.7143
F_2	a_c	3.0	1.8	1.58824	1.5283	1.50932	1.5	1.5	1.5
	c_2	27	34.7143	39.7636	41.968	42.7788	43.1999	43.2	43.2

$$F_1 \equiv \beta_l = \begin{cases} \dfrac{3}{4^l}, & l \neq n, \\ \dfrac{1}{4^{n-1}}, & l = n, \end{cases} \tag{44}$$

$$F_2 \equiv \beta_l = \begin{cases} \dfrac{2}{3^l}, & l \neq n, \\ \dfrac{1}{3^{n-1}}, & l = n. \end{cases} \tag{45}$$

We calculate the values of the critical sensitivity a_c and the propagation velocities c_1, c_2 by use of Eqs. (44) and (45). They are shown in Tables I and II.

Table I shows the propagation velocity c_1 is a constant, while Table II shows the propagation velocity c_2 is variable and increases with increasing n. In both models, the critical sensitivities decrease with the increase of n, and the stability regions are enlarged for the two models. As n raises up to a certain value, the critical sensitivities a_c and the propagation velocities c_2 will not change again, and the system is in a stable state. In fact, only the former three terms play an important role to the stability. We may consider this state as the optimal state. The information of this state is enough for a driver.We also find that as the ratios β_l/β_{l-1} of weighting function are bigger, the whole stability is better. As $n=1$, which corresponds to the first value of weighting function is 1 and the others are 0, the stability region is the smallest. So considering, the cooperative driving system will stabilize the traffic flow.

The neutral stability lines in the parameter space (ρ, a) are shown in Fig. 1 for model A and the optimal function is described by Eqs. (44) in the model.

VI. SUMMARY

We have proposed two lattice hydrodynamic models of traffic flow for the purpose of constructing a driving system for freeway traffic and given a form of optimal velocity function taking into account the nonlocal effect. The traffic nature has been analytically analyzed by using the linear stability theory and the nonlinear analysis. It has been shown that there exist critical points in the two models and obtained the neutral stability lines, which demonstrate that multivehicle consideration could further stabilize traffic flow, and considering three sites in front is an optimal state. We have derived the mKdV equations to describe the traffic jam near the critical points, respectively, and obtained the kink-antikink soliton solutions related to the traffic density waves. Moreover, two examples with different optimal velocity functions are calculated to show the results clearly. As $n=1$, the two models reduce to the original lattice version of the continuum models on a single-lane highway, and the results are identical.

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