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Devil's-staircase-like behavior of the range of random time series with record-breaking fluctuations

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We propose insight into the analysis of the record-breaking fluctuations in random time series, which permits to distinguish between the self-organized criticality and the record dynamics (RD) scenarios of system evolution, using a finite time series realization. Performed analysis of the time series associated with the historical prices of different commodities has shown that the evolution of commodity markets is controlled by the record-breaking fluctuations as it is outlined by the RD. Furthermore, we found that the sizes of record-breaking fluctuations follow a fat-tailed distribution and the devil's-staircase-like records of price ranges are multiaffine and persistent, nevertheless, the high moments $(q>q_C>2)$ of their q-order height-height correlation functions behave logarithmically.

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The evolution of many complex systems occurs in terms of sudden outburst activity separated by periods of relative quiescence [1,2]. The intermittent activity has been observed in a great variety of systems studied in physics [3], biology [4,5], geosciences [6], and econophysics [7]. It was suggested that this behavior may be caused by self-organization of the system into a "critical state" in which the system exists in a state of punctuated equilibrium [1,8]. This is known today as the self-organized criticality (SOC). The essential feature of SOC systems is that they evolve, slowly driven by means of an external force, into a *stationary critical state* in which the distribution of intermittent events (quakes or avalanches) follows a power law [9]. However, the evolution of a great variety of physical, biological, economic, and social systems is essentially nonstationary, because the quakes gradually change both the physical and statistical properties of the system [2]. In many cases these intermittent changes are characterized by the decelerating rate associated with the log-Poisson statistics, since the aging tends on the average to increase the stability of subsequent metastable states [2,4]. The relevant examples range from the evolution in rugged fitness landscapes [10] to biological macroevolution, [11] and from the aging in spin glasses [12] to magnetic relaxation in type-II superconductors [2]. The coarse-grained aging dynamics ruling the system through a sequence of gradually deeper attractors was outlined by the paradigm of record dynamics (RD), which deals with the largest values of relevant parameters assumed by up to the current time t [2].

In contrast to the SOC scenario, the RD is concerned with the non-steady-state statistics of a system in which the relevant macroscopic variables slowly change in time at a decelerating rate [4,10–12]. RD makes no statement about the quake size distribution, since a system ruled by the record-breaking fluctuations may either achieve or not achieve the critical state [2]. So, to model a system displaying intermittent activity, first of all we need to distinguish between two fundamentally different scenarios of system evolution.

In many cases, the analysis of time series of appropriate observables, $p(t)$, has been shown to give important information regarding the underlying processes responsible for the observed macroscopic behavior [13]. Specifically, in the case of SOC, the time series is expected to be stationary, whereas the RD is associated with the logarithmic increase (decrease) in the mean and variance of the analyzed signal [2]. However, in many cases the problem of characterizing and quantifying a system dynamics from a finite realization of a time series is not a trivial one. For example, the moving averages and variances of prices of many commodities measured in constant dollars [see Fig. 1(a)] fluctuate only slightly around their mean values [see Fig. 2(a)], nevertheless, the time series range increases with time [see Fig. 1(c)]. Furthermore, we are not always able to detect the RD from finite realizations of time series, because the record signal $M(t)=\max p(t)$, as it is defined in the tangled nature and restricted occupancy models of RD [2], remains constant during a long time period, comparable with the time series realization length, T [see Fig. 1(a)].

Fortunately, in many cases the analysis of time series fluctuations can give important additional information regarding the underlying processes responsible for the observed macroscopic behavior of the system. The fluctuations of any time series can be characterized by the magnitude (absolute value) of changes and their direction (sign) [14]. The magnitude series relates to the nonlinear properties of the original time series, while the sign series relates to the linear properties [13]. It was found that the magnitude of fluctuations of many apparent random time series follows a fat-tailed distribution and exhibits long-range power-law correlations, characterized by the so-called Hurst exponent ζ [13,14]. The "sign time series" also displays the scale-invariant dynamics, but with a different scaling exponent ζ_{sign} [13]. Moreover, the scaling properties of negative and positive changes of real-world time series may be different [15,16]. This asymmetry should be reflected in the scaling behavior of the time series range, $R(t)=\max_{0\leq t\leq T} p(t)-\min_{0\leq t\leq T} p(t)$, and so, the range of such a time series seems to be an appropriate characteristic of the record-breaking fluctuations.

In this work, we analyzed the time series of historical

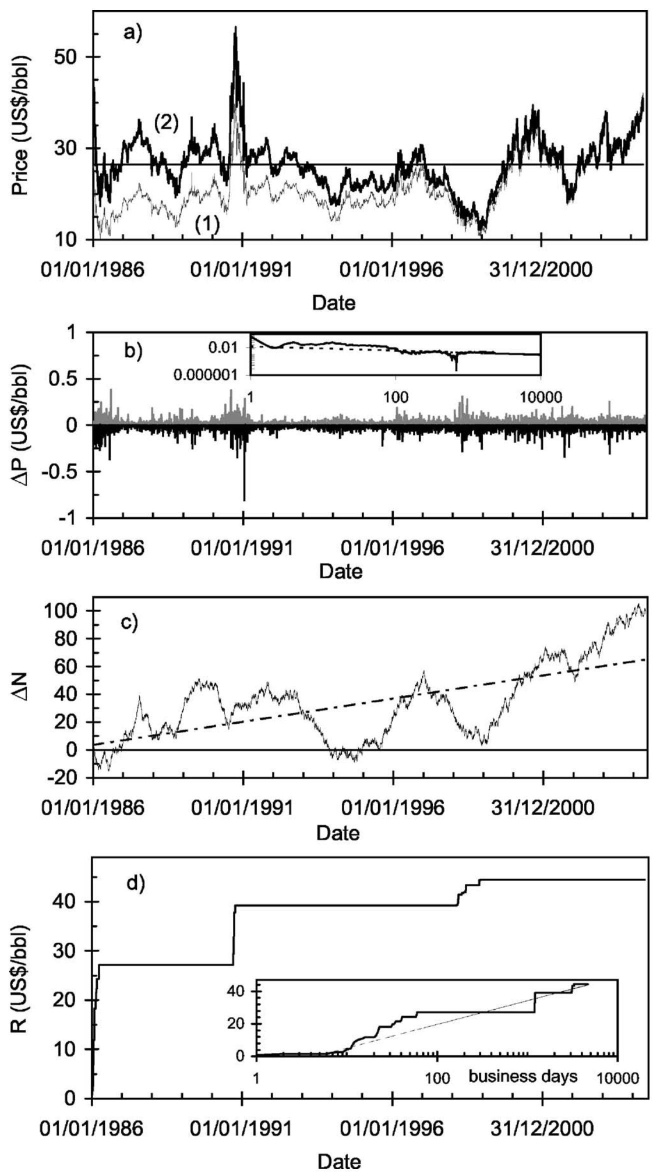

FIG. 1. (a) Time series of West Texas Intermediate crude oil spot price in the current (1) and in the 2003 constant (2) dollars per barrel, \$/bbl (source: Bloomberg database [17]). (b) Time series of price changes (inset: Δn vs n in the log-log coordinates). (c) The graph of ΔN vs the calendar time. (d) Time series of price range [the inset shows the logarithmic trend of $R(t)$].

prices for some commodities [17] (crude oil, natural gas, and gold). Previously, the fluctuations in these time series were studied in Refs. [18–20]. It was found that the magnitude of price changes $|\Delta P(t,\tau)|$ exhibit the long-range power-law correlations, nevertheless, the price $P(t)$ and the price changes $\Delta P(t,\tau)=P(t+\tau)-P(t)$ are uncorrelated beyond rather short time scales [16,18–20]. Furthermore, it was noted that the distributions of negative and positive changes of prices are fat tailed and characterized by slightly different exponents [14]. Hence, one may expect that the analysis of the range records of these time series permits to distinguish between the SOC and RD scenarios of system evolution and gives additional information about the correlations in the market dynamics.

Accordingly, in this work we were focused on the scaling behavior of $R(t)$ and its relation to the scaling behavior of the sign and magnitude of price changes. To test the correlations in the analyzed time series, we studied the autocorrelation function, $C(\tau)=\langle p(t+\tau)p(t)\rangle_T/\langle p^2(t)\rangle_T$, where the angle brackets denote the time average. The scaling properties of (nonstationary [21]) time series and their ranges were studied by calculating the q-order height difference correlation function

$$\sigma_q(\tau)=\langle|p(t)-p(t+\tau)|^q\rangle_T^{1/q}\propto\tau^{\zeta_q}, \tag{1}$$

for $0.01<q\leq 100$; here ζ_q is the spectrum of scaling exponents [22]. Furthermore, the Hurst exponent, $\zeta=\zeta_2$, of each time series was also determined from the scaling behavior of the power spectrum, $S(\omega)=\langle\hat{p}(\omega)\hat{p}(-\omega)\rangle\propto\omega^{-(2\zeta+1)}$, where $\hat{p}(\omega)=T^{-1/2}\int dx[p(t)-\langle p(t)\rangle_T]\exp(i\omega t)$ is the Fourier transform of $p(t)$.

Figures 1(a) and 1(b) show the daily records of the spot prices, $P(t)$, and price changes, $\Delta P(t)=P(t+1)-P(t)$, from the West Texas Intermediate crude oil price listings [17]. To avoid the effect of inflation, we analyzed the crude oil price in constant 2003 US dollars over the period from 2 January 1986 to 28 May 2004 representing $T=4652$ observations (weekends and business holidays are excluded). Earlier we found [19] that the autocorrelation function of the price time series decays exponentially as $C\propto\exp(-\tau/\tau_0)$ with a characteristic time $\tau_0=120$ business days (about half the business year). Furthermore, we found that $\zeta_q=\zeta_2=\zeta=0.5\pm0.02$ [19]. So, there are no long-range correlations in the time series of crude oil prices, as well as in the time series of prices of other studied commodities [18–20]. This is consistent with the finding that the prices of commodities follow the symmetric logistic distribution [see Fig. 2(b)].

At the same time, we noted that the absolute values of negative price changes, $|\Delta P_-|$, are generally larger than positive changes, $\Delta P_+>0$ [see Fig. 1(b)], while the number (frequency) of positive changes $N_+(t)$ are slightly larger then the number of negative changes $N_-(t)$. Moreover, we found that the difference $\Delta N(t)=N_+-N_-$ possesses a linear trend [see Fig. 1(c)], whereas the difference between the absolute values of consecutive-ordered negative and positive changes scales as $\Delta(n)=|\Delta P_-|-\Delta P_+\propto n^{-0.3}$ [see inset in Fig. 1(b)]. As a result of these "leverage effects," the price range $R(t)$ displays a stepwise increase with a logarithmic trend [Fig. 1(d)].

Furthermore, we found that the ratio $r=(t_i-t_{i-1})/t_i$ of the waiting time between quakes (price range increments), $y=t_i-t_{i-1}$, and the time of the $(i-1)$th quake, $x=t_{i-1}$, remains nearly constant over about four orders of magnitude [see inset in Fig. 2(c)]. The cumulative distribution of this ratio displays a logarithmic trend over about two decades [see Fig. 2(c)], while the distribution of log waiting times, $z=\ln(t_i/t_{i-1})$, is exponential [see Fig. 2(d)], which suggests [23] that the "quakes" follow the log-Poisson distribution.

Therefore, the data presented in Figs. 1(d), 2(c), and 2(d) provide strong evidence of the RD nature (see Refs. [2,4]) of crude oil market evolution; nevertheless, the quake sizes [range increments, $\Delta R=R(t_i)-R(t_{i-1})$, see Fig. 3(a)] follow the fat-tailed log-logistic distribution [Fig. 3(b)] with Lévy

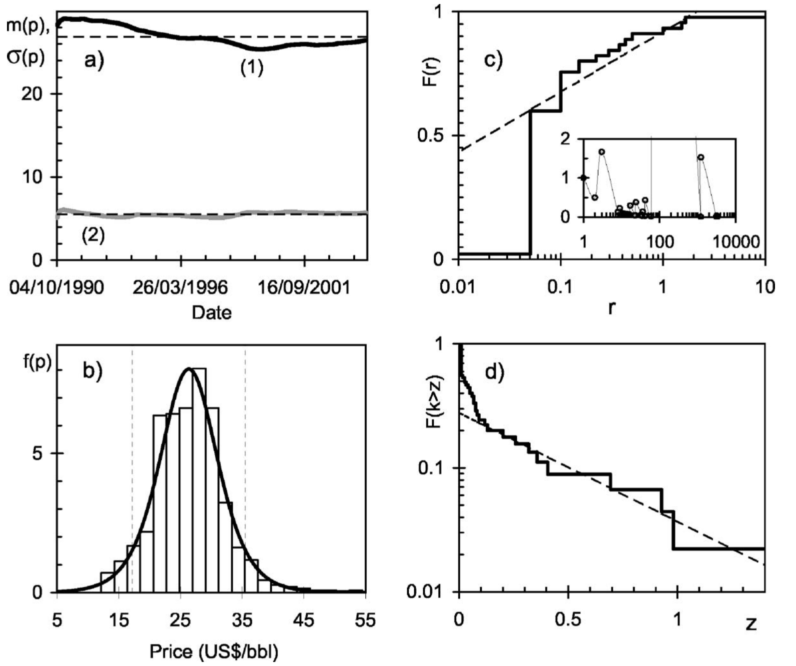

FIG. 2. (a) The moving average (1) and standard deviation (2), and (b) the conditional probability distribution of crude oil price in constant dollars. (c) The cumulative distribution $F(r)$ of the ratio $r=(t_i-t_{i-1})/t_i$ in semilog coordinates [the inset shows the ratio $r=(t_i-t_{i-1})/t_i$ of the waiting time between quakes vs the time of the $(i-1)$th quake, $x=t_{i-1}$]. (d) The distribution $F(k>z)$ of the log waiting times, $z=\ln(t_i/t_{i-1})$, in semilog coordinates.

index $\mu=2.58$ outside the Lévy stable range $(0<\mu<2)$, which indicates the presence of the long-range correlations in the price range behavior.

Furthermore, the scaling analysis shows [see Figs. 3(c)–3(e) that $R(t)$ has a nontrivial scaling behavior. Specifically, we found that the high-order moments of height-height correlations depend logarithmically on τ [see Fig. 3(e)], e.g.,

$$\sigma_q(\tau) = b(q)\ln \tau - a(q) \quad \text{for } q > q_C, \tag{2}$$

where $a(q)$ and $b(q)$ are decreasing functions of q, and q_C is the critical order. The lower moments $(q \leqslant q_C)$ display a multiaffine power-law scaling (1) [see Figs. 3(d) and 3(e)] characterized by the following spectrum of scaling exponents [see Fig. 3(f)]:

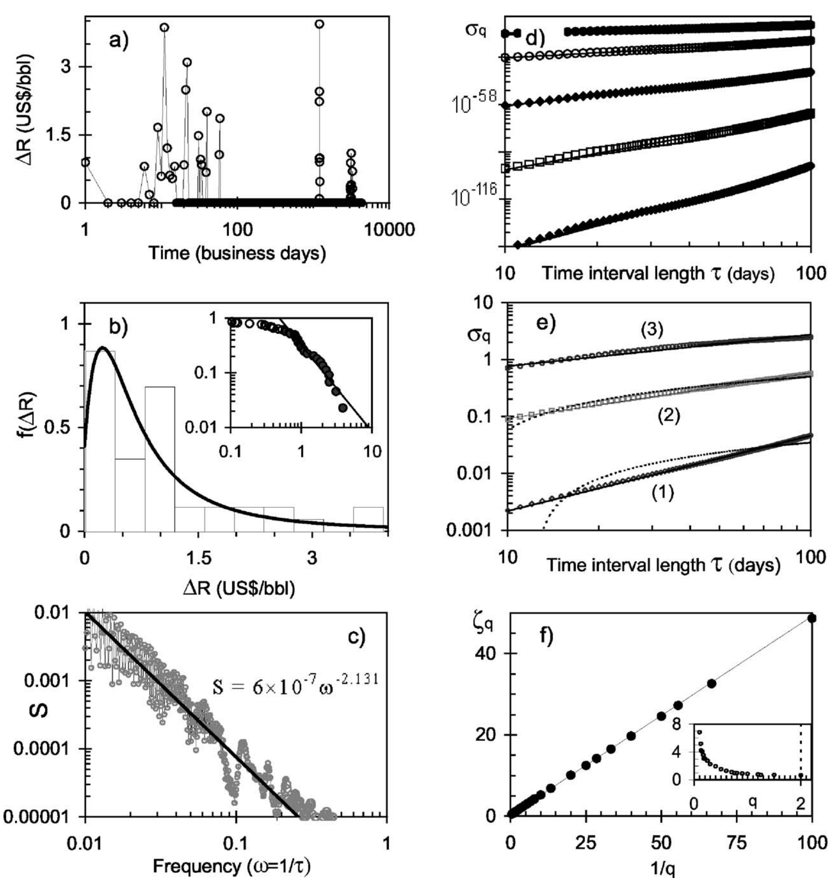

FIG. 3. (a) Time series of price range changes ΔR (quakes). (b) Conditional probability distribution of ΔR (inset: the distribution trend in the log-log coordinates). (c) Power spectrum of the time series $R(t)$ shown in Fig. 1(d). (d) and (e) Graphs of $\sigma_q(\tau)$ in the log-log coordinates: (d) from bottom to top $q=0.01$, 0.015, 0.025, 0.05, 0.1; (e) $q=0.5$ (1), $q=1$ (2), and $q=2.2$ (3) [solid lines—the power-law fits: (1) $\sigma_{0.5}=0.0001\tau^{1.234}$, $R^2=0.9995$; (2) $\sigma_1=0.017\tau^{0.745}$, $R^2=0.9969$; (3) $\sigma_{2.2}=0.2741\tau^{0.492}$, $R^2=0.9695$; and pointed lines-the logarithmic fits: (1) $\sigma_{0.5}=0.02\ln\tau-0.056$, $R^2=0.8741$; (2) $\sigma_1=0.214\ln\tau-0.474$, $R^2=0.9689$; (3) $\sigma_2=0.787\ln\tau-1.129$, $R^2=0.9986$. (f) Graph of ζ_q vs $1/q$ [dots-experimental data, solid line-data fit by Eq. (3) for $0.01 \leqslant q \leqslant 2.15$]; the inset shows the graph of ζ_q vs q.

$$\zeta_q = \zeta_* \left(1 + \frac{\alpha}{q}\right) \quad \text{for } q \leq q_C. \tag{3}$$

The value q_C is defined from the behavior of the square of the correlation coefficient for data fitting using Eqs. (2) and (3): $R^2(1) \geq R^2(2)$, when $q \leq q_C$, and $R^2(1) < R^2(2)$, when $q > q_C$ [see Fig. 3(e)].

For the crude oil price range, $R(t)$, we found $\zeta_* = 0.31 \pm 0.01$, $\alpha = 1.58 \pm 0.02$, and $q_C = 2.15$ [see Figs. 3(e) and 3(f)], e.g., $\zeta_2 = 0.56 \pm 0.02$ [see also Fig. 3(c)]; i.e., $R(t)$, displays persistence. It should be emphasized that the same results were also obtained for different parts of length ($T = 3650$ observations) of the original time series ($T = 4652$ observations).

Furthermore, we found that the price ranges of all studied commodities (natural gas and gold) display the multiaffine devil's-staircase-like behavior characterized by the spectrum of scaling exponents (3) with $2 < q_C < 3$. We also found that the range increments for all studied commodity prices follow the fat-tailed distribution with the Lévy index $2 < \mu < 3$ outside the Lévy stable range. Moreover, we found that for all studied commodities the scaling exponents satisfy the empirical relation

$$\alpha = \mu - 1, \tag{4}$$

and ζ_* varies in the range $0.25 < \zeta_* \leq 0.5$. Detailed results of these studies will be published elsewhere.

We expect that our findings are applicable to a wide variety of systems with dynamics controlled by the record-breaking fluctuations.

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[1] M. Paczuski, S. Maslov, and P. Bak, Phys. Rev. E **53**, 414 (1996).

[2] P. E. Anderson, H. J. Jensen, L. P. Oliveria, and P. Sibani, Complexity **10**, 49 (2004).

[3] P. Sibani and K. M. Andersen, Phys. Rev. E **64**, 021103 (2001); A. S. Balankin, O. Susarrey, and A. Bravo, *ibid.* **64**, 066131 (2001); B. J. Kim *et al.*, *ibid.* **66**, 021907 (2002); Y. F. Contoyiannis, F. K. Diakonos, and A. Malakis, Phys. Rev. Lett. **89**, 035701 (2002); L. Staron, J.-P. Vilotte, and F. Radjai, *ibid.* **89**, 204302 (2002); E. Altshuler *et al.*, *ibid.* **91**, 014501 (2003); Q. Chen *et al.*, *ibid.* **90**, 214503 (2003); Ch. Lee, K. Yeo, and J.-I. Choi, *ibid.* **92**, 144502 (2004).

[4] M. Hall, K. Christensen, S. A. Collobiano, and H. J. Jensen, Phys. Rev. E **66**, 011904 (2002): K. Christensen, S. A. Collobiano, M. Hall, and H. J. Jensen, J. Theor. Biol. **216**, 73 (2002).

[5] P. A. Rikvold and R. K. P. Zia, Phys. Rev. E **68**, 031913 (2003).

[6] Z. Olami, H. J. S. Feder, and K. Christensen, Phys. Rev. Lett. **68**, 1244 (1992); S. Lise and M. Paczuski, Phys. Rev. E **63**, 036111 (2001); V. N. Livina *et al.*, *ibid.* **67**, 042101 (2003).

[7] A. Krawiecki, Y. A. Hołyst, and D. Helbing, Phys. Rev. Lett. **89**, 158701 (2002); M. Ausloos and K. Ivanova, Phys. Rev. E **68**, 046122 (2002).

[8] S. Boettcher and M. Paczuski, Phys. Rev. Lett. **76**, 348 (1996); **79**, 889 (1997).

[9] P. Bak, *How Nature Works* (Oxford University Press, Oxford, U.K., 1995).

[10] P. Sibani and A. Pedersen, Europhys. Lett. **48**, 346 (1999); P. Sibani, Phys. Rev. Lett. **79**, 1413 (1997); P. E. Anderson, L. Tsimring, and V. Vinokur, *ibid.* **79**, 3298 (1997).

[11] P. Sibani, M. Schmidt, and P. Alstrom, Phys. Rev. Lett. **75**, 2055 (1995).

[12] P. Sibani and J. Dall, Europhys. Lett. **64**, 8 (2003).

[13] Y. Chen, M. Ding, and J. A. Scott Kelso, Phys. Rev. Lett. **79**, 4501 (1997); E. Koscielny-Bunde *et al.*, *ibid.* **81**, 729 (1998); L. A. N. Amaral, A. L. Goldberger, P. Ch. Ivanov, and H. E. Stanley, *ibid.* **81**, 2388 (1998); R. B. Govindan *et al.*, *ibid.* **89**, 028501 (2002); A. Bunde *et al.*, *ibid.* **92**, 039801 (2004); M. Argollo de Menezes and A.-L. Barabási, *ibid.* **92**, 028701 (2004).

[14] Y. Ashkenazy *et al.*, Phys. Rev. Lett. **86**, 1900 (2001).

[15] J.-P. Bouchaud, A. Matacz, and M. Potters, Phys. Rev. Lett. **87**, 228701 (2001).

[16] K. Matia, L. A. N. Amaral, S. P. Goodwin, and H. E. Stanley, Phys. Rev. E **66**, 045103 (2002).

[17] URL (http://www. bloomberg.com).

[18] K. Ohashi, L. A. N. Amaral, B. H. Natelson, and Y. Yamamoto, Phys. Rev. E **68**, 065204 (2003).

[19] A. S. Balankin *et al.*, Phys. Rev. E **69**, 036121 (2004).

[20] T. C. Mills, Physica A **338**, 559 (2004).

[21] J. W. Kantelhardt *et al.*, Physica A **316**, 87 (2002).

[22] A.-L. Barabási and T. Vicsek, Phys. Rev. A **44**, 2730 (1991); A.-L. Barabási *et al.*, *ibid.* **45**, R6951 (1992); J. Asikainen, S. Majaniemi, M. Dubé, and T. Ala-Nissila, Phys. Rev. E **65**, 052104 (2002).

[23] P. Sibani and P. B. Littlewood, Phys. Rev. Lett. **71**, 1482 (2002).