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# Simple acoustic multiplexer

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Simple structures enabling the multiplexing of acoustic waves are presented. Such structures are constructed out of two monomode acoustic wires and two masses bound together, and to the wires by springs. We show analytically that these simple structures can transfer with selectivity and in one direction one acoustic wavelength from one wire to the other, leaving neighbor acoustic wavelengths unaffected. We give closed-form relations enabling to obtain the values of the relevant physical parameters for this multiplexing phenomena to happen at a chosen wavelength. Finally, we illustrate this general theory by an application.

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The directional ejection from one wave guide to another was considered before for elastic waves in slender tubes [1] and for optical phonons in atomic chains [2]. Such transfer processes are particularly important in wavelength multiplexing and in telecommunication routing devices [3,4].

A device enabling a directional ejection of an elastic wave of a given wavelength from one acoustic wire to the other should leave the other neighbor wavelengths, travel without perturbation in the input wire. At the same time this wave of one selected and well-defined wavelength is expected to be transferred to the other wire with a phase shift as the only admitted distortion. To meet the above requirements as closely as possible an appropriate coupling geometry should be designed.

In the present paper we describe a simple and general system, which, under certain conditions, makes possible the directional transfer of one acoustic wave with a very good selectivity and directivity. The system is depicted in Fig. 1.

The structure consists of two wires conducting predominantly the in-plane transverse waves. The wires are characterized by their linear mass density $\rho_l$ and the speed $c$ of the transverse acoustic waves propagating along them. Two identical masses, undergoing a constrain to move perpendicularly to the wires by appropriate frictionless rails, are coupled to the motionless support by a force constant $K$ for these, and only these, transverse waves. The masses are additionally coupled with each other by a spring whose free length is slightly longer than the distance $d$ between the rails. As can be seen from a classical Taylor expansion of a radial potential, this assures an effective harmonic force constant $\beta_2 = \varphi'(d)/d$ between the masses, where $\varphi'(r)$ is the first derivative of the potential energy of the spring linking the masses. The coupling of the wave motion in the wires with the motion of the masses is assured by four springs of the harmonic force constants $\beta_1$ (see Fig. 1). The wires go, respectively, through points (1, 2) and (3, 4) and the masses are at points 5 and 6. The distances between points (1, 2), (3, 4), and (5, 6) are $d$. As can be also seen from a classical Taylor expansion of a radial potential, for such a geometry there is no nonzero harmonic force constants coupling the in-plane transverse motions defined above to the other types of displacements. The system shows two perpendicular mirror symmetry planes. Moreover, the simple system presented here can be solved in closed form. This enables to determine easily all the parameters necessary for its fabrication.

Many other systems and geometries of this type can be devised, which differ in some details but show generally the same multiplexing property. On the other hand, an analogous geometry realized on an atomic or nanometric scale offers even a richer variety of multiplexing systems.

The acoustic dispersion relation of the transverse modes of the wires is

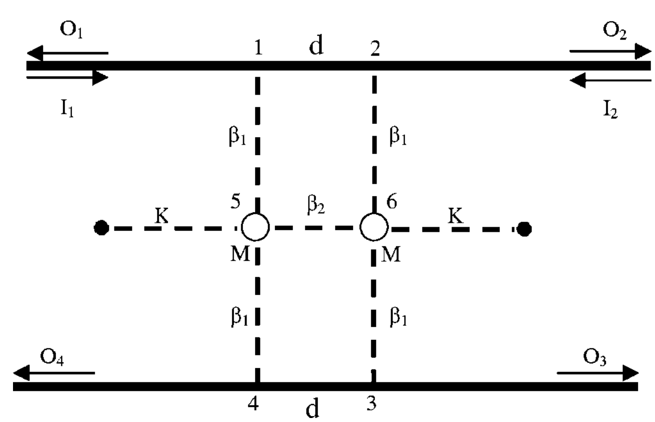


FIG. 1. Sketch of one possible geometry of the considered simple acoustic multiplexer. It consists of two wires and two masses $M$. The two masses $M$ are bounded to two fixed points by two springs having an harmonic force constant $K$ and are restricted by an appropriate device to move only in the direction parallel to the wires. They are bound also between themselves by a spring with an harmonic force constant $\beta_2$ and to points (1-4) of the wires by four springs out of equilibrium with harmonic force constant $\beta_1$. We consider one input $I_1$ or two inputs $I_1 = I_2$ of longitudinal acoustic waves and four outputs $O_1$, $O_2$, $O_3$, and $O_4$.

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$$\omega = ck, \tag{1}$$

where $k=2\pi/\lambda$ is the propagation vector of the acoustic wave of wavelength $\lambda$.

In general, any incident acoustic wave launched onto the coupling structure, e.g., from the input gate 1, generates, as a result of scattering processes, the reflected wave at node 1, along with the three transmitted waves at nodes 2, 3, and 4 (cf. Fig. 1). The corresponding reflection ($t_{11}$) and transmission ($t_{1n}, n=2,3,4$) amplitudes can be expressed in terms of the elements of the Green's function $g$ of the system via [1]

$$t_{11} = 2iFg(1,1) - 1 \tag{2a}$$

and

$$t_{1n} = 2iFg(1,n), \quad (n=2,3,4). \tag{2b}$$

In Eqs. (2), $i=\sqrt{-1}$ stands for the imaginary unit, while

$$F = c\rho_l\omega. \tag{3a}$$

The necessary Green's-function elements can be readily obtained taking into account the symmetry of the system. Consequently, for acoustic waves incoming through gate 1, the expressions for the reflection and transmission wave-function amplitudes can be conveniently written as

$$t_{11} = z_1 + z_2 + z_3 + z_4 - 1, \tag{4a}$$

$$t_{12} = z_1 + z_2 - z_3 - z_4, \tag{4b}$$

$$t_{13} = z_1 - z_2 + z_3 - z_4, \tag{4c}$$

and

$$t_{14} = z_1 - z_2 - z_3 + z_4, \tag{4d}$$

where

$$z_n = \frac{i}{2(i+y_n)} \quad (n=1,2,3,4), \tag{5}$$

with the $y_n$'s determined by the particular resonant-coupling structure under consideration [1]. For the structure of Fig. 1,

$$y_1 = y_2 - \frac{2\beta_1^2}{F(M\omega^2 - K - 2\beta_1)}, \tag{6}$$

$$y_2 = \tan\left(\frac{kd}{2}\right) - \frac{\beta_1}{F}, \tag{7}$$

$$y_3 = -\left[\tan\left(\frac{kd}{2}\right)\right]^{-1} - \frac{\beta_1}{F}, \tag{8}$$

and

$$y_4 = y_3 - \frac{2\beta_1^2}{F(M\omega^2 - K - 2\beta_1 - 2\beta_2)}. \tag{9}$$

For an incoming acoustic wave intensity $I_1(kd)=1$, the outgoing acoustic wave intensities $O_j(kd)$ are given by

$$O_j(kd) = |t_{1j}|^2, j=1,2,3,4. \tag{10}$$

The total acoustic wave transfer from a single input at gate 1 to the output 3, i.e., $I_1(kd)=1$, $O_1(k_0d)=0$, $O_2(k_0d)=0$, $O_3(k_0d)=1$, and $O_4(k_0d)=0$ can be realized exactly at the angular frequency $\omega_0=ck_0$, when the following conditions are fulfilled:

$$(k_0d)^2 = \frac{(d/c)^2}{M}[K + 2\beta_1 + \beta_2], \tag{11}$$

$$\cos(k_0d) = -\frac{\beta_2}{2\beta_1}, \tag{12}$$

and

$$\frac{\sin(k_0d)}{(k_0d)} = \frac{\rho_l c(c/d)\beta_2}{\beta_1^2}. \tag{13}$$

The transferred wave has some width in $kd$ around $k_0d$. If one wishes the corresponding peak in $O_3(kd)$ to be symmetric, then one obtains another condition [1,2], namely

$$k_0d = (1+4n_0)\frac{\pi}{2}, \quad n_0 = 0,1,2,\ldots \tag{14}$$

However, this condition and the ones given by Eqs. (12) and (13) are only fulfilled for $\beta_1=0$ and $\beta_2=0$. So in what follows, we will tolerate a small dissymmetry of the peak in $O_3(kd)$ and a small imprecision on the condition given by Eq. (14).

Let us also define the quality factor associated with the linewidth of the transferred signal by

$$Q(k_0d) = \frac{k_0d}{\Delta(k_0d)}, \tag{15}$$

where $\Delta(k_0d)$ is the width of this signal for $O_3(kd)=0.5$.

An approximated value of this quality factor is found to be

$$Q(k_0d) = (1+4n_0)^2\frac{\pi^2}{4}\frac{M}{\beta_2}\frac{c^2}{d^2}. \tag{16}$$

To give an illustrative and at the same time realistic example complying with the above assumptions we consider $n_0=0$, $M=0.001$ kg, $c=10$ m/s, $\rho_l=0.07$ kg/m, $d=0.02$ m, $K=475$ kg/s$^2$, $\beta_1=110$ Kg/s$^2$, and $\beta_2=22$ kg/s$^2$. This set of parameters corresponds to typical elastic wires and macroscopic springs able to transmit acoustic waves.

Figure 2 presents the transmission coefficients $O_3(kd)$ (solid line), $O_2(kd)$ (dashed line), $O_4(kd)$ (dotted line), and $O_1(kd)$ (dotted-dashed line) as functions of the reduced wave vector $kd$. One remarks that the dissymmetry with respect to $kd=\pi/2$ is negligible. The peak in the transmission coefficient $O_3(kd)$ shows a width at half maximum of the order predicted by Eq. (16). In this figure $O_2(kd)$ is basically constant and equal to 1 after $kd=\pi/2$. This result comes from the parameters used in this calculation, but remains for other possible parameter sets as long as the analytical conditions given above are satisfied with a good precision and the chosen quality factor is not too low.

Now, with two inputs of intensity $I_1(kd)=I_2(kd)=1$ at

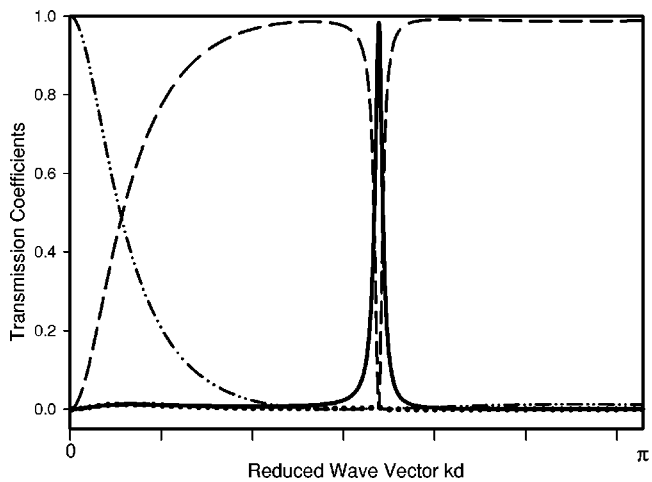


FIG. 2. The transmission coefficients $O_3(kd)$ (solid line), $O_2(kd)$ (dashed line), $O_4(kd)$ (dotted line), and $O_1(kd)$ (dotted-dashed line) as functions of $kd$ for the structure of Fig. 1 for $n_0=0$, $M=0.001$ kg, $c=10$ m/s, $\rho_l=0.07$ kg/m, $d=0.02$ m, $K=475$ kg/s$^2$, $\beta_1=110$ kg/s$^2$, and $\beta_2=22$ kg/s$^2$, for one single input $I_1=1$.

gates 1 and 2, the output transmission probabilities are

$$O_1(kd) = O_2(kd) = |2(z_1 + z_2) - 1|^2 \tag{17a}$$

and

$$O_3(kd) = O_4(kd) = |2(z_1 - z_2)|^2. \tag{17b}$$

In other words, two transverse acoustic waves of particular propagation vector $k_0$ are cross transferred through the structure to gates 3 and 4, respectively. This "cross-talk" effect is illustrated in Fig. 3.

The results of the present paper show that the simple structure presented in this brief report can realize transverse acoustic wave multiplexing and also cross transfer of two acoustic waves, respectively, from gate 1 to gate 3 and from gate 2 to gate 4. Moreover, the above derived closed-form expressions enable to find easily the optimal parameters for the desired device, enabling one to engineer it at will for specific applications.

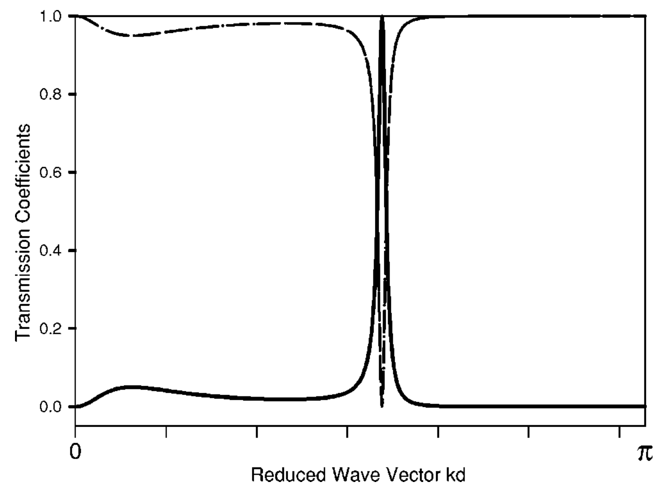


FIG. 3. Output-signal intensities $O_1(kd)=O_2(kd)$ (dashed line) and $O_3(kd)=O_4(kd)$ (solid line) as functions of $kd$ for the structure of Fig. 1 for the same parameters as in Fig. 2 when two inputs of intensity $I_1(kd)=I_2(kd)=1$ at gates 1 and 2 are simultaneously present.

This simple acoustic multiplexer is expected to stimulate further research.

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