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Retrieval of the effective constitutive parameters of bianisotropic metamaterials

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We propose a method to retrieve the effective constitutive parameters of a slab of bianisotropic metamaterial composed of split-ring resonators from the measurement of the *S* parameters. Analytical inversion equations are derived for homogeneous lossless bianisotropic media, and a numerical retrieval approach is presented for the case of lossy bianisotropic media. The method is verified both analytically and numerically, and it is shown that the results for various split-ring resonator metamaterials qualitatively corroborate the conclusions found in published papers. The proposed retrieval method can be used as a valuable tool for the study of anisotropic and bianisotropic properties of metamaterials.

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I. INTRODUCTION

Left-handed materials have been realized as metamaterials, which most commonly consist of infinite metallic wires and split-ring resonators (SRRs) [1–3]. While the infinite metallic wires behave like a plasma, thus providing a negative permittivity (ϵ), periodic arrays of SRRs give a negative permeability (μ). Since the metamaterials present bulk properties when the spacing between the rings is much smaller than the wavelength, it is justified to look for their effective constitutive parameters. The constitutive parameters can be obtained either by numerically calculating the ratios of the electromagnetic fields [4], which is easy to use in numerical simulation but hard to apply in an experimental situation, or by some approximate analytical models [5–7], which give insight into the relationship between the physical properties and geometrical properties of the metamaterial but are not straightforward to use in metamaterials with complicated structures. Another common method to retrieve the constitutive parameters is to use the reflection and transmission coefficients (or *S* parameters) [8–11], which can be applied to both simple and complicated structures, and can use both numerical and experimental data.

The retrieval methods published so far deal with isotropic permittivities and permeabilities. However, it is known already that the metamaterials are intrinsically anisotropic because of the orientations of the rings and rods in space, and that they are also possibly bianisotropic because of the specific properties of their split rings. For example, it has been shown in [5,6] that the original concentric split ring exhibits a bianisotropic behavior, directly due to its geometry. Consequently, the existing retrieval algorithms need to be improved to take into account these additional properties.

In this paper, we extend the work presented in [11] and present a methodology to retrieve bianisotropic parameters as well. Although our approach is general, we derive it here for the specific retrieval of the bianisotropic term expected from the original concentric split-ring resonator [2], shown in Fig. 1(a). The SRR structure is made of two concentric rings, each interrupted by a small gap. For convenience, we refer to this SRR structure as the edge-coupled SRR [5]. It has been pointed out [6,12] that this structure presents bianisotropy: A magnetic field in the $\hat{y}$ direction induces an electrical dipole in the $\hat{z}$ direction due to the asymmetry of the inner and outer rings, and an electric field in the $\hat{z}$ direc-

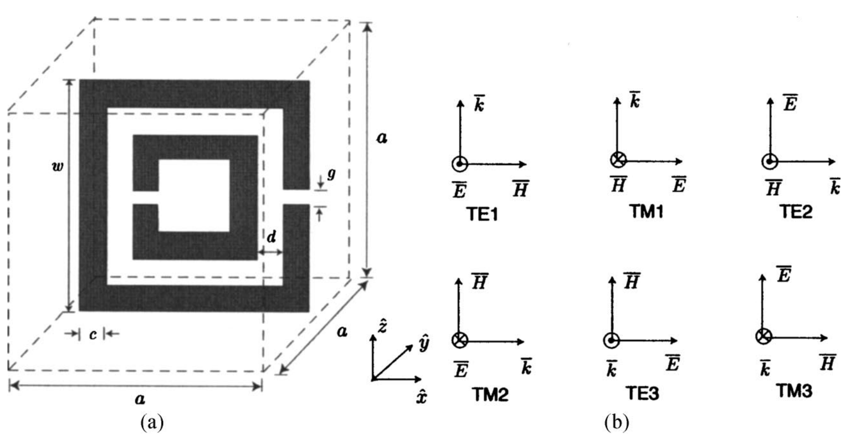


FIG. 1. Unit cell of a metamaterial composed of the edge-coupled SRR upon which six incidences are used to obtain the *S* parameters from finite-difference time-domain simulations. (a) Structure of the edge-coupled SRR: The unit cell is a cube of edge a=5.25 mm. The parameters of the SRR are g=d=0.125 mm, c=0.25 mm, and w=3.75 mm. The background medium is free space. (b) Modes of multiple incidences: When the wave is incident normally on the unit cell, the unit cell is periodically repeated in the plane perpendicular to the incidence direction, thus forming a slab.

TABLE I. Dispersion relationship, redefined impedance, and redefined refractive index for each incidence of Fig. 1(b).

Case	Dispersion relationship	z	n	Solved components
TE1	$k_z^2=k_0^2\epsilon_y\mu_x$	$\sqrt{\mu_x/\epsilon_y}$	$\sqrt{\epsilon_y\mu_x}$	ϵ_y,μ_x
TM1	$k_z^2=k_0^2(\epsilon_x\mu_y-\epsilon_x/\epsilon_z\xi_0^2)$	$\frac{\epsilon_x}{\sqrt{\epsilon_x\mu_y-\epsilon_x/\epsilon_z\xi_0^2}}$	$\sqrt{\epsilon_x\mu_y-\epsilon_x/\epsilon_z\xi_0^2}$	ϵ_x
TE2	$k_x^2=k_0^2(\epsilon_z\mu_y-\xi_0^2)$	$\frac{\mu_y}{\sqrt{\epsilon_z\mu_y-\xi_0^2}+i\xi_0}$	$\sqrt{\epsilon_z\mu_y-\xi_0^2}$	
TM2	$k_x^2=k_0^2\epsilon_y\mu_z$	$\sqrt{\epsilon_y/\mu_z}$	$\sqrt{\epsilon_y\mu_z}$	ϵ_y,μ_z
TE3	$k_y^2=k_0^2\epsilon_x\mu_z$	$\sqrt{\mu_z/\epsilon_x}$	$\sqrt{\epsilon_x\mu_z}$	ϵ_x,μ_z
TM3	$k_y^2=k_0^2(\epsilon_z\mu_x-\mu_x/\mu_y\xi_0^2)$	$\frac{\epsilon_z}{\sqrt{\epsilon_z\mu_x-\mu_x/\mu_y\xi_0^2}}$	$\sqrt{\epsilon_z\mu_x-\mu_x/\mu_y\xi_0^2}$	ϵ_z

tion, producing an unbalanced current distribution in the rings, induces a magnetic dipole in the $\hat{y}$ direction. Supposing that the medium is reciprocal [6,13–15], the constitutive relationships can be written as

$$\overline{D}=\bar{\bar{\epsilon}}\cdot\overline{E}+\bar{\bar{\xi}}\cdot\overline{H}, \tag{1}$$

$$\overline{B}=\bar{\bar{\mu}}\cdot\overline{H}+\bar{\bar{\zeta}}\cdot\overline{E}, \tag{2}$$

where

$$\bar{\bar{\epsilon}}=\epsilon_0\begin{pmatrix}\epsilon_x & 0 & 0\\ 0 & \epsilon_y & 0\\ 0 & 0 & \epsilon_z\end{pmatrix},\quad \bar{\bar{\mu}}=\mu_0\begin{pmatrix}\mu_x & 0 & 0\\ 0 & \mu_y & 0\\ 0 & 0 & \mu_z\end{pmatrix},$$

$$\bar{\bar{\xi}}=\frac{1}{c}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & -i\xi_0 & 0\end{pmatrix},\quad \bar{\bar{\zeta}}=\frac{1}{c}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & i\xi_0\\ 0 & 0 & 0\end{pmatrix}, \tag{3}$$

where ϵ_0 and μ_0 are the permittivity and permeability of free space respectively, and c is the speed of light in free space. Note that $\epsilon_x,\epsilon_y,\epsilon_z,\mu_x,\mu_y,\mu_z$, and ξ_0 are all dimensionless quantities. Since there are seven complex unknowns to be determined, at least seven equations are required. In order to obtain these, we resort to multiple incidences as shown in Fig. 1(b), where each incidence gives two complex equations, one for reflection (S_{11}) and and the other one for transmission (S_{21}).

We propose a method to retrieve the above-mentioned constitutive parameters of a homogeneous material from the measured S parameters. The analytical inversion equations are proposed for homogeneous lossless bianisotropic media, and a numerical retrieval approach is presented for the case of lossy bianisotropic media. Both methods are verified by numerical examples, where analytical ϵ,μ, and ξ_0 are supposed and are retrieved from the S parameters. Finally, we use the retrieval method to study the properties of various SRR-based metamaterials. The retrieval results corroborate the conclusions found in previously published work [5,6,12,16].

Although the retrieval method proposed in this paper is used to retrieve the constitutive parameters of media with the bianisotropy in the yz position [see Eq. (3)], it can be easily generalized to retrieve the constitutive parameters of a medium with the bianisotropy coupling other field components. Therefore, a general analysis tool can be constructed for the study of bianisotropic properties of metamaterials.

II. RETRIEVAL METHODS

In this section, we present the retrieval equations for media described by Eq. (3) from the knowledge of the S parameters. Two main cases are identified: if the medium is lossless or if it is lossy. In the former case, the retrieved constitutive parameters can be obtained analytically while in the latter case, although the equations are analytical, their solution has to be obtained numerically. Among the six incidences we need as shown in Fig. 1(b), three are TE modes and three are TM modes. Note that the S parameters are defined in terms of the electric and magnetic fields for the TE and TM incidences, respectively. We see from Eqs. (1) and (3) that only the y component of $\overline{H}$ contributes to $\overline{D}$, and similarly only the z component of $\overline{E}$ contributes to $\overline{B}$. Thus, among the six incidences, only TM1, TE2, and TM3 see the bianisotropy, while the other three waves are propagating as if the material were isotropic. The incidence TE2, containing both H_y and E_z, is more complicated than all the other incidences and the retrieval method in this case has to be studied independently.

A. Incidences other than TE2

As mentioned above, the incidence TE2 is very particular, and we shall study it in the next section. We show here that all the other incidences share the same retrieval equations, upon properly defining the effective impedance and the refractive index. For each incidence, the dispersion relationship, together with the redefined impedance and refractive index, is listed in Table I.

Since the incidences TE1, TM2, and TE3 do not contain H_y or E_z components that cause the bianisotropy, they behave as if the medium were isotropic. In order to retrieve the constitutive parameters in these cases, we use the previously published retrieval methods that deal with isotropic media [8–11]. The S parameters for a plane wave incident normally on a slab of an isotropic medium are expressed by [13,17]

$$S_{11} = \frac{R_{01}(1 - e^{i2nk_0 d})}{1 - R_{01}^2 e^{i2nk_0 d}}, \tag{4a}$$

$$S_{21} = \frac{(1 - R_{01}^2)e^{ink_0 d}}{1 - R_{01}^2 e^{i2nk_0 d}}, \tag{4b}$$

where k_0 denotes the wave number of the incident wave in free space, d is the thickness of the slab, n is the refractive index, $R_{01}=(z-1)/(z+1)$ is the half-space reflection coefficient, and z is the impedance and the admittance for TE and TM waves, respectively. The retrieval equations are [11]

$$z = \pm \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}}, \tag{5a}$$

$$e^{ink_0 d} = X \pm i\sqrt{1 - X^2}, \tag{5b}$$

where $X=1/2S_{21}(1-S_{11}^2+S_{21}^2)$. The sign in Eq. (5a) and (5b) is determined by the conditions for passive media,

$$z' \geqslant 0, \tag{6a}$$

$$n'' \geqslant 0, \tag{6b}$$

where $(\cdot)'$ and $(\cdot)''$ denote the real part and imaginary part operators, respectively. The issues of the effective boundaries and the branch cut of the real part of n are solved in the way described in [11].

For incidences TM1 and TM3, we use the method proposed in [18–20] to calculate the S parameters, and find that they take the same form as in the isotropic case, provided that the impedance z and the refractive index n are properly redefined as shown in Table I. Consequently, z and n for incidences TM1 and TM3 can also be retrieved using Eq. (5a) and (5b).

B. Incidence TE2

Since the incidence TE2 contains both H_y inducing an electric dipole in the $\hat{z}$ direction, and E_z inducing a magnetic dipole in the $\hat{y}$ direction, it shows stronger bianisotropy than the incidences TM1 and TM3. The S parameters for incidence TE2 can be obtained by using the method presented in [18,19], and their analytical expressions are as follows:

$$S_{11} = \frac{\dfrac{\mu_y - k_x/k_0 - i\xi_0}{\mu_y + k_x/k_0 + i\xi_0}(1 - e^{i2k_x d})}{1 - \dfrac{(k_x/k_0 - \mu_y)^2 + \xi_0^2}{(k_x/k_0 + \mu_y)^2 + \xi_0^2}e^{i2k_x d}}, \tag{7a}$$

$$S_{21} = \frac{\left(1 - \dfrac{(k_x/k_0 - \mu_y)^2 + \xi_0^2}{(k_x/k_0 + \mu_y)^2 + \xi_0^2}\right)e^{ik_x d}}{1 - \dfrac{(k_x/k_0 - \mu_y)^2 + \xi_0^2}{(k_x/k_0 + \mu_y)^2 + \xi_0^2}e^{i2k_x d}}, \tag{7b}$$

where k_x is the wave number in the incidence direction inside the medium, and the dispersion relationship is given in Table I. In general, we cannot define an impedance z and a refractive index n in order to simplify Eq. (7a) and (7b) and solve z and n analytically, as in the cases of incidences TM1 and TM3. Therefore, we resort to a numerical approach to solve for μ_y and ξ_0 in Eq. (7a) and (7b).

As mentioned previously, using six incidences yields 12 equations for seven unknowns to be solved. Five of the unknowns are therefore solved twice in this overdetermined problem. We see from Table I that ϵ_x, ϵ_y, and μ_z are each retrieved twice. Also since ϵ_z is obtained from the incidence TM3, we solve for μ_y and ξ_0 twice using the following two methods.

1. Method 1

From the incidences TM1 and TM3, we obtain the expressions of μ_y and ξ_0:

$$\mu_y = \frac{\epsilon_x \mu_x z_{\mathrm{TM3}}^2}{\epsilon_z z_{\mathrm{TM1}}^2}, \tag{8}$$

$$\xi_0^2 = \epsilon_z \mu_y \left(1 - \frac{\epsilon_z}{\mu_x z_{\mathrm{TM3}}^2}\right), \tag{9}$$

where z_{TM3} denotes the redefined impedance in the case of incidence TM3 (other variables with the incidence mode in the subscript are defined similarly). There exist two roots of ξ_0^2, and the one that yields a better match between the calculated [by Eq. (7a) and (7b)] and the measured (or simulated) S parameters is identified as the correct ξ_0 value of the medium.

2. Method 2

We use an optimization approach to obtain ξ_0. For a given ξ_0, we have the following relationship from the incidence TM3:

$$\mu_y = \frac{\xi_0^2}{\epsilon_z[1 - \epsilon_z/(\mu_x z_{\mathrm{TM3}}^2)]}. \tag{10}$$

Thus, the S parameters for the case TE2 can be calculated using Eq. (7a) and (7b) for the given ξ_0. The value of ξ_0 is obtained by optimizing its real and imaginary parts so that the calculated S parameters match the measured (or simulated) ones. The optimization method we are using here is the differential evolution algorithm [21].

While method 1 uses the TM1, TE2, and TM3 incidences to solve for μ_y and ξ_0, method 2 uses only TE2 and TM3 incidences. Yet we expect these two different mathematical approaches to yield the same retrieval results, which will be shown later in the numerical verification.

C. Incidence TE2: Lossless media

It is worth mentioning that there is an analytical approach to solve for μ_y and ξ_0 in a special case. When the wave number k_x and the constitutive parameters are real numbers, which refers to a propagating wave inside a lossless medium, it can be shown that the S parameters in Eq. (7a) and (7b) reduce to

$$S_{11} = \frac{R_{01}(1 - e^{i2nk_0d})}{1 - |R_{01}|^2 e^{i2nk_0d}}, \tag{11a}$$

$$S_{21} = \frac{(1 - |R_{01}|^2) e^{ink_0d}}{1 - |R_{01}|^2 e^{i2nk_0d}}, \tag{11b}$$

where the refractive index n and the impedance z are redefined as in Table I. For convenience, we call the retrieval method in this case a lossless retrieval, while the retrieval method in the previous section is referred to as a lossy retrieval. In what follows, we solve for n and z by inverting Eq. (11a) and (11b) as

$$\frac{S_{11}}{S_{21}} = (e^{ink_0d} - e^{-ink_0d}) \frac{R_{01}}{|R_{01}|^2 - 1}, \tag{12a}$$

$$\frac{1}{S_{21}} = e^{ink_0d} + (e^{ink_0d} - e^{-ink_0d}) \frac{1}{|R_{01}|^2 - 1}. \tag{12b}$$

Eliminating $e^{ink_0d} - e^{-ink_0d}$, we find

$$e^{ink_0d} = \frac{R_{01} - S_{11}}{S_{21} R_{01}}. \tag{13}$$

Substituting Eq. (13) into Eq. (12a), and using $|R_{01}|^2 = R_{01} R_{01}^*$, we eventually obtain the value of z as

$$z' = \sqrt{\frac{DC'' - B'}{A'} - \frac{D^2}{4}}, \tag{14a}$$

$$z'' = \frac{1}{2} D, \tag{14b}$$

where

$$A = 2S_{11} - (S_{11}^2 + 1 - S_{21}^2), \tag{15a}$$

$$B = 2S_{11} + (S_{11}^2 + 1 - S_{21}^2), \tag{15b}$$

$$C = S_{11}^2 + S_{21}^2 - 1, \tag{15c}$$

$$D = \frac{A''B' - A'B''}{A''C'' + A'C'}. \tag{15d}$$

Once z is obtained, n can be solved via Eq. (13). Similarly to the lossy case, we have two different methods to solve for μ_y and ξ_0 in the lossless case.

1. Method 1

The component μ_y is calculated from the incidences TM1 and TM3, as shown in Eq. (8). From z_{TE2} listed in Table I, we obtain

$$\xi_0 = \frac{1}{2i} \left(\frac{\mu_y}{z_{\mathrm{TE2}}} - \epsilon_z z_{\mathrm{TE2}} \right). \tag{16}$$

2. Method 2

From the results for the cases TE2 and TM3, we get

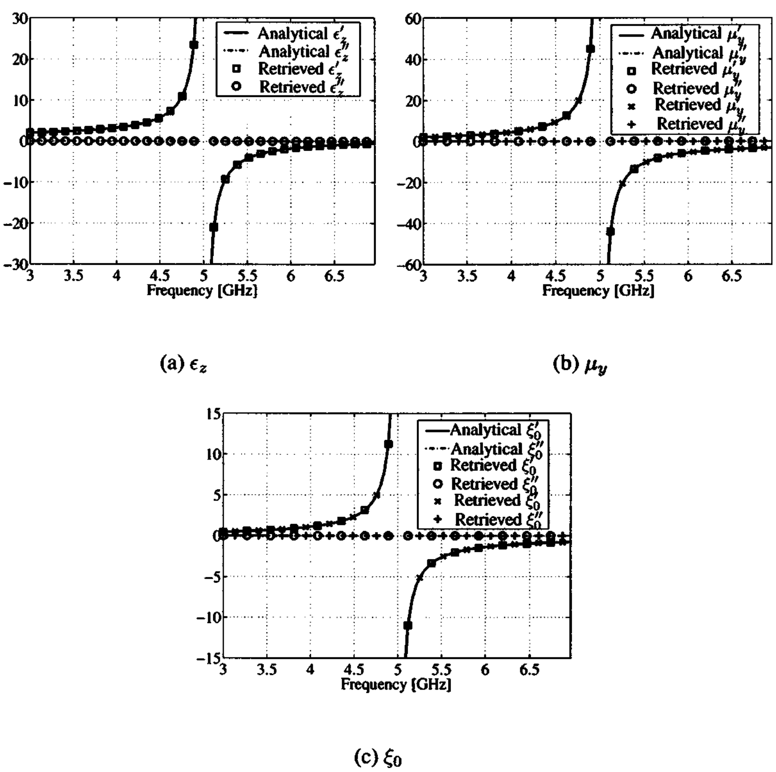


FIG. 2. Comparison of the analytical and the retrieved results for a lossless homogeneous medium. The curves with □ and ○ are the retrieval results using method 1, and the curves with × and + are from method 2. Note that the markers in the figure are hard to distinguish because the results are almost identical for the two methods.

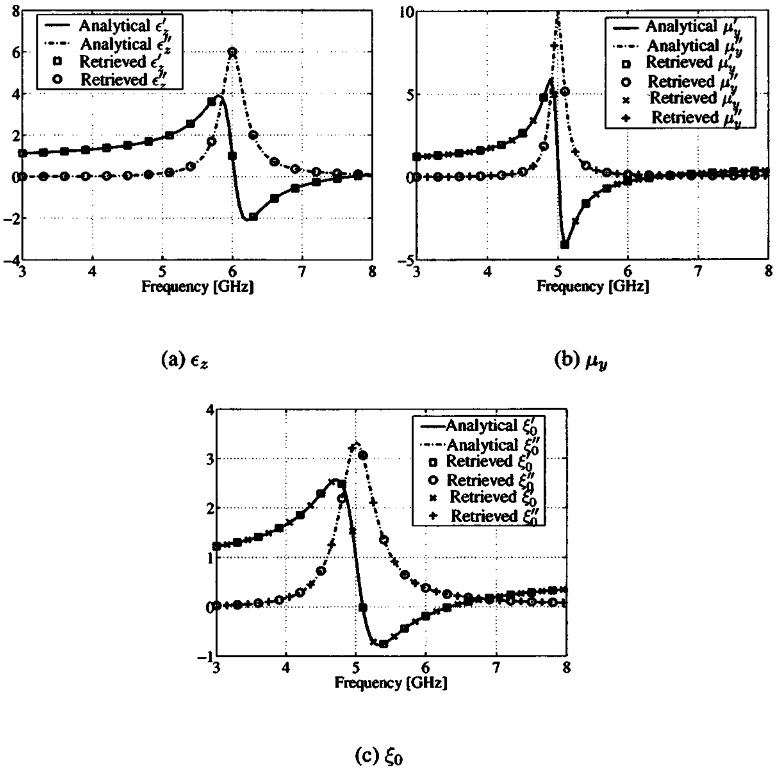


FIG. 3. Comparison of the analytical and the retrieved results for a lossy homogeneous medium. The curves with □ and ○ are the retrieval results using method 1, and the curves with × and + are from method 2. Note that the markers in the figure are hard to distinguish because the results are almost identical for the two methods.

$$\mu_y = \frac{n_{\mathrm{TE2}}^2 \mu_x z_{\mathrm{TM3}}^2}{\epsilon_z^2}. \tag{17}$$

Consequently, the value of ξ_0 is calculated using Eq. (16).

III. NUMERICAL VALIDATION

The retrieval equations presented in the previous section have first been validated using analytical models. In this process, a slab of a given thickness is assigned the constitutive parameters of Eq. (3), where each component of the constitutive tensors is described by a frequency dispersive model (or by a positive constant in some cases), either lossy or lossless. The *S* parameters are computed analytically, and are used as input to the retrieval algorithms. The retrieved parameters are obviously expected to match exactly the input functions.

A. Retrieval for lossless media

For a lossless medium, the constitutive parameters are chosen to follow the form proposed in [6] (note the difference in the coordinate systems used in this paper and in [6]),

$$\epsilon_x(f) = C_1,$$

$$\epsilon_z(f) = \epsilon_x + C_2(f_0^2/f^2 - 1)^{-1},$$

$$\mu_y(f) = 1 + C_3(f_0^2/f^2 - 1)^{-1},$$

$$\xi_0(f) = C_4 f_0/f(f_0^2/f^2 - 1)^{-1},$$

$$\epsilon_y(f) = \mu_x(f) = \mu_z(f) = 1,$$

where the coefficients are chosen arbitrarily to be $C_1=1.5$, $C_2=1.0$, $C_3=2.0$, $C_4=0.5$, and $f_0=5.0$ GHz.

The constitutive parameters are retrieved using the lossless retrieval method presented in the previous section. For the nondispersive components, the retrieval results agree exactly with the above given values. For the dispersive components, the retrieved results are compared with the analytical ones in Fig. 2. The retrieval results near the resonance are divergent and become numerically unstable; thus they are not shown here. Slightly away from this resonant region, however, the retrieved results are in perfect agreement with the input functions, which validates the proposed lossless retrieval method.

B. Retrieval for lossy media

In order to prove the validity of the proposed retrieval method for lossy media, we retrieve the constitutive parameters of the following homogeneous lossy bianisotropic medium

$$\epsilon_x(f) = C_1,$$

$$\epsilon_z(f) = 1 - F_e f^2/(f^2 - f_e^2 + i\gamma_e f),$$

$$\mu_y(f) = 1 - F_m f^2/(f^2 - f_m^2 + i\gamma_m f),$$

$$\xi_0(f) = 1 - F_\xi f^2/(f^2 - f_\xi^2 + i\gamma_\xi f),$$

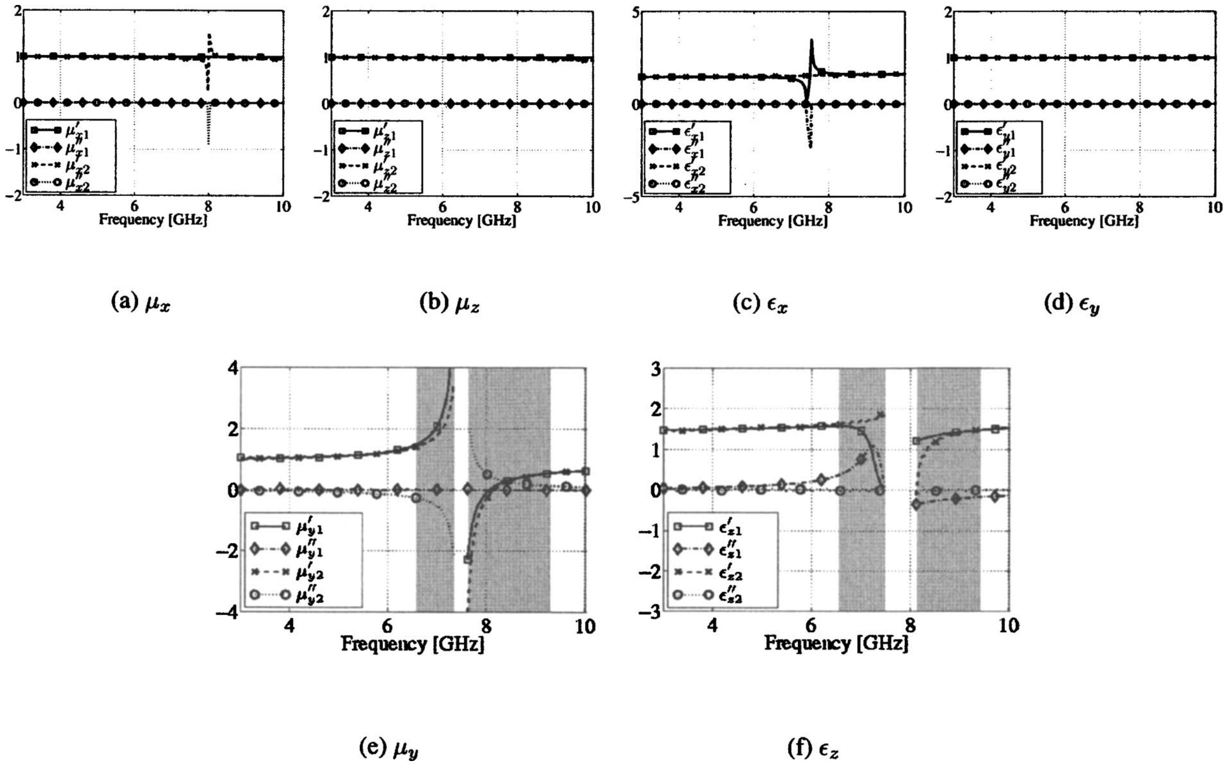


FIG. 4. Retrieval results for a lossless edge-coupled SRR metamaterial whose unit cell is shown in Fig. 1(a), using a retrieval method not considering the bianisotropy. The retrieved μ_x (a) and ϵ_x (c) show negative imaginary parts around the resonance, which violates physical laws [15] and therefore indicates that the results are not reliable in the corresponding region. Those results difficult to read within the resonance band are not shown in (e) and (f). The shaded region indicates the frequency range where the mismatch of either μ_y or ϵ_z exceeds the threshold ($M_R > 0.25$).

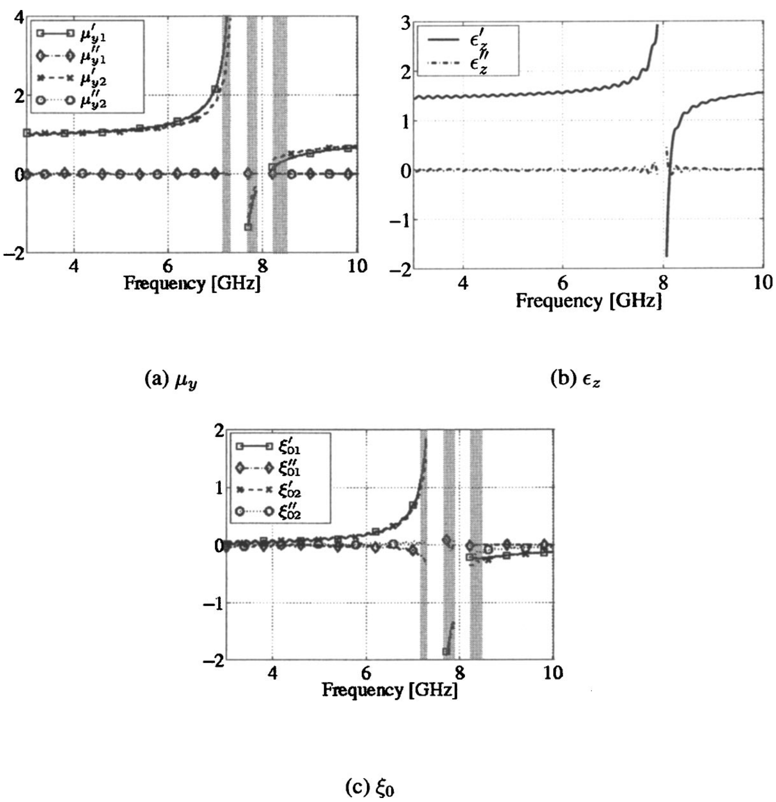


FIG. 5. Retrieval results for a lossless edge-coupled SRR metamaterial whose unit cell is shown in Fig. 1(a), using lossless retrieval for bianisotropic media. The subscripts 1 and 2 denote the results obtained from the proposed methods 1 and 2, respectively. Those results difficult to read within the resonance band are not shown here. The shaded region indicates the frequency range where the mismatch of either μ_y or ξ_0 exceeds the threshold ($M_R > 0.25$).

$$\epsilon_y(f) = \mu_x(f) = \mu_z(f) = C_2.$$

Here we choose arbitrarily $C_1=2.0$, $C_2=1$, $f_e=6.0$ GHz, $f_m=f_\xi=5.0$ GHz, $\gamma_e=0.4$ GHz, $\gamma_m=0.2$ GHz, $\gamma_\xi=0.6$ GHz, and $F_e=F_m=F_\xi=0.4$.

The constitutive parameters are retrieved using the lossy retrieval method proposed previously. For the dispersive components, the retrieved results are compared with the analytical ones in Fig. 3, where a perfect agreement can be seen. It is worth mentioning that the losses avoid the divergence of ϵ_z, μ_y, and ξ_0, which is advantageous for the retrieval algorithm.

IV. RETRIEVAL RESULTS FOR SRR-BASED METAMATERIAL

The edge-coupled SRR-based metamaterial shown in Fig. 1(a) has been studied and proven to exhibit bianisotropy [5,6], which has been corroborated by the studies in [12,16]. In this section, we apply our retrieval method to a metamaterial composed of edge-coupled SRRs in order to quantify rigorously the magnitude of the bianisotropic term as a function of frequency. The retrieval results show that the edge-coupled SRR structure indeed presents a strong bianisotropy, while a slight modification of it exhibits no bianisotropy, which agrees with the conclusion found in [5,6].

A. Why bianisotropy is needed

In order to illustrate the necessity of using a retrieval method in which bianisotropy is considered, we first retrieve the effective constitutive parameters of an edge-coupled SRR metamaterial shown in Fig. 1(a) using the isotropic retrieval method [11]. Each component of the permittivity and the permeability tensor is retrieved exactly twice since an isotropic retrieval is carried out for each of the six incidences, where the S parameters are obtained using the periodic finite-difference time-domain method [22]. The retrieved results are shown in Fig. 4. While we observe good agreements in the retrieved μ_x, μ_z, ϵ_x, and ϵ_y (except around the resonance), there is a noticeable mismatch between the two retrieval methods for the parameters ϵ_z and μ_y, which indicates that the anisotropic model is not sufficient to describe the homogeneity of an edge-coupled SRR metamaterial shown in Fig. 1(a). Therefore, a better model is needed. Since the y component of the magnetic field produces an electric dipole in the $\hat{z}$ direction and the z component of the electric field produces a magnetic dipole in the $\hat{y}$ direction, the bianisotropy terms in Eq. (3) cannot be neglected [6]. It is no surprise to see the unsuccessful retrieval of ϵ_z and μ_y, since ξ_0 is coupled with both of them in the incidences TM1, TE2, and TM3 (see Table I).

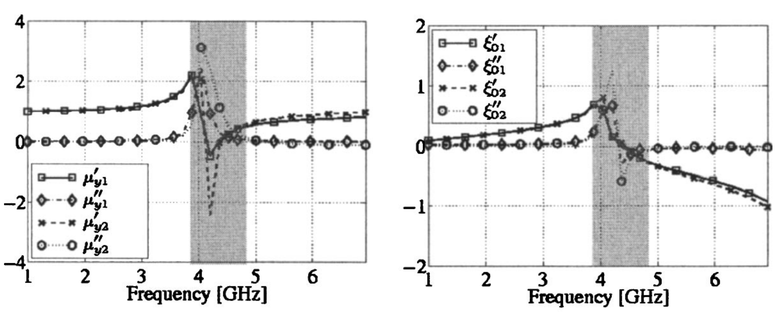
TABLE II. Frequency ranges (in GHz) of unsatisfactory match for the retrieved μ_y, ϵ_z, and ξ_0.

	For μ_y	For ϵ_z or ξ_0	Total range
Fig. 4	6.9–9.3	6.6–8.5	6.6–9.3
Fig. 5	7.2–7.9, 8.1–8.5	7.2–7.5, 8.1–8.5	7.2–7.9, 8.1–8.5
Fig. 6	3.9–4.8	4.1–4.5	3.9–4.8
Fig. 7	5.9–6.7	6.0–6.5	5.9–6.7

When a lossless retrieval for bianisotropic media is applied, the retrieved μ_x, μ_z, ϵ_x, and ϵ_y are identical to the ones shown in Fig. 4, while the retrieved ϵ_z, μ_y, and ξ_0 are shown in Fig. 5. Unlike in the situation when an anisotropic retrieval is used, we observe a good match between the two retrieved values for both μ_y and ξ_0, except around the resonance frequencies, a range known to be hard to deal with in the retrieval of metamaterial parameters [10,11]. To show quantitatively a better match in Fig. 5, we define the relative mismatch (M_R) of μ_y as

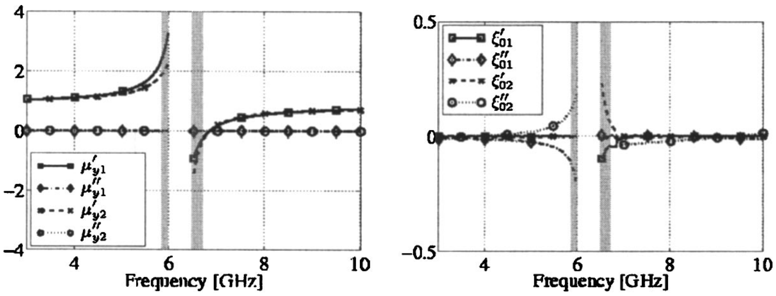
$$M_R(\mu_y) = \begin{cases} 0 & \text{if } \dfrac{|\mu_{y1}| + |\mu_{y2}|}{2} < \alpha, \\ \dfrac{|\mu_{y1} - \mu_{y2}|}{\frac{1}{2}(|\mu_{y1}| + |\mu_{y2}|)} & \text{otherwise}, \end{cases} \tag{18}$$

where α is a small positive number. The smaller M_R, the better the matching between the two results. The relative mismatch for ϵ_z and ξ_0 can be defined similarly. We refer to the frequency where M_R is larger than a constant β as the unsatisfactory matching frequency. For the parameters $\alpha=0.25$ and $\beta=0.25$, the unsatisfactory matching frequency ranges for μ_y, ϵ_z in Fig. 4 and μ_y, ξ_0 in Fig. 5 are listed in Table II. The total unsatisfactory frequency ranges in which



(a) μ_y (b) ξ_0

FIG. 6. Retrieval results for a lossy edge-coupled SRR metamaterial, whose unit cell is the same as that in Fig. 1(a) except that the background material is lossy ($\sigma=0.042$ S/m, $\epsilon_r=3.4$). The shaded region indicates the frequency range where the mismatch of either μ_y or ξ_0 exceeds the threshold ($M_R>0.25$).



(a) μ_y (b) ξ_0

FIG. 7. Retrieved results for a broadside-coupled SRR metamaterial, where a negligible ξ_0 is observed (except around the resonance 5.9 GHz–6.7 GHz; see Table II.) Those results difficult to read within the resonance band are not shown here. The shaded region indicates the frequency range where the mismatch of either μ_y or ξ_0 exceeds the threshold ($M_R > 0.25$).

either μ_y or ϵ_z (or ξ_0) does not match well are also given in Table II. We find that the unsatisfactory matching frequency range in Fig. 5 is narrower than that in Fig. 4.

Compared to [5,6], we see that the shapes of the retrieved μ_y and ϵ_z agree with the models proposed in [5,6], but that the resonances of ϵ_z and μ_y are not equal to each other (μ_y at 7.5 GHz, ϵ_z at 8.0 GHz). The fact that the two retrieval results now match well for both μ_y and ξ_0 and that the retrieved ξ_0 is not negligible proves the existence of the bianisotropy in the edge-coupled SRR metamaterial.

B. Lossy retrieval

Next, we apply the lossy retrieval method to retrieve the effective constitutive parameters of a lossy metamaterial. The SRR structure and the unit cell are same as shown in Fig. 1(a), but the whole unit cell is filled with a lossy material with the relative permittivity $\epsilon_r = 3.4$ and the conductivity $\sigma = 0.042$ S/m (yielding an imaginary part of ϵ of $0.01\epsilon_0$ at 7.5 GHz, the resonance frequency of the SRR structure, which is shown in Fig. 5). The retrieval results are shown in Fig. 6, where we observe a good match between the two retrieved values for both μ_y and ξ_0 (except around the resonance 3.9–4.8 GHz; see also Table II). We also see noticeable imaginary parts in the retrieved parameters.

C. Retrieval of broadside-coupled SRR metamaterial

The proposed retrieval method is a tool for studying not only the properties of bianisotropic media, but also anisotropic media in which the retrieved bianisotropic term is expected to be close to zero. We retrieve here the effective constitutive parameters of a broadside-coupled SRR metamaterial, which is anisotropic as proposed in [5,6]. The edge-coupled SRR shown in Fig. 1(a) can be slightly modified to be a broadside-coupled SRR (see Fig. 3. of [6]) by increasing the inner SRR to the size of the outer one and by separating the two rings by a certain distance (0.125 mm in our simulation). For this anisotropic structure, we expect to retrieve a zero or negligible bianisotropy term using the proposed retrieval method.

Since there is no loss in the system, we apply the lossless retrieval method to obtain the results shown in Fig. 7. It is seen that the retrieved ξ_0 is close to zero in most frequencies except around the resonance, which agrees with the argument in [5,6] that the broadside-coupled SRR does not present bianisotropy due to the symmetry of the electric charges and the currents. The successful retrieval results show that although the proposed retrieval method was initially constructed for the retrieval of bianisotropic media, it can also be applied to anisotropic media.

V. CONCLUSIONS

A useful tool is proposed to study the properties of bianisotropic metamaterials by retrieving their effective constitutive parameters from measurements of the S parameters. Analytical inversion equations are proposed to retrieve the constitutive parameters of homogeneous lossless bianisotropic media, while a numerical approach is proposed for lossy bianisotropic media. Both methods have been validated first using analytical functions as input values for the constitutive parameters and second, using simulated S parameters of real split-ring structures. The retrieval results qualitatively corroborate the conclusions of previously published articles, proving the existence of bianisotropy in edge-coupled SRR metamaterials, but not in broadside-coupled SRR metamaterials.

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