Finite-size effects and intermittency in a simple aging system

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We study the intermittent dynamics and the fluctuations of the dynamic correlation function of a simple aging system. Given its size L and its coherence length ξ, the system can be divided into N independent subsystems, where $N=(L/\xi)^d$, and d is the dimension of space. Each of them is considered as an aging subsystem which evolves according to an activated dynamics between energy levels. We compute analytically the distribution of trapping times for the global system, which can take power-law, stretched-exponential or exponential forms according to the values of N and the regime of times considered. An effective number of subsystems at age t_w, $N_{eff}(t_w)$, can be defined, which decreases as t_w increases, as well as an effective coherence length, $\xi(t_w) \sim t_w^{(1-\mu)/d}$, where $\mu<1$ characterizes the trapping times distribution of a single subsystem. We also compute the probability distribution functions of the time intervals between large decorrelations, which exhibit different power-law behaviors as t_w increases (or N decreases), and which should be accessible experimentally. Finally, we calculate the probability distribution function of the two-time correlator. We show that in a phenomenological approach, where N is replaced by the effective number of subsystems $N_{eff}(t_w)$, the same qualitative behaviour as in experiments and simulations of several glassy systems can be obtained.

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I. INTRODUCTION

The dynamics of glassy materials such as spin-glasses, structural glasses or amorphous soft materials like gels, pastes or foams has been a subject of considerable study [1–3]. Considerable effort has been made in order to understand and quantify the out-of-equilibrium character of their temporal relaxation, in particular the absence of time translational invariance (aging), through the study of dynamic correlation functions. On the theoretical side, global dynamic correlations have been described at a mean-field level in disordered systems such as spin-glasses [4], or at a phenomenological level in models such as the random energy model [5,6]. Both approaches neglect all spatial properties of the system, and are therefore likely to miss any spatial correlations that arise during the dynamics. Alternatively, a phenomenological picture, the droplet model, has been proposed, that focuses on the spatial properties as a key to understand the slow dynamics and the critical properties of spin-glasses [7,8].

In recent years, interest in the spatial properties of glassy systems has been growing. The size of excitations in finite-dimensional spin-glasses has been studied numerically [9]. In simulations of kinetically constrained glassy systems [10,11], and of supercooled liquids [12,13], cooperativity lengths have been identified, and related to the presence of heterogeneities in the dynamics [14,15].

More generally, a lot of physical questions remain; a crucial one being: in what manner, at a microscopic level, does a glassy system evolve, both spatially and temporally? What are the spatial configurations of the typical rearrangements experienced by a glassy system during its relaxation and how are they affected by aging? Are there any common relaxation mechanisms of glassy systems, though there also exist specificities to given materials? Recently, new results have been obtained in this direction. Focusing on the spatial aspect of glassy relaxation, cooperative rearrangements events have been evidenced, both experimentally [16,17] and through computer simulations [18], stimulating new research on the challenging question of coherence lengths and cooperativity in glasses. On the temporal side, beautiful experiments have shown evidence of temporal intermittency in colloidal gels and micellar polycrystals [19], and in polycarbonate glasses [20]. It seems now well established that in glasses and gels, relaxation takes place in a discontinuous way, involving sudden rearrangements followed by periods of arrest where almost nothing happens. The precise experimental determination of the distribution of time lags between rearranging events will give insight into the characterictic "trapping times" of the system. In experiments on glasses, this distribution seems to be close to a power law [20], which is consistent with a trap model with an exponential distribution of energies [6], whereas it has been found in simulations of supercooled glasses to correspond to a model of traps with either an exponential distribution of energies, or Gaussian distribution of energies [21,22]. Therefore, quantities of interest are not just average quantities, but also fluctuations, and in particular the full probability distributions of correlations. The study of fluctuations in glassy systems may contain subtle information, as was already realized by Israeloff and Weissman [23], who analyzed carefully mesoscopic noise in spin-glasses in an attempt to discriminate between a model of droplets, and a scenario of hierarchical dynamics.

Recently, probability distributions of two-time correlation functions in gels and glasses have been shown to exhibit a non-Gaussian behaviour [19,20]. These non-Gaussian features have been found also in numerical simulations of disordered systems and kinetically constrained models [24–26]. They have been tentatively explained using the analogy between glassy dynamics and critical dynamics [24], for which universal, non-Gaussian features can be expected [27].

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In this study, we will not advocate any similarity with critical dynamics, but we will rather try to deduce the non-Gaussian behavior merely from finite-size effects in a simple out-of-equilibrium glassy model.

We will consider a system that can be divided into N independent subsystems. Each subsystem is supposed to represent an independent model of glassy relaxation between energy traps. Such a model of traps has been studied extensively [6,28]. We will show that the superposition of the N subsystems will have the same average dynamical correlation as one individual subsystem; however, its probability distribution function will depend strongly on N; in the limit $N \to \infty$, one has to recover the Gaussian distribution, according to the central limit theorem. Moreover, the distribution of time intervals between relaxation events or between decorrelations will depend crucially on the number of subsystems, i.e., on the value of the internal coherence length.

The paper is organized as follows. In Sec. II, we introduce the model and recall the main results of [6]. In Sec. III, we calculate the distribution of time intervals between successive events in the whole system. In Sec. IV, we calculate the distribution of time intervals between successive decorrelations in the system, the same way they can be measured in time resolved correlation experiments on soft glassy materials [19]. Section V is devoted to the probability distribution of the two-time correlation function. Finally, in Sec. VI, we summarize and discuss our results.

II. DEFINITION OF THE MODEL

Let us consider a simple model of dynamics of a system between energy levels. In a generic disordered system, it is reasonable to assume that low-lying energy levels are exponentially distributed: this is the case for example for the lowest energy levels (nonextensive corrections to the ground state energy) of the random energy model (REM) [5,29] or in spin-glasses [30]. Experimental determination of energy barriers in low-temperature glasses also seem to support this exponential distribution [31].

More precisely, let us call E an energy barrier, which is the difference between a reference energy level which is taken as the origin for the energies, and a negative energy level. E is hence a positive quantity. We choose the distribution of barriers as $\rho(E)=(1/E_0)e^{-E/E_0}$.

Changes in configurations in a disordered or glassy system are often attributed to thermally activated events over energy barriers [21,22], although other mechanisms exist, such as kinetically constrained models [10], which do not require an energy landscape, and are able to reproduce some of the features of glassy materials. In the language of activated events, a trapping time τ corresponding to a barrier E can be defined as $\tau=\tau_0 e^{E/k_B T}$, where τ_0 is a microscopic time scale, k_B the Boltzmann constant, and T the temperature. For an exponential distribution of barriers, the distribution of trapping times is equal to $\psi(\tau)=\mu(\tau_0^\mu/\tau^{1+\mu})$, where $\mu = k_B T/E_0$.

The dynamics of a system evolving in such an energy landscape has been studied by different authors, according to which choice of transition rates between energy levels is made [6,29,32,33]. In the following, we will consider only the case where the transition rate from barrier E to any other barrier E' is $W(E \to E')=(1/\tau_0)e^{-E/k_B T}$ [6,29], which means that the escape from the initial trap is the limiting process, whatever the destination. Note that this family of models does not include any kind of spatial structure, since energy is not related here to spatial configurations, and that it is mean field in nature since transitions to all levels are allowed with the same probability. Extensions of these models to finite dimensionalities have however been attempted in [34,35]. We will see in the following that the superposition of several of such systems can actually introduce (though rather artificially) a relevant length scale.

Dynamical properties of the model have been studied in detail in [6]. In particular, when $\mu<1$, the model exhibits aging (absence of time translation invariance of the correlations), whereas it is time-translationally invariant for $\mu>1$. In this paper, we will focus on the case $\mu<1$. For a given trajectory of the system, the two-time correlation function $C(t_w, t_w+\tau)$ is defined as $C(t_w, t_w+\tau)=1$ if the system has remained in the same energy trap between t_w and $t_w+\tau$, $C(t_w, t_w+\tau)=0$ if between t_w and $t_w+\tau$, the system has left the energy trap it was in at t_w.

Averaging over all barrier configurations (which we denote by $\langle\ \rangle$), one obtains the average two-time correlation function:

$$\Pi(t_w, t_w+\tau) = \langle C(t_w, t_w+\tau)\rangle.$$

It was shown in [6] that for large t_w, this function is given by the following formula:

$$\Pi(t_w, t_w+\tau) \simeq \frac{\sin \pi\mu}{\pi} \int_{(\tau/t_w)(1+\tau/t_w)}^{1} dv (1-v)^{\mu-1} v^{-\mu},$$

which we will use in Secs. III and V.

A. The sprinkling density $S(t)$

Another important quantity for our study is the sprinkling density of events at time t, $S(t)$. An "event" has to be understood as a jump from an energy level to another one. The sprinkling density of events $S(t)$ is defined as the time density distribution of having an event at time t, whatever the number of events before t, and given that there was one event at $t=0$. This is a standard quantity defined in the context of renewal theory [36]. For any distribution of trapping times $\psi(\tau)$ where the trapping times are independent random variables—and given that it does not depend on the age, i.e., on the choice of the time origin—and independently of the type of dynamics used, the following formula holds:

$$S(t) = \psi(t) + \int_0^t dt_l S(t_l)\psi(t-t_l), \tag{1}$$

where t_l stands for the time of the last event to have taken place before t; the first term $\psi(t)$ corresponds to the special case $t_l=0$.

In the following, we will use the following trapping times distribution: $\psi(\tau)=\mu\tau_0^\mu/(\tau_0+\tau)^{1+\mu}$, in order to ensure that τ

can take values from 0 to ∞. We will also need the large time behavior of the corresponding sprinkling density $S(t)$. Following the lines of [37], this can be easily computed using Laplace transforms. Using the notation $\hat{f}(z)$ to denote the Laplace transform of a function $f(t)$, Eq. (1) is equivalent to

$$\hat{S}(z) = \frac{\hat{\psi}(z)}{1 - \hat{\psi}(z)}.$$

The Laplace transform $\hat{\psi}(z)$ can be computed as

$$\hat{\psi}(z) = \int_0^\infty dt\, e^{-zt}\psi(t) = \mu\tau_0^\mu e^{\tau_0 z} z^\mu \int_{\tau_0 z}^\infty dv \frac{e^{-v}}{v^{-1-\mu}}.$$

Two cases have to be considered before taking the limit $\tau_0 z \to 0$.

If $\mu < 1$, $\hat{\psi}(z) = 1 - \Gamma(1-\mu)(\tau_0 z)^\mu + [1/(1-\mu)]\tau_0 z + o(\tau_0 z)$. Then $\hat{S}(z) \simeq [1/\Gamma(1-\mu)][1/(\tau_0 z)^\mu]$, and for $t \gg \tau_0$, $S(t) \simeq c(\mu)(t^{\mu-1}/\tau_0^\mu)$; with $c(\mu) = 1/\Gamma(1-\mu)\Gamma(\mu) = \sin(\pi\mu)/\pi$. In this regime, $S(t)$ decreases with time; the decrease in time of the density of events results in the aging of the correlation function $\Pi(t_w, t_w+\tau)$, which characteristic time scale is proportional to the age t_w.

On the other hand, if $\mu > 1$, $\hat{\psi}(z) = 1 + [1/(1-\mu)]\tau_0 z + o(\tau_0 z)$. This implies that $\hat{S}(z) \simeq (\mu-1)/\tau_0 z$, and that for $t \gg \tau_0$, $S(t) \simeq (\mu-1)/\tau_0 = 1/\langle\tau\rangle$. In this nonaging regime, the sprinkling density is uniform in time and simply equal to the inverse trapping time $\langle\tau\rangle = \int_0^\infty d\tau\, \tau\psi(\tau) = \tau_0/(\mu-1)$.

B. Definition of the system of study as a superposition of *N* subsystems

Let us now turn to our system of interest. This new system is defined as the superposition of N subsystems, identical to the one introduced previously, each of which is defined by the same trapping times distribution: $\psi(\tau) = \mu\tau_0^\mu/(\tau_0+\tau)^{1+\mu}$. The subsystems are assumed to be independent of each other. One can give an interpretation of such a model in a real space representation: given a system of size L in d dimensions, we assume that one can divide this system into N independent subsystems of length ξ, where $N = (L/\xi)^d$. ξ is the typical coherence length of the system, and is considered constant during the time relaxation. However, we will see that this quantity is susceptible to evolve during aging.

During the dynamical evolution, each system relaxes independently, and hence contributes to some extent to the relaxation of the whole system. We will make the following assumptions: (i) all events occurring in a subsystem are also defined as individual events for the whole system; (ii) all events contribute equally to the relaxation of the whole system.

This translates into the following definitions:

(i) The analog for the whole system of $\psi(\tau)$ will be denoted as $P_N(\tau)$: it is the distribution of time intervals between all events (i.e., trapping times). We will see in the next section that this quantity depends in general on the age t_w; we will then call it $P_N(\tau, t_w)$.

(ii) The correlation function of the whole system is defined as

$$C(t_w, t_w+\tau) = \frac{1}{N}\sum_{i=1}^N C_i(t_w, t_w+\tau),$$

where $C_i(t_w, t_w+\tau)$ is the correlation function of subsystem i.

Before turning to a detailed calculation of $P_N(\tau, t_w)$, we can invoke an argument of statistics of extremes. If τ is a trapping time of the whole system, then it seems natural to say that $\tau = \min\{\tau_i\}_{i=1,\ldots,N}$, where τ_i is a trapping time of each subsystem i. However, this is true only if all subsystems undergo one event at some time origin, and that one computes the first trapping time of the whole system from this time origin. Hence this argument definitely excludes aging effects, because it neglects any memory effects in the dynamics of the subsystems.

Having made this approximation, one can follow a standard calculation of statistics of extremes, and one can find the distribution of time intervals $P_N(\tau)$:

$$P_N(\tau) = N\psi(\tau)\left[\int_\tau^\infty dt'\,\psi(t')\right]^{N-1} = N\mu\frac{\tau_0^{\mu N}}{(\tau_0+\tau)^{1+\mu N}}.$$

By expanding around the most probable value $\tau = 0$, and setting $u = \mu N\tau/\tau_0$, one finds the limiting exponential distribution $P(u) = e^{-u}$ for $u \ll 1$ (i.e., $\tau \ll \tau_0/\mu N$). This corresponds in fact to the convergence of the probability distribution of extremes towards the Weibull distribution, in the case where the elementary distribution $\psi(\tau)$ has a finite value for its minimum time $\tau = 0$ [38].

In this approximation, the time distribution of events of the whole system simply follows a Poisson process, with a rate proportional to the number of subsystems. In a Poisson process, the conditions of the experiment are supposed to remain constant in time, and all events are independent of each other. However, in this model, although single events in all subsystems are indeed independent of each other, the dynamics is not invariant under time translations [as can be inferred from the sprinkling density $S(t)$]. As we shall see in the next section, this will give rise to more complicated laws for $P_N(\tau, t_w)$.

Note finally that we have not been able to find a suitable argument of statistics of extremes for $C(t_w, t_w+\tau)$ (the statistics of this quantity in the framework of nonequilibrium dynamics have been related to the Gumbel distributions [24,27], which are one of the "universal" families of probability distributions of extremes). Instead, we will gain information (see Sec. V) by studying $C(t_w, t_w+\tau)$ as the sum of N random variables, reinforcing the idea that it is not an extremal quantity, but rather originates in the contribution of many individual events (as was already pointed out in [27]).

III. DISTRIBUTION OF TIME INTERVALS BETWEEN ALL EVENTS

Let $P_N(\tau, t_w)$ be the probability that an event takes place at $t_w+\tau$ if one took place at t_w, in the system composed of N independent subsystems.

In this section, it will be more practical for the computation to work with the cumulative probability distribution $P_N^C(\tau,t_w)=\int_\tau^\infty d\tau' P_N(\tau',t_w)$. By definition, it is the probability that the time difference between two successive events is larger than τ. Similarly, we will use $Q(\tau)=\int_\tau^\infty d\tau' \psi(\tau')=\tau_0^\mu/(\tau_0+\tau)^\mu$, which is the probability for a trapping time of a subsystem to be larger than τ, i.e., the probability for a subsystem not to change trap during the period of time τ.

We now call i the subsystem in which one event has taken place at t_w. Then let $\{t_j\}_{j\neq i}$ be the $(N-1)$ times of the last events before t_w in the other subsystems j. The next event to take place in the whole system will either happen in subsystem i or in any other subsystem. In order for this next event to occur after a time τ, one requires the following conditions: (i) subsystem i has to remain trapped between t_w and $t_w+\tau$, with probability $Q(\tau)$, and (ii) the other subsystems j have to remain trapped between t_j and $t_w+\tau$, with probability $Q(t_w+\tau)+\int_0^{t_w}dt_j S(t_j)Q(t_w+\tau-t_j)$, $Q(t_w+\tau)$ being the contribution for the special case $t_j=0$. This last probability is in fact equal to $\Pi(t_w,t_w+\tau)$, the probability for a system to remain trapped between times t_w and $t_w+\tau$ (see Sec. II).

Hence

$$P_N^C(\tau,t_w)=Q(\tau)[\Pi(t_w,t_w+\tau)]^{N-1},$$

$$P_N(\tau,t_w)=-\frac{\partial}{\partial\tau}[Q(\tau)[\Pi(t_w,t_w+\tau)]^{N-1}].$$

In the following, we will always consider the case of large t_w: $t_w \gg \tau_0$. We now treat separately two different regimes for τ: (i) $\tau\sim\tau_0$, and (ii) $x=\tau/t_w$ finite and smaller than 1.

A. Case $\tau\sim\tau_0$

In the regime of interest where $t_w \gg \tau_0$, $S(t_w)\simeq(1/\tau_0^\mu)\times[c(\mu)/t_w^{1-\mu}]$ (see Sec. II), so that the leading term in t_w is

$$\Pi(t_w,t_w+\tau)\simeq 1-\tau S(t_w)\simeq e^{-\tau S(t_w)}.$$

Then, we get the result

$$P_N^C(\tau,t_w)\simeq Q(\tau)e^{-(N-1)[c(\mu)\tau/\tau_0^\mu t_w^{1-\mu}]} \tag{2}$$

and

$$P_N(\tau,t_w)\simeq[\psi(\tau)+(N-1)S(t_w)Q(\tau)]e^{-(N-1)\tau S(t_w)}.$$

We note that for $N=1$ the result $P_N(\tau,t_w)=\psi(\tau)$ is recovered. As N increases, the exponential part in (2) becomes dominant, introducing a rate that is dependent on the waiting time: $\rho=c(\mu)(N-1)/\tau_0^\mu t_w^{1-\mu}$.

This can be interpreted as an effective Poisson process, where the number of instances N is replaced (for large N) by

$$N_{eff}(t_w)=N\left(\frac{\tau_0}{t_w}\right)^{1-\mu}.$$

In other words, the scaling of (2) suggests that computing at age t_w the distribution of events of a system composed initially of N subsystems is the same as computing the distribution of events of a "young" system composed of $N_{eff}(t_w)$ subsystems. In the case of (2), one can see that $N_{eff}(t_w)$ decreases explicitly with the age; in the limiting case where aging disappears ($\mu\to 1$), N_{eff} is simply a constant equal to N. The idea of a number of independent subsystems decreasing with the age in nonequilibrium systems is not new. It is intimately related to the concept of a growing lengthscale in an aging system. If one defines a typical length ξ of a subsystem by $N=(L/\xi)^d$, one has $\xi_{eff}=LN_{eff}^{-1/d}$, and the dependence of N_{eff} on t_w induces the following power law for ξ_{eff}:

$$\xi_{eff}(t_w)=LN^{-1/d}\left(\frac{t_w}{\tau_0}\right)^{(1-\mu)/d}.$$

B. Case $x=\tau/t_w \ll 1$

In the regime most accessible experimentally, $x=\tau/t_w \ll 1$, and $\Pi(t_w,t_w+\tau)\simeq 1-c(\mu)(\tau/t_w)^{1-\mu}$, which leads to the result

$$P_N^C(\tau,t_w)\simeq Q(\tau)e^{-(N-1)c(\mu)(\tau/t_w)^{1-\mu}} \tag{3}$$

and

$$P_N(\tau,t_w)\simeq\left[\psi(\tau)+\frac{(N-1)c(\mu)(1-\mu)}{t_w^{1-\mu}\tau^\mu}Q(\tau)\right]\times e^{-(N-1)c(\mu)(\tau/t_w)^{1-\mu}}.$$

Again the limiting case $P_N^C(\tau,t_w)=\psi(\tau)$ for $N=1$ is recovered. In this regime, when N increases, the distribution $P_N^C(\tau,t_w)$ evolves towards a stretched exponential in τ, with a characteristic time proportional to the age t_w. But as in the case of (2), the scaling of (3) suggests that the system at t_w is equivalent to a "young" system composed of $N_{eff}(t_w)$ subsystems with $N_{eff}(t_w)=N(\tau_0/t_w)^{1-\mu}$.

Let us now be more precise concerning the relevance of the quantity $N_{eff}(t_w)$. The physical meaning of the effective number of subsystems and of the effective coherence length is the following. Consider the system at time t_1, with a coherence length ξ_1. This system is hence made of N_1 independent subsystems of size ξ_1, by definition of the model; and each subsystem is a trap model that can hop in different states in time, with average hopping rate $S(t_1)$. Each subsystem ages, so that at some later time t_2, the hopping rate decreases and is equal to $S(t_2)$. This also means that per unit time, less subsystems have hopped than at age t_1. In a real space picture, the subsystems that hop are then more sparse and far away from each other. This induces some kind of enhanced spatial correlation. Then one can make a coarse graining of subsystems and define bigger subsystems, of size ξ_2, in such a way that the hopping rate of these new subsystems is the same as at time t_1, which is possible, precisely because they are bigger. Hence the total system at time t_2 is now a collection of N_2 subsystems of size ξ_2, each of which hops at the same rate as the N_1 subsystems of size ξ_1 at time t_1. This can actually be quantified by a simple argument. The average hopping rate of one subsystem at time t_1 is $S(t_1)$. At time t_2, one defines a coarse-grained subsystem as composed of N_1/N_2 of the former subsystems. Then the average hopping rate of one coarse-grained subsystem at time t_2 is

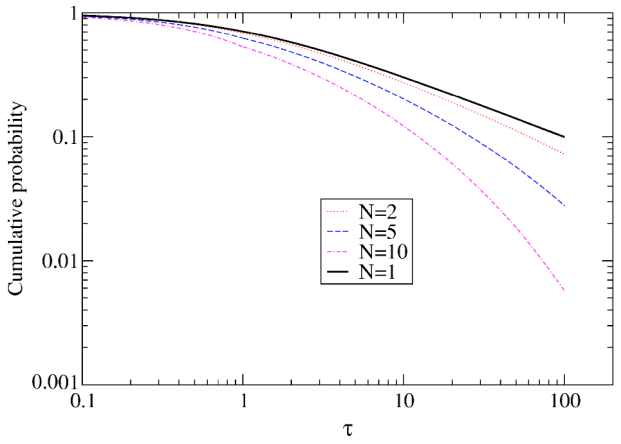

FIG. 1. (Color online) $P_N^C(\tau,t_w)$ in the two regimes studied for $N=1,2,5,10$; the values of the parameters are $\tau_0=1$, $\mu=0.5$, and $t_w=100$. As N is increased, the departure from the power-law is observed.

$S(t_2)N_1/N_2$. The hopping rates are chosen to be equal, which leads to $N_2=N_1S(t_2)/S(t_1)$. In the special case of $t_1=\tau_0$ and $t_2=t_w$, one finds: $N(t_w)=N(\tau_0/t_w)^{1-\mu}$, which is exactly the relation for $N_{eff}(t_w)$ found from the previous calculations.

To conclude this section, the study of the two cases investigated above show that for a small system, $P_N^C(\tau,t_w)$ [and $P_N(\tau,t_w)$] will still be very close to the power law characterizing one single subsystem. For a very large system, $P_N^C(\tau,t_w)$ crosses over from an exponential form to a stretched exponential form at larger τ.

In general, there will be a crossover in $P_N(\tau,t_w)$ from an exponential times a power-law, to a stretched exponential times a power-law, as τ increases. In all cases, the distributions become fatter with the age, which allows one to define an effective number of subsystems $N_{eff}(t_w)=N(\tau_0/t_w)^{1-\mu}$, or equivalently an effective coherence length $\xi_{eff}(t_w)=LN^{-1/d}(t_w/\tau_0)^{(1-\mu)/d}$.

For illustration, we plot on Fig. 1 the cumulative probability distribution $P_N^C(\tau,t_w)$ in the two regimes studied for different values of N; the values of the parameters are $\tau_0=1$, $\mu=0.5$, and $t_w=100$.

IV. DISTRIBUTIONS OF TIME INTERVALS BETWEEN DECORRELATIONS. APPLICATION TO EXPERIMENTALLY ACCESSIBLE DATA

A. Definition of the quantities of interest

When one is not able experimentally to identify individual rearrangement events, it may be easier to turn to the study of the fluctuations of global quantities such as correlation functions. More precisely, in the scattering experiments of [19], nonaveraged correlation functions called $c_I(t_w,\tau)$ are computed. $c_I(t_w,\tau)$ represents the degree of correlation between the speckle field scattered by the sample at time t_w, and the one scattered at time $t_w+\tau$. The time lag τ can be given a fixed value during the analysis of the data, and one computes the time series of $c_I(t_w,\tau)$ as a function of time, starting from time t_w [see, for example, Fig. 3(a) in [19]]. This allows one to compute the probability distribution of c_I, $P(c_I)$, for a given τ and a given age t_w. This is exactly what we compute in Sec. V, if we assume that the function $C(t_w,t_w+\tau)$ of our model can be identified with $c_I(t_w,\tau)$. The main experimental finding, namely that $P(c_I)$ is a negatively skewed distribution for a value τ small compared to t_w in a nonequilibrium system (a colloidal gel), is recovered in Sec. V.

In the analysis of the time series of $c_I(t_w,\tau)$, one can also compute the distribution of time intervals between significant decorrelations of the system, i.e., big downward jumps of $c_I(t_w,\tau)$, which is accessible experimentally. Note that these big jumps do not necessarily correspond to the same individual "events" studied in Sec. II and Sec. III, and actually correspond to the superposition of several of them. However these jumps are the only visible manifestation of the individual "events" from an experimental point of view, unless new techniques allow one to visualize in detail and record the spatial rearrangements of the particles in real space.

Typically one would like to compute the distribution of time intervals between the smallest values of $c_I(t_w,\tau)$—or $C(t_w,t_w+\tau)$ in our model—, a threshold value C_{th} being fixed. One can write that $C(t_w,t_w+\tau)=1-n(t_w,t_w+\tau)/N$, where $n(t_w,t_w+\tau)$ is the number of subsystems that changed trap between t_w and $t_w+\tau$. In this section, we will keep the notation N for the number of subsystems, since we will always place ourselves at a given age t_w. The influence of the dependence of the number of subsystems on t_w will be discussed in the conclusion and is not crucial here.

The threshold C_{th} can be chosen such that one selects only the situations where at least k subsystems have changed trap between t_w and $t_w+\tau$, so that $C_{th}=1-k/N$, and the values of $C(t_w,t_w+\tau)$ considered will be less than C_{th}. The distribution of time intervals between successive jumps of the correlation of this kind will be denoted $\mathcal{P}^{(k)}(\tau,t_w,T)$, where T is the time interval variable.

Before considering the general case, we will first focus on two simpler cases.

(i) We will first compute $\mathcal{P}^{(N)}(\tau,t_w,T)$, which corresponds to the distribution of times between successive decorrelations of maximum intensity ($k=N$). We will see that the age t_w is not relevant for this quantity, and it will be denoted $\mathcal{P}^{(N)}(\tau,T)$.

(ii) We will also compute $\mathcal{P}^{(k,k)}(\tau,t_w,T)$, the distribution of time intervals between successive decorrelations of the same intensity, i.e., which correspond to the case where k subsystems have changed trap between t_w and $t_w+\tau$, and k subsystems have changed trap between t_w+T and $t_w+T+\tau$, for the first time since t_w.

B. Special case of the largest decorrelations

In this section we want to compute $\mathcal{P}^{(N)}(\tau,t_w,T)$. Since we consider only the largest decorrelations, we say that an event has occurred at t_w if $C(t_w,t_w+\tau)$ first reaches the value 0 at t_w. We define $\mathcal{P}_0^{(N)}(\tau,t_w,T)$ as the probability per unit time τ that such an event takes place at time t_w+T, knowing that such an event happened at t_w. This quantity will be helpful in all the following in order to calculate $\mathcal{P}^{(N)}(\tau,t_w,T)$.

In the case of $\mathcal{P}^{(N)}(\tau,t_w,T)$, the decorrelations in the intervals $[t_w;t_w+\tau]$ and $[t_w+T;t_w+T+\tau]$ are two successive total decorrelations; whereas in the case of $\mathcal{P}_0^{(N)}(\tau,t_w,T)$, they may not be successive in time. Formally speaking, $\mathcal{P}^{(N)}(\tau,t_w,T)$ plays the role of $\psi(T)$ and $\mathcal{P}_0^{(N)}(\tau,t_w,T)$ the role of $S(T)$, where $\psi(T)$ and $S(T)$ have been introduced in Sec. II.

Throughout the whole section, we will consider the regime where $\tau \ll T$. In this case, a good approximation is

$$\mathcal{P}_0^{(N)}(\tau,t_w,T) \simeq \frac{1}{\tau}[\tau S(T)]^N.$$

Therefore, this quantity does not depend on age, and, as in Sec. II, one has the relation

$$\mathcal{P}_0^{(N)}(\tau,T) = \mathcal{P}^{(N)}(\tau,T) + \int_0^T dt' \mathcal{P}^{(N)}(\tau,t')\mathcal{P}_0^{(N)}(\tau,T-t').$$

Equivalently, if z is the Laplace variable conjugated to T, the Laplace transforms are related according to

$$\hat{\mathcal{P}}^{(N)}(\tau,z) = \frac{\hat{\mathcal{P}}_0^{(N)}(\tau,z)}{1+\hat{\mathcal{P}}_0^{(N)}(\tau,z)}.$$

In the case of interest ($\mu<1$), we use the fact that $S(T) \simeq c(\mu)(T^{\mu-1}/\tau_0^{\mu})$ for $T \gg \tau_0$, so that

$$\hat{\mathcal{P}}_0^{(N)}(\tau,z) \simeq \frac{1}{\tau}\left[\frac{c(\mu)\tau}{\tau_0^{\mu}}\right]^N \frac{1}{z^x}\int_{\tau_0 z}^{\infty} du\, e^{-u}u^{x-1},$$

where $x=1-N(1-\mu)$, and we have introduced the lower cutoff τ_0 for the case where the integral is divergent at the origin.

According to the value of N, we will see below through the study of the different cases that $\mathcal{P}^{(N)}(\tau,T)$ is a power law with an exponent depending on N.

1. Case $0<x<1$: $N(1-\mu)<1$

In this case, the former integral converges when $\tau_0 z \to 0$; therefore,

$$\hat{\mathcal{P}}_0^{(N)}(\tau,z) \simeq \frac{1}{\Gamma(1-x)}\frac{1}{(T_0 z)^x},$$

where we have defined $T_0^x = \tau[\tau_0^{\mu}/c(\mu)\tau]^N[1/\Gamma(x)\Gamma(1-x)]$. Hence, if $T_0 z \ll 1$, $\hat{\mathcal{P}}^{(N)}(\tau,z) \simeq 1-\Gamma(1-x)(T_0 z)^x$, which leads to

$$\mathcal{P}^{(N)}(\tau,T) \simeq \frac{xT_0^x}{(T_0+T)^{1+x}}.$$

Note that in the special case $N=1$, one has $x=\mu$ and $T_0=\tau_0$, and one recovers the power law with the initial exponent $1+\mu$.

2. Case $0<y<1$: $1<N(1-\mu)<2$

The integral is divergent at the origin in the case where $x<0$, and instead of x, we use for convenience $y=-x=N(1-\mu)-1>0$.

For $0<y<1$, or $1<N(1-\mu)<2$, we use the following expansion when $\tau_0 z \to 0$:

$$\int_{\tau_0 z}^{\infty} du \frac{e^{-u}}{u^{1+y}} \simeq \frac{1}{y}\frac{1}{(\tau_0 z)^y} - \frac{1}{y}\Gamma(1-y).$$

We define the time T_1 such that $T_1^y = (1/\tau)[c(\mu)\tau/\tau_0^{\mu}]^N$, and we find that $\hat{\mathcal{P}}_0^{(N)}(\tau,z) \simeq (1/y)(T_1/\tau_0)^y - (1/y)\Gamma(1-y) \times (T_1 z)^y$, and

$$\hat{\mathcal{P}}^{(N)}(\tau,z) \simeq \frac{\frac{1}{y}\left(\frac{T_1}{\tau_0}\right)^y}{1+\frac{1}{y}\left(\frac{T_1}{\tau_0}\right)^y}\left[1-\frac{\Gamma(1-y)}{1+\frac{1}{y}\left(\frac{T_1}{\tau_0}\right)^y}(\tau_0 z)^y\right].$$

Hence, $\mathcal{P}^{(N)}(\tau,T)$ is again a power law at large T:

$$\mathcal{P}^{(N)}(\tau,T) \simeq \frac{\frac{1}{y}\left(\frac{T_1}{\tau_0}\right)^y}{1+\frac{1}{y}\left(\frac{T_1}{\tau_0}\right)^y}\frac{yT_2^y}{(T_2+T)^{1+y}},$$

where $T_2 = \tau_0/[1+(1/y)(T_1/\tau_0)^y]^{1/y}$.

Note that the constant in front of the power law is not exact [one would need the expression for all T of $S(T)$ to get its exact expression]. Moreover, this constant is smaller than 1, which means that the distribution $\mathcal{P}^{(N)}(\tau,T)$ is not normalized. This comes from the fact that the total number of events [which is equal to $\hat{\mathcal{P}}_0^{(N)}(\tau,z=0)$] is finite; therefore there is a nonzero probability that the time interval between two events is infinite, so that $\mathcal{P}^{(N)}(\tau,T)$ is not normalized to unity.

3. Case $y>1$: $N(1-\mu)>2$

Now the expansion for $\tau_0 z \to 0$ of the integral reads

$$\int_{\tau_0 z}^{\infty} du \frac{e^{-u}}{u^{1+y}} \simeq \frac{1}{y}\frac{1}{(\tau_0 z)^y} - \frac{1}{y-1}\frac{1}{(\tau_0 z)^{y-1}} + \frac{1}{y}\frac{1}{1-y}\Gamma(2-y)$$

in the case where $1<y<2$. In general, if $n=E(y)$ is the integer part of y, the constant part of the expansion is proportional to $\Gamma(n+1-y)$ and is followed by a $(\tau_0 z)^{n+1-y}$ term. Then, following the case $1<y<2$, we find that $\hat{\mathcal{P}}_0^{(N)}(\tau,z) \simeq (1/y)(T_1/\tau_0)^y - [1/(y-1)]T_1^y\tau_0^{1-y}z + (1/y)[1/(1-y)]\Gamma(2-y) \times (T_1 z)^y + o(z^2)$. In general, there will always be a singular term in z^y in between two polynomial terms z^n and z^{n+1}. Again in the special case where $1<y<2$, we find

$$\hat{\mathcal{P}}^{(N)}(\tau,z) \simeq \frac{\frac{1}{y}\left(\frac{T_1}{\tau_0}\right)^y}{1+\frac{1}{y}\left(\frac{T_1}{\tau_0}\right)^y}\left(1-\frac{1}{y-1}yT_2^y\tau_0^{1-y}z + \frac{1}{y-1}\Gamma(2-y)(T_2 z)^y + o(z^2)\right).$$

As a reminder, let us note that the Laplace transform for small z of a power-law distribution $\psi(\tau)$ introduced in Sec. II is, for $1<\mu<2$, $\hat{\psi}(z) = 1-[1/(\mu-1)]\tau_0 z + [1/(\mu-1)]\Gamma(2$

$-\mu)(\tau_0 z)^{\mu}+o((\tau_0 z)^2)$**)**. Hence $\hat{\mathcal{P}}^{(N)}(\tau,z)$ cannot be mapped exactly on this type of function, and one can for example add an exponential function to a power law in order to recover the correct expansion up to order z^2. This introduces some indeterminacy in the determination of $\mathcal{P}^{(N)}(\tau,T)$. However, the behavior at very large T will be

$$\mathcal{P}^{(N)}(\tau,T) \simeq \frac{\alpha}{T^{1+y}},$$

where α is an indetermined coefficient. This final power-law behavior might be in fact rather hard to observe in reality.

C. Special case of the decorrelations with same value

In this section we compute the quantity $\mathcal{P}^{(k,k)}(\tau,t_w,T)$. Like for the previous case, we define $\mathcal{P}_0^{(k,k)}(\tau,t_w,T)$ corresponding to $\mathcal{P}^{(k,k)}(\tau,t_w,T)$, which measures the probability that k subsystems change traps in the interval $[t_w+T;t_w+T+\tau]$, knowing that k subsystems (not necessarily the same) have changed traps in the interval $[t_w;t_w+\tau]$. More precisely, in this section, we say that an event has occurred at t_w if $C(t_w,t_w+\tau)$ first reaches the value $1-k/N$ at t_w. Then $\mathcal{P}_0^{(N)}(\tau,t_w,T)$ is the probability per unit time τ that such an event takes place at time t_w+T, knowing that an event happened at t_w. In this case ($k\neq N$), all quantities will depend on t_w and a relation of the type of (1) is *a priori* not correct. However, in the limit that we consider, where both T and τ are small compared to t_w, and if k is not too far from N (experimentally, the threshold will be taken low enough in order to get rid of experimental noise), we will assume that such a relation can still be valid. We will write

$$\mathcal{P}_0^{(k,k)}(\tau,t_w,T) = \mathcal{P}^{(k,k)}(\tau,t_w,T) + \int_0^T dt' \mathcal{P}^{(k,k)}(\tau,t_w,t') \times \mathcal{P}_0^{(k,k)}(\tau,t_w,T-t'). \quad (4)$$

As in the previous section, we will first compute $\mathcal{P}_0^{(k,k)}(\tau,t_w,T)$ and deduce $\mathcal{P}^{(k,k)}(\tau,t_w,T)$ from its Laplace transform thanks to the relation

$$\hat{\mathcal{P}}^{(k,k)}(\tau,t_w,z) = \frac{\hat{\mathcal{P}}_0^{(k,k)}(\tau,t_w,z)}{1+\hat{\mathcal{P}}_0^{(k,k)}(\tau,t_w,z)}. \quad (5)$$

In order to compute $\mathcal{P}_0^{(k,k)}(\tau,t_w,T)$, we call n the number of subsystems that changed traps in the interval $[t_w,t_w+\tau]$, but stayed trapped in the interval $[t_w+T,t_w+T+\tau]$. Hence $k-n$ subsystems change traps both in the intervals $[t_w,t_w+\tau]$ and $[t_w+T,t_w+T+\tau]$; n subsystems change traps in the interval $[t_w+T,t_w+T+\tau]$ but where trapped in the interval $[t_w,t_w+\tau]$; finally, $N-k-n$ subsystems were trapped in both the intervals $[t_w,t_w+\tau]$ and $[t_w+T,t_w+T+\tau]$.

Therefore, if $k_{sup}=\min(k,N-k)$,

$$\mathcal{P}_0^{(k,k)}(\tau,t_w,T) = \frac{C_N^k}{\tau}\sum_{n=0}^{k_{sup}} C_k^n C_{N-k}^n [\tau S(T)]^{k-n}[1-\tau S(T)]^n \times [\tau S(t_w+T)]^n [1-\tau S(t_w+T)]^{N-k-n}.$$

Such a finite sum being analytically untractable, we will take its continuous limit, by defining $x=k/N$ and $y=n/N$, and by replacing $\Sigma_{n=0}^{k_{sup}}$ by $N\int_0^{x_{sup}}$ in the limit where N is large.

First, if we consider the lowest order in τ only, we reduce the expression to

$$\mathcal{P}_0^{(k,k)}(\tau,t_w,T) = \frac{C_N^k}{\tau}\sum_{n=0}^{k_{sup}} C_k^n C_{N-k}^n [\tau S(T)]^{k-n}[\tau S(t_w+T)]^n.$$

Then by using the Stirling formula $N!\sim N^N e^{-N}\sqrt{2\pi N}$, and taking the continuous limit, we get

$$\mathcal{P}_0^{(k,k)}(\tau,t_w,T) \simeq \frac{\tau^{k-1}}{(2\pi)^{3/2}N^{1/2}}\int_0^{x_{sup}} dy \exp[-N\{(x-y)\ln(x-y) + 2y\ln y + (1-x-y)\ln(1-x-y) - (x-y)\ln S(T) - y\ln S(T+t_w)\}]. \quad (6)$$

By using a saddle point method, we find that the former integral is maximized for y^* such that

$$(x-y^*)(1-x-y^*) = y^{*2}\gamma(t_w,T), \quad (7)$$

where $\gamma(t_w,T)=S(T)/S(T+t_w)$.

We keep the positive solution, having checked that we have both $y^*\leqslant x$ and $y^*\leqslant 1-x$:

$$y^* = \frac{1}{2(\gamma-1)}[-1+\sqrt{1+4x(1-x)(\gamma-1)}].$$

Replacing y^* in the last expression for $\mathcal{P}_0^{(k,k)}(\tau,t_w,T)$ leads to a complicated expression that can be simplified in our regime of interest, $\tau_0 \ll T \ll t_w$.

First, by using the expression for $S(T)$ at large times and $\mu<1$, we find that, for $\tau_0\ll T\ll t_w$, $\gamma(t_w,T)\simeq(t_w/T)^{1-\mu}$, and $y^*(T,t_w)\simeq[x(1-x)/\gamma(t_w,T)]^{1/2}$. Then, if t_w/T is large enough, we can always assume to have $y^*\ll\min(x,1-x)$, so that Eq. (6) can be expanded in y^*. Finally, we find the result

$$\mathcal{P}_0^{(k,k)}(\tau,t_w,T) \simeq \frac{C_N^k}{2\pi\tau}[\tau S(T)]^k e^{2\sqrt{k(N-k)}(T/t_w)^{(1-\mu)/2}}.$$

The Laplace transform with respect to T reads

$$\hat{\mathcal{P}}_0^{(k,k)}(\tau,t_w,z) \simeq \frac{C_N^k}{2\pi\tau}\left[\frac{c(\mu)\tau}{\tau_0^{\mu}}\right]^k I(t_w,z),$$

where

$$I(t_w,z) = \int_0^{\infty} dT \frac{e^{-zT}}{T^{(1-\mu)k}} \exp\left[K(k)\left(\frac{T}{t_w}\right)^{(1-\mu)/2}\right],$$

and $K(k)=2\sqrt{k(N-k)}$. $I(t_w,z)$ is a convergent integral for $k(1-\mu)<1$, which is the first case we consider.

1. Case $k(1-\mu)<1$

Since we are interested in the case $T\ll t_w$, we will make an expansion for $zt_w\gg 1$.

Then we need to consider two separate cases for the evaluation of $\hat{\mathcal{P}}^{(k,k)}(\tau,t_w,z)$ from relation (5). We report here the main results, and refer to Appendix A for the calculations.

2. Case $0<k<(1+\mu)/2(1-\mu)$

$\mathcal{P}^{(k,k)}(\tau,t_w,T)$ is a power law:

$$\mathcal{P}^{(k,k)}(\tau,t_w,T) \simeq \frac{C_N^k}{2\pi\tau}\left[\frac{c(\mu)\tau}{\tau_0^\mu}\right]^k \frac{At_w^\alpha}{\Gamma(-\beta)}\frac{1}{T^{1+\beta}}, \tag{8}$$

with $\alpha=k(1-\mu)^2/(1+\mu)$ and $\beta=2k(1-\mu)/(1+\mu)-1$.

3. Case $(1+\mu)/2(1-\mu)<k<1/(1-\mu)$

$$\mathcal{P}^{(k,k)}(\tau,t_w,T) \simeq e^{\cos(\pi/(1+\beta))[\beta\mathcal{K}(t_w,\tau)]^{1/(1+\beta)}(T/t_w)^{\beta/(1+\beta)}} \tag{9}$$

which is a stretched exponential since $\cos(\pi/(1+\beta))<0$.

4. Case $k(1-\mu)>1$

$\mathcal{P}^{(k,k)}(\tau,t_w,T)$ is simply an exponential, with a weak dependence on t_w (which vanishes for large t_w):

$$\mathcal{P}^{(k,k)}(\tau,t_w,T) \simeq \mathcal{C}(t_w,\tau)e^{-\mathcal{C}(t_w,\tau)T} \tag{10}$$

where $\mathcal{C}(t_w,\tau)=(C_N^k/2\pi\tau)[c(\mu)\tau/\tau_0^\mu]^k[e^{K(k)}(\tau_0/t_w)^{(1-\mu)/2}/\tau_0^{(1-\mu)k}]$.

We note here that it is not possible to make a clear continuity between the cases $k=N$ and the case k different from N treated in this section. This is due to the fact that we use a saddle-point approximation of $I(t_w,z)$, which is defined only if $K(k)$ is different from 0, hence k is different from N and 0. In this section, the approximations are more numerous and are susceptible to be valid only for some intermediate values of k. Moreover, we are interested in the aging regime and now the correct expansion is to be made for $zt_w \gg 1$, and not anymore for $z\tau_0 \ll 1$.

D. Generalization

The most general case, as introduced in the beginning of Sec. IV, consists in computing $\mathcal{P}^{(k)}(\tau,t_w,T)$ the distribution of time intervals between successive decorrelations larger than k/N.

As before, we will use the $\mathcal{P}_0$ quantities relative to the $\mathcal{P}$ quantities we are looking for, and we will assume that relations between them similar to Eq. (4) still hold.

Therefore, we will start by computing

$$\mathcal{P}_0^{(k)}(\tau,t_w,T) = \sum_{k'\geqslant k}\sum_{k''\geqslant k}\mathcal{P}_0^{(k',k'')}(\tau,t_w,T), \tag{11}$$

where $\mathcal{P}_0^{(k',k'')}(\tau,t_w,T)$ is the probability that k' subsystems have changed traps in the interval $[t_w,t_w+\tau]$ and k'' subsystems have changed traps in the interval $[t_w+T;t_w+T+\tau]$. We refer the reader to Appendix B for technical details of the computation, which is similar to the one explained in Sec. IV C.

From the calculation, we get

$$\mathcal{P}_0^{(k)}(\tau,t_w,T) = \mathcal{P}_0^{(k^*)}(\tau,t_w,T),$$

$$\mathcal{P}^{(k)}(\tau,t_w,T) = \mathcal{P}^{(k^*)}(\tau,t_w,T),$$

where k^* is a growing function of k, and is equal to k if $\tau S(T)<1$ and $k>N/2$.

Therefore the results of the previous section can be used. We conclude that, according to the value of k^* (which will be equal to k in most cases of interest), $\mathcal{P}^{(k)}(\tau,t_w,T)$ can either take a power-law form [Eq. (8)], a stretched exponential form [Eq. (9)], or an exponential form [Eq. (10)], where k has to be replaced by k^*.

V. PROBABILITY DISTRIBUTION OF THE TOTAL CORRELATOR

In Sec. III, we used the notion of "events," and calculated their time distribution, because it was the most natural quantity to compute in the framework of the model presented here. However, it may be difficult in an experiment to identify such events, or even to find a reasonable definition of individual events.

A well-defined quantity that is more readily accessible is the distribution probability of a correlator $C(t_w,t_w+\tau)$, as has been recently investigated in numerical simulations [24].

In our model, each subsystem i is a two-level system, where the correlation $C_i(t_w,t_w+\tau)$ can only take the values 0 or 1. More precisely, the probability distribution of the correlation of a subsystem i is

$$P(C_i) = f\delta(C_i-1) + (1-f)\delta(C_i),$$

where the parameter f coincides with the average value of the correlation for the values of t_w and τ considered: $f=\Pi(t_w,t_w+\tau)=\langle C_i(t_w,t_w+\tau)\rangle$. In the following we will omit temporarily the dependence of f on t_w and τ, for simplicity in the notations.

For the whole system (the superposition of the N independent subsystems), the definition of $C(t_w,t_w+\tau)$ has been given in Sec. II. One can actually rewrite this quantity as

$$C(t_w,t_w+\tau) = \frac{m(t_w,t_w+\tau)}{N} = 1 - \frac{n(t_w,t_w+\tau)}{N},$$

where $m(t_w,t_w+\tau)$ is the number of subsystems that remained in the same energy trap between t_w and $t_w+\tau$, and $n(t_w,t_w+\tau)$ the number of subsystems that changed trap between t_w and $t_w+\tau$.

Then, in order to find the value C for the correlation between times t_w and $t_w+\tau$, one has to draw m subsystems among N instances that are trapped during this time interval, with probability f, and $N-m$ subsystems that changed trap during this time interval, with probability $1-f$. Hence, the probability distribution of C, $P(C)$, is simply the binomial distribution: $P(C)=NP(m)$, with

$$P(m) = C_N^m f^m (1-f)^{N-m},$$

and C_N^m is the binomial coefficient.

Note that in the limit of a large number of subsystems N, the central limit theorem holds and the limiting distribution is a Gaussian:

$$P(C) \to \sqrt{\frac{N}{2\pi f(1-f)}}\exp\left[-\frac{N(C-f)^2}{2f(1-f)}\right].$$

We show on Fig. 2 the distributions $P(C)$ for different values of the average correlation: $\langle C\rangle=0.1,0.5,0.9$. The full

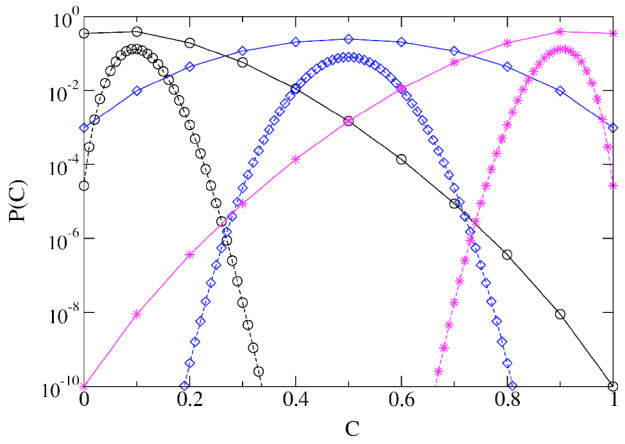

FIG. 2. (Color online) Probability distribution $P(C)$ for different values of the average correlation: $\langle C\rangle = 0.1$ (circles), 0.5 (diamonds), and 0.9 (stars). The full lines correspond to $N=10$ and the dotted lines to $N=100$.

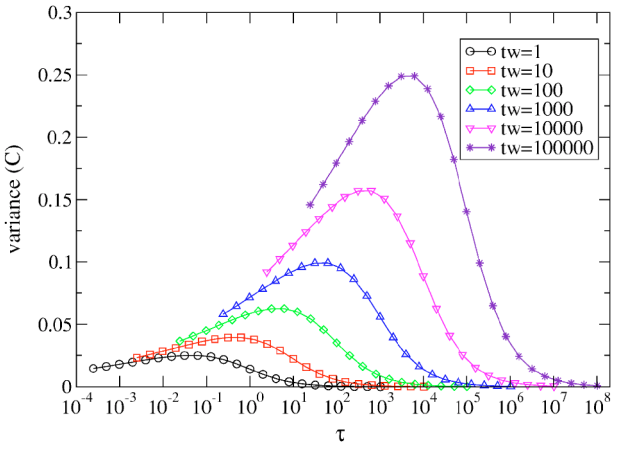

FIG. 3. (Color online) Variance of the probability distribution $P(C)$ for $\mu=0.8$ and $N=10$ independent subsystems initially, for different values of t_w.

lines correspond to $N=10$ and the dotted lines to $N=100$, showing a closer resemblance to a Gaussian distribution for $N=100$.

For comparison with experiments or numerical simulations, we computed the variance $\sigma^2(C)$, the skewness $s(C)$ and the kurtosis $\kappa(C)$ of the distribution $P(C)$. They can be easily computed by noticing that the cumulants of independent random variables are additive quantities:

$$\sigma^2(C) = \frac{1}{N} f(1-f),$$

$$s(C) = \frac{1}{\sqrt{N}} \frac{1-2f}{\sqrt{f(1-f)}},$$

$$\kappa(C) = \frac{1}{N} \frac{6f^2 - 6f + 1}{f(1-f)}.$$

At this stage, one would like to plot the previous quantities as a function of τ for different values of the age t_w, in order to compare with existing results in the literature [19,24,39]. For given τ and t_w, $f=\Pi(t_w, t_w+\tau)$ is explicitly defined using the relation of Sec. II. Moreover, we use the results of Sec. III that suggest that at age t_w, the system can actually be considered as a superposition of $N_{eff}(t_w) = N(\tau_0/t_w)^{1-\mu}$ independent subsystems. Hence in the preceding formulae for $\sigma^2(C)$, $s(C)$, $\kappa(C)$, we replace N by $N_{eff}(t_w)$. We want to stress that it is not proved here that this mapping can be applied to the case of the full probability of the correlation functions, it is only a phenomenological attempt to try to extend this coarse-graining to the calculation of other quantities than the distribution of times. This argument actually leads to results compatible to what is found in experiments.

In Figs. 3–5, $\sigma^2(C)$, $s(C)$, and $\kappa(C)$ are shown as a function of τ for different values of t_w, for $\mu=0.8$ and $N=10$. As observed in recent experiments on foams [19,39], one can see that the variance is maximum at a time τ of the order of the relaxation time (which in our model is proportional to the age t_w), and goes to 0 at large τ; this maximum increases with the age. The skewness is negative at small τ, crosses the origin at intermediate times when $P(C)$ becomes symmetric and then becomes positive; the negative skewness is more and more pronounced when t_w increases. The kurtosis is positive at small times, then negative, and positive again at large τ, while the negative part becomes more and more pronounced with the age. All these results are compatible with recent numerical simulations [24], and seem to be compatible with preliminary experiments on colloidal gels [19]. Note again that the use of $N_{eff}(t_w)$ contains crucial information: if N was kept constant, the curves would simply be superimposed by a τ/t_w rescaling, in particular, the maxima and minima of the variance, skewness and kurtosis would not depend on the age.

Finally, we computed the probability distribution $P(C)$ for a given τ/t_w ratio, i.e., a given value of the average correlation $\langle C\rangle$, at different ages, therefore for different $N_{eff}(t_w)$; the system is made initially of $N=1000$ independent subsystems. Figure 6 shows such probability distributions, for $\mu=0.8$, f

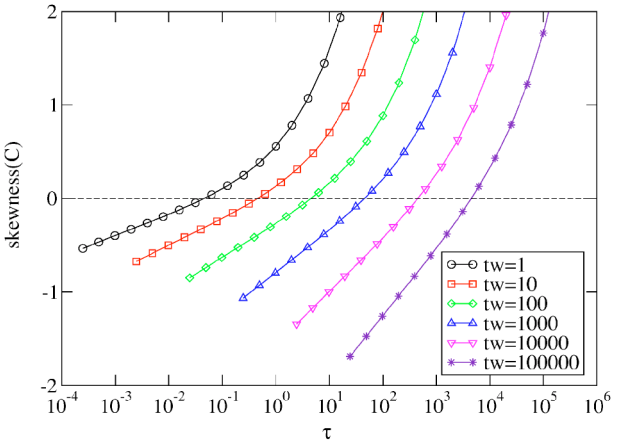

FIG. 4. (Color online) Skewness of the probability distribution $P(C)$ for $\mu=0.8$ and $N=10$ independent subsystems initially, for different values of t_w.

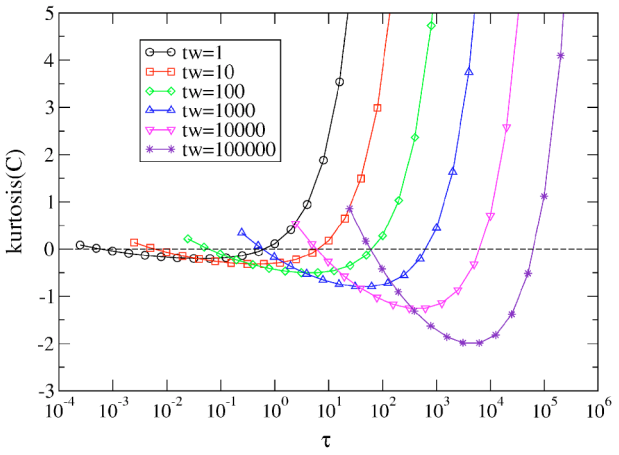

FIG. 5. (Color online) Kurtosis of the probability distribution $P(C)$ for $\mu=0.8$ and $N=10$ independent subsystems initially, for different values of t_w.

$=\langle C\rangle=0.8$, $N=1000$, plotting $P(C)\sigma(C)$ versus $(C-\langle C\rangle)/\sigma(C)$. One can see that even for $N=1000$, systematic deviations exist, though they may be hard to distinguish when an experimental noise is present. Deviations are systematic as t_w increases, which is not surprising since $N_{eff}(t_w)$ decreases with the age; the central limit theorem is less and less valid as t_w increases. Note that we tried to fit these curves with Gumbel distributions like in [24], but we did not find a very precise agreement in our case.

Finally, let us note that our results fail to reproduce the expected zero limit of the skewness and kurtosis of $P(C)$ when $\tau/t_w\to\infty$. These two quantities actually diverge in our case for any finite N, because $P(C)$ tends to a delta function $\delta(C)$ and $\sigma(C)\to 0$. However, the limit $\tau/t_w\to\infty$ is formally equivalent to $N_{eff}(t_w)\to\infty$, which restores the correct limit because of the central limit theorem. This problem would be absent if $P(C_i)$ for one individual system had a nonzero variance when $C_i\to 0$. In practice, this will be always the case, because of the presence of white thermal noise.

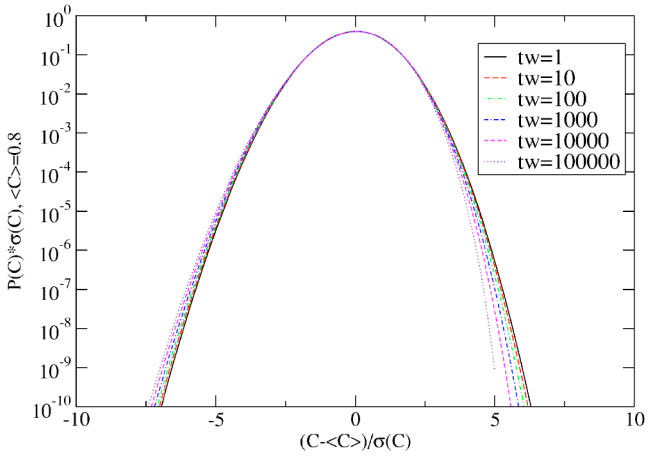

FIG. 6. (Color online) Rescaled probability distributions $P(C)$ for $\mu=0.8$, $N=1000$ independent subsystems initially, $\langle C\rangle=0.8$, and different values of t_w.

VI. DISCUSSION AND CONCLUSION

We have studied a system made of the superposition of N subsystems characterized individually by a slow dynamics and a power-law distribution of times between jumps. This system can in turn be characterized by its distribution of time intervals between events $P_N(\tau,t_w)$. For small N, the power-law behavior of a single subsystem is the dominant behavior. For large N, at very small time intervals, this distribution is exponential and crosses over at larger time intervals to a stretched exponential. In the two regimes, a natural scaling between N and t_w appears, which allows one to define an age dependent effective number of subsystems $N_{eff}(t_w)$ which decreases with t_w, or equivalently, an effective coherence length which grows slowly with the age: $\xi(t_w)\sim t_w^{(1-\mu)/d}$. It would be interesting to see whether experimental results also display the different regimes calculated here. In particular, one could observe an exponential or a stretched exponential distribution for a young system, and a power-law distribution for an aged system. As explained in Sec. III, the interpretation of $\xi_{eff}(t_w)$ comes from the aging of all individual subsystems: the systems that hop are more rare when t_w increases, and hence further apart from each other; this introduces a relevant length scale $\xi(t_w)$ that grows with t_w.

We also computed $\mathcal{P}^{(k)}(\tau,t_w,T)$ the distribution of time intervals between successive decorrelations over an interval τ, larger than k/N. This quantity, like $P(C)$, is accessible through scattering experiments. Although the derivation of this quantity is a bit technical, the results can be summarized in the following way. In the extreme case where one only counts the largest decorrelations ($k=N$), the distribution of time intervals between successive decorrelations crosses over from a power law with a small exponent at small N (or large t_w) to a power law with a large exponent at large N (or small t_w). It is hoped that in experiments these regimes will be observed, when one varies the age or other parameters. For example, if the temperature is decreased in a glass, or the density is increased in a gel, or if one increases the quench rate, one expects that the coherence length increases, i.e., the number of independent subsystems decreases. For $k\neq N$, the results are more difficult to obtain, due to technical complications, and are likely to be reliable only for intermediate values of k. If k is small (i.e., one counts only the decorrelations corresponding to a few subsystems), the distribution $\mathcal{P}^{(k)}(\tau,t_w,T)$ is a power law; if k is larger, it is a stretched exponential and for k large it is an exponential. Equivalently, since one expects the number of subsystems to decrease with the age, $\mathcal{P}^{(k)}(\tau,t_w,T)$ is typically an exponential at small t_w, then a stretched exponential and finally a power law at large t_w, for a given value of the correlation threshold $C_{th}=1-k/N$.

Finally, we also computed analytically the probability distribution of the correlator $P(C)$, which reproduces many features of the experimental findings of [19] and of the numerical results of [24]. We used the fact (not shown rigorously) that at t_w the system can be described as a collection of $N_{eff}(t_w)$ independent subsystems. No rescaling to some time-evolving Gumbell distribution seems to be relevant here. Since the full probability is calculated here, it would be in-

teresting to compare it more quantitatively to experimental results. In particular, we find that the variance grows with the age like $\sigma^2 \sim N_{eff}^{-1} \sim t_w^{1-\mu}$.

Although these results are derived from a simplified theoretical model, we think that such a description captures the essential intermittent character of the relaxation of a realistic glassy system, together with the existence of a coherence length scale growing with the age, due to the decrease of the rate of events in time. We hope that these findings will be helpful in analyzing more quantitatively some experimental results. In particular, the results of Sec. IV and Sec. V might be of interest for the time resolved correlation experiments of [19]. However, since the individual trap model is probably a very crude model for the relaxation of realistic materials [$\langle C(t_w, t_w+\tau)\rangle$ is *a priori* unlikely to coincide with the experimental average relaxation function of colloidal gels], it may be interesting in a further study to replace it with a more realistic dynamical model for jammed colloidal gels [40]. Concerning the intermittency dynamics studied in [20], the quantities calculated in Sec. III are maybe the most relevant ones to compare with the experimental results.

The model studied here has the major disadvantage that the relevant coherence length of the system is introduced by hand through the superposition of several subsystems. However, we were able to extract useful information from this picture. It would be more satisfactory to solve a model where an aging coherence length builds up during the dynamics, resulting from microscopic dynamics. Candidates could be special cases of kinetically constrained models, though analytical calculations will be quite hard [41]. However, numerical simulations provide now a useful alternative tool for the precise investigation, in microscopic models of glasses, of the quantities calculated in this paper [24].

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APPENDIX A

1. Case $k(1-\mu)<1$

In this case, we can perform a saddle-point calculation in order to evaluate $I(t_w,z)$, which leads to

$$I(t_w,z) \simeq A t_w^{\alpha} z^{\beta} e^{-B/(zt_w)^{\gamma}},$$

with $A=[(1-\mu)K(k)/2]^{-2k(1-\mu)/(1+\mu)}$, $B=[(1-\mu)K(k)/2]^{2/(1+\mu)}$, $\alpha=k(1-\mu)^2/(1+\mu)$, $\beta=2k(1-\mu)/(1+\mu)-1$, and $\gamma=(1-\mu)/(1+\mu)$.

Since we are interested in the case $T \ll t_w$, we expand $I(t_w,z)$ for $zt_w \gg 1$: $I(t_w,z) \simeq A t_w^{\alpha} z^{\beta}[1-B/(zt_w)^{\gamma}]$. Then we need to consider two separate cases for the evaluation of $\hat{P}^{(k,k)}(\tau,t_w,z)$ from relation (5).

2. Case $0<k<(1+\mu)/2(1-\mu)$

For $0<k<(1+\mu)/2(1-\mu)$ (i.e., $\beta<0$), one gets for $zt_w \gg 1$: $\hat{\mathcal{P}}^{(k,k)}(\tau,t_w,z) \simeq A(C_N^k/2\pi\tau)[c(\mu)\tau/\tau_0^{\mu}]^k t_w^{\alpha} z^{\beta}$. Therefore, $\mathcal{P}^{(k,k)}(\tau,t_w,T)$ is a power law:

$$\mathcal{P}^{(k,k)}(\tau,t_w,T) \simeq \frac{C_N^k}{2\pi\tau}\left[\frac{c(\mu)\tau}{\tau_0^{\mu}}\right]^k \frac{A t_w^{\alpha}}{\Gamma(-\beta)} \frac{1}{T^{1+\beta}}.$$

3. Case $(1+\mu)/2(1-\mu)<k<1/(1-\mu)$

In this case, we find that $\hat{\mathcal{P}}^{(k,k)}(\tau,t_w,z) \simeq \exp[-\mathcal{K}(t_w,\tau)/(zt_w)^{\beta}]$, with $\mathcal{K}(t_w,\tau)=(2\pi\tau/C_N^k)[\tau_0^{\mu}/c(\mu)\tau]^k (1/t_w^{\alpha-\beta})$.

The evaluation of the inverse Laplace transform leads to

$$\mathcal{P}^{(k,k)}(\tau,t_w,T) \simeq e^{\cos(\pi/(1+\beta))(\beta\mathcal{K}(t_w,\tau))^{1/(1+\beta)}(T/t_w)^{\beta/(1+\beta)}},$$

which is a stretched exponential since $\cos(\pi/(1+\beta))<0$.

4. Case $k(1-\mu)>1$

Finally, we consider the case $k(1-\mu)>1$: then we have to introduce the lower cutoff τ_0 in $I(t_w,z)$, which leads to

$$I(t_w,z) \simeq z^{(1-\mu)k-1} \int_{\tau_0 z}^{\infty} \frac{du}{u^{(1-\mu)k}} e^{-u} e^{K(k)(u/zt_w)^{(1-\mu)/2}}.$$

The saddle point solution is $u^*=[(1-\mu)K(k)/2]^{2/(1+\mu)} \times (zt_w)^{(\mu-1)(1+\mu)}$. In the limit of very large zt_w, one will at some point reach the situation where $u^*<\tau_0 z$. Hence, for $zt_w \gg 1$, the integral will be best evaluated by its lower cutoff value: $I(t_w,z) \simeq (1/z)(1/\tau_0^{(1-\mu)k}) e^{-\tau_0 z} e^{K(k)(\tau_0/t_w)^{(1-\mu)/2}}$. Since $T \gg \tau_0$, we finally get $\hat{\mathcal{P}}^{(k,k)}(\tau,t_w,z) \simeq \mathcal{C}(t_w,\tau)/z$, where $\mathcal{C}(t_w,\tau)=(C_N^k/2\pi\tau)[c(\mu)\tau/\tau_0^{\mu}]^k (e^{K(k)(\tau_0/t_w)^{(1-\mu)/2}}/\tau_0^{(1-\mu)k})$.

Finally, $\mathcal{P}^{(k,k)}(\tau,t_w,T)$ is simply an exponential, with a weak dependence on t_w (which vanishes for large t_w):

$$\mathcal{P}^{(k,k)}(\tau,t_w,T) \simeq \mathcal{C}(t_w,\tau) e^{-\mathcal{C}(t_w,\tau)T}.$$

APPENDIX B

For the computation of $\mathcal{P}_0^{(k',k'')}(\tau,t_w,T)$, we will proceed in the same way as in the last section for $\mathcal{P}_0^{(k,k)}(\tau,t_w,T)$. Using again combinatorial arguments, we can write that

$$\mathcal{P}_0^{(k',k'')}(\tau,t_w,T) = \frac{C_N^{k'}}{\tau} \sum_{n=n_{inf}}^{n_{sup}} C_{k'}^{n} C_{N-k'}^{k''-k'+n} [\tau S(T)]^{k'-n} \times [1-\tau S(T)]^{n} [\tau S(t_w+T)]^{k''-k'+n} \times [1-\tau S(t_w+T)]^{N-k''-n},$$

where $n_{inf}=\sup(0,k'-k'')$ and $n_{sup}=\inf(k',N-k'')$.

As before, we keep only the leading order in τ and go to the large N limit by introducing $x'=k'/N$, $x''=k''/N$, and $y=n/N$. This leads to

$$\mathcal{P}_0^{(k',k'')}(\tau,t_w,T) \simeq \frac{\tau^{k''-1}}{(2\pi)^{3/2}N^{1/2}}\int_{x_{inf}}^{x_{sup}} dy\, e^{-N\mathcal{S}(x',x'',y)},$$

where

$$\begin{aligned}\mathcal{S}(x',x'',y) = {} & y\ln y + (x''-x+y)\ln(x''-x'+y)\\ & + (x'-y)\ln(x'-y) + (1-x''-y)\ln(1-x''-y)\\ & - (x'-y)\ln S(T) - (x''-x'+y)\ln S(t_w+T).\end{aligned}$$

We use a saddle-point approximation to minimize $\mathcal{S}(x',x'',y)$: $(\partial\mathcal{S}/\partial y)|_{y^*}=0$ leads to

$$(x'-y^*)(1-x''-y^*) = y^*(x''-x'+y)\,\gamma(t_w,T),$$

which is the generalization of Eq. (7).

The positive solution is therefore

$$\begin{aligned}y^* = {} & \frac{1}{2(\gamma-1)}\big[-1-(\gamma-1)(x''-x')\\ & + \sqrt{[1+(\gamma-1)(x''-x')]^2+4x'(1-x'')(\gamma-1)}\big]\end{aligned}$$

and it is easy to check that $x_{inf}\leqslant y^*\leqslant x_{sup}$. Note that in this equation and in the following, we drop the (t_w,T) dependence of $\gamma(t_w,T)$ for convenience.

We now come back to the calculation of $\mathcal{P}_0^{(k)}(\tau,t_w,T)$ through Eq. (11). For large N, we have

$$\mathcal{P}_0^{(k)}(\tau,t_w,T) \simeq N^2\int_x^1 dx'\int_x^1 dx''\mathcal{P}_0^{(k',k'')}(\tau,t_w,T).$$

Direct integration is not possible here, so we use again a saddle point treatment of the double integral. The saddle points x'^* and x''^* are solutions of the set of equations formed by $(\partial\mathcal{S}/\partial x')|_{(x'^*,x''^*)}=0$ and $(\partial\mathcal{S}/\partial x'')|_{(x'^*,x''^*)}=0$, or

$$\frac{\partial y^*}{\partial x'}\ln\frac{y^*(x''-x'+y^*)\gamma}{(1-x''-y^*)(x'-y^*)} = \ln\frac{\gamma(x''-x'+y^*)}{(x'-y^*)}, \quad \text{(B1)}$$

$$\frac{\partial y^*}{\partial x''}\ln\frac{y^*(x''-x'+y^*)\gamma}{(1-x''-y^*)(x'-y^*)} = \ln\frac{\tau S(T+t_w)(1-x''-y^*)}{x''-x'+y^*}. \quad \text{(B2)}$$

Here, $(\partial y^*/\partial x')|_{(x'^*,x''^*)}$ and $(\partial y^*/\partial x'')|_{(x'^*,x''^*)}$ can be calculated using the expression for y^*.

At this point, taking the $T/t_w\ll 1$ limit enables us to make simplifications in Eqs. (B1) and (B2). A rather lengthy calculation leads to the conclusion that there is no couple of solutions (x'^*,x''^*) compatible with Eqs. (B1) and (B2), and such that $x'^*\neq x''^*$.

Consequently, we keep only the terms such that $k'=k''$ in the sum of Eq. (11), and try to find the value for k' that maximizes this sum, knowing the expression of $\mathcal{P}_0^{(k',k')}(\tau,t_w,T)$ from Sec. IV C:

$$\begin{aligned}\mathcal{P}_0^{(k)}(\tau,t_w,T) \propto {} & \int_x^1 dx'\,\exp\Bigg\{N\Bigg[-x'\ln x'-(1-x')\ln(1-x')\\ & + x'\ln S(T) + 2\left(\frac{x'(1-x')}{\gamma(t_w,T)}\right)^{1/2}\Bigg]\Bigg\}.\end{aligned}$$

The saddle-point equation for x'^* is now

$$\ln\left(\frac{x'^*}{1-x'^*}\right) = \frac{1-2x'^*}{\sqrt{x'^*(1-x'^*)}\gamma(t_w,T)} + \ln[\tau S(T)].$$

If $\tau S(T)=1$, this equation has the only solution $x'^*=\frac{1}{2}$. For $\tau S(T)>1$, there is a solution $x'^*>\frac{1}{2}$, and for $\tau S(T)<1$, there is a solution $x'^*<\frac{1}{2}$. In general, $x^*=\sup(x,x'^*)$ will maximize the expression for $\mathcal{P}_0^{(k)}(\tau,t_w,T)$, and can be estimated numerically. In particular, if $\tau S(T)<1$ and $k>N/2$, we have $x^*=x$.

Finally,

$$\mathcal{P}_0^{(k)}(\tau,t_w,T) = \mathcal{P}_0^{(k^*)}(\tau,t_w,T),$$

$$\mathcal{P}^{(k)}(\tau,t_w,T) = \mathcal{P}^{(k^*)}(\tau,t_w,T),$$

where $k^*=\sup(k,k'^*)$.

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