Longitudinal impedance and shielding effectiveness of a resistive beam pipe for arbitrary energy and frequency

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(Received 9 September 2004; published 23 February 2005)

The longitudinal coupling impedance of a cylindrical beam pipe for arbitrary relativistic γ_0 and mode frequency is obtained analytically for finite wall conductivity and finite wall thickness. Closed form expressions for the electromagnetic fields excited by a beam perturbation are derived analytically. General expressions for the resistive-wall impedance in the presence of a metallic shield and for the rf shielding effectiveness of the beam pipe have been obtained and then compared with approximate expressions. The results are applied to the GSI synchrotron SIS, where the thickness of the vacuum chamber in the dipole magnets is much smaller than the skin depth at injection energy.

DOI: 10.1103/PhysRevE.71.026501 PACS number(s): 29.27.Bd, 29.20.−c, 84.37.+q

I. INTRODUCTION

The concept of coupling impedance in accelerators was introduced for studying the instabilities in the ISR at CERN [1]. For the design of accelerators it is desired to reduce the coupling impedance of the beam to its environment in order to prevent beam instabilities. The longitudinal coupling impedance, which includes the space-charge and resistive-wall impedances, is an important physical quantity for understanding and modeling of the longitudinal dynamics of charged particle beams and the corresponding longitudinal beam instabilities. A beam of charged particles can excite electromagnetic fields in its environment and periodic excitations can occur depending on the coupling of the beam to its environment at a particular frequency. These excitations can perturb the beam dynamics and lead to beam instabilities [2–5].

In the case that the wall of the beam pipe is not perfectly conducting, part of the electromagnetic field excited by the beam penetrates into the pipe wall. The penetration depth is given by the skin depth δ_s. The current induced in the wall leads to heating due to the finite wall conductivity. Expressions for the corresponding resistive-wall impedance for arbitrary relativistic γ_0 and frequency can be found in the literature [6–8]. Recently, Zimmermann and Oide [9] obtained an expression for the resistive-wall impedance using the wake field approach. Lowest order corrections to longitudinal and transverse resistive-wall impedances have been derived, discussed and compared with those well known expressions obtained for ultrarelativistic beams.

When the skin depth is larger than the wall thickness the beam induced electromagnetic fields can penetrate through the wall and the impedance depends on the structures outside the pipe. In this situation, in addition to the impedance, detailed calculations of the shielding effectiveness of the pipe are necessary, also in order to estimate the currents that could be induced in hardware components behind the pipe. The shielding effectiveness (SE) of a conducting layer, $10\log_{10}(P_i/P_t)$, is measured in decibels (dB) and defined as the ratio of the transmitted power P_t to incident power P_i or in terms of the electric fields $\mathrm{SE}=20\log_{10}(E_i/E_t)$ (see, e.g., Refs. [10–14]). For commercial applications a shielding effectiveness greater than 40 dB is usually adequate for use in electronics housing (FCC Class B requirements). Requirements for military applications are significantly higher, in the range between 80 dB and 100 dB. For the shielding of beam generated rf fields in accelerators the requirements depend on the detailed accelerator environment and also on the beam current. During the bunch compression foreseen in the proposed heavy ion synchrotron SIS 100 at GSI Darmstadt [15], for example, peak currents approaching 100 A should be reached. Assuming 40 dB shielding effectiveness of the beam pipe still 1% of the induced fields or 1 A of the peak image (displacement) current could in principle "leak through the pipe."

In accelerator physics literature the shielding of beam generated rf fields by thin conducting layers was considered in a number of works (see, e.g. [16,17]). The well known ability of a thin layer of thickness d less than the skin depth δ_s to shield electromagnetic fields [18] produced by a particle beam was considered in Refs. [19,20], where approximate expressions for the impedance and for the shielding effectiveness of thin layers and also of wire cages in the limit of low frequencies or high γ_0 were found.

The shielding by a beam pipe which is thin as compared to the skin depth is of relevance for the SIS 18 heavy ion synchrotron at GSI as well as for the design of the new SIS 100 as part of the FAIR project [21]. The SIS 18 magnets can be ramped with 10 T/s, the superconducting SIS 100 magnets shall be ramped with 4 T/s. In order to reduce eddy current effects, the stainless steel beam pipe of SIS 18 is only 0.3 mm thick. The skin depth at injection (11.4 MeV/u) is 1 mm. For the stainless steel or titanium beam pipe in the new SIS

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100 a thickness of a few 0.1 mm will be required [22]. Because SIS 100 will be a "cold" machine, the heating of the thin pipe due to large image currents can be important.

The impedance and the shielding effectiveness of thin beam pipes is also of importance for other fast ramping synchrotron projects, like the rapid cycling 3 GeV synchrotron for J-PARC [23]. Because of the relevance of the issue for the design of high current ring machines it is important to have closed form expressions for the resistive wall impedance and for the shielding effectiveness covering the relevant range of frequencies, beam energies and wall thicknesses. In the present work these expression will be derived and compared with approximations.

The paper is organized as follows: In Sec II, we present the derivation of the electromagnetic fields associated with a particle beam moving in a beam pipe of finite wall conductivities. Accounting for finite surface currents within the wall by using the Leontovich boundary condition to find the fields outside the metallic wall, we present in Sec. III a review on the calculation of the total longitudinal coupling impedance for a beam pipe with a thick wall and with finite wall conductivity. The calculation of the longitudinal coupling impedance will be done by directly matching the fields at the inner wall surface. In Sec. IV the excited electromagnetic fields and the corresponding coupling impedance will be obtained and discussed in the presence of a metallic shield. Shielding happens via a thin metallic cylindrical layer of thickness d, where expressions like resistive-wall impedance, shielding effectiveness, and wall losses via attenuation of fields behind a good conducting shielding layer will be calculated. In Sec V we apply our results to the SIS heavy ion synchrotron. Finally, we present our conclusions in Sec. VI.

II. MODEL EQUATIONS: ELECTROMAGNETIC FIELDS IN A CYLINDRICAL PIPE

The general wave equations satisfied by the magnetic $\vec{B}$ and electric $\vec{E}$ fields in a conducting medium of conductivity S, permittivity ϵ_0 and permeability μ_0 are obtained from Faraday's and Ampere's laws [7,24–26], namely,

$$\nabla^2\vec{B}(\vec{r},t) - \mu_0\epsilon_0\frac{\partial^2\vec{B}(\vec{r},t)}{\partial t^2} - \mu_0 S\frac{\partial\vec{B}(\vec{r},t)}{\partial t} = -\mu_0\vec{\nabla}\times\vec{j}(\vec{r},t), \tag{1}$$

$$\nabla^2\vec{E}(\vec{r},t) - \mu_0\epsilon_0\frac{\partial^2\vec{E}(\vec{r},t)}{\partial t^2} - \mu_0 S\frac{\partial\vec{E}(\vec{r},t)}{\partial t} = \mu_0\frac{\partial\vec{j}(\vec{r},t)}{\partial t} + \frac{\vec{\nabla}\rho_c(\vec{r},t)}{\epsilon_0}, \tag{2}$$

where ρ_c and $\vec{j}$ are the external (free) charge and current densities, respectively, which obey the following continuity equation:

$$\frac{\partial\rho_c(\vec{r},t)}{\partial t} + \vec{\nabla}\cdot\vec{j}(\vec{r},t) = 0. \tag{3}$$

Assuming that a particle beam in a form of circular lamina of radius a and an axially symmetric transverse charge distribution $\sigma(r)$ is moving in a cylindrical pipe of radius b with a constant longitudinal velocity $\vec{v}=\beta c\hat{z}$ along the z axis, the beam charge and current densities are as follows:

$$\rho_c(\vec{r},t) = \sigma(r)\delta(z-vt), \tag{4}$$

$$Q = 2\pi\int_0^a \sigma(r) r dr, \tag{5}$$

$$\vec{j}(\vec{r},t) = \rho_c(\vec{r},t)\vec{v} = \sigma(r)\beta c\,\delta(z-vt)\hat{z}, \tag{6}$$

where Q is the total charge associated with the charge distribution $\rho_c(\vec{r},t)$. For a uniformly charged lamina, the surface charge density distribution in the transverse direction is $\sigma = Q/\pi a^2$. Accordingly, the Fourier time-transformed charge and current densities in Eqs. (4) and (6) are

$$\rho_c(r,z,\omega) = \frac{Q}{\pi a^2\beta c}e^{ik_z z}, \tag{7}$$

$$j_z(r,z,\omega) = \frac{Q}{\pi a^2}e^{ik_z z}, \tag{8}$$

where $\omega = k_z v$ has been used and k_z is the wave number in the direction of beam propagation.

Due to the symmetry of the particle beam (source) under consideration, only transverse-magnetic (TM) cylindrical waveguide modes couple to the propagating beam such that $B_z=0$. All other field components are obtained from $E_z(r,z,\omega)$ via Maxwell's equations, where $E_\theta(r,z,\omega)$ and $B_r(r,z,\omega)$ vanish identically because of the axial symmetry of the beam. We assume normal mode solution for the Fourier time transformed electric field such that $E_z(r,z,\omega) = E_z(r,\omega)e^{ik_z z}$. Upon Fourier transforming Eq. (2) in time and in transverse space coordinates, and making use of $\rho_c(r,z,\omega)$ and $j_z(r,z,\omega)$ in Eqs. (7) and (8), respectively, we get the following equation for the longitudinal electric field component within the beam for $0 \leqslant r \leqslant a$,

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{k_z^2}{\gamma_0^2}\right]E_z(r,\omega) = i\frac{Q}{\pi a^2}\frac{k_z}{\epsilon_0\gamma_0^2\beta c}, \tag{9}$$

where $\gamma_0^{-2} = 1-\beta^2$ was introduced. In the region of the beam pipe, $a \leqslant r \leqslant b$, which is free of charges, $\rho_c = 0$, Eq. (9) will be used to find the electric field in that region by setting its right-hand side to zero. However, within a conducting metallic region of conductivity S which includes no bulk free charges, the following equation will be used to determine the electric field within the metallic region, namely,

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{k_z^2}{\underline{\gamma}^2}\right]E_z(r,\omega) = 0, \tag{10}$$

$$\frac{1}{\underline{\gamma}^2} = \frac{1}{\gamma_0^2} - i\frac{\mu_0 S\omega}{k_z^2}, \tag{11}$$

and $\delta_s = \sqrt{2/\mu_0 S\omega}$ is the skin depth for the frequency ω. For the TM modes in the cylindrical pipe with azimuthal symmetry, the electromagnetic field components $B_\theta(r,z,\omega)$ and $E_r(r,z,\omega)$ are needed for matching the solutions at the different interfaces involved in the problem under consideration and are obtained from $E_z(r,z,\omega)$ via Maxwell equations as follows:

$$E_r(r,z,\omega) = -i\frac{\gamma^2}{k_z}\frac{\partial E_z(r,z,\omega)}{\partial r}, \tag{12}$$

$$B_\theta(r,z,\omega) = \left(\frac{\beta}{c} + i\frac{\mu_0\beta c S}{\omega}\right)E_r(r,z,\omega), \tag{13}$$

where γ in Eq. (12) stands for γ_0 in the nonconducting regions and for $\underline{\gamma}$ in regions of finite conductivity. In the following sections we will solve Eqs. (9) and (10) and then find the corresponding expressions for the total coupling impedance for different beam-pipe-wall structures. We also investigate the beam shielding effectiveness using a conducting cylindrical shield of thickness d. Results will be valid for particle beams of arbitrary β and of finite size, for arbitrary mode wavelengths, for a pipe wall of finite conductivity, and at any point r from the beam axis.

III. IMPEDANCE OF A THICK PIPE WALL

For an axially uniform transverse beam charge distribution in a beam pipe with a thick conducting wall, the beam-pipe-wall structure essentially involves three regions where Maxwell's equations should be solved with appropriate boundary conditions on the various interfaces. It is important to find the unique solution for the electromagnetic field components excited by the beam in a given beam-pipe structure to account for the correct boundary conditions on metallic surfaces and interfaces usually present in accelerators. For perfectly conducting media such that $S=\infty$, tangential and normal field components vanish identically on a given perfectly conducting surface. On the other hand, when the conductivity is large but finite, surface currents will flow on the conducting surface leading to energy dissipation via Joule heating produced in the wall. The net energy flux into the conducting wall is nonzero and can be characterized by a resistive-wall impedance [8].

The conductivity of most metals is very large but finite, and usually it is a function of temperature [27]. At very low temperatures, the conductivity may become infinite for direct currents if the metal becomes superconducting. However, it always remains finite for alternating currents. The characteristic penetration depth of electromagnetic fields into the metallic wall is the skin depth δ_s which is in most cases smaller than the beam-pipe radius b and the thickness of the beam-pipe wall. The fields close to the surface therefore behave like plane waves [27], and accordingly, the tangential electric ($\vec{E}_t$) and magnetic field ($\vec{H}_t$) components have to satisfy the Leontovich boundary condition [7,24–27], namely,

$$\vec{E}_t = Z_m \hat{n} \times \vec{H}_t, \tag{14}$$

$$Z_m = \sqrt{\frac{j\mu_0\omega}{S}} = (1+j)\sqrt{\frac{\mu_0\omega}{2S}} = \frac{1+j}{S\delta_s}, \tag{15}$$

where $\hat{n}$ is a unit normal to the surface pointing into the metallic wall and Z_m is the complex surface impedance of the metallic surface in electrical engineering conventions such that $j=-i$ [7]. Using physical conventions, the surface impedance Z_m will be such that $Z_m \sim 1-i$ [28], a result which is a consequence of the Fourier transform in time using the following physical conventions, namely, $f(\omega)=\int_{-\infty}^{\infty} f(t)e^{i\omega t}dt$. Since $\vec{E}_t$ and $\vec{H}_t$ are continuous, their values outside the metal near the surface must be related in the same way. It has been pointed out by Leontovich that Eq. (14) may be used as a boundary condition in determining the fields outside the metal without considering the fields inside [29].

Al-khateeb *et al.* investigated the problem of longitudinal space charge and resistive wall impedances in a smooth cylindrical beam pipe using the Leontovich boundary condition to relate the tangential field components at the metallic surface [8]. At any point from the beam axis, an expression for the total longitudinal coupling impedance (space-charge and resistive-wall) was obtained from the volume integral over the beam distribution. A second approach starting from the flux of the Poynting vector at the pipe wall was also considered for the calculation of the resistive wall impedance which was found to match the expression obtained from the volume integral over the beam distribution for $k_z\delta_s \ll 1$. Using the wake potential approach, Zimmermann and Oide recently obtained approximate expressions for the resistive-wall impedance that are valid in the limit $k_z\delta_s \ll 1$ [9], and were found to match the results of Ref. [8] in this limiting case.

For $k_z\delta_s \ll 2/\beta^2\gamma_0$, which is mostly satisfied in metallic regions, the resistive-wall impedance $Z_\parallel^{(\mathrm{rw})}(\omega)$ is given by the following expression [8]:

$$Z_\parallel^{(\mathrm{rw})}(\omega) \approx (1-i)\frac{nZ_0\beta\delta_s^*}{2\sqrt{nb}}\frac{4I_1^2(\sigma_0 a)}{\sigma^2 a^2 I_0^2(\sigma_0 b)} \equiv (1-i)\frac{nZ_0\beta\delta_s^*}{2\sqrt{nb}}g^{(\mathrm{rw})}. \tag{16}$$

Here $g^{(\mathrm{rw})}$ is an effective resistive wall geometry factor, $\sigma_0 = k_z/\gamma_0$ and $\delta_s^* = \sqrt{2/\mu_0 S\omega_0}$ is the skin depth at the revolution frequency $\omega_0 = \beta c/R$. For all situations of practical interest in ring accelerators we can approximate $I_1(\sigma_0 a) \approx \sigma_0 a/2$, $I_0(\sigma_0 b) \approx 1$ and therefore $g^{(\mathrm{rw})} \approx 1$. Accordingly, Eq. (16) reduces into the well known expression for the resistive thick wall impedance [7,30].

In the following analysis, we directly solve Maxwell's equations in each region of the beam-pipe-wall structure for TM cylindrical waveguide modes, and then match the fields at the beam-vacuum and vacuum-wall interfaces. The Leontovich boundary condition (or the impedance boundary condition) accounts for the finite surface current at the metallic interface. The longitudinal resistive-wall impedance in Eq. (16) was derived by making use of the Leontovich boundary condition for calculating the electromagnetic field components outside the metallic wall. Without making use of the Leontovich boundary condition, a self-consistent expression for the coupling impedance will be derived.

The z component of the electric field in each region will satisfy the following differential equation:

$$\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{k_z^2}{\gamma_0^2}\right]E_z^{(1)}(r,\omega)=i\frac{Q}{\pi a^2}\frac{k_z}{\epsilon_0\gamma_0^2\beta c},\quad r\leqslant a, \tag{17}$$

$$\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{k_z^2}{\gamma_0^2}\right]E_z^{(2)}(r,\omega)=0,\quad a\leqslant r\leqslant b, \tag{18}$$

$$\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{k_z^2}{\underline{\gamma}^2}\right]E_z^{(3)}(r,\omega)=0,\quad b\leqslant r<\infty. \tag{19}$$

The general solution for the electric field in the z direction is

$$E_z(r,\omega)=\begin{cases}A_1I_0(\sigma_0 r)-i\dfrac{Q}{\pi a^2\epsilon_0k_z\beta c}, & 0\leqslant r\leqslant a,\\ A_2I_0(\sigma_0 r)+A_3K_0(\sigma_0 r), & a\leqslant r\leqslant b,\\ A_4K_0(\underline{\sigma}r), & b\leqslant r,\end{cases} \tag{20}$$

where $\underline{\sigma}=k_z/\underline{\gamma}$ and I_0 and K_0 are modified Bessel functions of first and second kinds, respectively. Upon matching the tangential field components E_z and H_θ at the beam surface $r=a$ and at the pipe wall at $r=b$, we obtain the following coefficients:

$$A_1=i\frac{Q\sigma_0 a}{\pi a^2\epsilon_0k_z\beta c}\left[K_1(\sigma_0 a)+F^{-1}I_1(\sigma_0 a)\right], \tag{21}$$

$$A_2=i\frac{Q\sigma_0 a}{\pi a^2\epsilon_0k_z\beta c}\frac{I_1(\sigma_0 a)}{F},\quad A_3=-i\frac{Q\sigma_0 a}{\pi a^2\epsilon_0k_z\beta c}I_1(\sigma_0 a), \tag{22}$$

$$A_4=-i\frac{Q\sigma_0 a}{\pi a^2\epsilon_0k_z\beta c}\frac{\eta}{F}\frac{I_1(\sigma_0 a)}{K_1(\underline{\sigma}b)}[I_1(\sigma_0 b)+FK_1(\sigma_0 b)], \tag{23}$$

$$F=\frac{I_0(\sigma_0 b)+\eta\dfrac{K_0(\underline{\sigma}b)}{K_1(\underline{\sigma}b)}I_1(\sigma_0 b)}{K_0(\sigma_0 b)-\eta\dfrac{K_0(\underline{\sigma}b)}{K_1(\underline{\sigma}b)}K_1(\sigma_0 b)},\quad \eta=\frac{\omega\epsilon_0\gamma_0}{i\underline{\gamma}(S-i\omega\epsilon_0)}. \tag{24}$$

We now calculate the corresponding total longitudinal coupling impedance as a volume integral over the transverse distribution of the beam [7],

$$Z_\parallel(r,\omega)=\frac{1}{Q^2}\int_{V_{\text{beam}}}d^3x'E_z(\vec{r}',\omega)j_z^*(\vec{r}',\omega). \tag{25}$$

Substituting for $j(r,z,\omega)$ from Eq. (8) and making use of the harmonic number n defined such as $n=k_zR$, the total longitudinal impedance at any point r from the beam axis becomes

$$Z_\parallel(\omega)=-i\frac{nZ_0}{2\beta\gamma^2}g^{(\text{total})}(r,a,b,d,k_z,g,\beta)$$
$$\equiv -in\chi_0 g^{(\text{total})}(r,a,b,d,k_z,g,\beta), \tag{26}$$

$$g^{(\text{total})}(r,a,b,d,k_z,g,\beta)=\frac{4\gamma_0^2}{k_z^2a^2}\left[\frac{r^2}{a^2}-2\frac{r}{a}I_1(\sigma_0 a)[K_1(\sigma_0 a)+F^{-1}I_1(\sigma_0 a)]\right], \tag{27}$$

where the total geometric factor $g^{(\text{total})}$ has been introduced. At the point $r=a$ on the beam axis the geometric factor becomes

$$\begin{aligned}g^{(\text{total})}&=\frac{4\gamma_0^2}{k_z^2a^2}\{1-2I_1(\sigma_0 a)[K_1(\sigma_0 a)+F^{-1}I_1(\sigma_0 a)]\}\\&=\frac{4\gamma_0^2}{k_z^2a^2}\left[1-2I_1(\sigma_0 a)\left(K_1(\sigma_0 a)+\frac{K_0(\sigma_0 b)}{I_0(\sigma_0 b)}I_1(\sigma_0 a)\right)\right]\\&\quad+\frac{8\gamma_0^2I_1^2(\sigma_0 a)}{k_z^2a^2}\left[\frac{K_0(\sigma_0 b)}{I_0(\sigma_0 b)}-F^{-1}\right]\\&\equiv g^{(\text{sc})}(S=\infty)+g^{(\text{rw})}.\end{aligned} \tag{28}$$

The geometry factor $g^{(\text{sc})}(S=\infty)$ characterizes the longitudinal space-charge impedance in a perfectly conducting beam-pipe wall, and the factor $g^{(\text{rw})}$ results from the assumption of a finite wall conductivity. It characterizes the longitudinal resistive-wall impedance. For a perfectly conducting wall, we have $\eta=0$ and then $g^{(\text{rw})}=0$. Substituting for F from Eq. (24), the resistive-wall geometry factor $g^{(\text{rw})}$ takes the following form:

$$\begin{aligned}g^{(\text{rw})}&=\frac{8\gamma_0^2I_1^2(\sigma_0 a)}{k_z^2a^2}\left[\frac{K_0(\sigma_0 b)}{I_0(\sigma_0 b)}-F^{-1}\right]\\&=\frac{8}{\sigma_0^2a^2}\frac{I_1^2(\sigma_0 a)}{\sigma_0 bI_0(\sigma_0 b)}\frac{K_0(\underline{\sigma}b)}{K_1(\underline{\sigma}b)}\frac{\eta}{I_0(\sigma_0 b)+\eta\dfrac{K_0(\underline{\sigma}b)}{K_1(\underline{\sigma}b)}I_1(\sigma_0 b)}.\end{aligned} \tag{29}$$

Equation (29) characterizes the resistive-wall impedance for any metallic conductor of finite conductivity S. Contributions from arbitrary conduction and displacement current densities to the resistive-wall impedance are included in Eq. (29). However, in the limit of very large conductivity S such that $k_z\delta_s\ll 1$, the argument of the modified Bessel functions $K_0(\underline{\sigma}b)$ and $K_1(\underline{\sigma}b)$ becomes very large. For large arguments, K_0 and K_1 behave to first order equally so that the ratio of $K_0(\underline{\sigma}b)$ and $K_1(\underline{\sigma}b)$ in Eq. (29) can be put to unity. Further, in the limit $k_z\delta_s\ll 1$ corresponding to good conducting wall, and by making use of Eq. (11) for $\underline{\gamma}$, the parameter η takes the following limiting value, namely,

$$\eta = \frac{\omega\epsilon_0\gamma_0}{i\underline{\gamma}(S - i\omega\epsilon_0)} \approx \frac{1+i}{2}\frac{\beta^2 k_z^2 \delta_s}{\sigma_0} = (1+i)\frac{\beta^2\gamma_0 k_z \delta_s}{2}. \tag{30}$$

Accordingly, the resistive-wall geometric factor $g^{(\mathrm{rw})}$ and the corresponding impedance become

$$g^{(\mathrm{rw})} \approx \frac{8}{\sigma_0^2 a^2}\frac{I_1^2(\sigma_0 a)}{\sigma_0 b I_0(\sigma_0 b)}\frac{1+i}{I_0(\sigma_0 b)}\frac{\beta^2 k_z^2 \delta_s}{2\sigma_0}, \tag{31}$$

$$Z^{(\mathrm{rw})}(\omega) = -i\frac{nZ_0}{2\beta\gamma^2}g^{(\mathrm{rw})} = (1-i)\frac{nZ_0\beta\delta_s^*}{2\sqrt{nb}}\frac{4I_1^2(\sigma_0 a)}{\sigma_0^2 a^2 I_0^2(\sigma_0 b)}, \tag{32}$$

where the skin depth δ_s^* at the revolution frequency has been used. Within the assumptions we introduced above, Eq. (32) recover the expression for the resistive-wall impedance we already obtained using the Leontovich boundary condition.

IV. LONGITUDINAL COUPLING IMPEDANCE FOR A THIN WALL

In this section we calculate the coupling impedance and the shielding effectiveness of a thin cylindrical pipe of thickness d. We consider the case of a finite size beam of radius a inside a thin metallic cylindrical pipe of thickness d extending from $r=b$ to $r=h=b+d$. Outside the pipe for $r>h$ is vacuum.

For TM modes in azimuthally symmetric beam-pipe structures, the z component of the electric field in the four regions involved will satisfy the following differential equations:

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{k_z^2}{\gamma_0^2}\right]E_z^{(1)}(r,\omega) = i\frac{Q}{\pi a^2}\frac{k_z}{\epsilon_0\gamma_0^2\beta c}, \quad r \leqslant a, \tag{33}$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{k_z^2}{\gamma_0^2}\right]E_z^{(2)}(r,\omega) = 0, \quad a \leqslant r \leqslant b, \tag{34}$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{k_z^2}{\underline{\gamma}^2}\right]E_z^{(3)}(r,\omega) = 0, \quad b \leqslant r \leqslant h, \tag{35}$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{k_z^2}{\gamma_0^2}\right]E_z^{(4)}(r,\omega) = 0, \quad h \leqslant r < \infty. \tag{36}$$

The general solution for the z component of the electric field is

$$E_z(r,\omega) = \begin{cases} A_1 I_0(\sigma_0 r) - i\dfrac{Q}{\pi a^2\epsilon_0 k_z\beta c}, & 0 \leqslant r \leqslant a, \\ A_2 I_0(\sigma_0 r) + A_3 K_0(\sigma_0 r), & a \leqslant r \leqslant b, \\ A_4 I_0(\underline{\sigma} r) + A_5 K_0(\underline{\sigma} r), & b \leqslant r \leqslant h, \\ A_6 K_0(\sigma_0 r), & h \leqslant r < \infty. \end{cases} \tag{37}$$

Requiring the continuity of the tangential field components E_z and H_θ at $r=a$, $r=b$, and $r=h$, we obtain the following coefficients:

$$F = \frac{I_1(\underline{\sigma}h)K_0(\sigma_0 h) + \eta K_1(\sigma_0 h)I_0(\underline{\sigma}h)}{K_1(\underline{\sigma}h)K_0(\sigma_0 h) - \eta K_1(\sigma_0 h)K_0(\underline{\sigma}h)}, \tag{38}$$

$$G = \frac{1}{\eta}\frac{I_1(\underline{\sigma}b) - FK_1(\underline{\sigma}b)}{I_0(\underline{\sigma}b) + FK_0(\underline{\sigma}b)}, \quad H = \frac{I_1(\sigma_0 b) - G\, I_0(\sigma_0 b)}{K_1(\sigma_0 b) + GK_0(\sigma_0 b)}, \tag{39}$$

$$A_1 = -i\frac{Q}{\pi a^2}\frac{\sigma_0 a}{\epsilon_0 k_z\beta c H}\left[I_1(\sigma_0 a) - HK_1(\sigma_0 a)\right], \tag{40}$$

$$A_2 = \frac{I_1(\sigma_0 a)}{I_1(\sigma_0 a) - HK_1(\sigma_0 a)}A_1, \quad A_3 = HA_2, \tag{41}$$

$$A_4 = \frac{I_0(\sigma_0 b) + HK_0(\sigma_0 b}{I_0(\underline{\sigma}b) + HK_0(\underline{\sigma}b)}A_2, \quad A_5 \equiv FA_4, \tag{42}$$

$$A_6 = \frac{FK_1(\underline{\sigma}h) - I_1(\underline{\sigma}h)}{\eta K_1(\sigma_0 h)}A_4. \tag{43}$$

We now calculate the corresponding longitudinal impedance as a volume integral over the transverse distribution of the beam as follows:

$$\begin{aligned} Z(r,\omega) &= \frac{1}{Q^2}\int_{V_{\mathrm{beam}}} d^3x' \vec{E}(r',z,\omega)\cdot\vec{j}^*(r',z,\omega) \\ &= -i\frac{nZ_0}{2\beta\gamma^2}g^{(\mathrm{total})}(r,a,b,d,k_z,g,\beta) \\ &\equiv -in\chi_0 g^{(\mathrm{total})}(r,a,b,d,k_z,g,\beta), \end{aligned} \tag{44}$$

where we introduced a generalized geometry factor $g(r,a,b,d,k_z,g,\beta)$ defined as follows:

$$\begin{aligned} &g^{(\mathrm{total})}(r,a,b,d,k_z,g,\beta) \\ &\quad = \frac{4\gamma_0^2}{k_z^2 a^2}\left[\frac{r^2}{a^2} - 2\frac{r}{a}I_1(\sigma_0 a)[K_1(\sigma_0 a) - H^{-1}I_1(\sigma_0 a)]\right]. \end{aligned} \tag{45}$$

At the beam surface $r=a$, the geometry factor $g^{(\mathrm{total})}$ takes on the following form:

$$g^{(\text{total})} = \frac{4\gamma_0^2}{k_z^2 a^2}\left[1 - 2I_1(\sigma_0 a)\left(K_1(\sigma_0 a) + \frac{K_0(\sigma_0 b)}{I_0(\sigma_0 b)}I_1(\sigma_0 a)\right)\right] + \frac{8\gamma_0^2}{k_z^2 a^2}I_1^2(\sigma_0 a)\left[\frac{K_0(\sigma_0 b)}{I_0(\sigma_0 b)} + H^{-1}\right] \equiv g_1 + g_2, \tag{46}$$

where g_1 is the geometric factor associated with the space charge only for the case of a perfectly conducting thick wall at $r=b$. The geometric factor g_2 in Eq. (46) accounts for the finite width and finite electric conductivity of the cylindrical shield. Substituting for H, the geometry factor g_2 can be written in terms of F as follows:

$$g_2 = \frac{8\gamma_0^2}{k_z^2 a^2}\frac{I_1^2(\sigma_0 a)}{\sigma_0 b I_0(\sigma_0 b)}\frac{1}{I_1(\sigma_0 b) - \dfrac{I_0(\sigma_0 b)}{\eta}\dfrac{I_1(\underline{\sigma} b) - F K_1(\underline{\sigma} b)}{I_0(\underline{\sigma} b) + F K_0(\underline{\sigma} b)}}. \tag{47}$$

In the following subsections we derive analytic expressions for the resistive-wall impedance and the transmission coefficient (shielding effectiveness) in the limit of well conducting shielding layer.

A. Resistive-wall impedance for a thin wall

For the case of a very well conducting cylindrical shield of finite thickness and for $\delta_s k_z \ll 1$, we have the following:

$$F \approx \frac{e^{2\underline{\sigma} h}}{\pi}\frac{K_0(\sigma_0 h) + \eta K_1(\sigma_0 h)}{K_0(\sigma_0 h) - \eta K_1(\sigma_0 h)}, \quad G \approx -\frac{1}{\eta}\frac{K_0(\sigma_0 h)\tanh \underline{\sigma} d + \eta K_1(\sigma_0 h)}{K_0(\sigma_0 h) + \eta K_1(\sigma_0 h)\tanh \underline{\sigma} d}, \tag{48}$$

$$g_2 = \frac{1}{\sigma_0 b}\frac{4I_1^2(\sigma_0 a)}{\sigma_0^2 a^2 I_0^2(\sigma_0 b)}\frac{(1+i)\beta^2\gamma_0 k_z \delta_s\left[1 + \eta\dfrac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)}\tanh \underline{\sigma} d\right]}{\tanh \underline{\sigma} d + \eta\left(\dfrac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)} + \dfrac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}\right) + \eta^2\dfrac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)}\dfrac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}\tanh \underline{\sigma} d}. \tag{49}$$

The presence of a well conducting finite thickness cylindrical pipe results in the following contribution to the longitudinal impedance, namely,

$$Z_2(\omega) = -i\frac{nZ_0}{2\beta\gamma^2}g_2 = (1-i)\frac{nZ_0\beta\delta_s^*}{2\sqrt{n}b}\frac{4I_1^2(\sigma_0 a)}{\sigma_0^2 a^2 I_0^2(\sigma_0 b)}\frac{1 + \eta\dfrac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)}\tanh \underline{\sigma} d}{\tanh \underline{\sigma} d + \eta\left(\dfrac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)} + \dfrac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}\right) + \eta^2\dfrac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)}\dfrac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}\tanh \underline{\sigma} d}. \tag{50}$$

For a cylindrical pipe of finite thickness ($d \neq 0$), the impedance $Z_2(\omega)$ in Eq. (50) represents the resistive-wall impedance, which is the total impedance minus the impedance Z_1 of a perfectly conducting beam pipe. We consider below two limiting cases of Eq. (50).

Assuming a thick very well conducting pipe such that $d \to \infty$ and $k_z\delta_s \ll 1$, Eq. (50) becomes

$$Z_2^{(\text{wall})}(\omega) \approx (1-i)\frac{nZ_0\beta\delta_s^*}{2\sqrt{n}b}\frac{4I_1^2(\sigma_0 a)}{\sigma_0^2 a^2 I_0^2(\sigma_0 b)}, \quad k_z\delta_s \ll 1. \tag{51}$$

For $\eta \ll 1$ or $k_z\delta_s \ll 1$ and for arbitrary pipe thickness d, the impedance in Eq. (50) takes on the following approximate form:

$$Z^{(\text{wall})}(\omega) \approx (1-i)\frac{nZ_0\beta\delta_s^*}{2\sqrt{n}b}\frac{4I_1^2(\sigma_0 a)}{\sigma_0^2 a^2 I_0^2(\sigma_0 b)}\coth \underline{\sigma} d. \tag{52}$$

From Eq. (52) we see that the effect of a very well conducting pipe is to modify the well known resistive-wall impedance by the factor coth $\underline{\sigma} d$. Such a modification by cosh $\underline{\sigma} d$ of the resistive impedance or resistive geometric factor has been reported at low frequencies in previous works [31,32], and is well known in the transmission line theory in the case of a thin conducting sheet with surface resistance η [18]. Geometry factors and the total impedance in the limiting case of a very thin cylindrical shield take the following forms, namely,

$$g_2 = \frac{8I_1^2(\sigma_0 a)K_0(\sigma_0 b)}{\sigma_0^2 a^2 I_0(\sigma_0 b)},$$

$$g^{(\text{total})} = g_1 + g_2 = \frac{4\gamma_0^2}{k_z^2 a^2}\left[1 - 2I_1(\sigma_0 a)K_1(\sigma_0 a)\right], \tag{53}$$

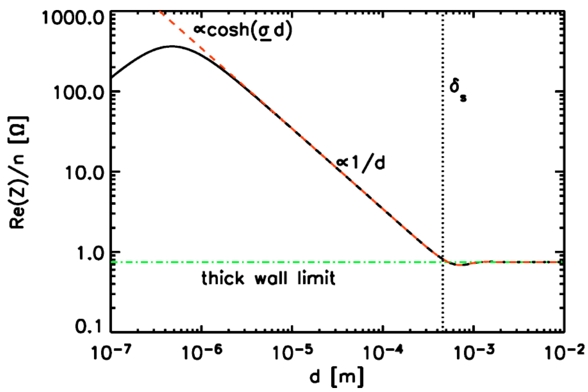

FIG. 1. Real part (solid line) of the resistive wall impedance for $n=1$ and $\gamma_0=2$ as a function of the wall thickness d. The parameters are (SIS 18) $C=216$ m, $a=1$ cm, and $b=10$ cm.

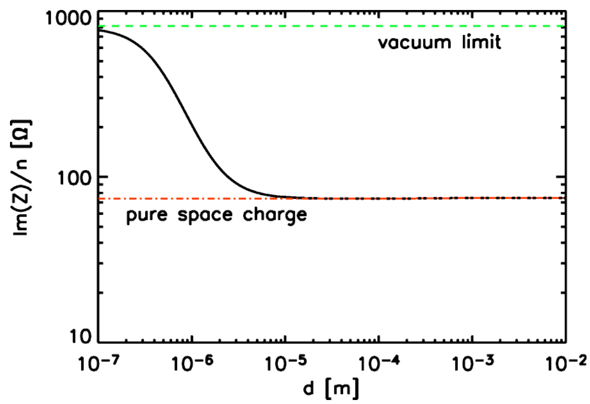

FIG. 2. Absolute value of the imaginary part (solid line) of the total (space charge and resistive wall) impedance for $n=1$ and $\gamma_0=2$ as a function of the wall thickness d. The parameters are (SIS 18) $C=216$ m, $a=1$ cm, and $b=10$ cm.

$$Z_{\parallel}^{(\mathrm{tot})}(\omega) = -i\frac{nZ_0}{2\beta\gamma_0^2}g^{(\mathrm{total})}$$

$$= -i\frac{nZ_0}{2\beta\gamma_0^2}\frac{4\gamma_0^2}{k_z^2a^2}\left[1-2I_1(\sigma_0 a)K_1(\sigma_0 a)\right]. \quad (54)$$

Equation (54) shows that the total longitudinal coupling impedance without cylindrical pipe ($d\to 0$) is purely imaginary. We see from Eq. (54) that the impedance becomes also independent of b. According to the beam-pipe structure treated here and for vanishing thickness of the pipe $d=0$, the interface at $r=b$ becomes a virtual one so that it can be moved to infinity without affecting the calculation results. The result in Eq. (54) is consistent with that obtained in the case of a smooth perfectly conducting beam pipe with b being moved to infinity ("vacuum space charge impedance").

In the following we apply our results to the SIS. In Fig. 1 we plot the real part of the resistive wall impedance [Eq. (50)] as a function of d for SIS parameters. As expected, the thick wall limit, Eq. (16), can be used for $d>\delta_s$. For $\gamma_0=2$ and $n=1$ the skin depth is slightly larger than the wall thickness (0.3 mm) in the dipoles. For $d<\delta_s$ the real part of the impedance is proportional to $1/d$. In both regimes Eq. (52) can be used to very good approximation. For very small d below 1 μm the exact impedance decreases towards zero. For SIS parameters the resistive wall impedance divided by the harmonic number remains well below 10 Ω. The total (space charge and resistive wall) imaginary part of the wall impedance is plotted in Fig. 2. For $d\gtrsim 1$ μm the imaginary part is dominated by the space charge impedance. For smaller d the imaginary part of the total wall impedance tends towards the vacuum result Eq. (53).

B. Transmission coefficient and shielding effectiveness

We will now find an expression for the transmission coefficient by assuming a good conducting pipe such that $\delta_s \ll 1$. We define the transmission coefficient τ as the ratio of A_6 to A_3 [19]. For good shielding, the coefficient τ should be as small as possible. The shielding effectiveness is defined as $\mathrm{SE}=10\log_{10}(|\tau|^{-2})$, where τ is the ratio of the electric (or magnetic) field transmitted through the shield to the electric (or magnetic) field incident on the shield. Every 20 dB increase in the SE represents a tenfold reduction in the electromagnetic field strength by passing through the shield. For beam pipes or other rf shields in accelerators we assume here a SE of 40 dB ($|\tau|=0.01$) as a sufficiently good value. The final classification of good shielding will depend on the detailed accelerator environment and on the beam intensity.

For a beam pipe the transmission coefficient is obtained from the electric fields [Eq. (38)] as

$$\tau = \frac{FK_1(\underline{\sigma}h)-I_1(\underline{\sigma}h)}{\eta HK_1(\sigma_0 h)}\frac{I_0(\sigma_0 b)+HK_0(\sigma_0 b)}{I_0(\underline{\sigma}b)+FK_0(\underline{\sigma}b)}$$

$$= \frac{\sqrt{\frac{b}{h}}}{\sigma_0 bK_1(\sigma_0 h)I_0(\sigma_0 b)}\frac{1}{\cosh\underline{\sigma}d+\frac{K_0(\sigma_0 h)}{\eta K_1(\sigma_0 h)}\sinh\underline{\sigma}d+\frac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}\left(\eta\sinh\underline{\sigma}d+\frac{K_0(\sigma_0 h)}{K_1(\sigma_0 h)}\cosh\underline{\sigma}d\right)}. \quad (55)$$

Equation (55) is valid for any pipe thickness d and for arbitrary β. In the special case of a conducting pipe and a skin depth that is much larger than the wall thickness of the pipe ($d\ll\delta_s$), Eq. (55) takes on the following form:

$$\tau = \sqrt{\frac{b}{h}}\frac{1}{\sigma_0 b K_1(\sigma_0 h) I_0(\sigma_0 b)}\frac{1}{1 - i\frac{2d}{\beta^2\gamma_0^2 k_z \delta_s^2}\frac{K_0(\sigma_0 h)}{K_1(\sigma_0 h)} + \frac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}\left(\beta^2\gamma_0 k_z d + \frac{K_0(\sigma_0 h)}{K_1(\sigma_0 h)}\right)}. \tag{56}$$

Further, in the limiting case $\sigma_0 b = k_z b/\gamma_0 \ll 1$ and $\sigma_0 h = k_z h/\gamma_0 \ll 1$ corresponding to not too high frequencies or to ultrarelativistic beam energies and for $b \approx h$, Eq. (56) becomes

$$\tau \approx \frac{1}{1 + \frac{\beta^2 k_z^2 b}{2} d + i\frac{2bd}{\beta^2\gamma_0^2\delta_s^2}\ln\frac{k_z b}{\gamma_0}}. \tag{57}$$

Note that in the equivalent formula of Ref. [19] the first sign in the denominator is a minus sign. The main contribution for $|\tau|$, however, comes from the term $(2d/\beta^2\gamma_0^2 k_z \delta_s^2) \times [K_0(\sigma_0 h)/K_1(\sigma_0 h)]$ in Eq. (56) which is much larger than unity. Accordingly, for $b \gg d$, we have the following shielding condition:

$$\frac{d}{\delta_s} \gg \frac{\beta^2\gamma_0 k_z \delta_s}{2b}\left|\frac{K_1(\sigma_0 b)}{K_0(\sigma_0 b}\right|. \tag{58}$$

Equation (58) reduces into the condition $d/\delta_s \gg (\frac{1}{2})\beta^2\gamma_0^2\delta_s/|\ln \sigma_0 b|$ for low frequencies or high energies $\sigma_0 b \ll 1$, a result which was derived previously by Gluckstern and Zotter [19].

V. SHIELDING EFFECTIVENESS IN SIS 18/100

The shielding of the beam generated rf fields by the beam pipe is an important issue for the SIS high current operation as well as for the design of the vacuum chambers for the proposed SIS 100/300 synchrotrons [15]. In the SIS the thickness of the stainless steel vacuum chamber in the dipoles is $d = 0.3$ mm. During the planned generation of intense, short (50 ns) U^{73} bunches peak currents exceeding 10 A will be reached in the SIS at a maximum energy of 1 GeV/u ($\gamma_0 \approx 2.0$). A corresponding peak image current will flow through the beam pipe. For insufficient shielding part of this image current can flow through structures outside the beam pipe, resulting in an undefined longitudinal impedance and possibly in perturbations of the accelerator hardware.

The skin depth δ_s^* at SIS injection energy (11.4 MeV/u) is 1 mm, at 1 GeV/u the skin depth of 0.4 mm will still be larger than the wall thickness in the dipoles. In the proposed SIS 100 synchrotron δ_s^* at injection ($\gamma_0 = 1.1$) will be as large as 1.5 mm, with a wall thickness in the dipoles of a few 0.1 mm. Therefore a detailed analysis of the shielding efficiency of the beam pipe in the existing SIS as well as in the new SIS 100 is of great importance.

In Fig. 3 we plot the absolute value of the transmission coefficient as a function of γ_0 for $d = 0.3$ mm, $S = 10^6\ (\Omega\text{m})^{-1}$ (stainless steel) and for two different harmonic numbers. The other SIS machine parameters of relevance are $2\pi R = 216$ m and $b \approx 0.1$ m. For $n = 1$ or wave lengths corresponding to the SIS circumference the transmission coefficient obtained from Eq. (55) reaches 1% (good shielding of 40 dB as defined in Sec. IV B) at $\gamma_0 \approx 6$. For $\gamma_0 = 2$ we obtain a sufficiently low value of ≈ 0.001 only. If we consider a shorter wave length corresponding to the final length of compressed bunches ($n \approx 20$) we obtain an even lower value well below 10^{-4}, but here one has to take into account that the compressed peak bunch current is more than 20 times higher.

The approximation Eq. (57) used in Ref. [19] overestimates the transmission coefficient for $n \gtrsim 10$. For $n = 1$ the approximation Eq. (57) agrees exactly with Eq. (55). Figure 4 shows the transmission coefficient for $n = 1$, $\gamma_0 = 2$ and SIS parameters as a function of the wall thickness d. Values ex-

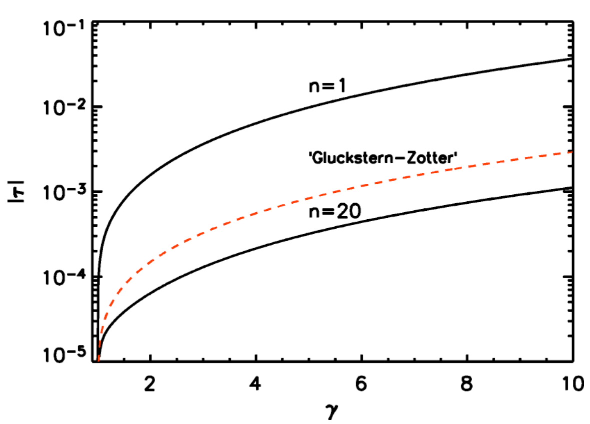

FIG. 3. Transmission coefficient vs relativistic γ_0 for the harmonic numbers $n = 1$ and $n = 20$. The other parameters are (SIS 18) $C = 216$ m, $b = 10$ cm, and $d = 0.3$ mm. The dashed line represents the transmission coefficient obtained from Eq. (57).

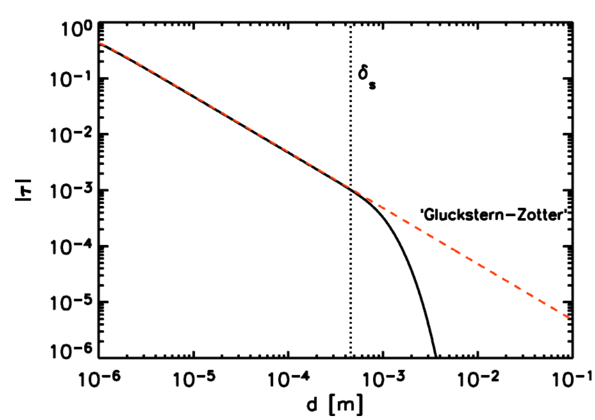

FIG. 4. Transmission coefficient vs d for $\gamma_0 = 2$ and $n = 1$. The other parameters are (SIS 18) $C = 216$ m and $b = 10$ cm. The dashed line represents the transmission coefficient obtained from Eq. (57).

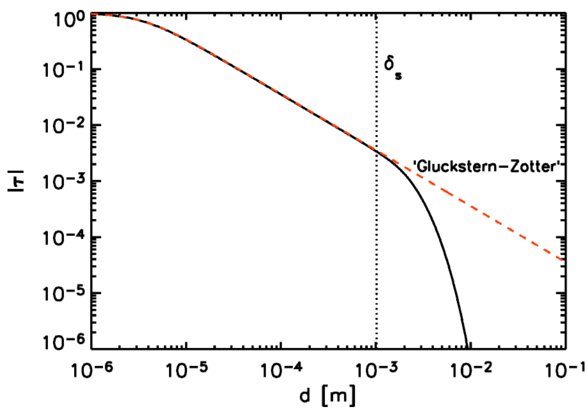

FIG. 5. Transmission coefficient vs d for $\gamma_0=2$ and $n=1$. The other parameters are (SIS 100) $C=1080$ m and $b=5$ cm. The dashed line represents the transmission coefficient obtained from Eq. (57).

ceeding 1% are obtained for d below 0.1 mm. The approximate expression Eq. (57) can be used for $d \lesssim \delta_s$. The transmission coefficient as a function of d for $\gamma_0=2$, for stainless steel and for the SIS 100 parameters is shown in Fig. 5. We see that for stainless steel a wall thickness of a few 0.1 mm might be sufficient in order to provide good shielding.

It is important to note that for the very small d of the order of a few μm the situation corresponds, e.g., to a ceramic beam pipe that is coated with a thin conducting film. The effect of the ceramic pipe on the total impedance and on the shielding effectiveness is very small. The impedance of rf-shielding wires inside a ceramic pipe was studied by Wang and Kurennoy [33]. They observed a weak dependence on the dielectric constant of the ceramic pipe.

A ceramic beam pipe coated with a thin (few μm) conducting film of, e.g., Cu $[S \approx 5\times 10^7\ (\Omega\mathrm{m})^{-1}]$ could provide transmission coefficients of the order of 10^{-3} for SIS 18. For SIS 100 parameters we obtain a "good" shielding coefficient of the order of 1% for standard operation.

We would like to point out that the radius of curvature has been assumed to be very large compared to the transverse dimensions of the vacuum chamber. In this case the curvature effects are negligible [34,35] and the beam-pipe system under consideration has been replaced by infinitely long and straight cylinder.

VI. SUMMARY

Closed form expressions for the resistive wall impedance and for the shielding effectiveness of a thin conducting beam pipe were derived. For the resistive wall impedance we compared the exact expression with the well known approximations for thick and for thin walls. In addition we obtained an approximate expression covering both regimes. In the limit of a very thin wall $d \ll \delta_s$ we recover the vacuum space charge impedance.

Concerning the shielding of beam induced rf fields by a thin conducting pipe we showed that the approximate expression obtained also previously in Refs. [19,20] underestimates the shielding effectiveness for the high mode numbers that are relevant, e.g., during bunch compression and for a wall thickness exceeding δ_s. Application of the derived analytical results to the GSI synchrotrons SIS 18 and SIS 100 (planned) shows that a shielding effectiveness of 40 dB (FCC Class B) for "good shielding" is required. This choice is somewhat arbitrary and will require more detailed investigations in the future.

ACKNOWLEDGMENTS

One of us (A.A.) thanks the High Current Beam Physics Group of GSI Darmstadt for the kind invitation. He also would like to thank the Council of Scientific Research of Yarmouk University, Irbid, Jordan, for supporting this work by the grant 4-2004.

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