Theory of dust and dust-void structures in the presence of the ion diffusion

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A dust void is a dust-free region inside the dust cloud that often develops for conditions relevant to plasma processing discharges and complex plasma experiments. A distinctive feature of the void is a sharp boundary between the dust and dust-free regions; this is manifested especially clear when dissipation in the plasma is small and discontinuity of the dust number density appear. Here, the structure of the dust void boundary and the distribution of the dust and plasma parameters in the dust structure bordering the void is analyzed taking into account effects of dissipation due to the ion diffusion on plasma neutrals. The sharp boundary between the dust and void regions exists also in the presence of the ion diffusion; however, only derivatives of the dust density, dust charge, electron density and electric field are discontinuous at the void boundaries, while the functions themselves as well as derivatives of the ion drift velocity and the ion density are continuous. Numerical calculations demonstrate various sorts of diffusive dust void structures; the possibility of singularities in the balance equations caused by the diffusion process inside the dust structures is investigated. These singularities can be responsible for a new type of shocklike structures. Other structures are typically self-organized to eliminate the singularities. Numerical computations in this case demonstrate a set of thin dust layers separated by high density thin dust clouds similar to the multiple-layer dust structures observed in the laboratory and in the upper ionosphere. The possibility for existence of a few equilibrium positions of the void boundary is discussed.

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I. INTRODUCTION

Recent observations [1,2] clearly demonstrate the presence of sharp boundaries separating the regions containing dust grains and the regions of the absence of dust grains—dust voids. A dust void appears as a dust-free region where the plasma ionization rate (comparatively higher than that in a dust region due to strong plasma dissipation on dust particles) supports an outward ion flow exerting an outward ion drag force on the dust particles, as sketched in Fig. 1. The sharp dust-void boundary presents a new type of discontinuities in a dusty plasma where the dust density can change rapidly while the ion and electron densities as well as the dust charge are continuous. The observed boundaries are often stationary, with the established balance of forces acting on dust particles.

First theoretical treatment of such boundaries complemented with the proof that these boundaries are sharp was done in [4] (see [5] for more details) where the model of self-organized dust sheath [6] influenced by the flow of plasma ions from both sides was considered. It was demonstrated [4] that a direct consequence of the Poisson's equation and the stationary force balance equations requires a sharp boundary and that the considered model can provide the dust density as well as the ion and the electron density distributions and the dust charge inside the dust sheath. It was also shown [4,5] that when the dust pressure is neglected, the parameter P [see Eq. (1) below] and the dust number density are discontinuous while other parameters such as the electron and ion densities are continuous.

Usually, dissipative processes tend to smooth a discontinuity (as, e.g., in the standard hydrodynamics) but in dusty plasmas the situation is significantly more complicated since the discontinuities themselves are created by dissipative processes related to plasma absorption by dust grains. In Refs. [4,5], such dissipative processes as the ion friction on dust grains due to Coulomb scattering by dust charges and due to the ion and electron absorption by dust particles were considered, and the drag of dust particles by the ion flow was calculated. It was obtained that these processes not only fail to smooth the boundaries, on the contrary, they create them. Thus in [3] and [7] the theory of dust void was proposed for the two limiting cases where the ion friction on the neutral gas atoms is assumed to be small [3] and in the case where the ion-neutral friction is dominating [7].

The first case refers to a "collisionless void" and the second is related to a "collision-dominated void." The collisionless void appears when its size is much less than the ion-

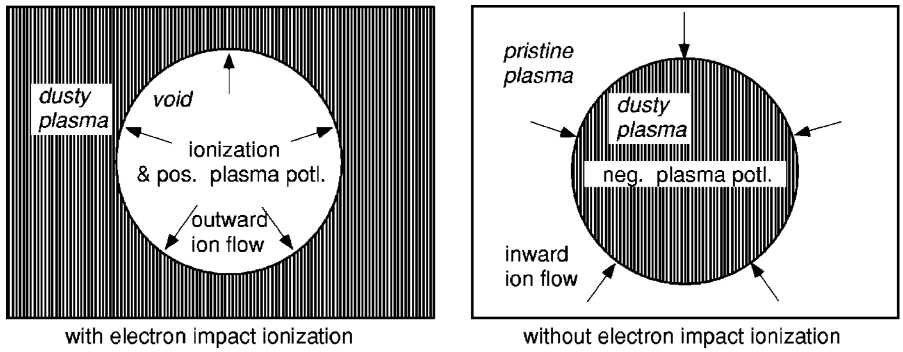

FIG. 1. Sketch of a void (left) and its converse (right). In the presence of the plasma ionization, a positive space potential develops, creating an outward ambipolar electric field that drives ions outward, applying an outward ion drag force, which can maintain a void. In the absence of the ionization, for example in a space plasma where plasma is generated far away, the dust cloud forms in the complementary shape of a spheroid, with its boundary sustained by an inward ion drag force driven by an inward electric field. Reprinted from Goree *et al.* [3].

neutral mean free path; in the opposite case, the void is collision-dominated. The difference between the developed theory [3,7] and the preliminary considerations [4,5] is that the ionization source was taken into account in Refs. [3] and [7] which (as it is practically observed in experiments) was assumed to be homogeneous and proportional to the plasma electron density (in Refs. [4] and [5], the ionization source was ignored). It was demonstrated that the presence of the plasma ionization is crucially important for the theory of voids since the ionization source produces the ion flow to the boundary thus supporting the void.

The model [3,7] has proved that the dust boundary is sharp for both the collision-dominated and collisionless cases, and the jump of the dust number density at the boundary was calculated explicitly. The boundary conditions at the void's surface were solved together with the condition that the dust density discontinuity creates a positive dust density in the dust region, otherwise (even if the boundary conditions are satisfied but the calculated dust density is negative) the solution (if exists) is nonphysical and such a void cannot be created. The void sizes predicted by the theory [7] correspond to those observed in experiments [1,2]. The calculated size is of the order of λ_{in}/τ, where λ_{in} is the mean free path for the ion-neutral collisions and $\tau = T_i/T_e$ is the ion to electron temperature ratio which in experiments is $(1-3)\times 10^{-2}$. Thus the calculated void size is significantly larger than the ion-neutral mean free path, the same as in the experimental observations [1,2]. This is the collision-dominated case.

Sharp dust boundaries were also investigated between the dust cloud and the wall (which is at the floating potential condition) [8] when so called near-wall voids appear. The equilibrium conditions requires that within the near-wall void the direction of the ion flow changes: it is directed to the dust cloud far from the wall (and near the dust boundary) while it is directed to the wall in the region closer to the latter and far from the dust. The peculiarity of these voids is that for the "ordinary" (i.e., not near-wall) voids the ion flow velocity becomes zero far from the dust boundary (namely, in the center of the voids discussed in Refs. [3] and [7]) while for the near-wall voids it crosses zero and reaches high opposite velocities (of the order of the Bohm velocity) at the wall. For the near-wall voids, the dust-void boundary is also shown to be sharp. A review of the void theory was given in Refs. [9] and [10]. Thus the problem of the structure of the dust-void boundaries in a dusty plasma is a hot topic of current and future experiments.

In this article, we investigate the dust void structures in the case when the ion diffusion on the neutral gas atoms plays an important role. We demonstrate that this dissipative process not only allows the existence of the dust voids (i.e., the absence of dust grains in the void region) but can create new types of discontinuities inside the dust structure.

The results are obtained for a one-dimensional model, with planar geometry that is symmetric about the center of the void, by numerical investigation of the stationary force balance equations for the void structure. When including the process of the ion diffusion, we investigate not only the position of the void boundary but also the distribution of the plasma parameters in the dust region next to the void region. We consider here only the dissipative process related to the ion diffusion on the neutral gas atoms and neglect the less significant processes such as the dust pressure effects. We note that the latter can lead to higher derivatives of the dust density, the electric field, and other dust and plasma parameters, and can in principle smooth such discontinuities at the dust boundaries as the dust density jumps as well as the discontinuities of the derivatives of the dust and plasma parameters.

In the considered model, we numerically investigate the non-linear balance equations and obtain novel unexpected results. Namely, we show that if the ion diffusion on neutral gas atoms is taken into account, the dust density has no discontinuity (according to the boundary conditions used) at the surface of the dust boundary; nevertheless, the ion diffusion process still creates sharp dust boundaries with complete absence of dust in the void region. It appears that not the dust density but the derivatives of various plasma parameters have discontinuities at the dust boundary. Note that in the presence of the ion diffusion the condition that the dust density is zero at the void boundary is automatically fulfilled when the boundary conditions are satisfied. We therefore loose the previous important condition that the dust density jump at the boundary is positive which was crucial for the selection of those surfaces that can describe the dust voids from other nonphysical surfaces satisfying the boundary conditions. Thus in order to construct the theory of dust voids in this case we have to revise the main concepts and procedures of the theoretical study of the dust void formation. This is done in the next section.

II. MAIN CONCEPTS, PROCEDURE, AND ASSUMPTIONS

First, we introduce here the concept of the dust charge in the absence of dust and call it the *virtual* dust charge. The virtual dust charge merely indicates that if a dust particle is placed in some region in a plasma, it will be charged corresponding to the virtual charge at that point. We note here that forces acting on the dust grain will move it if the chosen position does not satisfy the equilibrium force balance condition (see, e.g., recent dedicated experiments [11]). The proposed concept is close to the concept of the field strength in electrodynamics: the force acting on a charge realizes only when the charge is actually placed at the point where the field exists.

In the absence of dust grains the void boundaries can be obtained as virtual boundaries. The model [3,7], dealing with the boundary conditions at the void surface and assuming the jump of the dust density at the void boundary, can predict the virtual void size; however, the virtual dust charge can be obtained only at the void surface in this approach. Here, we use the concept of the virtual dust charge to calculate not only the void boundaries, but the actual dust density distribution.

If to relax the condition of the dust density discontinuity at the boundary, some solutions reveal negative dust density in the region occupied by dust. We thus need to perform further investigation in order to exclude the potential presence of nonphysical negative dust densities in the dust region. It appears that this new condition should stand for the condition of the positive dust density jump at the boundary used in the previous theories [3,7]. We therefore perform here a complete investigation of the void and dust regions. We obtain numerically that indeed some solutions of the balance equations in the dust region give the nonphysical result if initial parameters in the void region do not set up properly. The procedure includes the following: In the center of the void, in addition to the ion density, we also need to fix the ratio of the electron to ion density. By varying and adjusting the latter, we can determine the critical value when the dust density starts to increase from the void's boundary. This condition appears to be the necessary condition for the boundary to exist and the structure of the dust region can then be (numerically) calculated. For such a boundary we can obtain the structure of the dust cloud behind the dust void boundary. We can also numerically determine possible *singularities* in the dust region. In the one-dimensional model considered here, they indicate the points where the ion flow velocity tends to zero or the electron density tends to zero. In these cases the size of the obtained dust cloud can be of the order of the ion-neutral mean free path, i.e., corresponding to a rather thin cloud. Note that sets of thin dust layers found theoretically in the present paper were also observed in the laboratory experiments [1] as well as in the lower ionosphere [12]. In this paper, we show that the presence of the singular points in the dust region requires such an adjustment of the dust and plasma parameters in this region which is probably occurring at the stage of formation of the structure which therefore self-organizes itself [13]. We also find that the dust and the ion densities steepen at these singular points.

In the present study, we ignore the dust pressure effects proportional to the gradient of the dust density; in principle, they can be important close to the singular points since the dust density gradients rapidly increase close to the singularity. Although these effect can smooth the singularities, the real thickness related to them (for the parameters of the existing experiments) is at least two or even three orders of magnitude less than that considered here. We therefore concentrate here on the possibility that the dust cloud is self-consistently adjusting its parameters at the points where the ion flow velocity is close to zero in such a way that the singularity vanishes. We perform an analysis showing that the ion to electron density ratio at the point with the zero ion flow velocity should achieve a certain value that makes it possible for the solution to pass this point and thus to describe the existence of the dust cloud. We thus start our numerical calculations at the center of the void region which corresponds to the point with the zero ion flow velocity and proceed up to the void boundary, finding possible nonlinear solutions of the balance equations taking into account both the ion pressure and the ion diffusion effects. This allows us to determine the void and the dust regions.

The diffusion of plasma electrons in the neutral gas is neglected here since it is typically smaller than the electron pressure effect; it is therefore assumed in our model that the plasma electrons can be treated adiabatically. The electric field is produced by the electrons, ions, and charged dust, and it is shown that this field differs from the ambipolar field. Furthermore, only stationary dust structures are analyzed. We do not exclude that non-stationary or "breathing" structures (like "hearthbeat" voids observed experimentally) can appear for those parameters when the stationary structures cannot exist. This problem is not considered here as well as other time-dependent problems. It also seems to be no surprise that even a simplified system of nonlinear equations used in our model here has a number singular points. Indeed, even systems of nonlinear equations of much simpler types are known to describe singularities like strange attractor. Thus further mathematical analysis of our nonlinear system, for example, to investigate appearance of strange attractors, is one of possible further developments of the present study.

The paper is organized as follows: First, we set up the full system of the balance equations for stationary structures taking into account the ion-neutral diffusion, the ion pressure, and the dust pressure effects. Its derivation is related to the force balance in the hydrodynamic approach and the standard OML approach for the dust charging. We write these equations in the dimensionless form, the same as that used in Ref. [7]. Then we derive equations (and numerically solve them) in two limiting cases, namely (1) when the ion pressure is taken into account but the ion diffusion on neutral

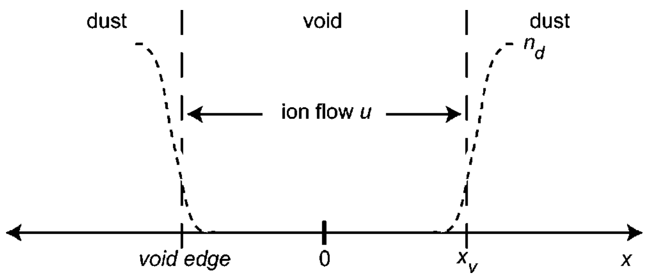

FIG. 2. Sketch of the one-dimensional simulation region. A dust cloud fills all space except for a void region of full width $2x_v$ with the center at $x=0$.

atoms is neglected, and (2) when the ion diffusion and the ion pressure are taken into account but the dust pressure is neglected. The calculated ion diffusion effect as compared to the ion pressure effect has only a numerical factor $1/3\sqrt{3}=0.236$ which (although not extremely small) can be regarded as small enough thus justifying the possibility of neglecting the ion diffusion in some approximations. Finally, in the last section of the paper, we discuss the main results.

III. GENERAL BALANCE EQUATIONS

We consider the one-dimensional model as sketched in Fig. 2. The overall approach assumes a planar geometry that is symmetric about the center of the void which is located at $x=0$. The ion drift velocity is zero at the center. When the void appears, its center is at $x=0$. The void's surface corresponds to x_v. The dust region corresponds to $x > x_v$ where the dust number density n_d is finite (and positive).

To describe collision-dominated structures with sizes which assumed to be much larger than the ion-neutral mean free path, we use (as in Ref. [7]) the dimensionless variables given by

$$n \to \frac{n_i}{n_{00}}, \quad n_e \to \frac{n_e}{n_{00}}, \quad P = \frac{n_d Z_d}{n_{00}}, \quad u \to \frac{u_i}{\sqrt{2} v_{Ti}},$$

$$E \to \frac{eE\lambda_{in}}{T_i}, \quad x \to \frac{x\tau}{\lambda_{in}}, \quad z = \frac{Z_d e^2}{aT_e}, \quad \tau_d = \frac{T_d e^2}{T_e^2 a}, \qquad (1)$$

where n_i, n_e, and n_d are the dimensional ion, electron, and dust number densities, respectively, Z_d is the dust charge in the units of the electron charge (the dust is assumed to be charged negatively), u_i is the ion drift velocity, $v_{Ti}=(T_i/m_i)^{1/2}$ is the ion thermal velocity, T_e and T_i are the electron and ion temperatures, T_d is the dust kinetic temperature in energy units (all temperatures are assumed to be homogeneous and constant), $\tau=T_i/T_e$ is the ion to electron temperature ratio assumed to be small, τ_d is the dimensionless dust temperature (also assumed to be small), E is the electric field, λ_{in} is ion-neutral mean free path, a is the dust size, and z is the dimensionless dust charge. Finally, n_{00} is the ion critical density used for normalization of the plasma electron and ion densities

$$n_{00} = n_0 \frac{\lambda_{id}\tau}{\lambda_{in}} = n_0 \frac{\lambda_{Di}^2}{a}\frac{\tau}{\lambda_{in}} = \frac{\tau T_i}{4\pi e^2 \lambda_{in} a}. \qquad (2)$$

The total system of balance equations is given by the following.

(1) The electron balance equation including the electron pressure force balanced by the electric field force,

$$\frac{dn_e}{dx} = -n_e E. \qquad (3)$$

(2) The ion balance equation including the ion pressure force balanced by the electric field force, the friction on the dust force and the friction on the neutral atoms force,

$$\tau\frac{dn}{dx} = n(E - \alpha_{dr}\, uzP - u(2 + \alpha_n|u|)), \qquad (4)$$

where $\alpha_{dr}=\alpha_{dr}(u,\tau/z)$ is the dust ion drag coefficient which is calculated taking into account both the capture force and the Coulomb scattering force [7]. In the limit $\tau \ll 1$, α_{dr} depends only on the ion drift velocity and is given by

$$\alpha_{dr}(|u|) = \left(\frac{\mathrm{erf}(|u|)}{2|u|^3} - \frac{\exp(-u^2)}{\sqrt{\pi}u^2}\right)\ln\Lambda, \qquad (5)$$

where $\ln\Lambda$ is a generalization of the Coulomb logarithm taking into account the collective plasma effects, the finite dust size, and scattering on large angles. Furthermore, α_n in Eq. (4) is a numerical coefficient describing the nonlinearity of the friction force on the neutral atoms which is typically of the order of 1 (thus taken equal to 1 in further numerical computations here). This value of α_n is based on the experimental dependence of the ion mobility on the electric field in a low-temperature plasma [14] which demonstrates that with the increase of the electric field E the mobility starts to depend on E and for large field $u \propto E$, i.e., the ion friction force on the neutral atoms $\propto u^2$.

(3) The dust balance equation (balancing the dust pressure force by the electric field force and the ion drag force)

$$\tau_d \frac{d}{dx}\left(\frac{P}{z}\right) = -P(E - n\alpha_{dr} uz). \qquad (6)$$

(4) The ion continuity equation determining the ion drift velocity and containing the ionization source and dissipation on the dust component

$$\frac{d\Phi}{dx} = \frac{n_e}{x_i} - \alpha_{ch} Pn, \qquad (7)$$

where Φ is the total dimensionless ion flux, x_i is the dimensionless ionization length (see [3,7]), and α_{ch} is the the capture coefficient appearing also in the dust charging equation (10) which in the limit $\tau \ll 1$ depends only on the ion flow velocity:

$$\alpha_{ch}(|u|) = \frac{1}{4|u|}\,\mathrm{erf}(|u|). \qquad (8)$$

(5) The ion flux relation including the convective flux and the diffusion flux

$$\Phi = nu - \tau\alpha_D \frac{dn}{dx}, \tag{9}$$

where α_D is the diffusion coefficient of the ions on the neutral particles; it given by $\alpha_D = 1/3\sqrt{2} = 0.236$ if estimated as $\lambda_{in} v_{Ti}/3$ for $T_i \approx T_n$ (T_n is the neutral gas temperature in energy units).

(6) The dust charging equation obtained from the balance of charging currents on the dust grains

$$\frac{1}{z}\frac{dz}{dx} = -\frac{1}{z+1}\left(E + \frac{1}{n}\frac{dn}{dx} + \frac{1}{\alpha_{ch}}\frac{d\alpha_{ch}}{du}\frac{du}{dx}\right). \tag{10}$$

Here, the charging coefficient α_{ch} and the drag coefficient α_{dr} in the limit $\tau \ll 1$ are functions of the ion drift velocity only

$$\alpha_{dr}(|u|) = \left(\frac{\mathrm{erf}(|u|)}{2|u|^3} - \frac{\exp(-u^2)}{\sqrt{\pi}u^2}\right)\ln\Lambda, \tag{11}$$

where $\ln\Lambda$ is generalization of the Coulomb logarithm taking into account the collective plasma effects, the finite dust size, and scattering on large angles.

(7) Poisson equation

$$\frac{dE}{dx} = \frac{1}{d^2}(n - n_e - P), \tag{12}$$

where

$$d^2 = \frac{a}{\lambda_{in}}. \tag{13}$$

Similar system of equations was used previously [3,7] by neglecting the dust pressure $\tau_d \to 0$ when Eq. (6) describes sharp boundaries ($P=0$ at one side of the boundary and $E = nuz\alpha_{dr}$ at the other side of the boundary), the ion diffusion $\alpha_D \to 0$ and/or the ion friction on neutrals. Note that the present system of the balance equations is written for the first time in this full form and an important point here is the explicit expression for τ_d containing no parameter able to change in the appearing structures.

There are two small parameters in the system, namely, $\tau \ll 1$ and $\tau_d \ll 1$. Indeed, in a typical complex plasma experiment (see, e.g. [2]) $\tau \sim 10^{-2}$ and $\tau_d \sim 10^{-3}$ for the electron temperature of a few eV, the dust size a few microns, and the dust kinetic temperature of the order of 10 eV (when in the crystal phase, the dust kinetic temperature is even less). In the above theory, these parameters appear at the derivatives of the corresponding functions which makes the full set to be of higher order in the derivatives, thus raising the known mathematical problem of the possibility of neglecting the terms at the next higher derivatives. Below we carefully analyze this system numerically, exclude nonphysical numerical instabilities, and point out the actual singular points in the system. We obtain that the real smoothing of the discontinuities in the described system is related only to the small parameter τ_d while the effects related to the parameter τ can even create new discontinuities. However, since for a typical experiment the parameter τ_d is much less than the parameter τ, sufficiently sharp discontinuities should be present in the real structures and their smoothness can be finally produced only by the dust pressure effects; in the present investigation we neglect the latter by putting $\tau_d = 0$.

IV. VOID STRUCTURE AND THE ION PRESSURE EFFECTS

We start with the equations where the ion diffusion and the dust pressure effects are neglected while the ion pressure effects are included. This requires $\alpha_D \ll 1$ which in fact is not extremely small: $\alpha_D = 0.236$. However, the use of this assumption is a good illustration of the problem since we can easily obtain both the void structure and the dust sheath structure having no problems with the point $u=0$ in this approximation. We assume that $x=0$ is in the center of the void and start numerical calculations from this point in the void region for $x>0$ to find the boundary x_v of the void. The equations used in the void region are

$$\frac{du}{dx} = -\frac{u}{\tau}[E - u(2+|u|)] + \frac{n_e}{nx_i}, \tag{14}$$

$$\frac{dn}{dx} = \frac{n}{\tau}[E - u(2+|u|)], \tag{15}$$

$$\frac{dn_e}{dx} = -n_e E, \tag{16}$$

$$\frac{dE}{dx} = \frac{1}{d^2}(n - n_e). \tag{17}$$

We solve these equations for the set of the parameters $\tau = 0.05$, $x_i = 2$, $d = 0.2$, and $n_0 = 2$. These values are close to typical parameters of complex plasma experiments such as [2]. The boundary conditions for the void surface (continuity of the electric field and the charging equation) are given by

$$E(x_v) = n(x_v) z_v u(x_v) \alpha_{dr}(|u(x_v)|) \tag{18}$$

and

$$\exp(-z_v) = \frac{2\sqrt{\pi}}{\tau m_i/m_e}\alpha_{ch}(|u(x_v)|) z_v \frac{n(x_v)}{n_e(x_v)}. \tag{19}$$

They give x_v and z_v and we therefore find $n_v = n(x_v)$, $n_{e,v} = n_e(x_v)$, $u_v = u(x_v)$, and $P_v = P(u_v, n_v, n_{e,v}, z_v)$. We obtain $x_v = 0.19004$, $z_v = 2.82089$, $n_v = 2.32804$, $n_{e,v} = 1.91211$, $E_v = 0.45031$, $u_v = 0.07948$. In the dust region the Poison equation gives the value of P as a function of other parameters in the dust region

$$P(u,n,n_e,z)=\frac{n-n_e+d^2\alpha_{dr}(|u|)u^2nz\left[\frac{\alpha_{dr}(|u|)nz}{z+1}+\frac{R(u,z)}{\tau}(nz-2-|u|)\right]}{1+R(u,z)d^2\alpha_{dr}(|u|)nz\left(\frac{u^2}{\tau}\alpha_{dr}(|u|)z-\alpha_{ch}(|u|)\right)}, \tag{20}$$

where the parameter $R(u,z)$ is related to the dependence of the dust drag α_{dr} and the charging α_{ch} coefficients on the ion flow velocity u

$$R(u,z)=1+u\frac{1}{\alpha_{dr}(|u|)}\frac{d\alpha_{dr}(|u|)}{du}-\frac{1}{(1+z)}\frac{u}{\alpha_{ch}(|u|)}\frac{d(\alpha_{ch}(|u|))}{du}. \tag{21}$$

Using the above found values at the void surface we can find the jump P_v of P at the surface, i.e the value P at the dust side of the surface. From Eqs. (20) and (21) we obtain $P_v=0.3686$ which is positive and since the condition $P_v>0$ is satisfied, the sharp dust void surface exists. All the parameters found at the surface can be used to solve the equations in the dust region up to the point $P=0$ together with the boundary conditions at the dust surface. We have

$$\frac{du}{dx}=\frac{R_1(u,n,n_e,z)}{R(u,z)}, \tag{22}$$

$$\frac{dn}{dx}=\frac{un}{\tau}\{\alpha_{dr}(|u|)z[n-P(u,n,n_e,z)]-2-|u|\}, \tag{23}$$

$$\frac{dn_e}{dx}=-unn_ez\alpha_{dr}(|u|), \tag{24}$$

$$\frac{dz}{dx}=R_2(u,n,n_e,z), \tag{25}$$

where

$$R_1(u,n,n_e,z)=\frac{\alpha_{dr}(|u|)znu^2}{1+z}-\frac{zu^2}{\tau(1+z)}\{\alpha_{dr}(|u|)z[n-P(u,n,n_e,z)]-2-|u|\}+\frac{n-n_e-P(u,n,n_e,z)}{d^2\alpha_{dr}(|u|)zn} \tag{26}$$

and

$$R_2(u,n,n_e,z)=-\frac{z}{z+1}\left[\frac{1}{\alpha_{ch}(|u|)}\frac{d\alpha_{ch}(|u|)}{du}\frac{R_1(u,n,n_e,z)}{R(u,z)}+\frac{u}{\tau}\alpha_{dr}(|u|)z[n-P(u,n,n_e,z)]+\alpha_{dr}(|u|)znu\right]. \tag{27}$$

The solution is presented in Fig. 3. We can see that at the dust boundary not only the parameter P is discontinuous but also its derivatives, as well as the jump occurs for dn/dx and its derivatives (although this might not be quite clearly seen from the figure but can be easily demonstrated from the corresponding equations). At the end of the structure, where $P=0$, the derivative of P is also discontinuous. It can be seen that the solution in the dust region has no singularity at $u=0$ and this is related to the fact that the electric field, being proportional to u (due to the dust balance equation), vanishes for $u=0$. Also, no point where $R=0$ is reached in the dust region. The expression R in denominators [see, e.g., Eq. (22)] appears when resolving the system of equations for du/dx and dn/dx. Note that in the two-dimensional case when the same system of equations with nonzero dust velocity describing formation of dust vortices has additional term $\mathbf{v}_d dz/d\mathbf{r}$, no such singularities appear for dust rotation. Thus $R=0$ in the one-dimensional case corresponds to the possibility of generation of dust vortices in the two-dimensional consideration. The absence of such a singularity for the results shown in Fig. 3 suggests that for the used set of parameters the dust vortices cannot be excited in the dust region. It is important that the sign of u changes inside the dust region. Mention that for another set of the initial parameters, the expression for R can reach zero inside the dust region (which can indicate start of dust convection in the two-dimensional case). In the case when the second surface is not a free surface, as was assumed for the point $P=0$ shown in Fig. 3, and the wall is present at the floating potential, the dust region is

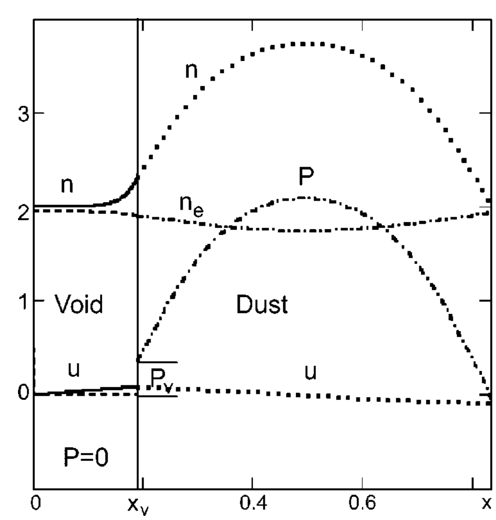

FIG. 3. Distribution of the main dimensionless parameters in the void and the dust regions in the case when the ion diffusion is neglected. Here, $\tau=0.05$, $x_i=2$, $d=0.2$, $n_0=2$, and $n_{0e}=0.98$. At the dust boundary, the parameter P as well as its derivatives are discontinuous. At the end of the structure, where $P=0$, the derivative of P is also discontinuous.

not continuing up to $P=0$ and before that point another jump of P can occur leading to a *near-wall void* where the ion drift velocity changes it sign once more and it is started to be directed to the wall. Note that only in this case the boundary conditions for the floating potential at the wall can be satisfied.

The above picture can serve as the reference starting point for further investigation of the dust-boundary structures. Mention that before starting the calculation we need to fix the values of the ion and electron densities in the center of the void. The ion density was chosen such that solutions of the boundary conditions exist and the electron density was found from the condition that at the center of the void the second derivative of the ion density is zero. The conditions for other parameters u and E were chosen from the asymptotics of the equations for $u \to 0$ and $E \to 0$ at the center of the void. The condition of vanishing of the second derivative of the ion density seems to be a bit artificial but it is required from the equation where the diffusion is taken into account assuming that close to the void center the ion flux is convective. Note that while ignoring the diffusion at this stage, we still keep the boundary condition that it introduces. This is not obligatory but for this value of the electron density the solution obviously exists. It should be also mentioned that the condition of local quasineutrality of the plasma at the void center cannot be used although the electrons and ions are created in pairs in equal amount, since the electrons have large thermal velocity and are rapidly redistributed in the void region under the action of the electrostatic potential appearing there.

Before proceeding further, we mention that for $d \to 0$ we have from expression (20)

$$P = n - n_e, \tag{28}$$

which is nothing else but the local quasi-neutrality condition. In the other limit $\tau \to 0$ for finite d we obtain

$$P = n - \frac{2 + |u|}{z\alpha_{dr}(|u|)}. \tag{29}$$

We use such relations (or similar to them) below.

V. VOID STRUCTURE WITH THE ION PRESSURE AND DIFFUSION

Here, we take into account both the ion pressure effects and the ion diffusion effects. By combining Eqs. (3) and (9) we find

$$P = n - \frac{2 + |u| + \dfrac{1}{\alpha_D}\left(1 - \dfrac{\Phi}{nu}\right)}{z\alpha_{dr}(|u|)}, \tag{30}$$

which differs from Eq. (29) by an additional term describing the difference between the total and the diffusion flux, i.e., the term determined by the diffusion flux. Relation (30) (where the diffusion flux is taken into account) is exact, contrary to relation (29) which is only an approximate one. Equation (30) can be converted to

$$\Phi = nu\{1 + \alpha_D[2 + |u| + (P - n)\alpha_{dr} z]\}. \tag{31}$$

We can now see that the total flux vanishes when $u \to 0$. As an independent new variable in the dust region we can choose either Φ or P. We mention this possibility for the purpose to check the absence of a numerical singularity when calculating the void boundary structure. For example, using Φ as an independent variable, numerical results can lead to accumulation of errors for Φ which will then not vanish when $u \to 0$ thus giving a numerical singularity. By using another variable P this will not happens. We completed numerical computations with both variables and checked the absence of the numerical singularity. Note also that close to $u=0$, expressions containing derivatives with respect to the velocity u in the ion drag and the dust charging are given as a ratio of expressions both of which tend to zero; thus again a possibility of a numerical error appears. We thus performed special calculations finding analytically the mentioned ratios and checked the absence of the numerical error at this point. After elucidating the numerical errors, we still obtain some physical singularities at $u \to 0$ in the dust region. Their presence is due to the necessity of an exact adjustment of the ratio of the electron to ion densities in the center of the void (with which we start the calculations). Here we provide an example using the same conditions in the center of the void as those in the case when the ion diffusion was neglected.

In the void region, $P=0$ and the electric field is not determined by the ion drag force. Using the basic equations, the flux Φ can be expressed through the electric field

$$\Phi = nu + \alpha_D n(u(2 + |u|) - E). \tag{32}$$

From the two last equations (31) and (32) follows that when the boundary conditions at the void surface are satisfied and $E = nuz\alpha_{dr}$, we have automatically at the surface $P_v=0$ which means that in this case the parameter P is continuously increasing from the surface. Due to the mentioned possibility of numerical errors we exclude Φ as a function to be determined from both the void region and the dust region, using there instead the functions E and P, respectively.

Equations in the void region take the form

$$\frac{du}{dx} = \frac{1}{2\alpha_D + 1}\left[-\frac{u}{\tau}[E - u(2 + |u|)] + \frac{\alpha_D}{\tau}[E - u(2 + |u|)]^2 + \frac{n_e}{nx_i} + \frac{\alpha_D}{d^2}(n - n_e)\right], \tag{33}$$

$$\frac{dn}{dx} = \frac{n}{\tau}[E - u(2 + |u|)], \tag{34}$$

$$\frac{dn_e}{dx} = -n_e E, \tag{35}$$

$$\frac{dE}{dx} = \frac{1}{d^2}(n - n_e). \tag{36}$$

The asymptotics of these equations in the center of the void where $u \to 0$ and $E \to 0$ gives $u = u_0 x$, $E = E_0 x$ which allows us to determine also the values of the second derivatives of the electron and ion densities in the center of the void. This is used to start calculations using these asymptotic expressions close to the center of the void (the initial point is $x_0=0.001$), namely

$$u=\frac{x_0}{2\alpha_D+1}\left[\frac{s}{x_i}+\frac{\alpha_D n_0(1-s)}{d^2}\right],$$

$$n=n_0\left[1+\frac{x_0^2}{2\tau(2\alpha_D+1)}\left(\frac{n_0(1-s)}{d^2}-\frac{2s}{x_i}\right)\right], \quad (37)$$

$$n_e=n_0 s\left[1-\frac{n_0x_0^2}{2d^2}(1-s)\right], \quad E=\frac{n_0x_0}{d^2}(1-s), \quad (38)$$

where n_0 is the value of the ion density in the center of the void and $s=n_e(0)/n_0$ is the ratio of the electron to ion densities in the center of the void. To be able to compare with the previous numerical results (where the ion diffusion was neglected) we give an example for $x_i=2$, but instead of using the condition that the second derivative of the ion density is zero (as it was used before) and thus $s=s_0$, where

$$s_0=\frac{1}{1+\dfrac{2d^2}{n_0x_i}}, \quad (39)$$

we here leave the parameter s as another independent initial parameter. It is easy to find that the total flux close to the center is a sum of the convective flux Φ_0^{conv} and the diffusion flux Φ_0^{diff},

$$\Phi_0=\Phi_0^{\mathrm{conv}}+\Phi_0^{\mathrm{diff}},$$

$$\Phi_0^{\mathrm{conv}}\approx n_0\frac{x_0}{2\alpha_D+1}\left[\frac{s}{x_i}+\frac{\alpha_D n_0(1-s)}{d^2}\right],$$

$$\Phi_0^{\mathrm{diff}}\approx-\alpha_D n_0\frac{x_0}{\tau(2\alpha_D+1)}\left[\frac{n_0(1-s)}{d^2}-\frac{2s}{x_i}\right], \quad (40)$$

and that the diffusion flux vanishes for $s=s_0$, thus demonstrating that the parameter s regulates the ratio of the diffusion to the convection flux at the void's center. We then try to calculate further by solving the boundary conditions (18) and (19) for the parameters in the center of the void used in the previous section, namely $n_0=2$ and $s=s_0$, and find that these equations have no solution. Then we can investigate the appearance of a solution keeping the condition $s=s_0$ but changing the parameter n_0 and find that the solution starts to appear at $n_0<1.1459$ with $x_v=0.24999$, $z_v=2.83909$, $u_v=0.13876$, $n_v=1.27132$, and $n_{ev}=1.06564$. We see that comparing to the case when the ion diffusion was ignored, the ion density at the void's center decreases, the dust void size increases and the ion drift velocity at the boundary of the void substantially increases (although still being much lower that the ion thermal velocity). With further decrease of the ion density in the void's center, solutions of the boundary equations exist showing a decrease of the void size with a decrease of the ion density in the center. This proceeds up to $n_0=0.8$ when the size of the void becomes small, about 0.064, and the ion drift velocity becomes very small such that the ion drag cannot sustain the void. By changing s and keeping $n_0=2$ we can find solutions for very small voids or very small dust charges. Thus the condition that the diffusion flux at the center is zero is the optimal for a *diffusive void* to be created.

With this possible range of parameters at the void's surface we investigate distribution of the densities, dust charges, etc., in the dust region. As a new function we chose the parameter P excluding Φ using relation (30). In this case, Eqs. (22)–(25) survive, with P not determined by expression (20) but considered as an independent additional function of the distance x and a new equation for P is derived by differentiating Eq. (30) with respect to x and using basic equations (7) and (12). We thus obtain the following system of equations in the dust region:

$$\frac{du}{dx}=\frac{R_1(u,n,n_e,z,P)}{R(u,z)}, \quad (41)$$

$$\frac{dn}{dx}=\frac{un}{\tau}[\alpha_{dr}(|u|)z(n-P)-2-|u|], \quad (42)$$

$$\frac{dn_e}{dx}=-unn_ez\alpha_{dr}(|u|), \quad (43)$$

$$\frac{dz}{dx}=R_2(u,n,n_e,z,P), \quad (44)$$

$$\frac{dP}{dx}=R_3(u,n,n_e,z,P)=R_3(u,n,0,z,P)-n_eR_4(u,n,z,P), \quad (45)$$

where

$$R_1(u,n,n_e,z,P)=\frac{\alpha_{dr}(|u|)znu^2}{1+z}-\frac{zu^2}{\tau(1+z)}[\alpha_{dr}(|u|)z(n-P)-2-|u|]+\frac{n-n_e-P}{d^2\alpha_{dr}(|u|)zn}, \quad (46)$$

$$R_2(u,n,n_e,z,P)=-\frac{z}{z+1}\left[\frac{1}{\alpha_{ch}(|u|)}\frac{d\alpha_{ch}(|u|)}{du}\frac{R_1(u,n,n_e,z,P)}{R(u,z)}+\frac{u}{\tau}\alpha_{dr}(|u|)z(n-P)+\alpha_{dr}(|u|)znu\right], \quad (47)$$

$$R_3 = \frac{1}{\alpha_D nuz\alpha_{dr}(u)}\left\{\frac{n_e}{x_i} - Pn\alpha_{ch}(u) + \frac{1}{d^2}\alpha_D n(n-n_e-P) + P\frac{\alpha_D}{1+z}[\alpha_{dr}(u)nuz]^2 + \left[\alpha_{dr}(u)zP\frac{\alpha_D}{z+1} + \alpha_D\alpha_{dr}(u)z(n-P) - [1+\alpha_D(2+|u|)]\right]\frac{u^2n}{\tau}[\alpha_{dr}(u)z(n-P)-2-|u|] + \left[\frac{\alpha_D}{1+z}P\alpha_{dr}(u)nuz\frac{1}{\alpha_{ch}(u)}\frac{d\alpha_{ch}(u)}{du} - \alpha_D nzP\frac{d}{du}[\alpha_{dr}(u)u] - n[1+2\alpha_D(1+|u|)]\right]\times\frac{R_1(u,n,n_e,z,P)}{R(u,z)}\right\}, \tag{48}$$

and

$$R_4(u,n,z,P) = \frac{1}{\alpha_D nuz}\left\{-\frac{1}{x_i} + \frac{\alpha_D n}{d^2} + \frac{1}{d^2\alpha_{dr}(u)znR(u,z)}\times\left[\frac{\alpha_D}{1+z}P\alpha_{dr}(u)nuz\frac{1}{\alpha_{ch}(u)}\frac{d\alpha_{ch}(u)}{du} - \alpha_D nzP\frac{d}{du}[\alpha_{dr}(u)u] - n[1+2\alpha_D(1+|u|)]\right]\right\}. \tag{49}$$

For the diffusion coefficient α_D, as mentioned before, we use the expression $1/3\sqrt{2}$ (for processes other than isothermal this expression is somewhat different but for simplicity we restrict our consideration by this simplest expression) and expression (21) remains for $R(u,z)$. The only difference between expressions (46) and (47) and expressions (26) and (27) is that P is an independent function of x in Eqs. (46) and (47), is not determined by Eq. (20) and should be found by solving the given system of equations (41)–(45). The possibility to write the second equality in Eq. (45) is a simple consequence of the fact that the expression for R_3 is a linear function of n_e.

For the function P, we have to define the boundary conditions. As mentioned before, the value obtained by solution of the boundary problem in the void region automatically gives $P=0$ at the boundary. However the value of P at the boundary should also have no jump for its derivative (when the ion diffusion is neglected such a jump an be clearly seen in Fig. 3), i.e. we assume that at the boundary

$$\frac{dP}{dx} = 0. \tag{50}$$

Note that this condition is also a consequence of the basic equation (6). From Eq. (45) we then find

$$n_{e,v}^{ds} = \frac{R_3(u_v,n_v,0,z_v,0)}{R_4(u_v,n_v,z_v,0)}, \tag{51}$$

where the superscript *ds* stands for the *dust side*. But the value of the electron density at the void side is already found and denoted as $n_{e,v}$. Therefore it is required that

$$n_{e,v} = n_{e,v}^{ds}. \tag{52}$$

For the numerical values of the parameters at the void boundary (the example given in this section for $n_0=1.1459$ and $z_v=2.83909$, $u_v=0.3876$, $n_v=1.27132$) we find $n_{e,v}^{ds}=1.291$ while $n_{ev}=1.06564$. Therefore we should adjust the initial parameters in the void region and particularly the value of ion density in the center of the void n_0 to obtain the coincidence of these two values. Indeed, we find that condition (52) is satisfied if the density in the center of the void is less than that taken before, namely, if $n_0=1.07136$ then $n_{ev}=n_{ev}^{ds}=0.9987$, $x_v=0.24045$, $n_v=1.14861$, $z_v=2.856552$, and $u_v=0.12796$, $P=0$ and $dP/dx=0$. The latter value should be used to find the structure of the dust region by solving the system of equations (41)–(45). There is no free parameter left. But one should have in mind that the parameter x_i is depending on the velocity distribution of plasma electrons in the dust region, which is changed by the dust in the charging process. Thus the parameter x_i is the dust region can be somewhat different and can be adjusted in numerical calculations. Figure 4 shows the result obtained for the case of equal x_i in the dust and void regions, while Fig. 5 shows the result obtained for the case when the ionization rate is twice smaller in the dust region as compared to the void region (in this case $n_e=n_{ev}^{ds}=0.9863$, $x_v=0.23824$, $n_v=1.12724$, $z_v=2.87$). Furthermore, Fig. 6 shows the result for the case where the ionization rate in the dust region is twice larger than in the void region (in this case $n_e=n_{ev}^{ds}=1.023$, $x_v=0.24451$, $n_v=1.10412$, $z_v=2.8585$).

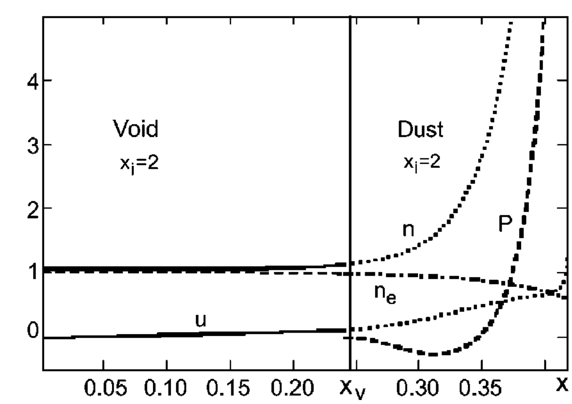

FIG. 4. Distribution of the main dimensionless parameters in the void and the dust regions in the case when the ion diffusion is taken into account. Here, the normalized ionization length is the same $x_i=2$ in the void as well as in the dust regions. Note the nonphysical domain of the negative values of P.

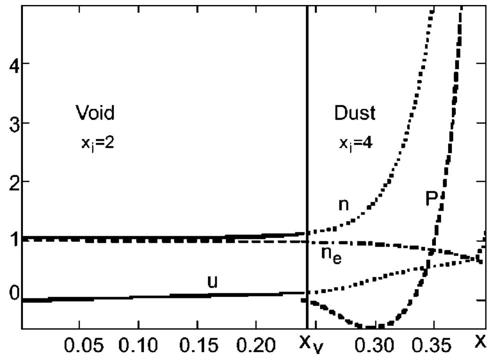

FIG. 5. The same as in Fig. 2 but for the normalized ionization length $x_i=2$ in the void region and $x_i=4$ in the dust region. Note the nonphysical domain of the negative values of P.

These figures demonstrate that in the cases when the ionization rate in then dust region is equal or less that the ionization rate in the void region there exist nonphysical domain of negative values of P, and the sharp increase of P is related with convection $R\to 0$ while for the case when the ionization rate in the dust region is larger than that in the void region the parameter P is everywhere positive, but the singularity appears at $u\to 0$ (the latter we discuss in more detail below). The parameters at $u=0$ can be properly adjusted but then the void is not a symmetric void (with its center in the center of the computation region) and should satisfy different boundary condition (for example the wall boundary conditions). In the case shown in Fig. 6, the size of the dust region is of the order or less than τ which means that it is of the order of the ion-neutral mean free path. We return to discussion of this point later. The results shown in Figs. 4 and 5 with the negative values of P suggest that although $dP/dx=0$ at the boundary, the second derivative of P is negative there. The only way to avoid this effect is to decrease the ion density in the center of the void, which in turn increases the derivative of P at the boundary. Note that a decrease of the ion density in the center in just a small value of order 10^{-3} makes already the derivative of P at the boundary to appreciate in about 0.2 but nevertheless soon in the dust region P is negative.

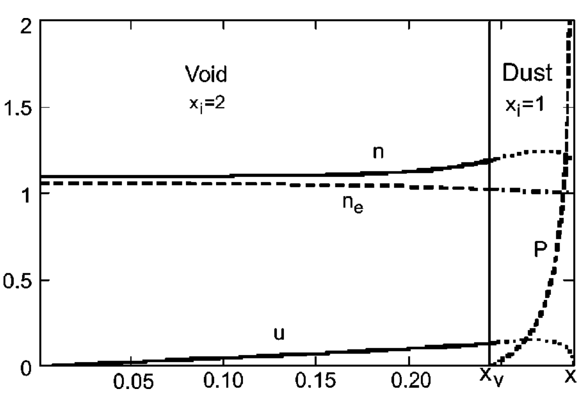

FIG. 6. The same as in Fig. 2 but for the normalized ionization length $x_i=2$ in the void region and $x_i=1$ in the dust region. The parameter P is everywhere positive. The size of the dust region is of the order or less than τ, i.e., it is of the order of the ion-neutral mean free path.

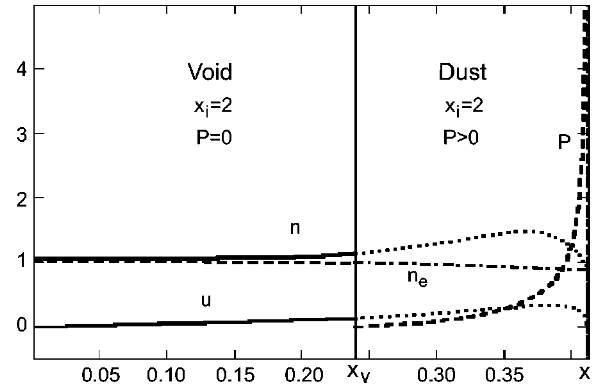

FIG. 7. Distributions of the dimensionless parameters in the void and dust regions for the (normalized) critical ion density in the center $n_0=1.065$.

Only by further decrease of the ion density in the center (to the value 5×10^{-3}) we can reach the condition where P becomes positive in the dust region. At this point the derivative of P at the boundary is equal to 1.15.

This numerical experiment conclusively shows that the ion diffusion *can eliminate the discontinuity of P at the boundary but not the discontinuity of its derivative*. Figure 7 shows distributions of the parameters in the void and dust regions for the critical ion density in the center $n_0=1.065$ when the parameter P starts to be positive in the whole dust region. Note that the singularity still appears in the region $u\to 0$ (similar to the results shown in Fig. 6). Since we clearly demonstrated that the ion diffusion does not smooth the derivatives of P at the boundary, we should allow such discontinuities of the derivatives and return to the result shown in Fig. 4 to establish this jump imposed by the boundary conditions at the void surface. We found then that at the boundary $dP/dx=-0.27861$, i.e., the solution is absent. The value of the derivative of P at the void boundary appears to be very sensitive on the value of the ion density in the center of the void. For example, by decreasing n_0 from the critical value $n_0=1.065$ up to $n_0=1$, we increase the discontinuity of the mentioned derivative from 1.15 up to 20.1 with a similar singularity appearing at significantly shorter distance 0.015 when $u\to 0$.

We also obtain that at $u\to 0$ such parameters in the dust cloud as the ion density and the ratio of the ion to the electron densities should satisfy certain relation for the singularity to be absent. We can start the calculations using these relations from the point $u=0$ and then reach the boundary $P=0$ and find the actual value of the dP/dx at the boundary, from which we can calculate the ion density in the center of the void. Such solutions have no singularity at $u=0$, as was demonstrated in Fig. 3 in the absence of the ion diffusion. Thus although the diffusion relates the parameters of the system stronger, it cannot eliminate all discontinuities of the parameters at the dust boundary. Also we checked all possible values of the ion and electron densities in the center of the void and found that the appearance of a singularity at $u=0$ is a quite general phenomenon.

The main conclusion that can be made from this study is (1) the ion diffusion can indeed remove the dust density discontinuity at the dust boundary but cannot remove the discontinuities of the dust density derivatives; (2) The values of these derivatives at the boundary in most cases are rather large (leading to very large but finite values and thus imitating the previously obtained jumps without taking into account the ion diffusion); (3) the calculation indicates the existence of regions with large values of dP/dx or even singular points with $dP/dx \to \infty$ where the dust pressure becomes important.

Note that only the dust pressure effect without effects associated with the ion diffusion cannot determine itself the thickness of the dust boundary since Eq. (6) is decoupled with other equations of the system. Similarly, the ion diffusion only, without the dust pressure taken into account, cannot determine the dust boundary thickness as we demonstrated in the previous section. Only the simultaneous effect of the dust pressure and the ion diffusion has to be taken into account to describe these effects.

VI. DUST STRUCTURES WITH THE CONTINUOUS CHANGE OF THE DIRECTION OF THE ION FLOW

The dust regions where the ion flow velocity changes its sign should satisfy certain relation. Particularly, from Eqs. (41)–(45) we find that if $u \to 0$ then the first derivatives of n and n_e equal zero (but the first derivatives of u and P are not zero) and the electron density at $u=0$ should be related to other parameters. Denoting the parameters at $u=0$ (i.e., at the point where the ion drift is zero) by the subscript $0d$ (not to be mixed with the subscript 0 used before for the parameters at the center of the void) we have

$$u_{0d} = u'_{0d}(x - x_{0d}), \quad P = P_{0d} + P'_{0d}(x - x_{0d}), \tag{53}$$

where

$$u'_{0d} = \frac{n_{0d} - n_{e,0d} - P_{0d}}{d^2 z_{0d} n_{0d} \alpha_{dr}(0)}, \tag{54}$$

and

$$n_{e,0d}(n_{0d}, P_{0d}, z_{0d}) = \frac{n_{0d} - P_{0d} + P_{0d} n_{0d} \alpha_{ch}(0) d^2 / F_{0d}}{1 + d^2 / x_i F_{0d}},$$

$$F_{0d} = \alpha_D (P_{0d} - n_{0d}) + \frac{1 + 2\alpha_D}{\alpha_{dr}(0) z_{0d}}. \tag{55}$$

According to the previous consideration, the derivative of u is negative at $u=0$ (see Fig. 3 especially, but also all other figures also demonstrate that this derivative is negative at the singular point). It is also natural from the physical point of view to consider this particular case. Equation (54) says that for $u'_{0d} < 0$ we have two possibilities, namely, either

$$P > P_{cr,1} = n - \frac{2 + \frac{1}{\alpha_D}}{\alpha_{dr}(0) z_{0d}}, \tag{56}$$

requiring

$$P > P_{cr,2} = \frac{n_{0d}}{1 + x_i \alpha_{ch}(0) n_{0d}}, \tag{57}$$

or

$$P < P_{cr,1}, \quad P < P_{cr,2}. \tag{58}$$

The difference in physics between these two cases can be found from relation (31), from which follows that $P = P_{cr,1}$ corresponds to the zero total flux at $u=0$. Thus the case (56) and (57) corresponds to the positive total flux at $u=0$, and the case (58) corresponds to the negative flux. We consider numerically both these cases.

The derivative of P at $u=0$ critically depends on the presence of the nonlinear term $u|u|$ in the ion-neutral friction force written as $u(2+|u|)$. Note that if the nonlinearity is neglected we obtain $P'_{0d}=0$. For the nonlinearity taken into account we find

$$P'_{0d} = u'_{0d}|u'_{0d}|\left(2 + \frac{1}{\alpha_D}\right)\left[\frac{\alpha_{ch}(0)}{\alpha_D} + \frac{P - P_{cr,1}}{n_{0d} d^2} + 2\alpha_{dr}(0) z_{0d} u'_{0d}\right]^{-1} \frac{x}{|x|}. \tag{59}$$

We start numerical calculations with these conditions. The value of the dust charge at $u=0$ is obtained using Eq. (55) in the charging equation

$$\exp(-z_{0d}) = \frac{1}{\sqrt{\tau m_i / m_e}} \frac{n_{0d}}{n_{e0d}(n_{0d}, P_{0d}, z_{0d})} z_{0d}. \tag{60}$$

We have two free parameters to begin with: n_{0d} and P_{0d}. Starting from some of them we need to satisfy either Eqs. (56) and (57) or (58). If we chose $P_{0d}=0.5$ we find that Eq. (58) is satisfied for n_{0d} of order or larger than 4 and Eqs. (56) and (57) for n_{0d} about 0.6. Between these values of n_0 neither Eq. (58) nor Eqs. (56) and (57) are satisfied (the parameter P_{0d} in this case appears to be larger than one of the critical values and lower than the other critical value). Thus there are two possible branches. For $n_{0d}=4$ we find from the above expressions that $ne_{0d}=3.714$ and $u'_{0d}=-0.5317$, $z_{d0}=2.91$, $P'_{0d}=-0.224$, i.e., we obtain a sufficiently large derivative of the ion flow velocity and a large dust charge. On the other hand, the absolute value of the ion flow velocity is not large, and the velocity changes its sign at $u=0$. With these values, we solve equations in the dust region in both directions until we reach the point where $P=0$ and then integrate equations in the void region until we reach (from the void boundary at $P=0$ to the left with respect to the point where we have started the calculations) the point where $u=0$. This corresponds to the center of the void and gives its size. The same procedure can be performed at the right point where $P=0$ when we reach the center of the void to the right with respect to the point we have started the calculations. Although we did not perform the latter calculation (to the right of the right point where $P=0$ having in mind that different boundary conditions can be used at the right boundary of the void)—we can still expect there the wall boundary condition or a next void and the next dust region. Everything depends on the relation of the size of the structure (size of the central

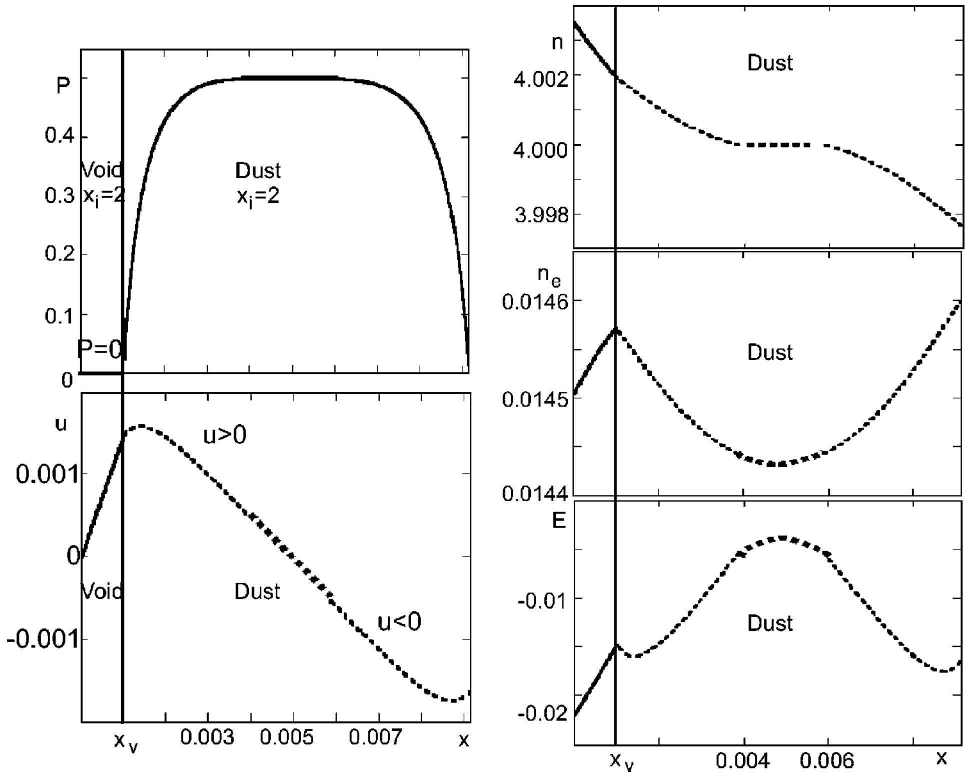

FIG. 8. Distributions of the dimensionless parameters in the void and dust regions with the continuous change of the direction of ion flow. The normalized ionization rate is $x_i=2$ both in the dust and in the void regions.

void plus the size of the dust structure) and the size between the electrodes. If the size between the electrodes is much larger than the combined size of the central void and the dust layer, we in principle can obtain a number of dust sheaths separated with dust voids. This is exactly the case of the complex structure found when solving the numerical problem for the parameters we obtained (the branch corresponding to $n_{0d}>4$) shown on Fig. 8 for the ionization rate $x_i=2$ equal in the dust and in the void regions and $\tau=0.05$. It is remarkable that the whole structure is of the order of or less than the ion-neutral mean free path (which in our dimensionless units is about 0.05). The dust layer appears to be not symmetric relative to the point where we start the calculations at $u=0$. This effect is related to the nonlinearity in the ion-neutral friction force (being in our dimensionless units equal to $u(2+\alpha_n|u|)$ when the second nonlinear term $\propto u|u|$ determines the derivative P'_{0d}. Also we can notice discontinuities of derivatives of P, n_e, and E at the void boundary supporting the previous statement that the ion diffusion is not removing them although discontinuity of the dust density is disappearing because of the ion diffusion. The discontinuity of the electric field does not mean that a surface charge is located at the void boundary since the electric field is continuous, but it rather means the presence of surface dipole moments at the void boundary. Also, the void size for the parameters we use here is smaller than the size of the dust region.

The calculations were also performed with another set of the initial parameters at the point $u=0$. Large size dust structures with a thin void as well as a thin dust structure with a large void (and intermediate cases) were observed. As an example, we present in Fig. 9 the dust structure for $x_i=8$ in the dust region and $x_i=2$ in the void region. The decrease of the ionization rate in the dust region is a natural effect since fast plasma electrons (mainly producing the gas ionization) are effectively absorbed by the dust grains and the ionization rate $1/x_i$ is determined by the integral of the ionization cross section and the electron distribution function. Other parameters taken for this example are $n_{0d}=3$ and $P_{0d}=0.3$. The calculations on the basis of expressions given in this section result in $z_{0d}=2.9$, $P_{cr,1}=0.525$, and $P_{cr,2}=0.385$; thus P_{0d} is larger than both $P_{cr,1}$ and $P_{cr,2}$ satisfying one of the possible necessary conditions. The calculations of the dust structure give (see Fig. 9) the size of the dust region 0.26 and the void region in front of it at the boundary $x_v=0.091$.

VII. SHOCKLIKE DISSIPATIVE DUST STRUCTURES IN THE DUST REGION

We already mentioned above that the singularity in the dust region is related to steepening of the ion density and the

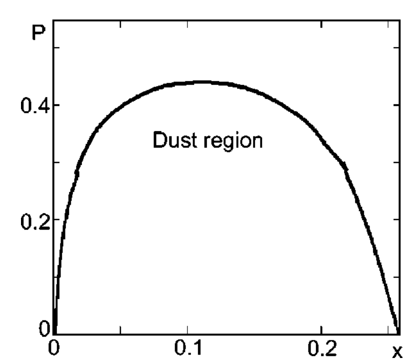

FIG. 9. Parameter P in the dust region for $x_i=2$ in the void region and $x_i=8$ in the dust region. The dust void (size $x_v=0.091$) is to the left (not shown). Other parameters are $z_{0d}=2.9$, $P_0d=0.3$ $<P_{cr,1}=0.525$, $P_{cr,2}=0.386$, $n_{e0d}=2.77$, and $n_{0d}=3$.

parameter P. These new singularities introduced by the ion diffusion can in principle create a new type of "standing shocklike" discontinuities where the ion flow velocity abruptly changes. Contrary to the usual well-known shocks where parameters on both sides of the shock are typically constant, the discontinuities in a dusty plasma cannot be of that kind due to numerous dissipative processes involved. Therefore one can only consider a local discontinuities where the parameters change in a complicated manner (according to the balance equations) on both sides of the discontinuity. Among these discontinuities can be the jumps with the change of the direction of the ion flow velocity. We now demonstrate that the number of equations for these "shock-like" jumps is in fact sufficient to find all the parameters (u_2, n_2, z_2, P_2) at one side of the discontinuity if the relevant set of parameters (u_1, n_1, z_1, P_1) is known at the other side of the jump. Note that the electric field E and the electron density n_e, according to the basic equations written above, cannot have such discontinuities. Below, we also include in the basic equations the ion ram pressure effects which correspond to the substitution

$$\frac{1}{n}\frac{dn}{dx} \to \frac{1}{n}\frac{dn}{dx} + \frac{du^2}{dx}. \tag{61}$$

The "Hugoniot-type" equations are related to conservation of the ram pressure and the ion pressure

$$u_1^2 + \ln(n_1) = u_2^2 + \ln(n_2), \tag{62}$$

conservation of the total flux

$$\Phi_1 = \Phi_2, \tag{63}$$

$$\Phi_1 = n_1 u_1 \{1 + \alpha_D[2 + |u_1| + \alpha_{dr}(|u_1|) z_1 (P_1 - n_1)]\},$$

$$\Phi_2 = n_2 u_2 \{1 + \alpha_D[2 + |u_2| + \alpha_{dr}(|u_2|) z_2 (P_2 - n_2)]\},$$

continuity of the dust density [see Eq. (6)]

$$\frac{P_1}{z_1} = \frac{P_2}{z_2}, \tag{64}$$

and continuity of the electron density together with the charging equation

$$\frac{\exp(-z_1)}{z_1 n_1} = \frac{\exp(-z_2)}{z_2 n_2}. \tag{65}$$

The set of four equations (62)–(65) is sufficient to find four values of the system variables at the shock boundary in the case when the corresponding solutions exist. This set of equations is merely a trivial consequence of the basic equations. These equations have obviously a trivial solution without any discontinuity, and it can be clearly seen that if the dust charges are not changing at the jump $z_2 = z_1$ then according to Eqs. (64) and (65) the ion density n and the parameter P are also continuous, which from Eq. (62) means that the ion velocity u does not change its value, and then from Eq. (63) we obtain that the direction of the ion velocity u is also not changing. This is a trivial solution. Thus the change in the dust charge is a necessary condition for such a discontinuity. We can call that "the dust charge jump." Note that for the dust charge discontinuity, the dust density is continuous and only its derivative can have jumps. With no change of the direction of the ion drift velocity, the set of equations (62)–(65) can have other nontrivial solutions and this should be investigated in detail separately. Here we pay attention only to such discontinuities where the direction of the ion flow velocity changes. The presence of the ion diffusion is crucial for these jumps since the change in the direction of the flow velocity means the change in the direction of the convection flux and the flux conservation requires that the diffusion flux should change in the opposite way at the boundary (the latter is quite probable). Thus these jumps have two important properties: (1) *the necessary change of the dust charge*, and (2) *the necessary change of the direction of the ion diffusion flux*. We can therefore call them the *dust-charge diffusive shocks*.

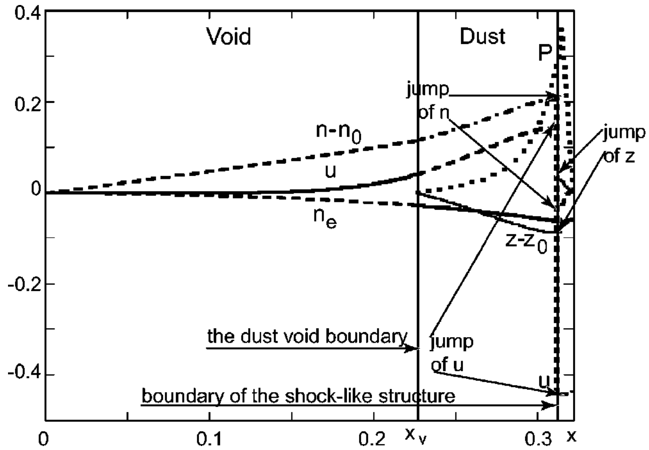

FIG. 10. Shocklike structure in the dust region with the change of the direction of the ion flow velocity. The normalized ionization rate $x_i = 2$ in the void and in the dust regions; $t = 0.05$. The unperturbed normalized density $n_0 = 1$ shown in the figure are $n - n_0$ and $z - z_0$. The normalized ion flow velocity u jumps from 0.2 to 0.44, the normalized ion density $n - n_0$ jumps from 1.38 to 0.97, the normalized dust charge $z - z_0$ jumps from 2.8 to 2.9, the parameter P jumps from 0.34 to 0.36, and the electron density and the electric field have only jump of derivatives.

We give here the result of numerical investigation showing that such dust-charge diffusive shocks can indeed be created in the void-dust structures in the presence of the ion diffusion on neutral atoms, Fig. 10. We see that the jumps of the dust and plasma parameters satisfying relations (62)–(65) with $u_1 > 0$ and $u_2 < 0$ exist in the dust region around the point where $u = 0$. In this example, we start the calculation with $n_0 = 1$ in the void region, find the void size and distributions of plasma parameters in the void region, and continue the calculations in the dust region until we reach the point where large gradients appear. Then we have to satisfy the above relations for the shocklike structure by putting $u_2 = -|u_2|$, i.e., assuming the change in the ion drift direction at the dust-charge diffusive shock boundary. By using Eqs. (62), (64), and (65), in Eq. (63), we obtain one equation to determine the dust charge behind the boundary z_2. All values denoted by the subscript 1 are given along the x direction as

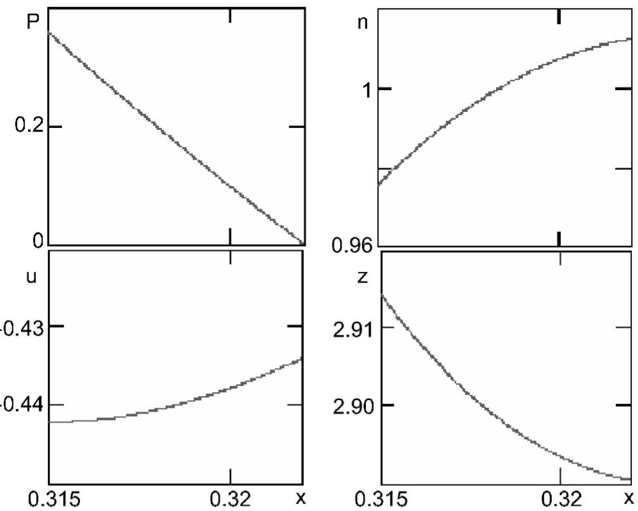

FIG. 11. Behavior of the main dimensionless parameters behind the dust-charge diffusive shock.

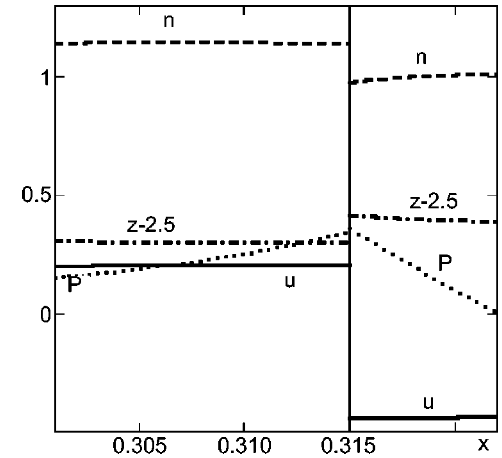

FIG. 12. The main dimensionless parameters in the vicinity of the dust-charge diffusive shock front.

solutions of the equations in the dust region. The next question is whether along this solution one can find a point where Eq. (63) is satisfied. We have found that such solutions indeed exist for our numerical example and that these points are in a narrow area around $x_{sh}=0.315$. We present numerical results for dust-charge diffusive shock for the value of x_{sh}. After that the jump of the dust charge z at the shock front is found; then we use Eqs. (62), (64), and (65) to find all other parameters after and before the discontinuity. The results are (1) the jump of the ion flow velocity is from 0.2 to −0.44, (2) the jump of the ion density is from 1.384 to 0.97593, (3) the jump of the dust charge is from 2.91 to 2.84, (4) the jump of the parameter P is from 0.3595 to 0.3449, and (5) the electron density is continuous (as it should be) but the derivative of the electron density also has a discontinuity. Together with the jumps of the values at the dust-charge diffusive shock front, the derivatives of the corresponding parameters also change. After we obtain all the parameters on the other side of the shock, we further integrate the balance equation in the dust region until we reach the point where $P=0$. Thus the result of the whole dust-void structure including the dust-charge diffusive shock is shown in Fig. 10, and the dependencies of the parameters behind the shock are shown in larger scales in Fig. 11, and the enlarged region of the shock front is shown in Fig. 12.

VIII. DISCUSSION

In the absence of the ion diffusion on neutral gas atoms, derivatives of the ion pressure are discontinuous at the void boundary which is expected since in these conditions the parameter P is discontinuous at the boundary and according to the ion balance relation the parameter P is directly related to the derivatives of the ion density describing the ion pressure effects. In the presence of the ion diffusion no jump of the ion density derivatives at the void boundary is observed. Simultaneously and naturally, there is no jump of the parameter P at the void boundaries. However, the discontinuity of derivatives of other parameters at the void boundary, such as, e.g., dP/dx, are shown to survive if we assume $\tau_d=0$.

We demonstrated that the ion diffusion can cause new discontinuities such as dE/dx and dn_e/dx for the structures which are self-adjusting their parameters preventing the steepening of P and n inside the dust region. These new type of discontinuities initiated by the ion diffusion appear only inside the dust region. They require finite values of the parameter P from both sides of the boundary, jumps of P, n, and u as well as of the dust charge z with the change of the direction of the ion drift velocity and the change of the direction of the diffusion flux. Such dust-charge diffusive shocks cannot appear at the void boundary; they exist only inside the dust region. Note that the dust density, the electron density, and the electric field are continuous at these shocks. The thickness of the discontinuity is determined by the dust pressure or the finite value of the parameter $\tau_d=T_d e^2/aT_e^2$ which for typical experiment data of the dust size of order 10 μm size grains, the electron temperature of the order of a few eV, and $\tau\approx 0.05$, $T_d\approx T_i$, gives $\tau_d\approx 3\times 10^{-6}$. One might expect that the sharpness of the singularities measured in distances λ_{in}/τ will be of the order of τ_d which for a typical experiment is much less than the interdust distance. If the latter is the case, the boundary singularities have a size of the order of the distance between dust particles. During the dust-crystal melting the temperature T_d can rise by three orders of magnitude but still τ_d is estimated to be rather small even in these conditions. We conclude that the presence in a dusty plasma of the sharp boundaries of the dust-charge diffusive shock type and of the type of the void boundaries is a *general property of a dusty plasma*. The ion diffusion does not smooth the singularities of the derivatives of the parameters at the void boundaries and even create a *new diffusive shock-like discontinuities* inside the dust region.

Note that finite dust pressure effect should be a subject of further investigation of the boundaries in dusty plasmas. Furthermore, investigations of sharp boundaries without change of the direction of the ion flow as well as more detailed investigation of the new Hugoniot equations for the dust-charge diffusive shocks could be the next step of research. We also note that the diffusion approximation can be questionable for the case where large gradients exist (as it is in the vicinity of the shock) since the characteristic scale of the gradients can become shorter than the mean free path for the ion-neutral collisions.

The structures investigated so far are self-organized dissipative structures. We found two types of them in the dust region. The first one is the structure which self-consistently adjusts its parameter in order to exclude the singularity; the second one is the dust-charge diffusive shock structure. Both of them can be probably realized experimentally, and the choice between them is determined by the time evolution process which we exclude from our present treatment. The latter can also be used to investigate the stability of the structures found so far.

The found qualitative effect of the presence of two type of solutions when $P > P_{cr,1}, P_{cr,2}$ and when $P < P_{cr,1}, P_{cr,2}$ in the dust region rise the problem that in the void region there can exist also two virtual void boundaries. In the case this is indeed true and in the case both of them are unstable, the real solution can be an oscillatory one bouncing between the two virtual void surfaces. Then can help to explain the observed "heart-beat" mode of the void oscillations. The future investigations should include considerations for finding possible two solutions for virtual void boundaries and their stability.

The effect of the presence of a set of thin dust layers separated by the dust void is the one that can be used for explanations of the observed stratification of the dusty plasma system observed in [1]. Also one can apply the effect found here for observations of the multiple thin dust layers in the upper atmosphere (lower ionosphere) [9]. Another point is that one cannot use the continuous description of thin dust layers when there is less than one dust grain on the unit length, and the set of thin layer divided by thin voids can be considered as a model for a one-dimensional dust crystal. If the calculations will be performed in three dimensions, one in principle should get a model of something similar to a three-dimensional dust-plasma crystal.

Finally, the ion diffusion does not prevent the void formation. It is shown that the latter with the absence of any dust grain in the void region exists, and can create a new type of void boundaries and new type of diffusive dissipative structures in dusty plasmas.

ACKNOWLEDGMENTS

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[1] D. Samsonov and J. Goree, Phys. Rev. E **59**, 1047 (1999).

[2] G. Morfill, H. Thomas, U. Konopka, H. Rothermel, M. Zuzic, A. Ivlev, and J. Goree, Phys. Rev. Lett. **83**, 1598 (1999).

[3] J. Goree, G. E. Morfill, V. N. Tsytovich, and S. V. Vladimirov, Phys. Rev. E **59**, 7055 (1999).

[4] V. N. Tsytovich, Comments Mod. Phys., Part C(Part C) **1**, 1 (2000).

[5] V. N. Tsytovich, Plasma Phys. Rep. **26**, 712 (2000).

[6] V. N. Tsytovich, S. V. Vladimirov, and S. Benkadda, Phys. Plasmas **6**, 2972 (1999).

[7] V. N. Tsytovich, S. V. Vladimirov, G. E. Morfill, and J. Goree, Phys. Rev. E **63**, 056609 (2001).

[8] G. E. Morfill and V. N. Tsytovich, Plasma Phys. Rep. **26**, 682 (2000).

[9] V. N. Tsytovich, Phys. Scr., T **T89**, 89 (2001).

[10] S. V. Vladimirov and K. Ostrikov, Phys. Rep. **393**, 175 (2004).

[11] C. Zafiu, A. Melzer, and A. Piel, Phys. Plasmas **10**, 1278 (2003).

[12] O. Havnes, T. Aslaksen, and A. Brattu, Phys. Scr., T **T89**, 133 (2001).

[13] A. Hasegawa, Adv. Phys. **34**, 1 (1985).

[14] L. S. Frost, Phys. Rev. **105**, 354 (1957).