Modified spontaneous emission from a two-dimensional photonic crystal

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Decay kinetic properties of a two-level atom near the band edges of a two-dimensional photonic crystal are studied based on the Green's function expression for the evolution operator. Our calculations show that the decay kinetics can be controlled by changing the atomic parameters. We also find that there is a quite wide lifetime distribution in the spontaneous emission process.

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Since the pioneering work of Yablonovitch [1] and John [2], the spontaneous emission properties of atoms or molecules in photonic crystals (PC's) have been extensively investigated both theoretically and experimentally. The previous studies show that the gap edge has great influence on the optical behavior of an atom in a photonic crystal, and many interesting effects have been discovered when the resonant transition frequency of the atom is near the photonic band edge, such as the appearance of enhanced quantum interference effects of spontaneous emission [3], coherent control of spontaneous emission [4], nonexponential spontaneous emission , non-Markovian effects [5], etc.

Theoretical studies have played an important role in the progress of this field over the past decade. An isotropic model developed by John and Wang [5] has been almost exclusively adopted to treat this problem. In this model, the dispersion relation of photons in a one-dimensional periodic multiplayer is extended directly to three-dimensional (3D) systems and the vectorial nature of electromagnetic (EM) fields is completely omitted. In the literature [5,6], an anisotropic model has been also introduced to improve the theoretical prediction. This model is reasonable only in the vicinity of the band edge and the vectorial nature of EM fields is also ignored as in the isotropic model. At the same time, some researchers have employed classic electrodynamics to study dipole radiation in 3D photonic crystals [7,8]; however, this classical model cannot account for the virtual physics concerning the spontaneous emission, where vacuum fluctuation plays a key role.

In this paper, we use the full vectorial quantum electrodynamics (QED) theory [9] developed recently to discuss the quantum theory of the spontaneous emission of two-level atoms in a 2D photonic crystal. We show that the in-plane band gap for a two-dimensional photonic crystal can be used to achieve a modification in the spontaneous emission lifetime even though it lacks a complete photonic band gap. The effect of the atomic position in the photonic lattice on the emission rate is also considered. Theoretically, a lifetime distribution function [10] for an assembly of atoms or molecules is numerically evaluated to reveal the dynamic decay processes.

For a two-level atom at $\boldsymbol{r}$ point in a 2D photonic crystal, using a method similar to that used in Ref. [10], we obtain the spontaneous emission lifetime

$$\tau(\boldsymbol{r},\omega_0) \approx 1/\Gamma(\boldsymbol{r},\omega_0); \tag{1}$$

here,

$$\Gamma(\boldsymbol{r},\omega) = 2\pi \sum_{n\boldsymbol{k}} |g_{n\boldsymbol{k}}(\boldsymbol{r})|^2 \delta(\omega - \omega_{n\boldsymbol{k}}),$$

$$g_{n\boldsymbol{k}}(\boldsymbol{r}) = i\omega_0 (2\epsilon_0 \hbar \omega_{n\boldsymbol{k}} V)^{-1/2} E_{n\boldsymbol{k}}(r) \cdot \boldsymbol{u}_d, \tag{2}$$

where ω_0 is the atomic transition frequency; $\omega_{n\boldsymbol{k}}$ is the frequency of the EM eigenmode $E_{n\boldsymbol{k}}(\boldsymbol{r}) = ic\nabla \times \boldsymbol{H}_{n\boldsymbol{k}}(\boldsymbol{r})/[\epsilon(\boldsymbol{r})\omega_{n\boldsymbol{k}}]$ in the PC's; $\boldsymbol{u}_d = u_d \hat{\boldsymbol{u}}$ is the dipole moment between two transition levels. And we define a lifetime distribution function as [11]

$$D(\tilde{\tau}) = \sum_i W_i \delta(\tilde{\tau} - \tilde{\tau}(\boldsymbol{r_i}, \omega_0)), \tag{3}$$

where $\tilde{\tau}(\boldsymbol{r},\omega_0) = \tau(\boldsymbol{r},\omega_0)/\tau_v(\omega_0)$, $\tau_v(\omega_0)$ being the spontaneous emission lifetime in vacuum, denotes the lifetime ratio, $\boldsymbol{r}_i$ denotes the position of the ith atom (molecule), and W_i is a weight factor. For the homogeneous distribution of the atoms (molecules) in space and the random orientation of the dipole moment, we have $W_i = 1$.

Using a nonorthogonal finite-difference time-domain (FDTD) method [12], we can calculate the Green's function. The local density of states (LDOS) is found from the imaginary part of the trace of $G^R(\omega)$ [14]:

$$\rho(\omega,\boldsymbol{r}) = -\frac{1}{\pi} \mathrm{Im}[\sum_j G^R_{jj}(\omega,\boldsymbol{r},\boldsymbol{r})]. \tag{4}$$

At a given frequency, the LDOS in a nonorthogonal FDTD calculation is proportional to $\tau(\boldsymbol{r},\omega)$ [13] and can be used to determine the spontaneous emission lifetime as

$$\left[\frac{\rho^{cry}}{\rho^{vac}}\right] = \frac{\tau^{vac}}{\tau^{cry}}, \tag{5}$$

where *cry* denotes the values for the PC's and *vac* denotes the values calculated for a vacuum.

Figure 1 shows an calculated example of LDOS for a single atom at various dipole positions using the method used by Ward and Pendry [12], in a two-dimensional array of dielectric cylinders placed in a triangular lattice. Owing to computational restrictions, a small 7×7 photonic crystal unit

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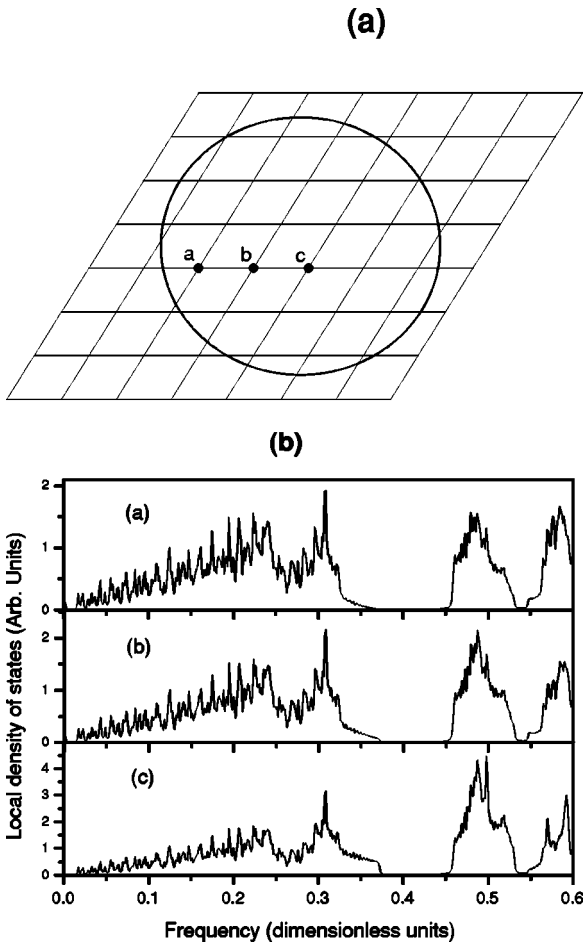

FIG. 1. Position dependence of the local density of states by use of a single atom at different locations in the photonic crystal structure. (a) Schematic indicating the calculated atomic locations in the triangular lattice. (b) Local density of states for the different atomic locations.

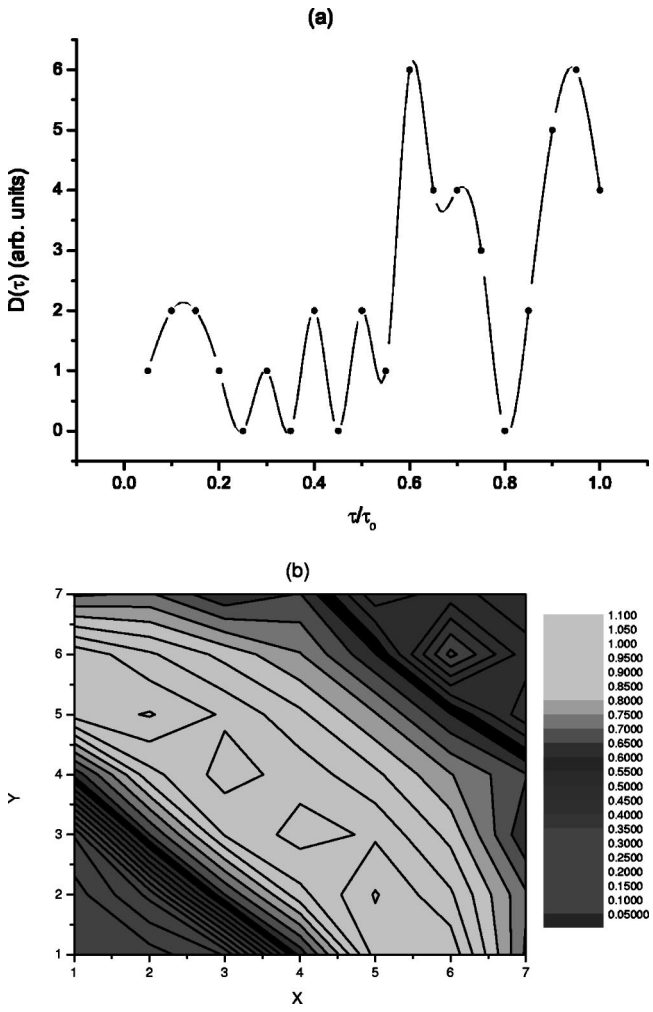

FIG. 2. (a) Lifetime distribution of the atoms in PC's. (b) Variation of the lifetime with atomic position in a unit of cell.

cell domainis used. The dielectric constant of the cylinder is set to 1. For the host medium, the dielectric constant is 13.0. The lattice constant is a_0 and the radius of each cylinder is $0.48a_0$. The LDOS is obtained by assuming a random orientation of the atomic transition dipole moment.

Figure 1(a) shows a schematic of the structure with three different locations, which correspond to the three LDOS in Fig. 1(b). The intervals between the sites we selected and the center of the dielectric cylinders are different, and site *c* is closest to the center. The LDOS turn out to vary slowly near both upper and lower band edges. We can see from Fig. 1(b) that the LDOS curve is remarkably different from the others, implying that the LDOS can be changed dramatically by the choice of position within the photonic lattice. This characteristic is not expected from scalar isotropic and anisotropic models. In all three cases, the band gap region can be seen approximately in the frequency range 0.38–0.45.

The lifetime distributions of the atoms are displayed in Fig. 2 and a very wide lifetime distribution is found. Figure 2(a) shows that the relative width of the lifetime distribution (RWOLD) $T_{rw}=(\tau_{max}-\tau_{min})/\tau_{min}$ is high up to 1400%, for transition frequencies ω_0 at the upper edges of the photonic band gap. Compared with the vacuum lifetime τ_0, the decay process for most of the atoms is remarkably accelerated. Figure 2(b) displays that variation of the lifetime with the atomic position in a unit cell. The red color indicates the maximum of the lifetime, and the black color indicates the minimum of the lifetime. We observe that the lifetime distribution has symmetry, and the lifetime in the high-index regions is higher than that in the low-index regions.

In summary, we have shown that a two-dimensional photonic band gap structure can strongly modify the spontaneous emission lifetime up to about one order of magnitude even though this structure lacks a complete three-dimensional band gap. The lifetime of the spontaneous emission predicted in a single two-level atom is strongly dependent on the choice of atomic position.

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