Role of wave interaction of wires and split-ring resonators for the losses in a left-handed composite

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In this work the analytical model explaining the main reason of transmission losses in the known two-phase lattice of Smith, Schultz, and Shelby representing a uniaxial variant of left-handed medium (LHM) at microwaves is presented. The role of electromagnetic interaction between split-ring resonators (SRRs) and straight wires leading to the dramatic increase of ohmic losses in SRRs within the band when the meta-material becomes a LHM is clarified. This paper explains why in this structure, rather high transmission losses are observed in the experimental data, whereas these losses in a separate lattice of SRRs and in a lattice of wires are negligible at these frequencies.

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I. INTRODUCTION

In his seminal work [1] Veselago summarized an extensive study of electromagnetic properties of media with real and negative parameters called left-handed materials (LHM). Since 2000 this topic has become a subject of an abundant discussion initiated by Pendry [2]. In 2001, the negative refraction (a feature of LHM that allows one to distinguish these media from conventional ones) was demonstrated in the microwave range by the group of Smith [3]. Since 2001 this result has been reproduced several times (see [4] and [5]) and is now considered reliable. The material designed in [3] is essentially a two-phase composite, where the two phases are respectively responsible for electric and magnetic polarization effects. The negative real part of effective permittivity $\epsilon_{\rm eff}$ was created by an array of parallel conducting wires (whose direction determines the optical axis x of the composite medium), which is known to behave similarly to a free-electron plasma at enough low frequencies. The negative value of the real part of permeability $\mu_{\rm eff}$ was provided by double split-ring resonators (SRRs), see also [6]. This negative permeability arises due to the resonance of the magnetic polarizability of SRRs within a very narrow subband which belongs to their resonant band. This subband lies in the wide frequency range where $\mathrm{Re}(\epsilon_{\rm eff})<0$ (more exactly the real part of a xx component of the permittivity tensor is negative). The meta-material becomes a LHM within this subband for waves polarized along x and propagating orthogonally to this axis. These electromagnetic waves suffer the magnetic and dielectric losses. There has been literature written about these losses since the work by Garcia and Nieto-Vesperinas [7]. In this work the structure tested in [3] is treated as opaque and an exotic explanation of negative refraction is presented. Comparing the results obtained by the group of Garcia ([7,8], etc.) or the results obtained by the group of Efros (e.g., in [9]) with the data of the group of Soukoulis (e.g., in [10]) one can see that different simulations of the same lattice give very different results for the transmission losses per unit thickness (from practical absence to huge values). Experimental data rarely fit with these simulations and give moderate values for transmittance in the LHM regime. The disagreement of the data from [10] with the experiment is referred to in this work as the influence of the dielectric board, but it is only a guess. In [11] the moderate transmission losses were obtained for a LHM in which the wires are located in free space and SRRs are positioned on very thin dielectric sheets. It has not been clearly stated which losses dominate in this meta-material: ohmic losses or dielectric losses. However, it is clear from [11] that the losses of LHM are rather significant in the microwave range and should be studied once more. Note that few alternative versions of a LHM at microwaves were suggested in the literature, e.g., [12] and [13]. In [12] the self-consistent analytical theory of the quasi-isotropic lattice of metal bi-anisotropic (Ω) particles has been presented, and the negative parameters predicted within the band 8.2–8.4 GHz. However, the losses were neglected in this work. In [13] the experimental testing of a LHM made from SRRs combined with capacitively loaded strips (instead of long wires) has been done. The aim of this work (as well as [12]) was to obtain an isotropic variant of LHM and to match this medium to free space. The losses in this structure are very high (the mini-passband corresponding to negative material parameters is almost invisible within the band of the resonant absorbtion of SRRs), and the band in which both material parameters extracted from measured data have negative real parts is rather narrow. A 50 MHz passband has been detected at 9.5 GHz and another one was located around 11 GHz (however, at these frequencies the lattice period becomes longer than $\lambda/4$ and the local constitutive parameters are not physically sound). In [14] the transmittance through the layer of a racemic medium from resonant chiral particles is calculated. This medium also exhibits the negative real part of

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constitutive parameters within the resonant band. However, the result for resonant transmission losses obtained in [14] was pessimistic.

Therefore in the present paper we return to the structure suggested by the group of Smith. We use a self-consistent analytical model for its material parameters taking into account ohmic and dielectric losses. The ohmic losses are calculated using the Landau model which takes into account the frequency dependence of the resistance per unit length of a metal ring unlike simulations in [10] and [11] where the ohmic losses were modeled through the frequency independent complex permittivity. The similar structure in its lossless variant has been already studied in [17] where the analytical model was presented for its material parameters. The difference of the structure suggested in [17] with that from [3] was another geometry of a SRR. We considered in [17] the SRRs suggested by Marques instead of SRRs of Pendry (coplanar SRRs suggested in [6]). The resonator of Marques is not bianisotropic, whereas the lattice of Smith, Schultz, and Shelby is in fact a weakly bianisotropic medium within the resonant band of SRRs (see [18]). In the present paper we compare the results obtained for both types of SRRs. The bianisotropy of Pendry's SRRs is neglected in our model.

The aim of [17] was to find the band-gap structure of the lossless meta-material. In the present paper we consider the SRRs of Pendry assuming that these are prepared from a copper wire with a round cross section. In [3] the SRRs were prepared from a thin metal strip. However, this is not a principal difference in what concerns the scattering properties of a SRR as was clearly shown in [19] and [20]. Our choice of the usual wire instead of a strip wire is explained by the absence of an analytical model of losses for curved strip wires. We consider at the first step the lattice of parallel SRRs and at the second step we study the meta-material from [3]. Comparing the result for the effective permeability of two meta-materials, SRRs with wires and SRRs only, we can see the role of the electromagnetic interaction in the two-phase material.

II. CALCULATIONS OF MAGNETIC LOSSES IN A LATTICE OF SRRS

In the model of the dense lattice of SRRs presented in [6], the calculation of ohmic losses did not take into account the curvature of wires. In this section we calculate these losses, using the Landau formula for a wire ring [15]. To find the permeability, we apply the rigorous model of the dipole lattice developed in [16] for electric dipoles and in [21] for nonreciprocal magnetic dipoles. The result we obtain in this section confirms the result from [6]: the resonant absorption in this lattice is rather small in the frequency band where $\mathrm{Re}(\mu_{\rm eff})<0$.

The reliable analytical model of a single SRR is needed to calculate a magnetic polarizability which enters into the dispersion equation of a lattice of magnetic dipoles derived in [21]. The magnetic polarizbility is defined as a relation of its magnetic moment m to the local magnetic field (polarized along the y axis, i.e., orthogonally to the SRR plane)

$$a_{mm} = \frac{m}{H^{\rm loc}}.$$

We consider SRRs consisting of two wires with a round cross section. The model of such SRRs was developed and validated by numerical simulations in our works [19,20] for the lossless case. The conductivity resistance of the ring can be calculated using the formula [16]

$$R_c = \mathrm{Re}\left(\frac{2\pi r}{\sigma' \pi r_0^2}\right). \tag{1}$$

Here r_0 is the radius of the wire cross section and σ' is the effective complex conductivity which is expressed through the metal conductivity σ as follows:

$$\sigma' = \frac{2\sigma J_1(\kappa r_0)}{\kappa r_0 J_0(\kappa r_0)}, \quad \kappa = \frac{(1-j)}{\delta},$$

where $\delta = 1/\sqrt{\sigma\omega\mu_0}$ is the skin depth. In this work we use the time dependence $\exp(j\omega t)$ commonly adopted in the theory of microwave composites. The model of a loss-less SRR from [20] is rather accurate but sophisticated. It takes into account the nonuniformity of the current induced in both rings and allows one to calculate not only the magnetic polarizability of a single SRR but also electric and magnetoelectric polarizabilties. These additional polarizabilities are related with the nonuniformity of the induced current around the rings of SRR. Let us neglect this nonuniformity assuming the electric and magnetoelectric resonances of a scatterer hold at different frequencies than the magnetic resonance. This assumption can be checked in the theory [20]. Then, the long formula (12) from [20] for magnetic polarizbility of a single SRR simplifies to a classical two-time derivative Lorentz relation [formula (19) from [20]]:

$$a_{mm} = -\frac{\omega^2 \mu_0^2 S^2}{L + M(\omega^2 - \omega_0^2) + j\omega\Gamma}. \tag{2}$$

Here $S = \pi(r_1^2 + r_2^2)/2$ is the averaged area of SRR, where r_1 and r_2 are radii of the outer and inner rings (which are assumed to be close to one another), $L = (L_1 + L_2)/2$ is the averaged proper inductance of rings and M is their mutual inductance.

Let us first consider the lossless case. Then the factor Γ in Eq. (2) determines radiation losses in the random medium of SRRs [19] and is proportional to the radiation resistance of the SRR, $\Gamma = \eta(k^2 S)^2/6\pi(L+M) = R_{\rm rad}/(L+M)$. Here η is the wave impedance and $k = \omega\sqrt{\epsilon_0\epsilon_m\mu_0}$ is the wave number of the host medium. Consequently, $\Gamma \sim \omega^4$ [see formulas (17) and (20) from [20]]. This result allows our model of SRR to satisfy the basic condition for any magnetic dipole (for electric dipoles this condition was introduced by Sipe and Kranendonk in [22]):

$$\mathrm{Im}\left\{\frac{1}{a_{mm}}\right\} = \frac{k^3}{6\pi\mu_0}. \tag{3}$$

Notice that the condition (3) (which is obvious for both classical and quantum scatterers and fits with the well-known Landau correction to the Lorentz theory of dispersion) was

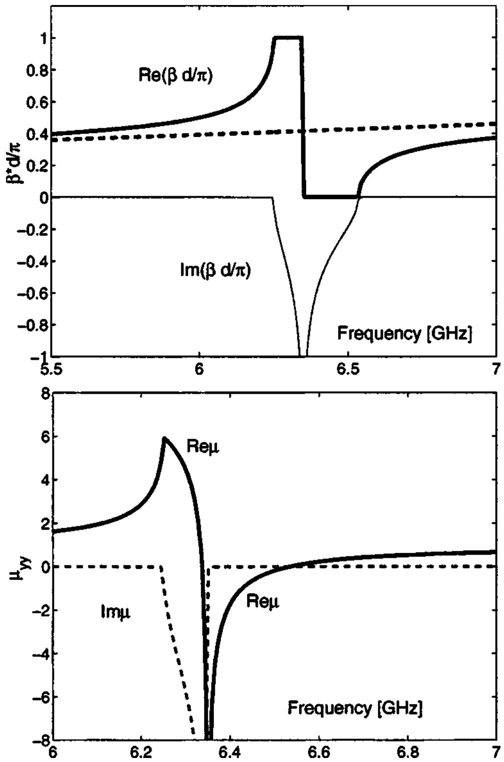

FIG. 1. Top: Dispersion plot of a cubic lattice of SRRs of copper. Bottom: Effective permeability of a lattice.

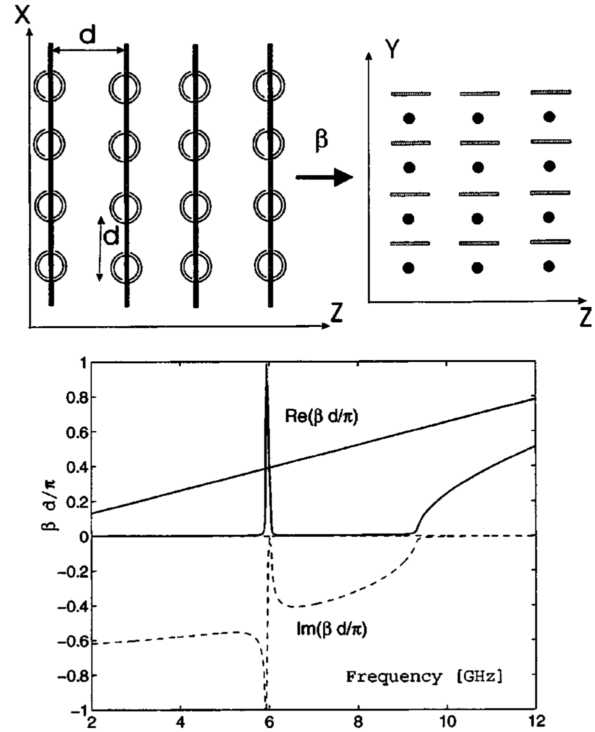

FIG. 2. Top: Structure under study. SRRs which are parallel to the plane $(y-z)$ are not taken into account since these are not excited by the mode propagating along z. Bottom: Dispersion plot of the structure. The straight line corresponds to the noninteracting mode with polarization $\mathbf{H}=H\mathbf{x}_0$, $\mathbf{E}=E\mathbf{y}_0$.

violated in the approximate model introduced in [6].

Let us now study the cubic lattice of parallel lossless SRRs with period d. To find $\mu_{\rm eff}$ of the lattice we use the relation $\mu_{\rm eff}=\sqrt{n}$, where n is the refraction index related with the propagation factor β of the basic propagating mode $n=\beta/k=\beta/\omega\sqrt{\epsilon_0\mu_0\epsilon_m}$. Factor β (within the basic Brillouin zone) can be found from the known dispersion equation for modes propagating along the x or z axes in a lattice of magnetic dipoles [21]. Equation (21) from [21] for a simple cubic lattice of parallel magnetic dipoles can be written in our notations as

$$\frac{\omega}{2\eta d^2}\frac{\sin kd}{\cos kd-\cos\beta d}=\mathrm{Re}\left(\frac{1}{a_{mm}}\right)-\frac{\omega}{4\eta d^2}\times\left(\frac{\cos kR}{kR}-\sin kR\right), \qquad (4)$$

where $\eta=\sqrt{\mu_0/\epsilon_0\epsilon_m}$ and $R\approx d/1.438$. The same equation can be obtained from [15] using the duality principle. This equation was obtained using the method of the local field. It gives the band-gap structure of the lattice with real β in passbands and negative imaginary β in stop bands. Within the resonant band of a scatterer when $|a_{mm}|\gg 2\eta d^2/\omega$ the well-known complex mode (inherent to metallic photonic crystals) appears and $\beta=\pi/d+j\,\mathrm{Im}\,\beta$. One can see from Eq. (4) that the radiation resistance of the scatterer does not influence the final parameters of a lattice. This results from the electromagnetic interaction in regular structures. The imaginary part of the interaction constant of arbitrary regular lattice exactly compensates the contribution of the radiation resistance into inverse polarizability of scatterers [15]. In Eq. (4) the term $\mathrm{Im}(1/a_{mm})$ has disappeared since it was exactly compensated by the imaginary part of the lattice interaction factor [21].

Now, let us consider the case when the total conductive resistance of the SRR $R_c^t=R_{c1}+R_{c2}$ is nonzero. In this case the relation (3) naturally generalizes to

$$\mathrm{Im}\left\{\frac{1}{a_{mm}}\right\}=\frac{R_{\rm rad}+R_c^t}{\omega\mu_0^2S^2}=\frac{k^3}{6\pi\mu_0}+\frac{R_c^t}{\omega\mu_0^2S^2}. \qquad (5)$$

The term with k^3 is totally compensated in the dispersion equation of a magnetodipole lattice by the interaction constant of the lattice, but the second term in the right-hand side of Eq. (5) remains and modifies Eq. (4). In this lossy case we should substitute into Eq. (4) the value $1/a'_{mm}$ instead of $\mathrm{Re}(1/a_{mm})$. Here a'_{mm} is given by Eq. (2) with the substitution $\Gamma=R_c^t/(L+M)=(R_{c1}+R_{c2})/(L+M)$. Resistances $R_{c1,c2}$ are given by Eq. (1) for both rings.

Our numerical example corresponds to the following parameters: relative permittivity of the host matrix $\epsilon_m=1.5-j0.002$, $r_1=1.5$ mm, $r_2=1.2$ mm, and wire radius $r_0=50$ mcm (it is still 80 times as large as δ at 6 GHz). Copper conductivity $\sigma=5.8\times10^7\ \Omega^{-1}\ \mathrm{m}^{-1}$. The period of a lattice of SRRs was taken equal, $d=8$ mm. The dispersion plot (see Fig. 1, top), shows the normalized propagation factor $\beta d/\pi$ versus frequency and contains two results. The first one (straight dashed line) corresponds to the polarization of the electric field along y and of the magnetic field in the plane $(x-z)$. Then the SRRs are not excited. The second one corresponds to another polarization of the mode, when the SRRs are excited. There is practically the lossless complex mode within the lower half of the resonant frequency band of SRRs and the usual stop band within its upper half. The difference with the lossless case exists but is not visible in the plot. In fact, the losses make $\beta d/\pi$ complex at all frequencies, however, this correction is maximally of the order 10^{-4}.

Therefore the result for $\mu_{\rm eff}$ presented in Fig. 1, bottom, is optimistic. Within the band 6.35–6.56 GHz we obtained $\mathrm{Re}(\mu_{\rm eff})<-0.1$ whereas $|\mathrm{Im}(\mu_{\rm eff})|<3\times10^{-4}$. Though we have considered SRRs from wires with a round cross section, we have chosen a very small value for r_0. We expect that our results correspond to a strip wire with a width of a few tenths of mm. However, we will see below that this result cannot be expanded to the lattice of SRRs and wires. Though the quasistatic interaction between SRRs and wires is absent in the lattice [3], the wave interaction exists. It was clearly demonstrated in [17] that this interaction (in the lossless case) strongly influences the real part of the material parameters. In the next section we will find its influence to their imaginary parts.

III. LATTICE OF WIRES AND SRRS

In this section we study the effective material parameters of the two-phase lattice of SRRs and wires. It is almost the same structure as in [17] but the metal of SRRs is not perfect (e.g., copper) and the background dielectric has small losses. We assume that the wires and SRRs are located in a uniform host medium (see Fig. 2, top). The analytical model does not allow one to calculate the true losses in the meta-material described in [3]. In our model the dielectric sheets on which the SRRs are located are replaced by a uniform host medium with relative permittivity ε_m. Since we neglect the influence of dielectric interfaces the comparison of our results with experimental data can be only qualitative. Our goal is to understand the impact of the electromagnetic interaction in the lattice on these losses. In fact, in [17] the structure uses the modified SRRs of Marques, whose electric polarizability is very small at the resonance of a_{mm}, and magnetoelectric polarizability is zero. In the present paper we consider the SRRs of Pendry, the ones used in [3]. However, it does not change the theory since we neglect the electric and magnetoelectric polarizabilities of SRRs. These are not negligible for quantitative calculations but are not important enough for our purposes. The only condition is critical: the resonant bands of the magnetic and electric polarizabilities should not overlap. This condition is respected in our studies. All we need in the theory is to add the term R_c^t to the radiation resistance of a SRR denoted as R_r in formula (10) of [17]. Then, following the theory [17] we come to the dispersion equation of the structure presented in Fig. 2 (top):

$$\cos^2\beta d(gp-1)-\cos\beta d[2gp\cos kd+(g-p)\sin kd] \\ -\sin^2 kd(1+gp)+gp+(g-p)\sin kd\cos kd+1=0. \tag{6}$$

This equation refers to a wave propagating along z (the wave vector has the only Cartesian component denoted as β) with electric polarization along x. In Eq. (6) the following notations are used:

$$g=\frac{2d^2\eta}{\omega}\left(\frac{1}{a'_{mm}}\right)-q,\quad p=\frac{kd}{\pi}\log\frac{d}{2\pi r_w},$$

$$q=\frac{1}{2}\left(\frac{\cos kR}{kR}-\sin kR\right).$$

Here r_w is the radius of straight wires. Relation (6) results of theory [17] with the only substitution $\mathrm{Re}(1/a_{mm})\to1/a'_{mm}$. Longitudinal components of the effective permittivity ϵ_{xx} and μ_{yy} are determined (respectively) by formulas (35) and (36) of [17].

In our numerical example, the lattice of SRRs is the same as in the preceding section and it is assumed that $r_w=r_0$. The result for permeability dramatically differs from the previous result; this is due to the electromagnetic interaction of SRRs and wires. This interaction is described by the parameter B in Eqs. (27) and (28) of [17]. If one formally puts $B=0$, the problem splits in two independent dispersion equations, one for the medium of SRRs and one for the wire medium. Then $\mu_{\rm eff}$ keeps the same values as in Fig. 1.

In Fig. 2 (bottom), we have shown the dispersion plot of the structure. This plot corresponds to the same permittivity of the host medium $\epsilon_m=1.5-j0.002$ as in the previous section. Comparing this plot with a similar plot for lossless structure (Fig. 5 of [17]) one finds that the complex mode (in Fig. 5 it corresponds to the lower half of the resonant band of SRRs) disappears. Instead, the forward wave with strong attenuation appears in this resonant subband. This forward wave corresponds to $\mathrm{Re}(\mu_{\rm eff})>0$ and $\mathrm{Re}(\epsilon_{\rm eff})<0$, and the propagation is possible due to the complexity of constitutive parameters. The attenuation factor is larger than the propagation one in this band. The upper half of the resonant band of SRRs contains the backward wave as well as in Fig. 5 of [17]. The ohmic losses broaden the frequency band of this backward wave, however, these also produce the visible imaginary part of the propagation factor in this band. This imaginary part is not so high as in the lower half of the resonant band and is related with ohmic losses in SRRs. It is practically not affected by the imaginary part of ϵ_m (until the threshold $\epsilon_m=1.5-j0.1$, when the influence of the dielectric losses in the matrix becomes visible in the dispersion plots). In this subband $\mathrm{Im}(\beta)$ decreases if one increases the wire radius r_0 and increases if one decreases σ.

In Fig. 3 we present the frequency behavior of $\epsilon_{\rm eff}$ and $\mu_{\rm eff}$ for the structure under study. Here $\epsilon_{\rm eff}$ means the xx component of tensor $\overline{\overline{\epsilon}}$, and similarly $\mu_{\rm eff}\equiv\mu_{yy}=\mu_{zz}$. On top the wide-band frequency dependence is shown for real parts of $\epsilon_{\rm eff}$ (thin line) and $\mu_{\rm eff}$ (thick line). One can see that both parameters are resonant. This is the result of the electromagnetic interaction between wires and SRRs (which also leads to the small shift of the resonant frequency for $\mu_{\rm eff}$ compared to that of a single lattice of SRRs). Though the resonance of permittivity is weak, it remains negative within the resonant band of SRRs. This resonance confirms once more that the very simplistic approach, in which the constitutive param-

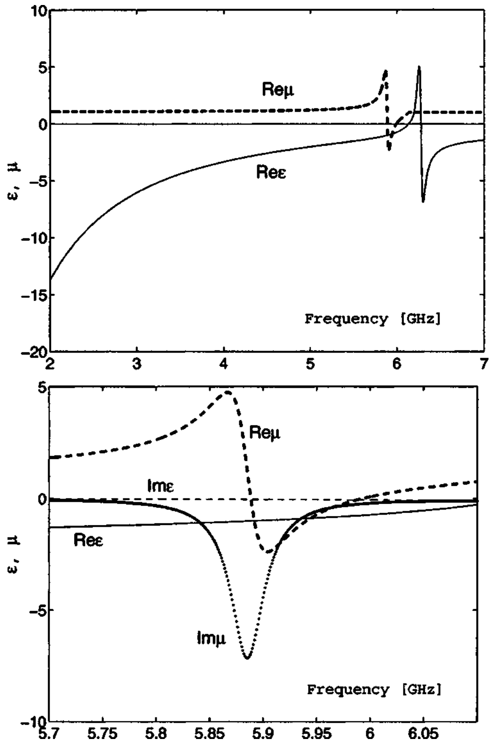

FIG. 3. Top: Real parts of $\epsilon_{\mathrm{eff}}, \mu_{\mathrm{eff}}$ in the wide frequency range. Bottom: Resonant frequency band. Real and imaginary parts of ϵ_{eff} and μ_{eff} vs frequency.

eters of the meta-material are obtained with the trivial superimposition of μ of SRRs and ϵ of wires, is not applicable in most of cases [17]. On the bottom of Fig. 3, both real and imaginary parts of these parameters are presented within the resonant band of SRRs. The band of the backward wave (the upper subband of the resonant band of SRRs) is the one in which the meta-material becomes the LHM. Unlike in the precedent section, in this subband the absolute value of $\mathrm{Im}(\mu_{\mathrm{eff}})$ is quite high. We obtained $|\mathrm{Im}(\mu_{\mathrm{eff}})| \approx (0.1\text{–}0.2)|\mathrm{Re}(\mu_{\mathrm{eff}})|$ within the frequency band of the backward wave (i.e., of the negative material parameters). This result qualitatively explains the moderate transmission losses obtained numerically in [11] and experimentally in [5].

Note that introducing the finite conductivity for straight wires practically does not change the result. The ohmic losses in straight wires do not pose a difficult problem since the negative permittivity of wire lattice is not resonant. But the permeability of the lattice of SRRs is resonant and it makes possible the strong influence of wires to SRRs. This is the reason for high magnetic losses in the backward-wave regime.

Additionally we have made calculations for the case when SRRs of Pendry are substituted by SRRs of Marques (parallel broken rings). An analytical model of the SRR has been presented in [17]. The structure under study is shown in Fig. 4 (top). The following sizes of SRRs were chosen: the outer diameter of rings 3.8 mm, radius of the wire $r_w=50$ or 200 μm, and the distance between the centers of parallel rings $h=0.84$ or 0.72 mm. This study was done in order to check the frequency behavior of the lattice effective permittivity and to understand the influence of the ring cross section to the transmission losses. The parameters of SRRs were picked up so that to obtain the resonant frequency close to 6 GHz as above.

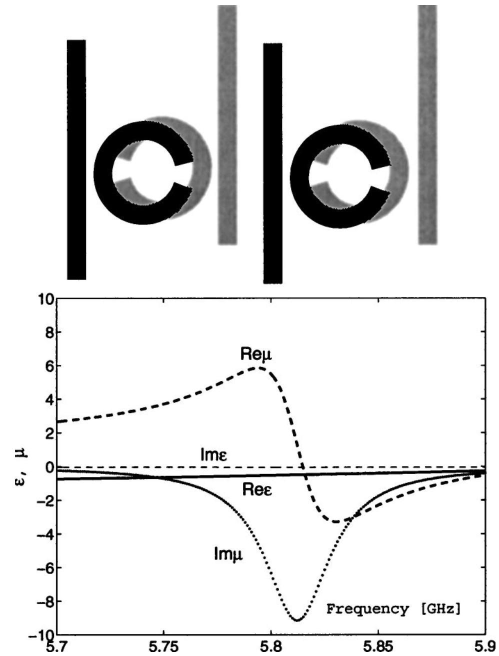

FIG. 4. Top: Structure under study. SRRs which are parallel to the plane $(y-z)$ are not shown since these are not excited. Bottom: Resonant frequency band for the case of thick rings ($r_w=0.2$ mm). Real and imaginary parts of ϵ_{eff} and μ_{eff} vs frequency.

The results for the case $r_w=50$ μm, $h=0.84$ mm are qualitatively the same as in Fig. 3. We can conclude that the design of SRRs is not very important for the imaginary part of permeability which determines the transmission losses. The resonant behavior of both types of SRRs is similar (two-time derivative Lorentz dispersion), This why the electromagnetic interaction of SRRs with straight wires leading to the resonance of the effective permittivity and to the increase of transmission losses is the same for both designs. However, the results for $r_w=0.2$ mm ($h=0.72$ mm) shown in Fig. 4 are different. Though we obtain the same level of magnetic losses [$\mathrm{Im}(\mu)$ is rather significant] the imaginary part of the normalized refraction index turns out much smaller than for the case of thin rings. Really, let the refraction index be defined as $n=\sqrt{\varepsilon_{\mathrm{eff}}\mu_{\mathrm{eff}}}=-n'-jn''$, where $\varepsilon_{\mathrm{eff}}=-\varepsilon_r-j\varepsilon_i$, $\mu_{\mathrm{eff}}=-\mu_r-j\mu_i$, and n', n'', ε_r, ε_i, μ_r, and μ_i are all positive values. This definition of the refraction index corresponds to the correct choice of the sign before the square root in the LHM region. Then we easily obtain

$$n'' = \left(\frac{\varepsilon_i\mu_i - \varepsilon_r\mu_r + \sqrt{(\varepsilon_r^2+\varepsilon_i^2)(\mu_r^2+\mu_i^2)}}{2} \right)^{1/2}.$$

Since ε_i is negligible in the case of thick rings (see Fig. 4) we can simplify this formula and obtain

$$n'' \approx \sqrt{\frac{\varepsilon_r(|\mu|-\mu_r)}{2}}.$$

Outside the resonance band ε_r is not affected by SRRs (thick and thin ones). However, within the resonance band

5.5–6.2 GHz we reduce not only ε_i but also ε_r, increasing the wire radius (compare Figs. 3 and 4). As a result, the refraction index becomes almost real $(n''/n' < 10^{-3})$ in the case of thick rings (r_w=0.2 mm). In spite of the electromagnetic interaction of wires and SRRs the small ohmic losses in magnetic scatterers dramatically reduce the transmission losses per unit length of the lattice [23]. This result qualitatively corresponds to the very small transmission losses observed in [24] for the case of rather thick rings.

Notice that we have studied additionally another arrangement of SRRs excited by the z-propagating wave (rings are parallel to the plane x-z). In this arrangement the layers of SRRs are shifted by $d/2$ along z with respect to the layers of wires. Then the electromagnetic interaction between the wires and SRRs becomes once more significant and the transmission losses become higher than in the case illustrated by Fig. 2. Also the resonant behavior of the effective permeability becomes different and the resonance bands of ε_{eff} and μ_{eff} become overlapping.

IV. CONCLUSION

In this paper we considered the problem of the influence of ohmic losses in the metal and dielectric losses in the host matrix to the magnetic and dielectric losses in the two-phase composite medium formed by a quadratic lattice of infinitely long parallel wires and a cubic lattice of SRRs symmetrically located between the wires. Periods of both lattices are equal to one another. It has been shown that the electromagnetic interaction between wires and SRRs together with ohmic losses in SRRs give rather significant magnetic losses. These losses are absent from the single lattice of SRRs. This way we suggest an explanation of the rather significant transmission losses in the structure from [3] which is considered as a uniaxial variant of left-handed material. We do not deny the significance of the losses in the dielectric board, which support the SRRs prepared from copper strips. However, the contribution of ohmic losses is also rather important. We have shown that the wave interaction of electric and magnetic components of the meta-material is the physical mechanism of the ohmic losses influence to the transmittance of LHM. Small ohmic losses in SRRs lead to rather high transmission losses due to this interaction. It is impossible to avoid it. The transmittance can be improved reducing the ohmic resistance of SRRs. Increasing the cross section the wire from which the SRR is prepared one can theoretically obtain the excellent transmission properties of the meta-material over the whole resonant band in spite of the electromagnetic interaction in the meta-material.

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