Closed fluid description of relativistic, magnetized plasma interacting with radiation field

R. D. Hazeltine and S. M. Mahajan
Institute for Fusion Studies, The University of Texas at Austin, Texas 78712, USA
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A closed set of averaged fluid equations for a relativistic plasma immersed, simultaneously, in a slowly varying magnetizing field and a sharply varying electromagnetic field (radiation field, for example) of arbitrary intensity is derived. The modifications due to the radiation field on the plasma stress tensor and the Lorentz force are explicitly displayed. The resulting equations include the effects of radiation reaction as well as radiation pressure.

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I. INTRODUCTION

Time scale separation

Many astrophysical systems, such as galactic and intragalactic jets, electron-positron ultrarelativistic beams in the pulsar atmospheres, relativistic plasmas in active galactic nuclei, and black hole magnetospheres [1] involve a plasma interacting with two kinds of electromagnetic field: a slowly varying field, sometimes called the background field, that serves to magnetize the plasma, and a radiation field. The time scale (typical period) of the radiation is usually very short compared to the plasma dynamics of interest. On the other hand the radiation field may not be a perturbation of the background field: The field strengths associated with the radiation can easily exceed those of the background.

Because of the separation of time scales, the radiation quickly equilibrates to a nearly thermodynamic equilibrium, at least in the case of an optically thick plasma. The radiation field then affects plasma dynamics on the longer time scale mainly in two ways: radiation reaction forces experienced by accelerated charged particles in the plasma and radiation pressure.

A closed fluid description of a magnetized relativistic plasma, without radiation, has been presented recently [2–4]. The effect of radiation reaction forces on this closure has also been considered [5]. The purpose of the present work is a comprehensive treatment of a plasma fluid interacting with a strong electromagnetic field, including the radiation field. Thus both radiation reaction and radiation pressure—the energy momentum tensor of the electromagnetic field—are included. In addition to the assumptions of time-scale separation and thermal equilibration of the radiation, our work is limited in two other ways. First, while we provide a closed description of the evolution of the (two-species) plasma including radiative effects, the slow-time-scale dynamics of the radiation itself is presumed to be given. The reason for this simplification is that much of the radiation in astrophysical contexts emanates from some region distant from that under consideration. Second, while the slow-time-scale plasma current is allowed to result from a combination of ion and electron motion, we assume that only the electrons participate in the fast dynamics. Since this assumption is based on the small electron-to-ion mass ratio, it may break down in extreme relativistic situations. We emphasize that our study does not assume any ordering between the energy densities of the plasma, the radiation, and the background field. Thus our equations pertain for arbitrary plasma β, the ratio of plasma pressure to background magnetic pressure, and for arbitrary magnitudes of the ratio (plasma pressure)/(radiation pressure).

Magnetized plasma

Including radiation in the energy-momentum balance of an optically thick plasma is straightforward and not new. In this work the modifications due to radiative effects, including radiation reaction, are systematically included in a closed set of fluid equations for a magnetized plasma. Thus the small gyroradius in the background field plays an essential role. The generalization of previous closure arguments, based on small gyroradius, to the case of multiple time scales is not entirely trivial.

Because of its rapid variation—generally including oscillation faster than the gyrofrequency—the radiation field plays no role in magnetizing the plasma. Yet in some cases this field is larger than the magnetizing background field, complicating the standard procedure for computing the plasma stress tensor, $T^{\mu\nu}$, in the small gyroradius limit. The point is that the limit of infinite electronic charge, which in the conventional case yields a simple formula for $T^{\mu\nu}$ [2], now includes a host of terms involving the radiation field. In the absence of a tractable, *local* evolution equation for the radiation, we must use a different scheme to compute $T^{\mu\nu}$.

We adopt the simplest solution to this difficulty, taking the energy momentum tensor to be given by its thermal equilibrium form. In other words $T^{\mu\nu}$ is assumed to be diagonal in the appropriate rest frame, and determined by the pressure and enthalpy. The resulting closed set of fluid equations, while strictly justified only in the collision-dominated limit, is usefully simple, while still containing such key physical processes as radiation reaction, radiation pressure, and their effects on plasma flow, in a covariant way.

The form of the most general energy-momentum tensor for a magnetized plasma is known [2]; it differs from the thermal equilibrium version used here in two ways. First, it allows for anisotropy of the stress, $p_{\parallel} \neq p_{\perp}$, where $p_{\parallel}$ and $p_{\perp}$ refer to pressures parallel and perpendicular to the magnetic field. Second, it includes parallel heat flow, $q_{\parallel}$, a quantity whose evolution is determined by high-order moment equations. Examination of the third-order moment equation reveals that heat flow is strongly modified in the presence of radiation; since it can be the dominant energy transport mechanism, especially at low collisionality, its absence in the present system is particularly regrettable. For this reason we intend to develop, in future work, a fluid closure which includes the radiative version of parallel heat flow.

Notation

Greek indices vary from 0 to 3; we occasionally use Roman indices for the three spatial components (1, 2, 3). Our convention for the Minkowski metric is

$$\eta_{\mu\nu} = \mathrm{diag}\{-1,1,1,1\}.$$

The Faraday tensor (or electromagnetic field-strength tensor) is denoted by $F^{\mu\nu}$. With regard to the plasma fluids, we use the notation of Ref. [2] in which Γ_a^μ is the four-vector particle flow (rest-frame density times fluid four-vector flow) of species a and $T_a^{\mu\nu}$ is the corresponding energy-momentum tensor. The a subscript is suppressed when it is not essential. The four-momentum moment of the collision operator C is denoted by

$$\mathcal{C}_a^\mu = \int d^3p p^\mu C_a,$$

where p^μ is the four-momentum coordinate in phase-space. The corresponding moment of the radiation reaction force, that is, the rate of momentum change of species a due to radiation reaction, is denoted by S_a^μ. Then energy-momentum evolution of plasma species a is governed by

$$\frac{\partial T_a^{\mu\nu}}{\partial x^\nu} - e_a F^{\mu\nu}\Gamma_{a\nu} = \mathcal{C}_a^\mu + S_a^\mu. \tag{1}$$

Here the second term on the left-hand side is the electromagnetic four-force, while the two terms on the right-hand side give the momentum change due to collisions and radiation reaction. It is often convenient to use an abbreviated notation in which tensor indices are omitted and Eq. (1) becomes

$$\partial \cdot T_a - e_a F \cdot \Gamma_a = \mathcal{C}_a + S_a. \tag{2}$$

Collisional momentum conservation implies

$$\mathcal{C}_i + \mathcal{C}_e = 0. \tag{3}$$

On the other hand the small electron-ion mass ratio allows us to neglect S_i. Thus the two single-species equations contained in Eq. (2) become

$$\partial \cdot T_i - eF \cdot \Gamma_i = -\mathcal{C}_e, \tag{4}$$

$$\partial \cdot T_e + eF \cdot \Gamma_e = \mathcal{C}_e + S. \tag{5}$$

The rest-frame particle density of each plasma species is denoted by n_R; a species subscript is omitted on the rest-frame density since we assume the plasma to be quasineutral. The pressure, as a Lorentz scalar, is denoted by $p_a = n_R T_a$, where T_a is the temperature. (The fact that temperature is denoted by the same symbol as the index-free energy-momentum tensor should not cause confusion.) As discussed in the Introduction, we assume that the energy-momentum tensors of both species have the well-known (see, for example, Ref. [6]) thermal equilibrium form

$$T_a^{\mu\nu} = p_a n^{\mu\nu} + h_a U_a^\mu U_a^\nu, \tag{6}$$

where $U_a^\mu = \Gamma_a^\mu / n_R$ is its four-vector flow velocity,

$$h_a = m_a n_R K_3(\zeta_a)/K_2(\zeta_a) \tag{7}$$

is the enthalpy density, K_n are MacDonald functions and $\zeta_a = m_a / T_a$ with m_a the particle mass.

II. PLASMA CURRENT

A. Temporal average

The simplification that allows straightforward incorporation of radiative effects into our fluid description is that the radiation frequencies are large compared to interesting rates of fluid evolution. Evidently this statement cannot hold in an arbitrary Lorentz frame. Rather it is based on the existence of a family of frames, connected by rotations and moderate Lorentz boosts, in which the two time scales are distinguishable, and in which the plasma is magnetized; for most astrophysical phenomena, this family includes the frame at rest with respect to neighboring stars. The use of this special family of reference frames does not preclude the derivation of fluid equations that are fully Lorentz covariant.

Thus we let t_r be a typical wave period of the radiation spectrum and t_s be the time scale for processes described by the fluid equations; we assume that there is an intermediate time period t_i such that

$$t_r \ll t_i \ll t_s. \tag{8}$$

It is then natural to define the temporal average of a physical quantity A,

$$\langle A \rangle = \overline{A} = t_i^{-1} \int_0^{t_i} A(t)\,dt, \tag{9}$$

and to use the conventional decomposition,

$$A = \overline{A} + \widetilde{A} \tag{10}$$

with

$$\langle \widetilde{A} \rangle = 0. \tag{11}$$

An example is the Faraday tensor (or electromagnetic field strength tensor) denoted by $F^{\mu\nu}$. Suppressing tensor indices, we associate $\overline{F}$ with the background electromagnetic field and $\widetilde{F}$ with the radiation.

B. Ion-electron plasma

The remainder of this work studies for simplicity a plasma consisting of electrons and a single species of ions. The current density J^μ is

$$J^\mu = e(\Gamma_i^\mu - \Gamma_e^\mu) \tag{12}$$

$$= \overline{J^\mu} + \widetilde{J}^\mu. \tag{13}$$

A key simplification of our analysis, discussed in Sec. I, is that only the electrons can respond quickly enough to participate in $\widetilde{J}$:

$$\widetilde{J}^\mu = - e\widetilde{\Gamma}_e^\mu. \tag{14}$$

Equivalently, we assume that

$$\Gamma_i^\mu = \overline{\Gamma_i^\mu}. \tag{15}$$

III. ENERGY-MOMENTUM EVOLUTION

A. Total energy-momentum evolution

We use a p subscript to distinguish the energy-momentum tensor of the entire plasma:

$$T_p \equiv T_i + T_e.$$

It is evident from Eqs. (4) and (5) that

$$\partial \cdot T_p - F \cdot J = S. \tag{16}$$

We recall that Maxwell's field equations imply that

$$F \cdot J = - \partial \cdot \Theta, \tag{17}$$

where

$$\Theta^{\mu\nu} = - F^\mu_{\ \alpha} F^{\alpha\nu} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$$

is the energy-momentum tensor of the electromagnetic field. Thus Eq. (16) has the familiar expression in terms of the total tensor $T_p + \Theta$. However, a distinct approach is needed to obtain a closed set of fluid equations. Thus we first consider the temporal average

$$\langle F \cdot J \rangle = \overline{F} \cdot \overline{J} + \langle \widetilde{F} \cdot \widetilde{J} \rangle \tag{18}$$

and then notice that

$$\langle \widetilde{F} \cdot \widetilde{J} \rangle = - \langle \partial \cdot \Theta_r \rangle = - \partial \cdot \langle \Theta_r \rangle, \tag{19}$$

where Θ_r is the energy-momentum tensor of the radiation field alone. In this way we express the average of Eq. (16) as

$$\partial \cdot (\overline{T_p} + \overline{\Theta_r}) = \overline{F} \cdot \overline{J} + \overline{S}. \tag{20}$$

This equation differs from the energy-momentum tensor without radiation in very simple ways and is therefore amenable, in the magnetized case, to the previously described [2] closure procedure. Before reviewing the closure scheme we turn to the energy-momentum conservation of the separate plasma species.

B. Energy-momentum tensors of individual species

The average of the ion equation, Eq. (4),

$$\partial \cdot \overline{T_i} - e\overline{F} \cdot \overline{\Gamma_i} = - \overline{\mathcal{C}}_e \tag{21}$$

does not involve the radiation field. It is effectively the same as the ion equation considered in previous work and requires no special discussion here.

The electron equation,

$$\partial \cdot \overline{T_e} + e\overline{F} \cdot \overline{\Gamma_e} + e\langle \widetilde{F} \cdot \widetilde{\Gamma}_e \rangle = \overline{\mathcal{C}}_e + \overline{S}$$

can be written as

$$\partial \cdot \overline{T_e} + e\overline{F} \cdot \overline{\Gamma_e} - \langle \widetilde{F} \cdot \widetilde{J} \rangle = \overline{\mathcal{C}}_e + \overline{S}$$

or

$$\partial \cdot (\overline{T_e} + \overline{\Theta_r}) = - e\overline{F} \cdot \overline{\Gamma_e} + \overline{\mathcal{C}}_e + \overline{S}. \tag{22}$$

The same result is obtained by subtracting Eq. (21) from Eq. (20).

While radiation effects are prominent in Eq. (22), both of the single-species equations are used to advance the pressures and parallel flows in a familiar way.

C. Field-strength ordering

It is convenient here to make our previous assumptions about the relative strength of the radiation field explicit. Our orderings are maximal in the sense that

(1) The electromagnetic energy-momentum tensor (radiation pressure) is allowed to be comparable to the plasma energy-momentum tensor:

$$\overline{\Theta_r} \sim \overline{T}. \tag{23}$$

(2) The radiation field strength is allowed to be comparable to the strength of the slowly varying, background field:

$$\widetilde{F} \sim \overline{F}. \tag{24}$$

The small parameter associated with our assumed scale-length separation may be identified with the ratio of a typical radiation wavelength, λ, to the scale-length L associated with the dynamics under investigation. Notice that the rapidly varying part of the energy-momentum evolution law requires that

$$\widetilde{T} \sim \frac{\lambda}{L}\overline{T}. \tag{25}$$

In other words, the response of the plasma to the rapidly varying components of the field is relatively small. It is easily confirmed that the three basic orderings (23)–(25) are mutually consistent.

IV. RADIATION REACTION

A. Energy-momentum loss

The energy-momentum loss due to radiation reaction was computed, using Rohrlich's expression for the force [7], in a previous work [5]. For Maxwellian electrons the loss term has the form

$$S^{\mu} = \frac{2}{3}\frac{e^3}{m^2}\left[(\partial_\lambda F^{\mu}_{\kappa})T^{\kappa\lambda} + \frac{2eW}{m}\frac{K_3}{K_2}T\Gamma^{\mu}\right] \tag{26}$$

with

$$W \equiv B^2 - E^2.$$

Recall that the radiation reaction is needed only for electrons; the species subscript is suppressed.

Notice that Eq. (26), like the energy-momentum evolution law Eq. (1) which it enters, consists of terms linear in $T^{\kappa\lambda}$ and in Γ^{μ}. Upon inserting Eq. (26) into (the electron version of) Eq. (1) and by appropriately grouping terms, we obtain the result

$$\left[\partial_\nu \eta^{\mu}_{\kappa} - \frac{2}{3}\frac{e}{m}r_0(\partial_\nu F^{\mu}_{\kappa})\right]T^{\kappa\nu} + \left(eF^{\mu\nu} + \frac{4r_0^2}{3}\frac{TK_3}{mK_2}W\eta^{\mu\nu}\right)\Gamma_\nu = \mathcal{C}^{\mu}, \tag{27}$$

where

$$r_0 = e^2/m$$

is the classical electron radius.

The law, Eq. (27), shows that radiative reaction has two effects.

(1) It corrects the gradient of the stress tensor with an additional term involving the field, analogous to a gauge correction.

(2) It corrects the Faraday tensor in the force term with an additional, symmetric tensor.

B. Relative magnitude

It is convenient to measure the electromagnetic field strength through a representative gyrofrequency,

$$\Omega_* \sim e[F^{\mu\nu}]/m$$

where $[F^{\mu\nu}] \sim E \sim B$ is the magnitude of a typical nonvanishing element of the Faraday tensor, including both the radiation and background fields. The true electron gyrofrequency $\Omega_e = e\overline{B}/m$ can be considered a special case of Ω_*. Recall that we allow the radiation field to be comparable to the background field.

Inspection of Eq. (27) now shows that each of the two corrections from radiation reaction contains a factor $\Omega_* r_0$ compared to the term it corrects. The Faraday correction contains in addition a relative factor of $(T/m)(K_3/K_2)$. In cgs units,

$$r_0 = 2.82 \times 10^{-13}\ \text{cm}$$

corresponding to a time interval

$$t_e = r_0/c \approx 10^{-23}\ \text{s}.$$

The case $\Omega r_0 \sim 1$ is close to the limits of validity of classical and quantum electrodynamics [8]. At much smaller values, $\Omega r_0 \sim 10^{-3}$, pair production, which our fluid model omits, becomes a dominating process. Therefore we assume

$$\Omega_* r_0 \ll 1 \tag{28}$$

and neglect the stress-tensor correction.

The correction to the Faraday tensor, containing the addition factor of $(T/m)(K_3/K_2)$, could be significantly larger, because

$$\frac{TK_3}{mK_2} \to \frac{2T^2}{m^2}$$

in the limit of large T/m. On the other hand very large values of T/m would imply massive pair production. Therefore we consider the product $\Omega_* r_0 (T/m)^2$ to be smaller than unity, but not negligible.

Thus Eq. (26) is replaced by the approximate form

$$S^{\mu} = \frac{4}{3}r_0^2\sigma(T)W\Gamma^{\mu}, \tag{29}$$

where

$$\sigma(T) \equiv \frac{T}{m}\frac{K_3}{K_2} \tag{30}$$

is a dimensionless function of temperature. In view of Eq. (25), the average of S^{μ} is easily computed:

$$\overline{S^{\mu}} = \frac{4}{3}r_0^2\sigma(\overline{T})\overline{W\Gamma^{\mu}}. \tag{31}$$

Notice that

$$\widetilde{B} = \widetilde{E}$$

implies

$$\overline{W} = \overline{B}^2 - \overline{E}^2, \tag{32}$$

determined entirely by the background fields.

V. CLOSED FLUID DESCRIPTION

A. Plasma current density

Now we construct a closed set of fluid equations for the plasma and the slowly varying part of the electromagnetic field. Since the argument is almost identical to that in previous work [2–4], it is presented with minimal discussion.

The starting point is equation Eq. (20), which is interpreted as an equation for the plasma current density $\overline{J}$. From here on we deal exclusively with the temporally averaged fluid variables and fields, suppressing the overbar; thus all variables in the sequel have implicit overbars:

$$\overline{J} \to J, \text{etc}.$$

We then introduce the perpendicular projector

$$e^{\mu\nu} \equiv -\frac{F^{\mu}_{\kappa}F^{\kappa\nu}}{W}$$

and multiply Eq. (20) by F^{μ}_{κ} to obtain

$$e^{\mu\nu}J_\nu = \frac{F^{\mu}_{\kappa}}{W}\left[\frac{\partial}{\partial x^{\nu}}(T^{\kappa\nu}_p + \Theta^{\kappa\nu}_r) - S^{\kappa}\right] \tag{33}$$

for the two components of J^{μ} that are transverse to the magnetic field. For the remaining components we use charge conservation

$$\frac{\partial J^{\nu}}{\partial x^{\nu}} = 0 \quad (34)$$

and the quasineutrality condition,

$$U_{\nu} J^{\nu} = 0, \quad (35)$$

where $U_{\nu} \equiv \Gamma_{\nu}/n_R$ is the four-vector flow velocity.

At this point we recall that the radiation energy-momentum tensor is presumed known, and that the radiation reaction is given in terms of Γ_e^{μ} by Eq. (31). Hence Eqs. (33)–(35) determine the current in terms of the four-vector particle flow and the plasma energy-momentum tensor. In other words, closure of the electromagnetic field equations depends upon calculation of the energy-momentum tensors for the individual plasma species.

B. Ion dynamics

As we have noted, ion evolution is not affected by radiation. In the magnetized limit, $e \to \infty$, Eq. (21) implies the familiar relation

$$F^{\mu\nu} \Gamma_{i\nu}^{(0)} = 0$$

which forces the lowest order ion flow $\Gamma_{i\nu}^{(0)}$ to have the magnetohydrodynamics (MHD) form

$$\Gamma_{i\nu}^{(0)} = \gamma(V) n_R (1, \mathbf{V}_{\parallel i} + \mathbf{V}_E). \quad (36)$$

Here $\mathbf{V} = \mathbf{V}_{\parallel i} + \mathbf{V}_E$ is the conventional MHD flow, with $\mathbf{V}_{\parallel i}$ an ion flow along the magnetic field and $\mathbf{V}_E = \mathbf{E} \times \mathbf{B}/B^2$ the usual electric drift. In what follows the (0) superscript will be suppressed:

$$\Gamma_{i\nu}^{(0)} \to \Gamma_{i\nu}.$$

The ion energy-momentum tensor is given by Eqs. (6) and (7) in terms of the ion pressure and the parallel flow. To determine the evolution of these quantities, we must eliminate the dominant, electromagnetic term in Eq. (21). There are two linearly independent four-vectors that "annihilate" this large term: the flow vector $U_{i\nu}$,

$$\Gamma_{i\mu} e F^{\mu\nu} U_{i\nu} = 0$$

because of the antisymmetry of the Faraday tensor, and the four-vector $k_{i\nu}$ defined by

$$k_{i\mu} \equiv \frac{\mathcal{F}_{\mu\nu} U^{i\nu}}{\sqrt{W}}.$$

Here $\mathcal{F}_{\mu\nu}$ is the dual Faraday tensor, defined by

$$\mathcal{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\kappa\lambda} F_{\kappa\lambda},$$

where $\epsilon^{\mu\nu\kappa\lambda}$ is the unit antisymmetrical tensor. The four-vector $k_{i\mu}$ is an approximate annihilator because of the familiar identity,

$$k_{i\mu} e F^{\mu\nu} \Gamma_{i\nu} = W^{-1/2} n_R E_{\parallel} B \quad (37)$$

and because the parallel electric field $E_{\parallel} = \mathbf{E} \cdot \mathbf{B}/B$ is small (first order in the gyroradius). This annihilator choice is slightly different from that of previous work [2] because of our use of a Maxwellian distribution.

From the first annihilator we find

$$U_{i\mu} \frac{\partial T_i^{\mu\nu}}{\partial x^{\nu}} = - U_{i\mu} \mathcal{C}_e^{\mu}. \quad (38)$$

This can be seen to yield an equation for the evolution of ion pressure. From the second (approximate) annihilator we find

$$k_{i\mu} \frac{\partial T_i^{\mu\nu}}{\partial x^{\nu}} + W^{-1/2} n_R E_{\parallel} B = - k_{i\mu} \mathcal{C}_e^{\mu}, \quad (39)$$

an equation for the evolution of the parallel flow. Hence the ion contribution to the plasma current is determined.

C. Electron dynamics

The electron flow $\Gamma_e^{\mu} = n_R U_e^{\mu}$ is determined in the same way as Γ_i^{μ} and has the same MHD form; it can differ only because we allow

$$V_{\parallel e} \neq V_{\parallel i}.$$

The electron pressure evolves according to the electron version of Eq. (38),

$$U_{e\mu} \frac{\partial}{\partial x^{\nu}} (T_e^{\mu\nu} + \Theta_r^{\mu\nu}) = U_{e\mu} (\mathcal{C}_e^{\mu} + S^{\mu}) \quad (40)$$

obtained from Eq. (22). Because S^{μ} is proportional to the electron flow, and because $U_{\mu} U^{\mu} = -1$, the radiation reaction enters Eq. (40) in an especially simple way:

$$U_{e\mu} S^{\mu} = - \frac{4}{3} r_0^2 n_R \sigma(T) (B^2 - E^2). \quad (41)$$

Finally the evolution of electron parallel flow is determined by the electron version of Eq. (39). Because $k_{\mu} U^{\mu} = 0$, the radiation reaction does not enter this equation at all. Thus S^{μ} affects fluid evolution only through its effect, given by Eqs. (40) and (41), on electron pressure evolution. We have

$$k_{e\mu} \frac{\partial}{\partial x^{\nu}} (T_e^{\mu\nu} + \Theta_r^{\mu\nu}) = U_{e\mu} \mathcal{C}_e^{\mu}. \quad (42)$$

Recall that Eqs. (33)–(35) provide the four-vector current density in terms of the energy-momentum, or stress tensors, of the two plasma species. Hence the electrodynamical evolution of the system is determined once the stress tensors are known. Since these tensors are determined by Eq. (6) and by Eqs. (38)–(42), together with Eq. (36) for the ion flow, we have a closed set of fluid equations for a magnetized plasma that includes both radiation pressure and radiation reaction in a systematic way.

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