23/9 dimensional anisotropic scaling of passive admixtures using lidar data of aerosols

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In buoyancy-driven flows, another dimensional quantity appears in addition to the energy flux. Classically, this leads to the prediction that at large scales, isotropic Bolgiano-Obukhov (BO) scaling can dominate isotropic Kolmogorov scaling. We investigate this in the atmosphere by using state-of-the-art high-powered lidar data. We examine simultaneous horizontal and vertical sections of passive scalar surrogates over the ranges 100 m to 120 km and 3 m to 4.5 km, respectively. Overall, this spans the crucial "mesoscale" and involves nearly 1000 times more data than the largest relevant experiments to date. Rather than a transition from one isotropic regime to another, we find that the two regimes always coexist in an anisotropic Corrsin-Obukhov law with the Kolmogorov holding in the horizontal, and the BO holding in the vertical. The stratification is quantified by an elliptical dimension D_{el} found to be equal to 2.55 ± 0.02. This anisotropic scaling is very close to that predicted by the 23/9 dimensional unified scaling model of the atmosphere and is consistent with observations of the horizontal wind.

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I. INTRODUCTION

Practically all theories of turbulence assume isotropy or at least local isotropy. In many dynamically driven laboratory flows, this may indeed be justified. However, in buoyancy driven flows, the justification is not obvious because gravity breaks the isotropy and acts at all scales. In such flows, the classical assumption is that gravity (and perhaps rotation) leads to a basic stably stratified state. The perturbations—usually treated with the Boussinesq approximation—are nevertheless assumed to be statistically isotropic. When considering the buoyancy driven atmosphere, there is a further challenge to isotropic theories. This comes from the fact that the scale height for the mean pressure is about 10 km, so that no isotropic three-dimensional turbulence can extend to the large "synoptic" scales. However, even in atmospheric applications, the now mostly outdated classical view (see, e.g., [1]) clings to isotropy by postulating an intermediate "mesoscale gap" followed at larger scales by qualitatively different, (quasi) two-dimensional—but still isotropic—turbulence. This isotropy is postulated *a priori*, and a mesoscale break (or "dimensional transition" [2]) is a purely theoretical consequence, inferred in order to save the isotropy hypothesis.

Today, although we still lack consensus about the full horizontal atmospheric statistics, the mesoscale gap is no longer taken seriously. Practically all the relevant experimental campaigns in the last 20 years (see the review in [3] of Refs. [4–11]) agree that (to within intermittency corrections) the horizontal wind is scaling in the horizontal direction with (Kolmogorov) exponent $\beta_h=5/3$ out to at least several hundred kilometers. In addition, the statistics of horizontal wind fluctuations along the vertical are also generally taken to be scaling but with a spectral exponent $\beta_v>\beta_h$. The scaling in the vertical direction and the value of β_v are explained in the existing literature predominantly by either buoyancy-driven $\beta_v=11/5$ (e.g., Bolgiano [12] and Obukhov [13]) and $\beta_v=3$ (e.g., Lumley [15] and Shur [14] gravity waves [16,17]). Note also the existence of quasigeostrophic [18] and shallow water equation [19] approaches, and other (quasi) two-dimensional (2D) theories in which there are no vertical shears.

If we take two different scaling relations to hold simultaneously in the horizontal and vertical directions, the atmosphere is anisotropic at all scales and, if $\beta_v>\beta_h$, it effectively becomes progressively flatter and flatter at larger and larger scales. The flattening can be characterized by an intermediate "elliptical dimension": $D_{\mathrm{el}}=2+(\beta_h-1)/(\beta_v-1)$ with $2\leqslant D_{\mathrm{el}}\leqslant 3$. The elliptical dimension quantifies the rate of increase in volume of nonintermittent structures [2]. In this framework, the atmosphere is therefore neither 3D isotropic at small scales nor 2D isotropic at large scales. $D_{\mathrm{el}}=3$ and 2 are the 3D and 2D isotropic cases, respectively. In terms of D_{el}—from the point of view of these anisotropic theories—the atmospheric debate is thus between buoyancy-driven flows with $D_{\mathrm{el}}=23/9$ and a gravity wave mechanism leading to $D_{\mathrm{el}}=7/3$. Of course, if the gravity wave explanation is

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correct, then the structures will not be spatially localized so that this characterization will not be so useful. In 2D theories based on quasigeostrophy [19] or in the shallow water equations (e.g., [20]), there is no vertical variability, so that, although β_h can take various values (notably $\beta_h=3$ for quasi-geostrophy, or $8/3$ and $13/3$ for the shallow water equations), we have $\beta_v=\infty$ so that $D_{\mathrm{el}}=2$.

The value $\beta_v=3$ is justified on the basis of gravity waves. However, this explanation is hardly fundamental since it requires the dynamics to be weakly nonlinear in order for a meaningful dispersion relation to be defined and, at the same time, to be strong enough for a horizontal turbulent forcing to exist. In addition, it requires a separate mechanism of unclear origin for the forcing. In fact, it is not well supported by existing data. In Ref. [3], a dozen or so empirical studies from Refs. [14–30] are reviewed and it is concluded that all the available atmospheric evidence (including the largest vertical sounding study to date [28]) are more compatible with the Bolgiano-Obukhov (BO) value $\beta_v=11/5$ which emerges from the conservation of the buoyancy variance flux in buoyancy-driven flows and is discussed further below.

In buoyancy-driven *laboratory* (e.g., Rayleigh-Bénard) flows, there is a corresponding debate about the scaling exponents. While this debate also involves BO scaling, to date, it has been between isotropic Kolmogorov and isotropic BO scaling. In this view, although the latter should theoretically be dominant at scales larger than the Bolgiano scale l_B (the scale at which, in the Boussinesq approximation, the buoyancy forcing dominates the viscous damping), there is still no concensus. For instance, in numerical convection modeling, there is no strong evidence for BO scaling [31], although Ref. [32] and Ref. [33] give it some support in 3D and 2D, respectively. According to Ref. [31], the problem may lie in the use of the Boussinesq approximation, although it is significant that the crucial statistics—of the horizontal velocity in the vertical direction—do not seem to have been considered carefully enough. On the empirical side, progress has been hampered due to experimental difficulties in measuring velocities in the presence of large temperature gradients. For example, the well-known "helium in a box" experiment [34] obtains Kolmogorov statistics for temporal temperature fluctuations for scales both smaller and larger than the Bolgiano scale [35]. However, recent technological advances (e.g., [36,37]) have led to improved data that apparently favor BO scaling although over short ranges of scale and for the vertical velocity in time.

II. THE UNIFIED SCALING MODEL

Although there are obvious differences between atmospheric and laboratory flows, fundamental anisotropic theories unifying horizontal and vertical statistics based on kinetic energy and buoyancy force invariants should apply—at least in some measure—to both cases. The anisotropic $D_{\mathrm{el}}=23/9$ "unified scaling model" [2] is the simplest and physically most appealing such unified theory. This model was proposed on the basis of (a) the observed atmospheric statistics of vertical shear of horizontal wind and (b) an anisotropic modification of the classical BO buoyancy subrange theory. This anisotropic model does not assume stable stratification nor does it require that any reference states (e.g., mean density or temperature profiles) play physical roles. Rather, it assumes that the buoyancy force variance flux $\phi=(g\Delta\ln\theta)^2/\tau$ (θ is the potential temperature, τ is a time scale for the transfer, and $\Delta\ln\theta$ is the difference over a layer thickness Δz) dominates the vertical statistics, while the standard energy flux ($\varepsilon=\Delta v^2/\tau$, $\tau=\Delta x/\Delta v$) dominates the horizontal statistics. From dimensional analysis, the scale corresponding to the classical Bolgiano length l_B (see [13]) is the "spheroscale" l_s:

$$l_s=\phi^{-3/4}\varepsilon^{5/4}. \tag{1}$$

However, unlike the Bolgiano scale—which is a transition between two different isotropic regimes—in the $(23/9)$D model, l_s simply denotes the scale at which the amplitudes of typical vertical and horizontal fluctuations are equal. Contrary to the case of l_B, at l_s, there is no qualitative change in behavior.

In the $(23/9)$D model, a scale function $\|\underline{r}\|$ is introduced, which is now the physically relevant notion of scale. In all the usual isotropic turbulent laws, $\|\underline{r}\|$ is used in place of the usual Euclidean distance $|\underline{r}|$. This physical scale satisfies

$$\|\lambda^{-G}\underline{r}\|=\lambda^{-1}\|\underline{r}\|, \tag{2a}$$

where G is the generator of the group scale changing operators. In the simplest case of linear general scale invariance [2], G is independent of position; it is a matrix. Note that the anisotropic contraction property [Eq. (2a)] allows us to define in a straightforward manner anisotropic fractional differential operators [46] and therefore a fractional differential stratification. Consider a 2D vertical $(x\text{-}z)$ cross section so that we may take

$$G=\begin{pmatrix}1 & 0\\ 0 & H_z\end{pmatrix} \tag{2b}$$

where H_z characterizes anisotropy. A simple example of a scale function for vertical stratification satisfying Eqs. (2a) and (2b) is

$$\|\underline{r}\|=l_s\left[\left(\frac{x}{l_s}\right)^2+\left(\frac{z}{l_s}\right)^{2/H_z}\right]^{1/2}. \tag{3}$$

The theoretical exponent $H_z=H_h/H_v=(1/3)/(3/5)=5/9$, where H_h and H_v are the theoretical real space scaling exponents in the horizontal and vertical directions, respectively, corresponding to β_h and β_v (within intermittency corrections). We can write a general anisotropic law for the horizontal velocity shear as a function of a separation $\|\underline{\Delta r}\|$ in an arbitrary direction:

$$\Delta v(\underline{\Delta r})\approx\varepsilon^{1/3}\|\underline{\Delta r}\|^{1/3} \tag{4}$$

As required, in the horizontal where $\underline{\Delta r}=(\Delta x,0)$, it reduces to Kolmogorov scaling and in the vertical where $\underline{\Delta r}=(0,\Delta z)$, it reduces to BO scaling.

The balance of existing empirical evidence is—in our view—in favor of the unified scaling model, based on the review in Ref. [3] of the experimental results in the horizontal and vertical given in Refs. [4–11] and Refs. [14–30], respectively. However, it has come almost exclusively from measurements made independently in the horizontal and in the vertical (a partial exception [28,38] involved both balloon and aircraft data from the same field experiment and provided the first multifractal characterization of horizontal wind anisotropy, however, it still did not use vertical cross-sections). Up to now, no direct measure of the anisotropy has been made on vertical and horizontal cross sections (the only exception was radar rain data [47] which had only a factor of 8 in the vertical). In order to perform such a direct empirical test, we analyzed high-resolution two-dimensional aircraft lidar data of the atmospheric aerosol backscatter ratio. These data are a surrogate of aerosol density, itself a good approximation for a passive scalar (see [3] for details).

The $(23/9)$D model predicts that the aerosol density statistics should follow the anisotropic Corrsin-Obukhov (CO) law for passive scalar advection, obtained from the isotropic CO law by once again substituting $|\underline{\Delta r}|$ by $\|\underline{\Delta r}\|$:

$$\Delta\rho(\underline{\Delta r}) \approx \chi^{1/2}\varepsilon^{-1/6}\|\underline{\Delta r}\|^{1/3}. \tag{5}$$

χ is the passive scalar variance flux. Equation (5) reduces to the standard CO law in the horizontal, but predicts a different scaling law in the vertical:

$$\Delta\rho(\Delta z) \approx \chi^{1/2}\varepsilon^{-1/2}\phi^{1/5}\Delta z^{3/5}. \tag{6}$$

Although the lidar measures only a surrogate of ρ, according to the $(23/9)$D model, any physical atmospheric field should have the same ratio of horizontal to vertical exponents. In addition, if one can express the backscatter ratio as a power of the aerosol density (see, for example, the treatment in [39]), then, to within intermittency corrections, any power of ρ will respect the anisotropic CO law. Therefore, a systematic comparison of the horizontal and vertical scalings of the *backscatter ratio* will still test the $(23/9)$D model, and the ratio of horizontal to vertical exponents should be H_z.

III. THE EXPERIMENT

This experiment was conducted using an airborne lidar platform (the Meteorological Service of Canada's AERosol Imaging Airborne Lidar–AERIAL [48]) flown at constant attitude over a series of flight legs up to 120 km in length in the Lower Fraser Valley, BC. Lidar remote sensing is a time-of-flight technique that uses laser radiation backscattered from atmospheric particulates to obtain range-resolved backscatter measurement fields. The commonly measured quantity is the backscatter ratio (B), which is the ratio of the aerosol backscatter coefficient to that of the background molecular scattering. The airborne lidar platform is a simultaneous up-down system mounted aboard the Canadian National Research Council Convair 580 aircraft. In this paper, only the data obtained from the downward pointing system were used. The downward laser operated at the fundamental wavelength of 1064 nm, suited for the detection of particles with diameter of the order of 1 μm and had a pulse repetition rate of 20 Hz.

B was measured continuously in a 2D vertical planar section bounded above at 4.5 km (the aircraft altitude). The data extended up to 120 km in the horizontal with a corresponding resolution of 100 m, leading to a total scale ratio $\lambda = 1200$. This resolution was set by the aircraft speed, the laser pulse repetition rate, and the 1 s pulse averaging required to improve the signal-to-noise ratio. The vertical extent of the data was typically 4500 m (i.e., the aircraft altitude) with a resolution of 3 m (i.e., the pulse length) leading to $\lambda = 1500$.

IV. RESULTS

Figure 1 shows a typical data set. In the results presented here, an ensemble average was taken over nine available data sets, treating the horizontal and vertical directions separately.

Taking into account the intermittency of the flux of the scalar variance and of the energy flux, defining the multiscaling exponent $K(q)$ for an arbitrary statistical moment q as [40]

$$\langle \chi_{\|\underline{\Delta r}\|}^{q/2}\varepsilon_{\|\underline{\Delta r}\|}^{-q/6}\rangle \propto \|\underline{\Delta r}\|^{-K(q)} \tag{7}$$

and taking the ensemble average of Eq. (5), one obtains for any arbitrary vector Δr

$$\langle|\Delta\rho(\underline{\Delta r})|^q\rangle = \|\underline{\Delta r}\|^{\zeta(q)} \text{ with } \zeta(q) = q/3 - K(q), \tag{8}$$

where $\Delta\rho(\underline{\Delta r}) = \rho(\underline{r}+\underline{\Delta r}) - \rho(\underline{r})$ and $\zeta(q)$ is the structure function exponent. Since $K(1)$ is small, $\zeta(1) \approx 1/3$. In the horizontal, we have $\Delta r = (\Delta x, 0)$ so that $\|\underline{\Delta r}\| \approx \Delta x$, with $\zeta_h(q) = \zeta(q)$. In the vertical, $\Delta r = (0, \Delta z)$, such that $\|\underline{\Delta r}\| \approx \Delta z^{1/H_z}$ with $H_z = (1/3)/(3/5) = 5/9$ [see Eq. (3)]; therefore defining the corresponding vertical scaling exponent $\zeta_v(q)$ with respect to Δz we expect $\zeta_h(q)/\zeta_v(q) = H_z = 5/9$.

Figure 2 shows the results for $q=1$. By regression, it was found that $H_h = 0.33 \pm 0.03$ and $H_v = 0.60 \pm 0.04$, while in theory $\zeta_h(1) \approx H_h \approx 1/3$ and $\zeta_v(1) \approx H_v \approx 3/5$ ("$\approx$" is used to account for small intermittency corrections). Over the range of roughly 200 m to 60 km along the horizontal, 6 m to 1 km in the vertical, the anisotropic CO law thus holds remarkably well.

The variation of the exponent ratio H_z for each of the nine data sets individually can also be checked. For the ensemble, $H_z = 0.55 \pm 0.02$. On a per realization basis, $0.31 \leqslant H_h \leqslant 0.39$, $0.59 \leqslant H_v \leqslant 0.69$, and $0.51 \leqslant H_z \leqslant 0.58$. By averaging the individual H_z estimates, the mean value is $H_z = 0.55 \pm 0.02$. A regression of $\zeta_h(q)$ versus $\zeta_v(q)$ for increasing q over the range $0 < q < 3$ also gives $H_z = 0.55 \pm 0.02$. We therefore conclude $D_{el} = 2 + H_z = 2.55 \pm 0.02$.

By extrapolating the lines in Fig. 2 until they intersect, we estimate the average spheroscale l_s at approximately 10 cm. Recall that it is the scale at which horizontal and vertical fluctuations are of equal amplitude. It was found to vary between 3 and 80 cm. As previously pointed out, structures larger than l_s will be horizontally stratified.

Turning to $q=2$ statistics, we can compute the 1D energy spectra (Fig. 3). Since the spectrum is the Fourier transform of the correlation, and taking into account intermittency corrections to second-order statistics, the spectral exponent β is $\beta = 1 + \zeta(2)$. This leads to $\beta_h = 5/3 - K_h(2)$ and $\beta_v = 11/5 - K_v(2)$. Once again, the theoretical predictions are very accurately followed. From log-log linear regression, β_h

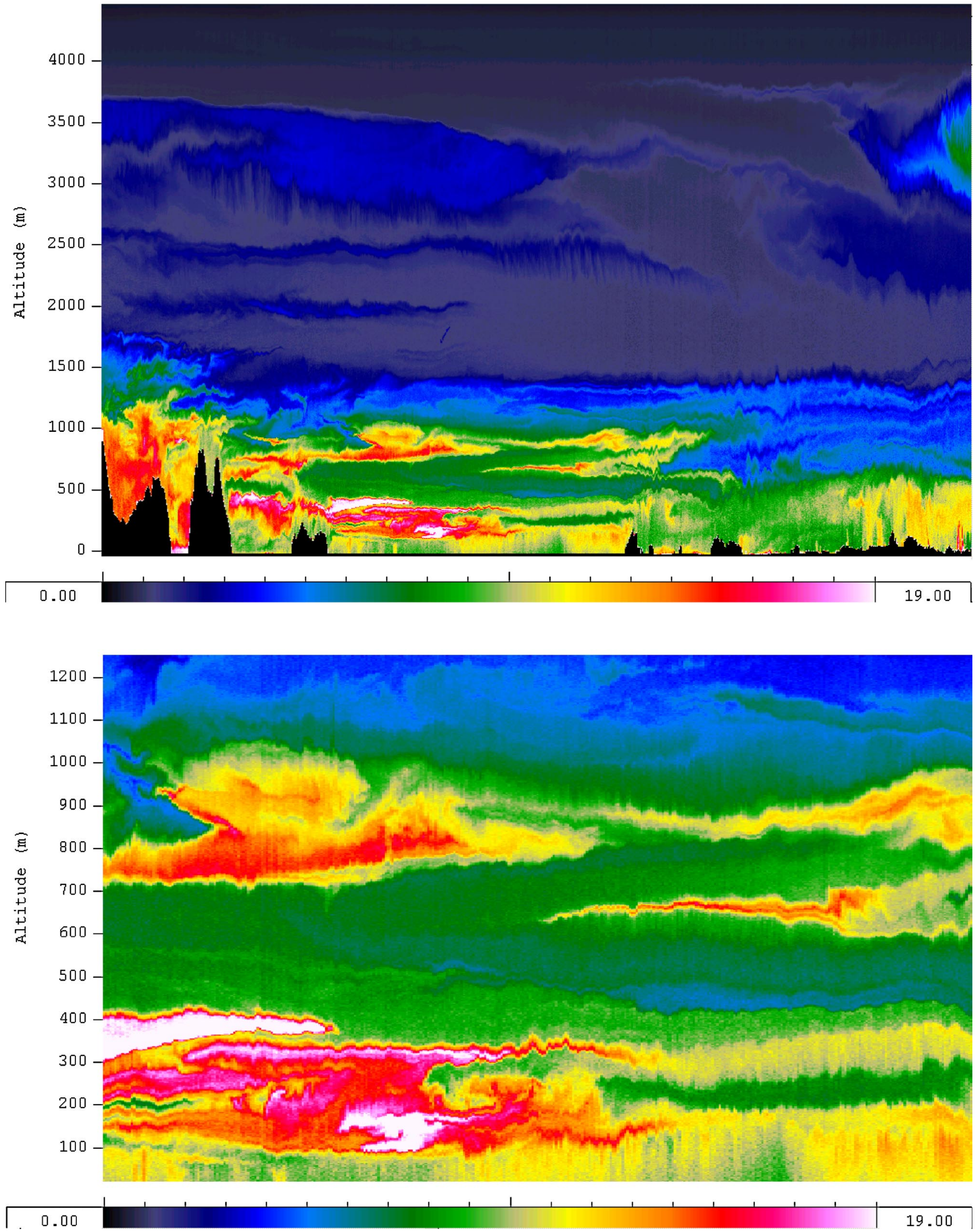

FIG. 1. (Color online) This is the full data taken on 15 August 2001. The scale on the bottom is a logarithmic color scale: darker is for smaller backscatter (aerosol density surrogate), lighter is for larger backscatter. In the first panel, the vertical is 4.5 km and the horizontal is 120 km. The horizontal resolution is 100 m and the vertical resolution is 3 m. The range of scales in this data set is 1200×1500. The black shapes along the bottom are mountains in the British Columbia region. There are no bad pixels in the image. The second panel is a zoom of the first panel; it is 30 km wide and 1300 m thick. This panel highlights the high spatial resolution and the wide dynamic range. There is no saturated signal and high sensitivity to low signal return.

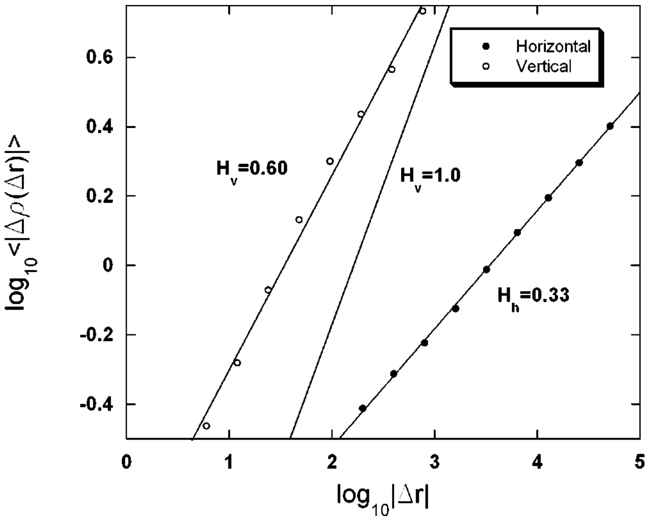

FIG. 2. The lower trace is the first-order $q=1.0$ structure function for the fluctuations in ρ as a function of horizontal distance Δr (in meters), and its line of best fit has slope $H_h=0.33$. The upper trace is the first-order structure function for the fluctuations in ρ as a function of vertical distance Δr with a line of best fit with slope $H_v=0.60$. An $H_v=1.0$ (corresponding to the k-space exponent $\beta_v=3$) was added for comparison with the prediction of gravity wave theories.

$=1.61\pm 0.05$ and $\beta_v=2.15\pm 0.05$. From the estimates made of $K_h(2)$ and $K_v(2)$ using the trace moment (TM) and double trace moment (DTM) techniques [41], we found $\beta_h=5/3-K(2)=1.60\pm 0.03$ and $\beta_v=11/5-K_v(2)=2.10\pm 0.04$.

For small k, deviations occur from the reference line of slope β because of poor statistics whereas for large k, the spectrum flattens out due to instrumental noise. The real space counterparts (the structure functions) give a less clear separation of scales. The small deviations above and below the reference line of slope H_v are therefore less localized but can be explained in the same way.

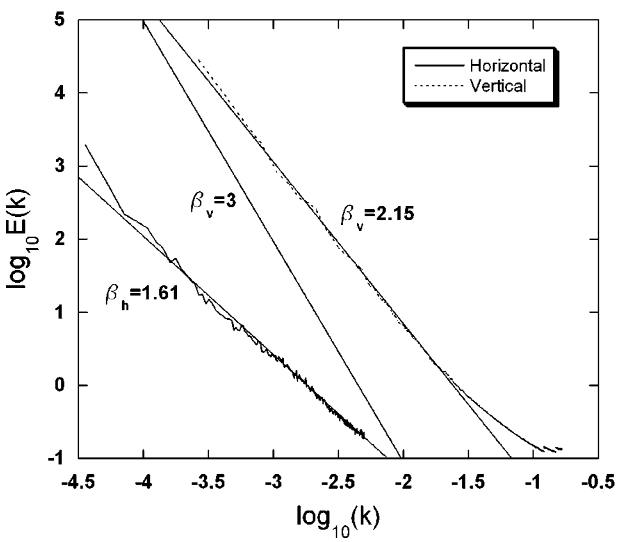

FIG. 3. The lower trace is the Fourier spectrum for the fluctuations in ρ as a function of horizontal wave number k (in m^{-1}) with a line of best fit with slope $\beta_h=1.61$. The upper trace is the Fourier spectrum for the fluctuations in ρ as a function of vertical number k with a line of best fit with slope $\beta_v=2.15$. A $\beta_v=3$ reference line was added for comparison with the prediction of gravity wave theories.

In principle, a full characterization of the intermittency would require knowledge of the entire $K(q)$ functions for $0\leq q<\infty$. Fortunately, multifractal processes have stable, attractive generators[1] leading to the following universal form [40]:

$$K(q)=\frac{C_1}{\alpha-1}(q^{\alpha}-q) \tag{9}$$

where $0\leq C_1\leq d$ is the codimension of the mean, d is the dimension of space, and $0\leq\alpha\leq 2$ is the Lévy stability index characterizing the generator. If the cascade is indeed anisotropic with a scale function such as Eq. (3), then it must be that $K_h/K_v=H_z$ such that $\alpha_h=\alpha_v$ and $C_{1,h}/C_{1,v}=H_z$. Applying the TM and DTM techniques on K_h and K_v gives $\alpha_h=1.82\pm 0.05$, $\alpha_h=1.83\pm 0.04$, $C_{1,h}=0.037\pm 0.006$, and $C_{1,h}=0.053\pm 0.007$. The exponents α_h and α_v are equal within error bars, supporting the expectation that $K_h/K_v=H_z$. On the other hand, the ratio of the C_1's is equal to 0.69 ± 0.2, also within the error bars of H_z. In addition, by comparing these values with those of other estimates for passive scalars (e.g., SF_6, H_2O [43]; H_2O [44]) for which the mean values $\alpha=1.65\pm 0.05$ and $C_1=0.085\pm 0.01$ were obtained, we find that α is a little larger while C_1 is a bit smaller. As a possible explanation, if it is assumed that B is a power of the aerosol density [39], $\rho=B^{\eta}$ where ρ is the true passive scalar density, and η is an exponent which accounts for the optical properties and particle size distribution, then for universal multifractals with a fixed α, $C_1=\eta^{\alpha}C_{\mathrm{lidar}}$ [45], so that in the present case $\eta=1.4$.

V. CONCLUSION

Fundamental theories of buoyancy-driven turbulence involve a quadratic invariant in addition to the energy flux. If only on dimensional grounds, this leads to a buoyancy-dominated Bolgiano-Obukhov scaling regime. Classically, the Bolgiano-Obukhov regime has been assumed to be isotropic, dominating the Kolmogorov scaling for scales larger than l_B. In laboratory experiments—where isotropy is at least tenable—the debate is indeed mostly between isotropic Kolmogorov and BO theories. Unfortunately, in spite of the fact that gravity acts at all scales, standard theories are isotropic. In the atmosphere the classical isotropic BO scaling has been convincingly shown to not exist. However (for the horizontal velocity), starting in the late 1960s, the BO scaling was consistently observed in the vertical direction while Kolmogorov scaling was observed in the horizontal. Nevertheless, due to the difficulties (in both laboratories and the atmosphere) of obtaining appropriate data, decisive evidence in favor of the model have been lacking. In this paper, rather than to statistically compare aircraft and radiosonde unsynchronized wind

[1] A log-Poisson model [42] is often used for fitting K(q), but it does not possess stable, attractive generators.

data, we use passive scalar surrogates from nine airborne lidar cross sections, thereby accessing both horizontal and vertical information virtually simultaneously. We consider the ensemble average results of the analysis of data from nine vertical cross sections, each spanning factors of over 1000 in scale in each direction. The overall data set is nearly 1000 times larger than the largest existing comparable experiment [28,38]. This explains the high accuracy of the exponents found here. We interpret the results in the framework of the unified scaling model (USM). In the USM, an anisotropic physical scale is introduced to replace standard isotropic scales, predicting a different anisotropic Corrsin-Obukhov law for passive scalars. We find that the theoretically predicted anisotropic extension of the CO law for passive scalars holds extremely accurately (to within 5% for the critical exponent $H_z = 5/9$). In confirming this prediction—and this includes intermittency corrections—we find $D_{\mathrm{el}} = 2.55 \pm 0.02$, very close to the theoretical value 23/9 and effectively ruling out the other theories, including quasi-2D turbulence or gravity wave dynamics. Contrary to the assumptions of standard isotropic turbulence theories and also contrary to meteorological parametrizations, passive scalars are thus differentially stratified over this range, a fact that has important consequences in the modeling of the atmosphere. In addition, the 23/9D model involves an anisotropic "physical scale" which allows the standard isotropic results (including cascades) to be mapped onto anisotropic ones. This aspect should be relatively easy to test in the laboratory.

Finally, since we find no scale breaks, it will be hard to reconcile our results with those of the Boussinesq and other related approximations, since they rely on postulating well-defined (physically relevant) vertical profiles.

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