# Density of modes and tunneling times in finite one-dimensional photonic crystals: A comprehensive analysis

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We present a unified treatment of density of modes and tunneling times in finite, one-dimensional photonic crystals. We exploit connections and differences between the various approaches used to calculate the density of modes, which include the Green function, the Wigner phase time, and the electromagnetic energy density, and conclude that the Green function is always the correct path to the true density of modes. We also find that for an arbitrary structure the density of modes can always be found as the ratio between the power emitted by a source located inside the structure and the power emitted by the same source in free space, regardless of absorption or dispersion.

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## I. INTRODUCTION

Structures in which scattering or diffracting elements are arranged in such a way that their mutual distances are comparable with the wavelength of the incident wave are often referred to as photonic crystals (PCs), or photonic band gap structures (PBGs). The field owes its birth to the pioneering works of Yablonovitch [1] and John [2], both of which focused on the study of spontaneous emission control and light localization. These contributions paved the way to a period of intense theoretical and experimental investigation of PBG structures that continues today. Over the years researchers have pointed out that PBG principles can be exploited, and many applications are possible. Here we mention photonic crystals fibers [3], photonic crystal circuits [4], transparent metal-dielectric stacks [5], highly efficient micron-sized devices for nonlinear frequency conversion [6–8]. An up to date review of recent advancements in the field of PBG structures may be found in Refs. [9,10].

One-dimensional (1D) PCs are made by arranging macroscopic dielectric and/or metallic unit cells into a periodic or quasiperiodic array, in order to affect the properties of the light in essentially the same way that semiconductor crystals affect the properties of electrons. The periodic arrangement results in allowed and forbidden frequency bands and gaps for the light, i.e., light can be either transmitted or reflected depending on its frequency, in analogy to energy bands and gaps of semiconductors.

Although the number of experimental and theoretical reports on 1D PCs is already quite large, in our view the issue of the density of modes (DOM), or density of states (DOS), regarding what one means by it, and its true and otherwise implied connections to other physical or measurable quantities, such as emitted energy and group velocity to name just two, is still far from being considered closed.

There are at least three different ways to calculate the DOM that are currently used in the literature. The first way consists of calculating the local density of modes (LDOM) as follows: $\rho_\omega(z)=(-2k_0/c)\mathrm{Im}[G_\omega(z,z)]$, where $G_\omega(z,z)$ is the electromagnetic Green function of a source located at $\xi=z$ inside a 1D structure, and which oscillates with a harmonic time dependence of the type $\exp(-i\omega t)$ [11–13]; $c$ is the speed of light in vacuum, and $k_0=\omega/c$ is the vacuum wave vector. The DOM is then defined as the weighted average of the LDOM over the length $L$ of the PC, i.e., $\rho_\omega^\varepsilon=(-2k_0/cL)\int_0^L\varepsilon_\omega^R(z)\mathrm{Im}[G_\omega(z,z)]dz$ [13], where $\varepsilon_\omega^R(z)$ is the spatially dependent, linear, real, relative dielectric function of the PC and plays the role of the weight function. The second way consists of calculating the LDOM (DOM) as the spatially averaged electromagnetic energy density stored inside the crystal. This approach has been discussed at length in Ref. [8], and we will return to it later. The third approach was first proposed in Ref. [14], where the DOM was defined as: $\rho_\omega^\varphi=(1/L)(d\varphi_t/d\omega)$, where $\phi_t(\omega)$ is the phase of the transmission function $\tilde{t}_\omega=|\tilde{t}_\omega|\exp[i\varphi_t(\omega)]$. In the literature $\tau_\omega^\varphi=(d\varphi_t/d\omega)=\rho_\omega^\varphi L$ is often referred to as the phase time [15–17], "group delay," and "Wigner time" [18], and it gives the time that the transmitted part of an incident, quasimonochromatic, unchirped pulse takes to traverse a 1D barrier [19,20]. We will refer to the DOM calculated this way as the "DOM calculated via the Wigner time."

In this paper we exploit connection and differences between the approaches outlined above in order to give the DOM a firmer theoretical footing when it comes to 1D crystals. We will show that the DOM can be directly linked to the energy emitted from the structure, which is clearly a measurable quantity, and we will clarify the links that have previ-

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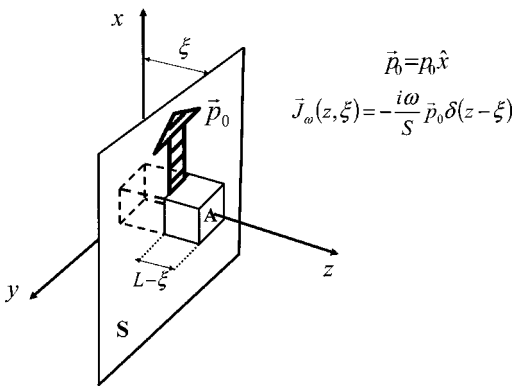


FIG. 1. Schematic representation of a dipole sheet of surface $S$ and dipole moment $\vec{p}(t)=\vec{p}_0\exp[-i\omega t]$ located along the plane $z=\xi$ and parallel to the surfaces A of a 1D PC of length $L$ located between $z=0$ and $z=L$.

ously been established between the concept of DOM and the tunneling times of quasimonochromatic incident pulses.

## II. DOM CALCULATED THROUGH THE GREEN's FUNCTION: THE TRUE DOM

Let us suppose that a dipole sheet, of surface $S$ and harmonically oscillating dipole moment: $\vec{p}(t)=(1/2)[\vec{p}_0\exp(-i\omega t)+\mathrm{c.c.}]$ oriented along $\hat{x}$, is located in the plane $z=\xi$ and it is positioned parallel to the surfaces $A$ of a PBG of length L. This situation is sketched in Fig. 1, where the structure is shown to occupy the space between $z=0$ and $z=$L. Due to its planar symmetry, the problem reduces to a 1D one. As a consequence, the electric field $\vec{E}_\omega(z,\xi)=E_\omega(z,\xi)\hat{x}$ generated in the PC by the current density $\vec{J}_\omega(z,\xi)=J_\omega(z,\xi)\hat{x}$ that oscillates along the plane S can be calculated through the scalar Helmholtz equation as:

$$\frac{\partial^2 E_\omega(z,\xi)}{\partial z^2}+\frac{\omega^2\varepsilon_\omega(z)E_\omega(z,\xi)}{c^2}=-\mu_0\omega^2\frac{p_0}{S}\delta(z-\xi), \quad (1)$$

where $\varepsilon_\omega(z)=1+\chi_\omega(z)=\varepsilon_\omega^R(z)+i\varepsilon_\omega^I(z)$ is the spatially dependent, complex dielectric function, $\chi_\omega(z)$ is the linear complex susceptibility of the medium, and $\delta(z\text{-}\xi)$ is the Dirac delta function. In Eq. (1) we suppose a nonmagnetic material, i.e. $\mu_\omega=1$. We seek solutions of Eq. (1) that satisfy boundary conditions of outgoing waves, i.e., the radiated energy from the dipole sheet leaves the structure never to return, and no energy is incident from outside, namely:

$$E_\omega(z,\xi)=-\omega^2\mu_0\frac{p_0}{S}G_\omega(z,\xi), \quad (2)$$

where $G_\omega(z,\xi)$ is the scalar Green's function that satisfies the following equation:

$$\frac{\partial^2 G_\omega(z,\xi)}{\partial z^2}+\frac{\omega^2\varepsilon_\omega(z)G_\omega(z,\xi)}{c^2}=\delta(z-\xi). \quad (3)$$

The way to construct the Green function for planar dielectric structures using the light-modes has been discussed at length in Refs. [21–23]. In 1D, the Green function has the following form (see Appendix A):

$$G_\omega(z,\xi)=\begin{cases}\dfrac{\Phi_\omega^{(+)}(z)\Phi_\omega^{(-)}(\xi)}{2ik_0\tilde{t}_\omega} & L\geqslant z\geqslant\xi\\ \dfrac{\Phi_\omega^{(+)}(\xi)\Phi_\omega^{(-)}(z)}{2ik_0\tilde{t}_\omega} & 0\leqslant z\leqslant\xi\ ,\end{cases} \quad (4)$$

where $\{\Phi_\omega^{(\pm)}\}$ are the left-to-right (LTR) and right-to-left (RTL) light-modes, $\tilde{t}_\omega=n_{0,1}t^{(-)}=n_{0,2}t^{(+)}$ is the transmission function, $n_{0,1\text{-}2}$ are the refractive indices of the materials surrounding the structure, $t_\omega^{(\pm)}$ are the LTR and RTL transmission functions (see Fig. 2), $k_0=\omega/c$ is the vacuum wave vector. LTR and RTL modes can be calculated using a standard linear matrix transfer technique, assuming a unitary electric field is incident on the structure from LTR for the $\Phi_\omega^{(+)}$ mode, and from RTL for the $\Phi_\omega^{(-)}$ mode, as shown in Fig. 2, and as first reported in Ref. [24]. For clarity, we report the details of the calculations which lead to Eq. (4) in Appendix A [25]. Note that Eq. (4) is valid for an arbitrary 1D, finite structure, one that may also include material absorption and dispersion. Now, using Eq. (2) and the expression for the current density, it can be shown that the mean electromagnetic power emitted by the dipole sheet embedded within the PC is given by:

$$\overline{W}^{(1D)}_{\text{emitted in PC}}\equiv-\frac{1}{2}\mathrm{Re}\int_V\vec{J}_\omega\cdot\vec{E}^*_\omega dV$$
$$=-\frac{\omega^3\mu_0|p_0|^2A}{2S^2}\mathrm{Im}[G_\omega(\xi,\xi)]. \quad (5)$$

Equation (5) tells us that the mean electromagnetic power emitted by a dipole sheet located at $z=\xi$ is proportional to the imaginary part of the scalar Green function calculated at $z=\xi$. We use the superscript "1D" to remark the fact that our approach is specific for electromagnetic problems that have planar symmetry, and can therefore be reduced to 1D problems. The mean electromagnetic power emitted by the same dipole sheet located in free space in the same volume $V=AL$ occupied by the PC is:

$$\overline{W}^{(1D)}_{\text{emitted in V,free space}}=\frac{\omega^3\mu_0|p_0|^2A}{4S^2k_0}. \quad (6)$$

From Eq. (5) and Eq. (6) we find:

$$\frac{\overline{W}^{(1D)}_{\text{emitted in PC}}(\xi)}{\overline{W}^{(1D)}_{\text{emitted in V,free space}}}=-2k_0\,\mathrm{Im}[G_\omega(\xi,\xi)]. \quad (7)$$

There are at least two physical conditions that our 1D LDOM should meet, i.e., that (i) it account for the modification of dipole sheet emission rates with respect to emission rates in vacuum; (ii) it give the correct limiting value for the DOM of free space when calculated for a 1D empty cavity whose dimensions go to infinity. The simplest way to satisfy these two requirements is to write the LDOM as:

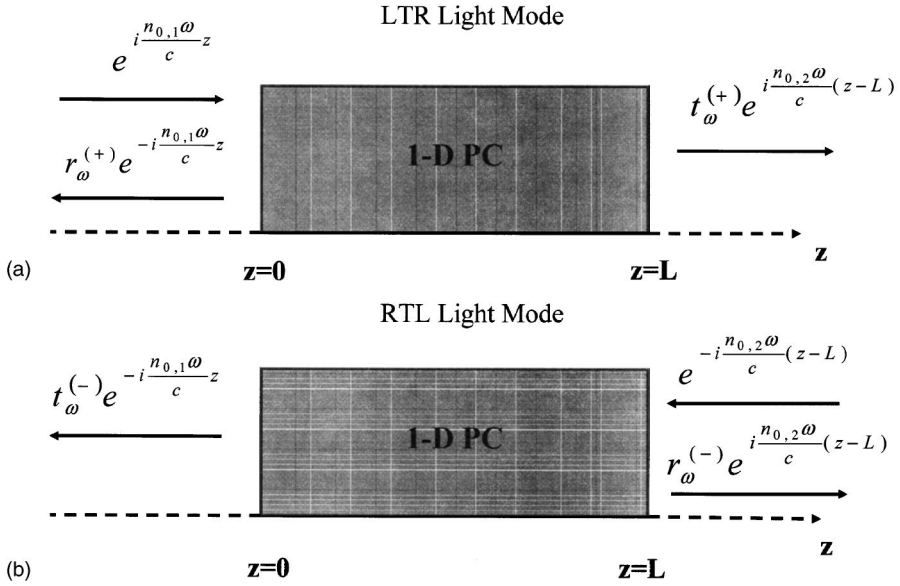


FIG. 2. Schematic representation of the boundary conditions imposed on: (a) LTR and (b) RTL light modes: $r_\omega^{(\pm)}$ are the LTR and the RTL reflection coefficients, respectively, and $t_\omega^{(\pm)}$ are the corresponding transmission coefficients. $n_{0,1}$and $n_{0,2}$ are the refractive indexes of the materials surrounding the PC. Note that $n_{0,2}t_\omega^{(+)}=n_{0,1}t_\omega^{(-)}$ as a consequence of time reversal symmetry.

$$\rho_\omega(\xi)\equiv\rho_{\omega,\text{free space}}^{(1D)}\frac{\overline{W}_{\text{emitted in PC}}^{(1D)}(\xi)}{\overline{W}_{\text{emitted in V,free space}}^{(1D)}}=-\frac{2k_0}{c}\text{Im}[G_\omega(\xi,\xi)], \tag{8}$$

where $\rho_{\omega,\text{free space}}^{(1D)}=1/c$ is the 1D DOM of the free space [26]. The DOM is then the average of the LDOM over the volume V:

$$\rho_\omega\equiv\rho_{\omega,\text{free space}}^{(1D)}\frac{\langle\overline{W}_{\text{emitted in PC}}^{(1D)}(z)\rangle}{\overline{W}_{\text{emitted in V,free space}}^{(1D)}}=-\frac{2k_0}{cL}\int_0^L\text{Im}[G_\omega(z,z)]dz, \tag{9}$$

where the integration variable $\xi$ has been relabeled $z$. From Eqs. (8) and (9), we note that the DOM is defined in an unambiguous way because it is related to the power emitted by a dipole sheet in a 1D PC. If the PC is composed of nonabsorbing materials, the mean power emitted by the dipole sheet is also equal to the mean power that flows through the surfaces A of the PC: $\overline{W}_{\text{emitted in PC}}^{(1D)}=\overline{W}_{\text{flowing through A}}$. So, we have arrived at an operational definition of the DOM and LDOM that can be directly linked to an experimental quantity, i.e., the emitted energy. As defined in Eq. (9), the DOM maintains its generality when absorption and dispersion are present. In fact, the idea can be generalized to 2D and 3D structures of finite size. While in the 1D case the source needs to have planar symmetry (infinite dipole sheet), in 2D the source should have cylindrical symmetry (infinite wire), and in 3D the source should have point symmetry (point source). In any case, the DOM can always be defined as the DOM of free space multiplied by the ratio between the spatial average of the mean power emitted by a source embedded within the PC and the mean power emitted by the source in the free space:

$$\rho_\omega\equiv\rho_{\omega,\text{free space}}^{(nD)}\langle\overline{W}_{\text{emitted in PC}}^{(nD)}(\vec{r})\rangle/\overline{W}_{\text{emitted in V,free space}}^{(nD),}$$

$$n=1,2,3,$$

where $\rho_{\omega,\text{free space}}^{(1D)}=1/c$, $\rho_{\omega,\text{free space}}^{(2D)}=\omega/c^2$, $\rho_{\omega,\text{free space}}^{(3D)}=\omega^2/(\pi c^3)$ are, respectively, the DOM of the free space in 1D, 2D, and 3D. The reader interested in the extension of Eq. (9) to the case of finite size, 3D structures can consult Ref. [27]. It is worth noting that the DOM calculated as *the average* of the LDOM over the volume V and the DOM calculated as the *weighted average* where the weight function is the real part of the dielectric function, $\varepsilon_\omega^R(z)$, are related through the following equation:

$$\rho_\omega=\frac{\rho_\omega^\varepsilon}{\langle\varepsilon_\omega^R\rangle}+\frac{2k_0}{cL\langle\varepsilon_\omega^R\rangle}\int_0^L\delta\varepsilon_\omega^R(z)\text{Im}[G_\omega(z,z)]dz, \tag{10}$$

where $\delta\varepsilon_\omega^R(z)=\varepsilon_\omega^R(z)-\langle\varepsilon_\omega^R\rangle$ represents the variation of the real part of the dielectric function with respect to its average value. In the case of structures with low index contrast ($|\delta\varepsilon_\omega^R(z)|\ll1$) the two definitions are proportional to each other through a constant scale factor: $(\rho_\omega^\varepsilon/\rho_\omega)\cong\langle\varepsilon_\omega^R\rangle$. We will discuss the physical meaning of $\rho_\omega^\varepsilon$ in Sec. III.

## III. DOM CALCULATED THROUGH THE WIGNER TIME AND DOM CALCULATED THROUGH THE DWELL TIME

The DOM calculated using the the Wigner time for 1D, finite, structures is defined as follows [14]:

$$\rho_\omega^\varphi = \frac{1}{L}\frac{d\varphi_t}{d\omega} = \frac{\tau_\omega^\varphi}{L} \tag{11}$$

where $\phi_t(\omega)$ is the phase of the transmission function and $\tau_\omega^\varphi = (d\varphi_t/d\omega)$ is the Wigner time [15–18] which gives the time that the transmitted part of an incident, quasimonochromatic, unchirped pulse takes to traverse a 1D, barrier [19,20]. Now, from Eq. (A3), another way of writing the transmission for LTR propagation is $t_\omega^{(+)} = \Phi_\omega^{(+)}(L)$; taking the derivative with respect to the frequency, $dt_\omega^{(+)}/d\omega = \partial\Phi_\omega^{(+)}(L)/\partial\omega$, and rewriting the transmission in terms of a phase and an amplitude, i.e., $t_\omega^{(+)} = |t_\omega^{(+)}|\exp[i\varphi_t(\omega)]$ [28], we obtain:

$$\frac{d|t_\omega^{(+)}|}{d\omega}\exp(i\varphi_t) + i\frac{d\varphi_t}{d\omega}|t_\omega^{(+)}|\exp(i\varphi_t) = \frac{\partial\Phi_\omega^{(+)}(L)}{\partial\omega}. \tag{12}$$

Using Eqs. (11) and (12), the DOM can be recast as follows:

$$\rho_\omega^\varphi = \frac{1}{L}\,\mathrm{Im}\left[\frac{1}{t_\omega^{(+)}}\frac{\partial\Phi_\omega^{(+)}(L)}{\partial\omega}\right]. \tag{13}$$

The term $\partial\Phi_\omega^{(+)}(L)/\partial\omega$ can be calculated using the Green function of Eq. (4) (see Appendix B):

$$\frac{\partial\Phi_\omega^{(+)}(L)}{\partial\omega} = -\frac{k_0}{c}\int_0^L \left[2\varepsilon_\omega(\xi) + \omega\frac{\partial\varepsilon_\omega(\xi)}{\partial\omega}\right] G_\omega(L,\xi)\Phi_\omega^{(+)}(\xi)d\xi. \tag{14}$$

From the expression for the Green function given in Eq. (4), we have:

$$G_\omega(L,\xi) = \frac{\Phi_\omega^{(+)}(L)\Phi_\omega^{(-)}(\xi)}{2ik_0\tilde{t}_\omega} = \frac{\Phi_\omega^{(-)}(\xi)}{2ik_0 n_{0,2}}, \tag{15}$$

and from Eqs. (13)–(15) we obtain:

$$\rho_\omega^\varphi = -\frac{k_0}{cL}\,\mathrm{Im}\left[\frac{1}{2ik_0\tilde{t}_\omega}\int_0^L \left(2\varepsilon_\omega(z) + \omega\frac{\partial\varepsilon_\omega(z)}{\partial\omega}\right) \times\Phi_\omega^{(+)}(z)\Phi_\omega^{(-)}(z)dz\right], \tag{16}$$

where the integration variable $\xi$ has been relabeled $z$. Equation (16) can be rewritten as follows:

$$\rho_\omega^\varphi = \rho_\omega^\varepsilon - \frac{2k_0}{cL}\int_0^L \varepsilon_\omega^I(z)\mathrm{Re}[G_\omega(z,z)]dz - \frac{k_0\omega}{cL} \times\int_0^L \left[\frac{\partial\varepsilon_\omega^R(z)}{\partial\omega}\,\mathrm{Im}[G_\omega(z,z)] + \frac{\partial\varepsilon_\omega^I(z)}{\partial\omega}\,\mathrm{Re}[G_\omega(z,z)]\right]dz, \tag{17}$$

where we have identified

$$\rho_\omega^\varepsilon = (-2k_0/cL)\int_0^L \varepsilon_\omega^R(z)\mathrm{Im}[G_\omega(z,z)]dz$$

as the DOM calculated using the real part of the relative dielectric function as the weight function. Equation (17) provides an illuminating link between the different definitions of DOM. For the sake of clarity we analyze three cases: (a) no absorption and no dispersion; (b) dispersion and negligible absorption; (c) absorption and dispersion.

### A. No absorption and no dispersion

In this case, from Eqs. (17) and (10) we obtain:

$$\rho_\omega^\varphi = \rho_\omega^\varepsilon = \langle\varepsilon_\omega^R\rangle\rho_\omega - \frac{2k_0}{cL}\int_0^L \delta\varepsilon_\omega^R(z)\mathrm{Im}[G_\omega(z,z)]dz. \tag{18}$$

Equation (18) tells us that in the absence of absorption and dispersion the DOM calculated through the Wigner time $\rho_\omega^\varphi$ is equal to the DOM calculated by averaging the Green function over the real part of the dielectric function. However, both $\rho_\omega^\varepsilon$ and $\rho_\omega^\varphi$ overestimate the true DOM by a factor that is equal to the average value of the grating, $\langle\varepsilon_\omega^R\rangle$, with the addition of a term that depends on the index contrast. In the case of no absorption, $\rho_\omega^\varepsilon$ can also be expressed in a form involving the dwell time (see Appendix C) as:

$$\rho_\omega^\varepsilon = \frac{\tau_D^{(+)} + \tau_D^{(-)}}{2L} + \frac{1}{2ck_0L}\,\mathrm{Im}(r_\omega^{(+)} + r_\omega^{(-)}), \tag{19}$$

where $\tau_D^{(+)} = (2L\langle U_\omega^{(+)}\rangle)/cn_{01}$ and $\tau_D^{(-)} = (2L\langle U_\omega^{(-)}\rangle)/cn_{02}$, are the LTR and RTL dwell times, respectively, and

$$U_\omega^{(\pm)} = \frac{1}{4}\left[\varepsilon_\omega^R(z)|\Phi_\omega^{(\pm)}(z)|^2 + \frac{c^2}{\omega^2}\left|\frac{d\Phi_\omega^{(\pm)}(z)}{dz}\right|^2\right]$$

are the corresponding LTR and RTL time-averaged electromagnetic energy densities stored in the PC, for incident fields that have a harmonic time dependence of the type $\exp(-i\omega t)$, and unitary amplitude (i.e., $\varepsilon_0|A_\omega^{(\pm)}|^2 = 1$, $\varepsilon_0$ is the vacuum dielectric constant, $A_\omega^{(\pm)}$ are the amplitudes of incident fields). The dwell time was first introduced for ballistic electrons, and was intended to measure the average time a quantum particle spends within a barrier, whether it is reflected and/or transmitted at the end of its stay [29,30]. In the case of electromagnetic radiation, the dwell time can be calculated by resorting to the electromagnetic energy density [31,32] as the ratio between the stored electromagnetic energy and the input power. Note that when $|r_\omega^{(\pm)}| = 0$ (i.e., at the peaks of transmission),

$$\rho_\omega^\varepsilon = \frac{\tau_D^{(+)} + \tau_D^{(-)}}{2L},$$

that is to say, $\rho_\omega^\varepsilon$ is exactly the average of the LTR and RTL dwell times divided the length L of the PC. We point out that in most cases $k_0L \gg 1$ (equivalent to saying that the typical structure is much longer than the incident wavelength), and so the extra term

$$\frac{1}{2ck_0L}\,\mathrm{Im}(r_\omega^{(+)} + r_\omega^{(-)})$$

nearly always gives a maximum correction of the order of $10^{-2}L/c$ inside the gap [33]. As also noted in Ref. [31], this extra term comes from the fact that in a finite structure the energy density is not equally shared between the electric and magnetic components of the field. For *symmetric or suffi-*

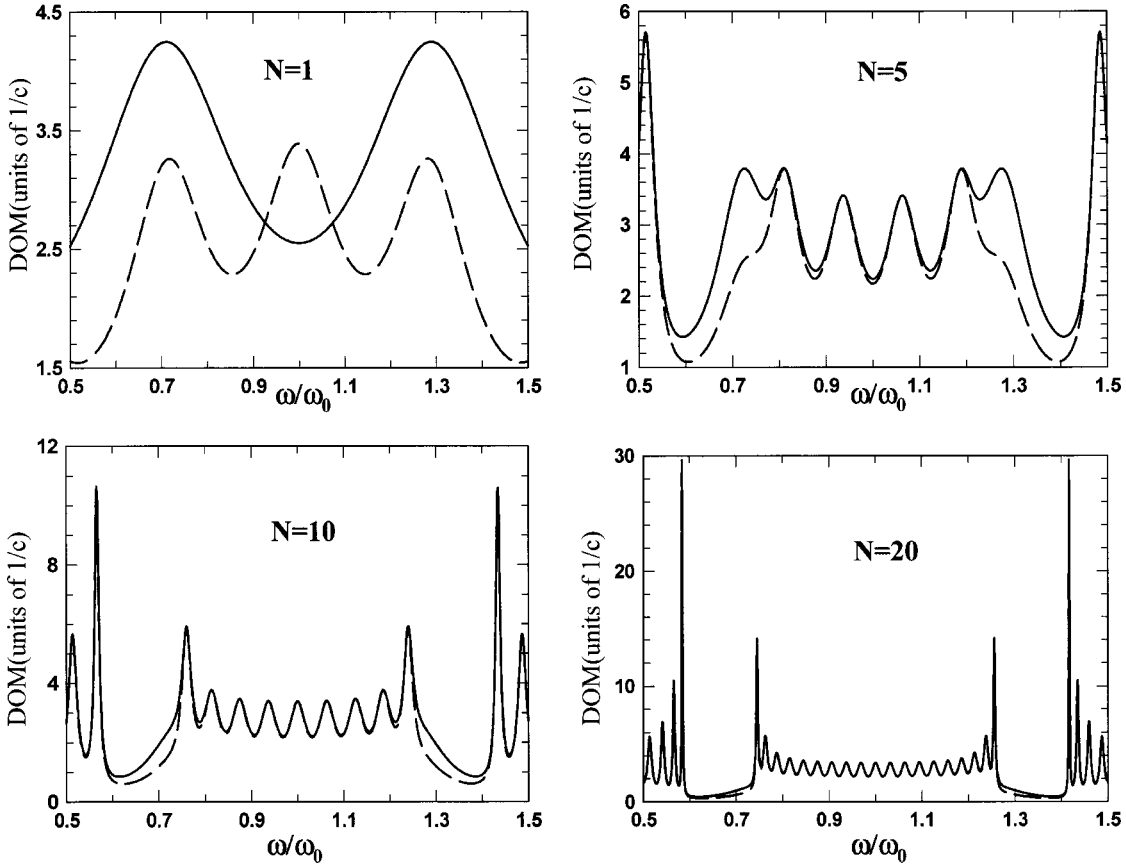


FIG. 3. $\rho_\omega^U$ (solid line) and $\rho_\omega^\varepsilon=\rho_\omega^\varphi$ (dashed line) vs $\omega/\omega_0$ ($\omega_0=2\pi c/\lambda_0$ and $\lambda_0=1$ $\mu$m) for PCs made by $N=1$, $N=5$, $N=10$, and $N=20$ periods, respectively. The elementary cell is composed of two layers of refractive index respectively $n_a=2$ and $n_b=3$. The thicknesses of the layers are $a=125$ nm and $b=166$ nm, respectively. The structure is surrounded by air.

*ciently long structures* embedded in symmetric environments [34] it is straightforward to verify that $\langle U_\omega^{(+)}\rangle=\langle U_\omega^{(-)}\rangle=\langle U_\omega\rangle$, and consequently $\rho_\omega^\varepsilon$ takes the following simple form:

$$\rho_\omega^\varepsilon=\rho_\omega^U+\frac{|r_\omega|}{k_0cL}\sin\varphi_r, \tag{20}$$

where

$$\rho_\omega^U\equiv\frac{1}{2Lcn_0}\int_0^L\left[\varepsilon_\omega^R(z)|\Phi_\omega|^2+\frac{c^2}{\omega^2}\left|\frac{d\Phi_\omega}{dz}\right|^2\right]dz=\frac{\tau_D}{L}$$

is the DOM as it was first defined in Ref. [8] in terms of the electromagnetic energy density. Equation (20) provides new insight into the profound link that effectively binds the DOM calculated by averaging the Green's function over the grating, and the electromagnetic energy density, at least for nonabsorbing structures. In addition, Eq. (20) provides a theoretical foundation for the numerical results reported in Ref. [8].

Some observations are now in order. We have shown that for a PC embedded in symmetric environments, $\rho_\omega^\varphi=\rho_\omega^\varepsilon$ is also approximately equal to $\rho_\omega^U$. In Fig. 3 we compare $\rho_\omega^\varphi=\rho_\omega^\varepsilon$ with $\rho_\omega^U$ for PCs made by the same elementary cell, repeated $N=1$, $N=5$, $N=10$, and $N=20$ times, respectively. The details of the structures are given in the figure caption. We note that when $N=20$, the PC is practically equivalent to a symmetric structure, and $\rho_\omega^\varphi=\rho_\omega^\varepsilon\cong\rho_\omega^U$. The small discrepancy inside the gap is mostly due to the extra term

$$\frac{|r_\omega|}{k_0cL}\sin\varphi_r,$$

and to a lesser degree to the small asymmetry built into the structure. Figure 4 compares $\rho_\omega^\varphi=\rho_\omega^\varepsilon$, $\rho_\omega^U$, and $\rho_\omega$ for 20-period structure. Again the figure reflects the fact that $\rho_\omega^\varphi=\rho_\omega^\varepsilon\cong\rho_\omega^U$.

The first nontrivial implications of our results affect the relation between the tunneling times. From Eqs. (11), (18), and (19) we obtain:

$$\tau_\omega^\varphi=\frac{\tau_D^{(+)}+\tau_D^{(-)}}{2}+\frac{1}{2\omega}\mathrm{Im}(r_\omega^{(+)}+r_\omega^{(-)}). \tag{21}$$

Equation (21) tells us that for a nonabsorbing, nondispersive structure the Wigner time and the average (RTL and LTR) dwell time differ by an amount proportional to the average imaginary part of the RTL and LTR reflection coefficient. We note that this difference is zero at all transmission resonances, because there the energy density is equally shared by the electric and magnetic fields. Equation (21) is a result that clarifies the link between the Wigner time and the dwell time. In related work on tunneling times [32], a link between the Wigner time and the dwell time has also been pointed

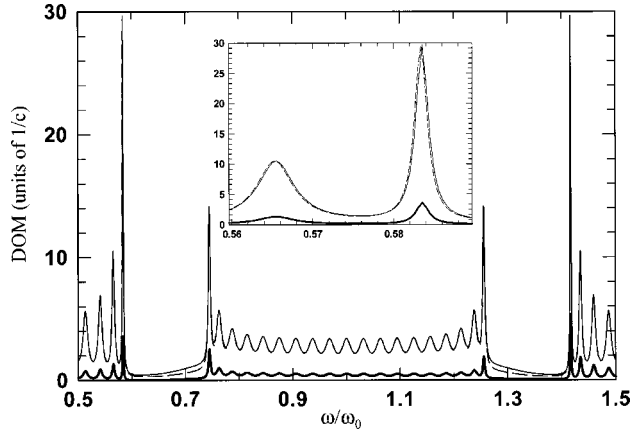


FIG. 4. $\rho_\omega^U$ (thin-solid line), $\rho_\omega^\varepsilon = \rho_\omega^\varphi$ (dashed line) and $\rho_\omega$ (thick-solid line) vs $\omega/\omega_0$ ($\omega_0 = 2\pi c/\lambda_0$ and $\lambda_0 = 1$ $\mu$m) for a PC made of $N=20$ periods. The elementary cell is the same described in the caption of Fig. 3. The structure is surrounded by air. Inset: Magnification of DOM at the two transmission resonance near the first band gap.

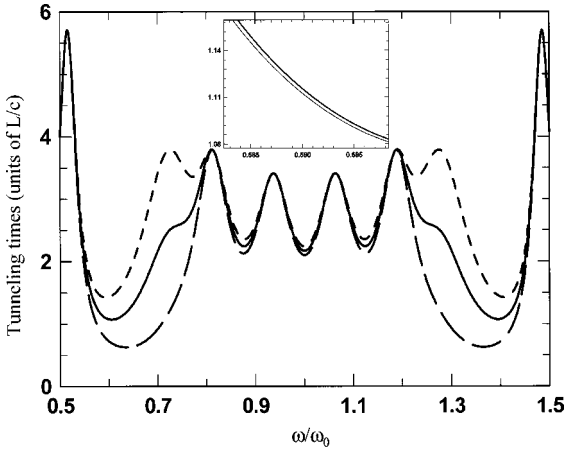


FIG. 5. LTR dwell time $\tau_D^{(+)}$ (short-dashed line), RTL dwell time $\tau_D^{(-)}$ (long-dashed line) and average dwell time $(\tau_D^{(+)} + \tau_D^{(-)})/2$ (thick solid line) vs $\omega/\omega_0$ ($\omega_0 = 2\pi c/\lambda_0$ and $\lambda_0 = 1$ $\mu$m) for a PC made of $N=5$ periods. $L/c$ in this case is 4.85 fs. The elementary cell is the same described in the caption of Fig. 3. The figure contains also the Wigner time $\tau_\omega^\varphi$ (thin solid line) but on this scale it is practically indistinguishable from the average dwell time. Inset: Magnification of the average dwell time (thick solid line) and the Wigner time (thin solid line). The difference between the two times is of the order of $10^{-2}$ fs.

out. However, in Ref. [32] only the case of symmetric structures in symmetric environments was addressed, which is a particular case of our more general Eq. (21). In fact, in the case of a symmetric structure located in a symmetric environment, $\tau_D^{(+)} = \tau_D^{(-)} = \tau_D$ and $r_\omega^{(+)} = r_\omega^{(-)} = r_\omega$, and from Eq. (21) we obtain:

$$\tau_\omega^\varphi = \tau_D + \frac{1}{\omega}\mathrm{Im}(r_\omega) \tag{22}$$

which is the result in Ref. [32] There, the term $\tau_i = \frac{1}{\omega}\,\mathrm{Im}(r_\omega)$ is referred to as "self-interference delay." Again we stress that Eq. (22) is not valid in general, because it was designed to handle symmetric structures located in symmetric environments. As a consequence, it does not predict the correct tunneling times for periodic structures having only a few periods.

To better clarify this point, in Fig. 5 we compare the Wigner time and the dwell times, i.e., Eqs. (21) and (22), for a 5-period structure. In this case the LTR and RTL dwell times differ from each other, and only their average value converges to the Wigner time, as predicted by our Eq. (21). As calculated by Eq. (21), the "self-interference delay" is of order $10^{-2}$ fs, a quantity that is hardly measurable in any experiment. Our results also suggest that the upper limit of the "self interference delay" available for any kind of structure can in fact be estimated from our Eq. (21), namely, $|\tau_i|_{\max} \leqslant \frac{1}{\omega}$, which means $|\tau_i|_{\max} \leqslant 0.5$ fs for $\lambda \cong 1$ $\mu$m, and $|\tau_i|_{\max} \leqslant 0.1$ fs for $\lambda = 0.2$ $\mu$m. In units of $(L/c)$, the upper limit of the available self-interference delay is $1/(k_0 L)$. In the optical regime ($\lambda \cong 1$–$0.2$ $\mu$m) for PCs only a few micrometers in length, $k_0 L \approx 10^{-2}$, the upper limit of the self-interference delay available is of the order of $10^{-2} L/c$. In the structure considered in Fig. 5, $L/c = 4.85$ fs, and the upper limit for the self interference delay in the range of frequency shown in the inset is approximately 0.8 fs. This is compatible with the value of $10^{-2}$ fs that is the difference between the average dwell time and the Wigner time in this case. In summary, our results show that in most cases of interest, the correction due to the self-interference delay is negligible in the optical regime, and that the Wigner time is for all intents and purposes approximately equal to the average of the LTR and RTL dwell times.

In Ref. [31] it was demonstrated that beginning with the definition of energy velocity as the ratio between the spatially averaged Poynting vector and the spatially averaged energy density, $V_E^{(\pm)} = \langle S_\omega^{(\pm)} \rangle / \langle U_\omega^{(\pm)} \rangle$, it is possible to arrive at a simple relation that links the energy velocity to the dwell time, namely, $V_E^{(\pm)} = |t_\omega^{(\pm)}|^2 (L/\tau_D^{(\pm)})$. Now, by using Eq. (21) and the connection of the dwell time with the energy velocity, we arrive at the following equivalence:

$$\tau_\omega^\varphi = \frac{L}{2}\frac{|t_\omega^{(+)}|^2 V_E^{(-)} + |t_\omega^{(-)}|^2 V_E^{(+)}}{V_E^{(+)} V_E^{(-)}} + \frac{1}{2\omega}\mathrm{Im}(r_\omega^{(+)} + r_\omega^{(-)}). \tag{23}$$

From Eq. (23), once again for the case of a symmetric structure embedded in a symmetric environment, and neglecting the corrective term $\frac{1}{2\omega}\,\mathrm{Im}(r_\omega^{(+)} + r_\omega^{(-)})$ on the right-hand side of Eq. (23), we obtain: $V_E \cong |t|^2 V_g$ [31], where $V_g = L/\tau_\omega^\varphi$ is the group or tunneling velocity. Therefore, our Eq. (23) confirms and extends the results first reported in Ref. [31].

### B. Case of dispersion with negligible absorption

In the case at hand, from Eq. (17) we obtain:

$$\rho_\omega^\varphi = \rho_\omega^\varepsilon - \frac{k_0 \omega}{cL}\int_0^L \left[\frac{\partial \varepsilon_\omega^R(z)}{\partial \omega}\,\mathrm{Im}[G_\omega(z,z)]\right] dz. \tag{24}$$

Using the explicit expression for $\rho_\omega^\varepsilon$, Eq. (24) can be recast in the following form:

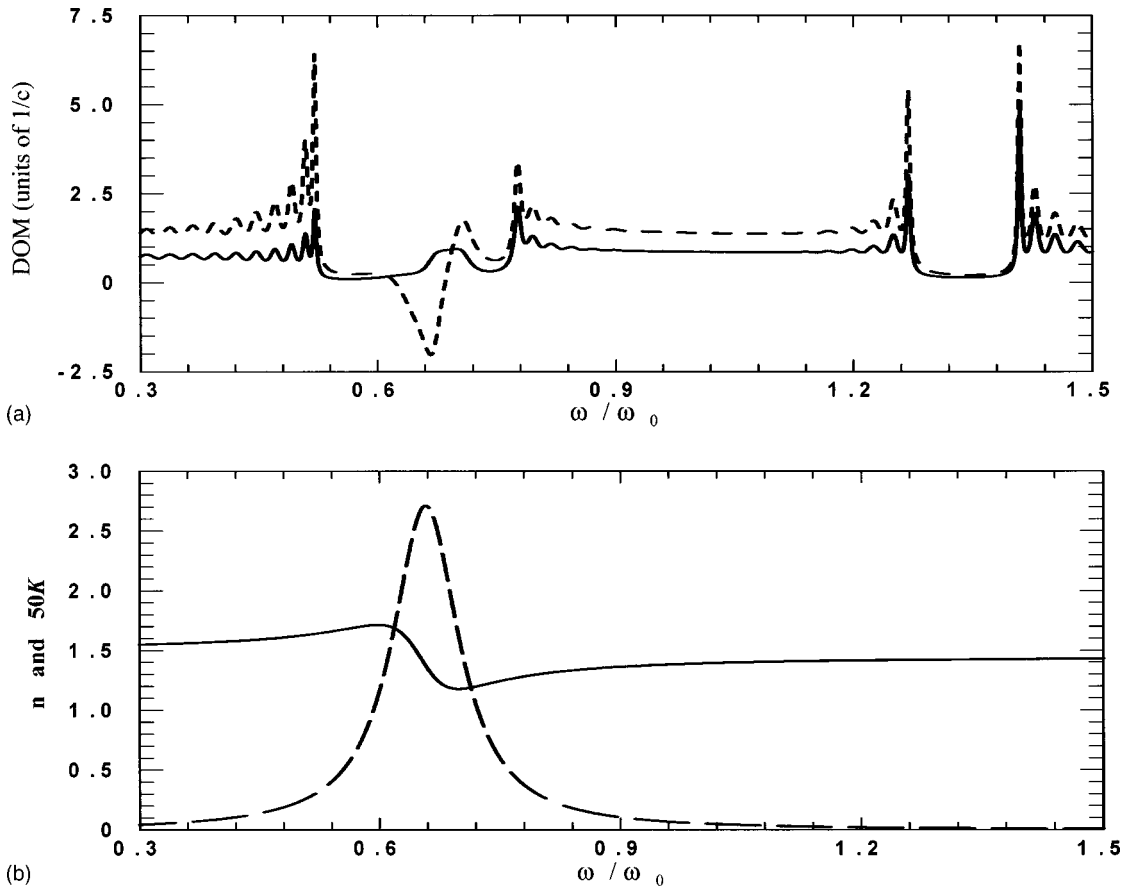


FIG. 6. (a) $\rho_\omega$ (solid line) and $\rho_\omega^\varphi$ (dashed line) vs $\omega/\omega_0$ ($\omega_0=2\pi c/\lambda_0$ and $\lambda_0=1$ $\mu$m). The structure is made by 40 alternating layers of a dielectric material and air. The dielectric material has a Lorentzian absorption line centered around $\omega/\omega_0=0.65$ and an index of refraction of approximately 1.42 in the visible range. The layers have thicknesses $a=350$ nm (dielectric material) and $b=250$ nm (air) for a total length $L=12$ $\mu$m; (b) refractive index ($n$) (solid line) and extinction coefficient ($K$) (dashed line) of the dielectric material vs $\omega/\omega_0$. By definition the refractive index and the extinction coefficient are related to the dielectric function for a nonmagnetic material by: $\sqrt{\varepsilon}=n+iK$. The extinction coefficient is magnified 50 times.

$$\rho_\omega^\varphi = (-2k_0/cL)\int_0^L \left[\varepsilon_\omega^R(z) + \frac{\omega}{2}\frac{\partial\varepsilon_\omega^R(z)}{\partial\omega}\right] \mathrm{Im}[G_\omega(z,z)]dz, \tag{25}$$

where Eq. (25) suggests that the DOM is calculated by averaging the imaginary part of the Green's function over the weight function:

$$\varepsilon_\omega^R(z) + \frac{\omega}{2}\frac{\partial\varepsilon_\omega^R(z)}{\partial\omega}.$$

In analogy with the definition of $\rho_\omega^\varepsilon$, we can define

$$\rho_\omega^{\varepsilon+(\omega/2)(\partial\varepsilon/\partial\omega)} \equiv (-2k_0/cL)\int_0^L \left[\varepsilon_\omega^R(z) + \frac{\omega}{2}\frac{\partial\varepsilon_\omega^R(z)}{\partial\omega}\right] \mathrm{Im}[G_\omega(z,z)]dz$$

and we can rewrite Eq. (25) in a more concise form as:

$$\rho_\omega^\varphi = \rho_\omega^{\varepsilon+(\omega/2)(\partial\varepsilon/\partial\omega)}. \tag{26}$$

Now, using the explicit expression of the imaginary part of the Green's function in terms of the light modes [see Eq. (C7) in Appendix C] and using the following relation:

$$\int_0^L \varepsilon_\omega^R(z)|\Phi_\omega^{(\pm)}|^2dz = (c^2/\omega^2)\int_0^L |d\Phi^{(\pm)}/dz|^2dz + (1/k_0)n_{01,2}\,\mathrm{Im}[r_\omega^{(\pm)}]$$

we can write $\rho_\omega^{\varepsilon+(\omega/2)(\partial\varepsilon/\partial\omega)}$ in a form that involves again the dwell times:

$$\rho_\omega^{\varepsilon+(\omega/2)(\partial\varepsilon/\partial\omega)} = \frac{\tau_D^{(+)}+\tau_D^{(-)}}{2L} + \frac{1}{2ck_0L}\,\mathrm{Im}(r_\omega^{(+)}+r_\omega^{(-)}), \tag{27}$$

where $\tau_D^{(+)}=(2L\langle U_\omega^{(+)}\rangle)/cn_{01}$ and $\tau_D^{(-)}=(2L\langle U_\omega^{(-)}\rangle)/cn_{02}$, are the LTR and RTL dwell times, respectively, and

$$U_\omega^{(\pm)} = \frac{1}{4}\left[\frac{\partial}{\partial\omega}[\omega\varepsilon_\omega^R(z)]|\Phi_\omega^{(\pm)}(z)|^2 + \frac{c^2}{\omega^2}\left|\frac{d\Phi_\omega^{(\pm)}(z)}{dz}\right|^2\right]$$

are the corresponding time-averaged electromagnetic energies calculated taking into account the dispersion of the medium [35]. Therefore all the connections between the Wigner

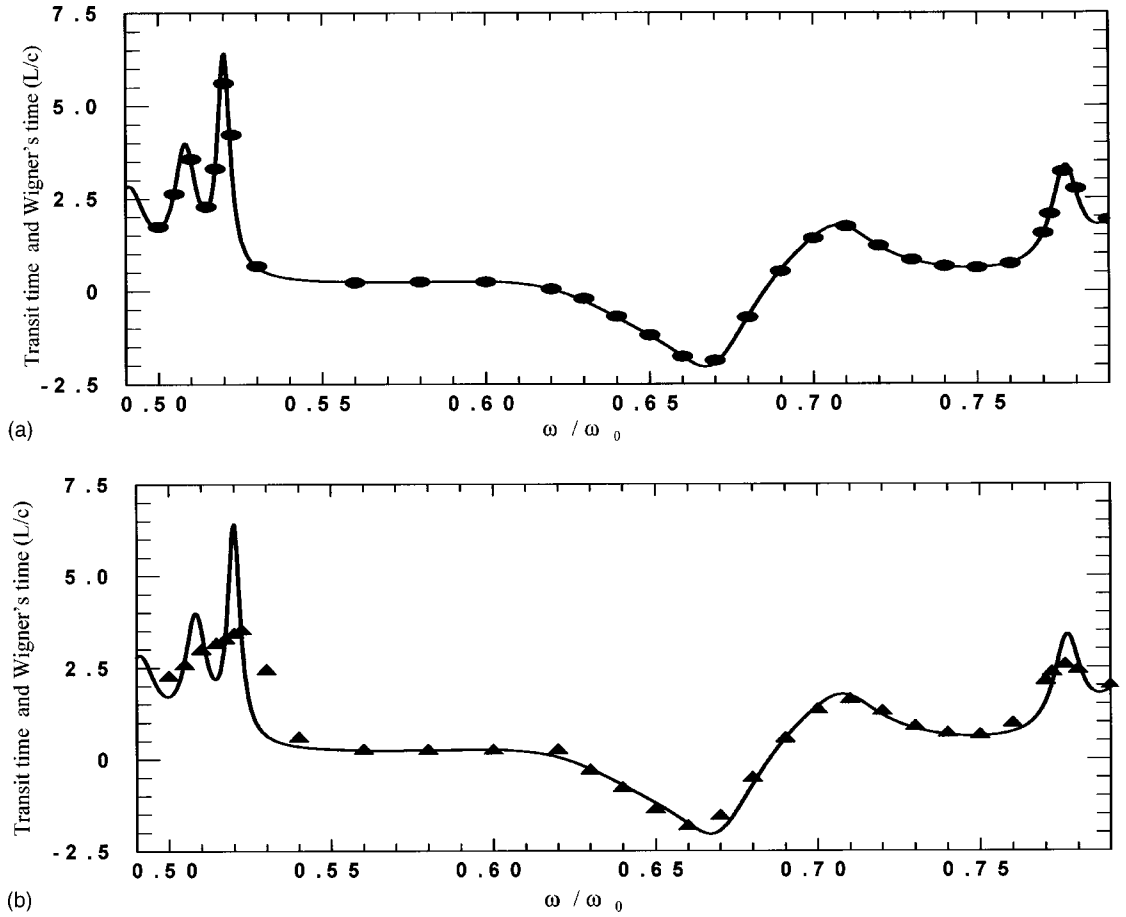


FIG. 7. (a) Transit time (solid circles) and Wigner time (solid line) vs $\omega/\omega_0$ ($\omega_0=2\pi c/\lambda_0$ and $\lambda_0=1$ $\mu$m) where $\omega$ is the carrier frequency of the input pulse. The structure is the same as that described in Fig. 7. Input pulses at $z=0$ ($z=0$ is the location of the input surface of the PC) have the following form: $A(z=0,t)=\exp[-(t^2/2\tau_0^2)-i\omega t]$ where $\tau_0=0.5$ ps. The transit time has been calculated as the time the peak of the transmitted part of the input pulse needs to exit the structure, and the reference time ($t=0$) is the time when the peak of the input pulse reaches the input surface of the PC; (b) transit time (solid triangles) and Wigner time (solid line) in the case of a Gaussian pulse with $\tau_0=0.1$ ps.

time and the dwell time and their relationships with the DOM that were demonstrated for the case of no absorption and no dispersion are still valid in the case of dispersion and negligible absorption, provided the energy density is calculated taking into account the dispersion of the medium.

### C. Case of absorption and dispersion

When the absorption of the material comes into play the DOM calculated using the Wigner time can become negative near the absorption line of the material. Therefore, it can no longer be interpreted as a DOM in the true sense of the word. On the other hand, the DOM defined through Eq. (9) continues to be a positive quantity, and maintains the physical meaning of a quantity proportional to the mean power emitted by a source located inside the PC, as outlined at length in Sec. II. In Fig. 6(a) we compare the DOM calculated using the Wigner time, (dashed line) and the DOM calculated using Eq. (9) (solid line) for a PC structure whose details are described in the figure caption. The high index layer is endowed with a Lorentzian absorption line centered around $\omega/\omega_0=0.65$, and a refractive index approximately of 1.42 in the visible range. The refraction index ($n$) and the extinction coefficient ($K$) of the high index layer are shown in Fig. 6(b). The figure shows that the DOM calculated via the Wigner time attains negative values near the center of the absorption line of the dielectric material ($\rho_\omega^\varphi \cong -2/c$ at $\omega/\omega_0 \cong 0.66$), while Eq. (9) always gives a positive DOM. While this shortcoming clearly implies that the DOM defined in terms of the Wigner time fails to maintain its physical meaning, nevertheless the Wigner time continues to be a good indicator of the tunneling time imparted to the peak of the transmitted part of an input, quasimonochromatic, unchirped pulse as it traverses a 1D barrier. In fact, it is well known that the transit time of pulses tuned near the absorption line of a dielectric material can become superluminal or even negative as in our case [36–38]. Of course, superluminal or negative transit times are not an indication that causality or relativity somehow breakdown, because the fact is that signal velocity always remains subluminal [39], and the peak of the transmitted pulse can always be found under the envelope of a

similar input pulse propagating for the same length of free space [20]. In Fig. 7(a) we compare the Wigner time and the transit time of a Gaussian pulse of unitary amplitude that traverses the structure. Input pulses at $z=0$ ($z=0$ is the location of the input surface of the PC) have the following form: $A(z=0,t)=\exp[-(t^2/2\tau_0^2)-i\omega t]$ where $\tau_0=0.5$ ps and $\omega$ is the carrier frequency. The transit time has been numerically calculated as the time the peak of the transmitted part of the input pulse needs to exit the structure, and the reference time ($t=0$) is the time at which the peak of the input pulse reaches the input surface of the PC. The figure shows that the transit time is well described by the Wigner time, even when strong absorption and dispersion come into play. In this case the spectral bandwidth of the input pulse is $\sim 6$ THz, which corresponds to $\sim 0.83$ ps FWHM of the pulse intensity, and the quasimonochromatic limit is approached (this limit is quickly reached because the spatial extension of a typical pulse is always much larger than the typical structure, which is only a few microns in length). The transmitted pulses come out only slightly distorted with respect to the input pulses, and attenuated by a factor that depends on the transmission properties of the PC.

If shorter pulses were used, as in Fig. 7(b), where the pulses have a duration in time of approximately 0.16 ps FWHM of the pulse intensity, the transit time then begins to differ significantly from the Wigner time. In this case higher order terms of the geometrical dispersion of the structure that are not accounted by the Wigner time come into play, and as a consequence pulses that tunnel through the structure undergo appreciable distortion.

## IV. CONCLUSIONS

In conclusion, we have highlighted the connections that exist between the DOM and tunneling times for 1D barriers. In the absence of absorption, the DOM calculated using the Wigner time is approximately equal to the average of the LTR and RTL dwell times, divided the length L of the structure. We have shown that the self-interference delay is generally negligible [Eqs. (19) and (20), and Fig. 5]. Both the Wigner and dwell-time DOMs *overestimate* the true DOM defined in our Eq. (9) by a factor roughly proportional to the average index of the barrier [Eq. (18), and Fig. 4]. Structures embedded in symmetric environments composed of a sufficient number of periods acquire the properties of symmetric structures, and the DOM calculated through the Wigner time is then approximately equal to the DOM calculated using the energy density [Eq. (20) and Fig. 3], as first proposed in Ref. [8] using heuristic arguments. In the presence of absorption, the Wigner DOM can become negative (see Fig. 6), while the true DOM remains always positive. In that case, the Wigner DOM ceases to be a valid representation of DOM, but can still be interpreted and is still a good measure of group velocity of unchirped, quasimonochromatic pulses. We conclude that the DOM in 1D barriers should always be calculated using our Eq. (9), and plenty of caution should be exercised when one makes connections between the DOM and tunneling times. Such a connection sometimes yields useful information about the system, such as group velocity, for example, but if the true DOM is sought the approach suggested by our Eq. (9) should always be preferred.

Finally, we point out that while the DOM defined through Eq. (9) can be straightforwardly generalized to multidimensional cases, the other definitions based on the tunneling times find a direct link with the DOM only in one dimension, and then only when there is no absorption.

## ACKNOWLEDGMENTS

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## APPENDIX A

We begin with Eq. (3):

$$\frac{\partial^2 G_\omega(z,\xi)}{\partial z^2}+\frac{\omega^2\varepsilon_\omega(z)G_\omega(z,\xi)}{c^2}=\delta(z-\xi), \tag{A1}$$

when $G_\omega(z,\xi)$ exists, it follows that $G_\omega(z,\xi)$ satisfies the homogeneous equation:

$$\frac{\partial^2 G_\omega(z,\xi)}{\partial z^2}+\frac{\omega^2\varepsilon_\omega(z)G_\omega(z,\xi)}{c^2}=0, \tag{A2}$$

at all points of the interval $0\leqslant z\leqslant L$ except at the point $z=\xi$. As discussed at length in Ref. [24], the light-modes $\{\Phi_\omega^{(\pm)}\}$ are a fundamental set of solutions of Eq. (A2) and they are subject to the following boundary conditions for a nonmagnetic material (see Fig. 2):

$$\begin{array}{ll}
\Phi_\omega^{(+)}(0)=1+r_\omega^{(+)} & \Phi_\omega^{(-)}(0)=t_\omega^{(-)} \\
\Phi_\omega^{(+)}(L)=t_\omega^{(+)} & \Phi_\omega^{(-)}(L)=1+r_\omega^{(-)} \\
[d\Phi_\omega^{(+)}/dz]_{z=0}=ik_0n_{0,1}(1-r_\omega^{(+)}), & [d\Phi_\omega^{(+)}/dz]_{z=L}=ik_0n_{0,2}t_\omega^{(+)} \\
[d\Phi_\omega^{(-)}/dz]_{z=0}=-ik_0n_{0,1}t_\omega^{(-)} & [d\Phi_\omega^{(-)}/dz]_{z=L}=ik_0n_{0,2}(r_\omega^{(-)}-1)
\end{array}. \tag{A3}$$

Consequently, we can express the most general solution of Eq. (A1) as:

$$G_\omega(z,\xi)=\begin{cases} C_1(\xi)\Phi_\omega^{(+)}(z)+D_1(\xi)\Phi_\omega^{(-)}(z) & L\geqslant z>\xi \\ D_2(\xi)\Phi_\omega^{(+)}(z)+C_2(\xi)\Phi_\omega^{(-)}(z) & 0\leqslant z<\xi. \end{cases} \tag{A4}$$

The four constants must now be determined. First, we impose the condition of "*outgoing waves*." This condition requires that the radiated energy from the point source located at $z=\xi$ leaves the structure, and no energy is incoming into the structure. This means that the coefficients $D_1$ of the incoming RTL wave and the coefficient $D_2$ of the incoming LTR wave must be zero. The constant $C_1$ and $C_2$ must be determined by imposing the continuity of $G_\omega(z,\xi)$ at $z=\xi$, while its derivative has a jump of magnitude one [25]. Following the above procedure we find:

$$\begin{cases} C_1(\xi)=\dfrac{\Phi_\omega^{(-)}(\xi)}{[W(\Phi_\omega^{(-)},\Phi_\omega^{(+)})]_{z=\xi}}, \\ D_1=0 \\ C_2(\xi)=\dfrac{\Phi_\omega^{(+)}(\xi)}{[W(\Phi_\omega^{(-)},\Phi_\omega^{(+)})]_{z=\xi}}, \\ D_2=0 \end{cases} \tag{A5}$$

where

$$W(\Phi_\omega^{(-)},\Phi_\omega^{(+)})\equiv\begin{vmatrix} \Phi_\omega^{(-)} & \Phi_\omega^{(+)} \\ d\Phi_\omega^{(-)}/dz & d\Phi_\omega^{(+)}/dz \end{vmatrix}$$

is the Wronskian of the fundamental set of solutions. In our case the Wronskian is a conserved quantity, i.e., $dW(\Phi_\omega^{(-)},\Phi_\omega^{(+)})/dz=0$ and it can be calculated by resorting to the boundary conditions in Eq. (A3). The result is

$$\begin{aligned}[W(\Phi_\omega^{(-)},\Phi_\omega^{(+)})]_{z=\xi}&=[W(\Phi_\omega^{(-)},\Phi_\omega^{(+)})]_{z=0}=2ik_0n_{0,1}t_\omega^{(-)} \\ &=[W(\Phi_\omega^{(-)},\Phi_\omega^{(+)})]_{z=L}=2ik_0n_{0,2}t_\omega^{(+)},\end{aligned}$$

from which one also derives that $n_{0,1}t_\omega^{(-)}=n_{0,2}t_\omega^{(+)}=\tilde{t}_\omega$. The Wronskian calculated at the point $z=\xi$ can be consequently expressed as:

$$W(\Phi_\omega^{(-)},\Phi_\omega^{(+)})=[W(\Phi_\omega^{(-)},\Phi_\omega^{(+)})]_{z=\xi}=2ik_0\tilde{t}_\omega. \tag{A6}$$

Equation (4) in the main text follows from Eqs. (A4)–(A6).

## APPENDIX B

We start by writing the Helmholtz equation for the field $\Phi_{\omega+\delta\omega}^{(+)}(z)$:

$$\frac{\partial^2\Phi_{\omega+\delta\omega}^{(+)}(z)}{\partial z^2}+\frac{(\omega+\delta\omega)^2\varepsilon_{\omega+\delta\omega}(z)\Phi_{\omega+\delta\omega}^{(+)}(z)}{c^2}=0. \tag{B1}$$

Let us expand the functions in Taylor series:

$$\Phi_{\omega+\delta\omega}^{(+)}(z)=\Phi_\omega^{(+)}(z)+\Gamma_\omega(z)\,\delta\omega+O(\delta\omega^2), \tag{B2.1}$$

$$(\omega+\delta\omega)^2=\omega^2+2\omega\delta\omega+O(\delta\omega^2), \tag{B2.2}$$

$$\varepsilon_{\omega+\delta\omega}(z)=\varepsilon_\omega(z)+\frac{\partial\varepsilon_\omega(z)}{\partial\omega}\delta\omega+O(\delta\omega^2), \tag{B2.3}$$

where $\Gamma_\omega(z)=[\partial\Phi_\omega^{(+)}(z)/\partial\omega]$. By substituting Eqs. (B2.1), (B2.2), and (B2.3) in Eq. (B1) we obtain:

$$\begin{aligned}&\frac{\partial^2\Phi_\omega^{(+)}(z)}{\partial z^2}+\frac{\omega^2\varepsilon_\omega(z)\Phi_\omega^{(+)}(z)}{c^2}+\left[\frac{\partial^2\Gamma_\omega(z)}{\partial z^2}+\frac{\omega^2\varepsilon_\omega(z)}{c^2}\Gamma_\omega(z)\right. \\ &\left.+\frac{\omega}{c^2}\Phi_\omega^{(+)}(z)\left(2\varepsilon_\omega(z)+\frac{\partial\varepsilon_\omega(z)}{\partial\omega}\omega\right)\right]\delta\omega+O(\delta\omega^2)=0.\end{aligned} \tag{B3}$$

From Eq. (B3), equating to zero the terms of the same order in $\delta\omega$ we obtain the following equations:

$$\frac{\partial^2\Phi_\omega^{(+)}(z)}{\partial z^2}+\frac{\omega^2\varepsilon_\omega(z)\Phi_\omega^{(+)}(z)}{c^2}=0, \tag{B4.1}$$

$$\frac{\partial^2\Gamma_\omega(z)}{\partial z^2}+\frac{\omega^2\varepsilon_\omega(z)}{c^2}\Gamma_\omega(z)+\frac{\omega}{c^2}\Phi_\omega^{(+)}(z)\left[2\varepsilon_\omega(z)+\frac{\partial\varepsilon_\omega(z)}{\partial\omega}\omega\right]=0. \tag{B4.2}$$

Equation (B4.1) is the Helmholtz equation for the field $\Phi_\omega^{(+)}$ and the solution is known, while Eq. (B4.2) is a second-order differential equation for the function $\Gamma_\omega(z)$ and it can be written in the following form:

$$\frac{\partial^2\Gamma_\omega(z)}{\partial z^2}+\frac{\omega^2\varepsilon_\omega(z)}{c^2}\Gamma_\omega(z)=-\frac{\omega}{c^2}\Phi_\omega^{(+)}(z)\left[2\varepsilon_\omega(z)+\frac{\partial\varepsilon_\omega(z)}{\partial\omega}\omega\right]. \tag{B5}$$

The solution of second order linear differential equations of the same type as that in Eq. (B5) can be written using the method of the Green's function [25]. In our case, the solution of Eq. (B5) with the boundary conditions corresponding to "outgoing waves" can be expressed in terms of the Green function calculated in Appendix A as follows:

$$\Gamma_\omega(z)=-\frac{\omega}{c^2}\int_0^L G_\omega(z,\xi)\Phi_\omega^{(+)}(\xi)\left[2\varepsilon_\omega(\xi)+\frac{\partial\varepsilon_\omega(\xi)}{\partial\omega}\omega\right]d\xi, \tag{B6}$$

by calculating Eq. (B6) for $z=L$ we obtain Eq. (14) in the main text.

We remark that the approach we have followed to arrive to Eq. (B6) is valid under general conditions. In fact: (a) both absorbtion and dispersion of the medium are taken into account because we consider the dielectric response $\varepsilon_\omega(z)$ as a complex function of the frequency; (b) no perturbative approach is used to arrive to Eq. (B6), but only the hypothesis that the functions can be expanded in Taylor series.

## APPENDIX C

Let us start from the definition of $\rho_\omega^\varepsilon$:

$$\rho_\omega^\varepsilon=(-2k_0/cL)\int_0^L\varepsilon_\omega^R(z)\mathrm{Im}[G_\omega(z,z)]dz. \tag{C1}$$

Equation (C1) can be rewritten in the following form:

$$\rho_\omega^\varepsilon = \frac{1}{cL|\tilde{t}_\omega|}\int_0^L \varepsilon_\omega^R(z)|\Phi_\omega^{(+)}||\Phi_\omega^{(-)}|\cos[\varphi_\omega^{(+)}+\varphi_\omega^{(-)}-\varphi_t]dz, \tag{C2}$$

where we have used the expression of the Green's function given by Eq. (4), we have written the LTR and RTL modes as: $\Phi_\omega^{(\pm)}(z)=|\Phi_\omega^{(\pm)}(z)|\exp(i\varphi_\omega^{(\pm)})$, and the transmission function of the PC as: $\tilde{t}_\omega=|\tilde{t}_\omega|\exp[i\varphi_t(\omega)]$. Now, equating the real and imaginary parts of Eq. (A6), we obtain:

$$|\Phi_\omega^{(-)}|\frac{d|\Phi_\omega^{(+)}|}{dz}-|\Phi_\omega^{(+)}|\frac{d|\Phi_\omega^{(-)}|}{dz}=2k_0|\tilde{t}_\omega|\sin[\varphi_\omega^{(+)}+\varphi_\omega^{(-)}-\varphi_t], \tag{C3.1}$$

$$|\Phi_\omega^{(+)}||\Phi_\omega^{(-)}|\left[\frac{d\varphi_\omega^{(+)}}{dz}-\frac{d\varphi_\omega^{(-)}}{dz}\right]=2k_0|\tilde{t}_\omega|\cos[\varphi_\omega^{(+)}+\varphi_\omega^{(-)}-\varphi_t]. \tag{C3.2}$$

Using Eq. (C3.2), we can recast Eq. (C2) in the following form:

$$\rho_\omega^\varepsilon = \frac{1}{2k_0cL|\tilde{t}_\omega|^2}\int_0^L \varepsilon_\omega^R(z)|\Phi_\omega^{(+)}|^2|\Phi_\omega^{(-)}|^2\left[\frac{d\varphi_\omega^{(+)}}{dz}-\frac{d\varphi_\omega^{(-)}}{dz}\right]dz. \tag{C4}$$

For a nonabsorbing PC, i.e., $\varepsilon_\omega^I(z)=0$, it can be shown that $|\Phi_\omega^{(\pm)}(z)|^2\ (d\varphi_\omega^{(\pm)}/dz)$ is a conserved quantity [31] and it can be calculated by resorting to the boundary conditions imposed on the LTR and RTL modes, i.e., Eqs. (A3), giving the following results:

$$|\Phi_\omega^{(+)}|^2\frac{d\varphi_\omega^{(+)}}{dz}=+\frac{k_0}{n_{02}}|\tilde{t}_\omega|^2, \tag{C5.1}$$

$$|\Phi_\omega^{(-)}|^2\frac{d\varphi_\omega^{(-)}}{dz}=-\frac{k_0}{n_{01}}|\tilde{t}_\omega|^2. \tag{C5.2}$$

From Eqs. (C5) and (C4) we obtain:

$$\rho_\omega^\varepsilon = \frac{1}{2cLn_{0,1}n_{0,2}}\int_0^L \varepsilon_\omega^R(z)[n_{0,2}|\Phi_\omega^{(+)}|^2+n_{0,1}|\Phi_\omega^{(-)}|^2]dz, \tag{C6}$$

and from Eqs. (C6) and (C1) we also arrive to a useful expression of the imaginary part of the Green's function in terms of the LTR and RTL light modes:

$$\mathrm{Im}[G_\omega(z,z)]=-\frac{1}{4k_0}\left[\frac{n_{0,2}|\Phi_\omega^{(+)}|^2+n_{0,1}|\Phi_\omega^{(-)}|^2}{n_{0,1}n_{0,2}}\right]. \tag{C7}$$

Now, using the relation:

$$\int_0^L \varepsilon_\omega^R(z)|\Phi_\omega^{(\pm)}|^2dz=(c^2/\omega^2)\int_0^L|d\Phi^{(\pm)}/dz|^2dz + (1/k_0)n_{01,2}\mathrm{Im}[r_\omega^{(\pm)}]$$

[8,31], Eq. (C6) can be recast in the following form:

$$\rho_\omega^\varepsilon = \frac{1}{4cL}\left[\frac{1}{n_{0,1}}\int_0^L\left(\varepsilon_\omega^R|\Phi_\omega^{(+)}|^2+\frac{c^2}{\omega^2}\left|\frac{d\Phi^{(+)}}{dz}\right|^2\right)dz+\frac{1}{n_{0,2}} \times\int_0^L\left(\varepsilon_\omega^R|\Phi_\omega^{(-)}|^2+\frac{c^2}{\omega^2}\left|\frac{d\Phi^{(-)}}{dz}\right|^2\right)dz+\frac{2}{k_0}\,\mathrm{Im}(r_\omega^{(+)}+r_\omega^{(-)})\right]. \tag{C8}$$

The dwell time is defined as the average electromagnetic energy density stored in the structure divided by the input power [31,32]. In our case the expression for the dwell times for a LTR and RTL input are, respectively,

$$\tau_D^{(\pm)}=\frac{1}{2cn^{(\pm)}}\int_0^L\left(\varepsilon_\omega^R|\Phi_\omega^{(\pm)}|^2+\frac{c^2}{\omega^2}\left|\frac{d\Phi^{(\pm)}}{dz}\right|^2\right)dz, \tag{C9}$$

where $n^{(+)}=n_{0,1}$ and $n^{(-)}=n_{0,2}$. Equation (19) is from Eqs. (C8) and (C9).


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