Control of on-off intermittency by slow parametric modulation

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(Received 7 January 2004; published 17 June 2004)

We study on-off intermittent behavior in two coupled double-well Duffing oscillators with stochastic driving and demonstrate that, by using slow harmonic modulation applied to an accessible system parameter, the intermittent attractors can be completely eliminated. The influence of noise is also investigated. Power-law scaling of the average laminar time with a critical exponent of −1 as a function of both the amplitude and frequency of the control modulation is found near the onset of intermittency, which is a signature of on-off intermittency.

DOI: 10.1103/PhysRevE.69.067203 PACS number(s): 05.45.Pq, 02.60.Cb, 05.45.Xt

Coexisting attractors and intermittency are common complex phenomena observed in many nonlinear dynamical systems. The intermittency route to chaos may be observed in a dynamical system when a control parameter passes through a critical value. The intermittent behavior is characterized by irregular bursts (turbulent phases) interrupting the nearly regular (laminar) phases. Different types of intermittency have been observed and classified into type I, type II, and type III of Pomeau-Manneville intermittency [1], on-off intermittency [2], and crisis-induced intermittency [3]. The type of intermittency depends on the type of bifurcation at the critical point. The type I and on-off intermittency are associated with saddle-node bifurcations, the type II and type III with Hopf and inverse period-doubling bifurcations, respectively, and crisis-induced intermittency with a crisis of chaotic attractors when two (or more) chaotic attractors simultaneously collide with a periodic orbit (or orbits) [4].

On-off intermittency differs from other types of intermittency because it requires a dynamical time-dependent forcing of a bifurcation parameter through a bifurcation point [5], whereas for other types of intermittency the parameters are fixed. Therefore, this type of intermittency is often called modulational intermittency [6]. In on-off intermittency one or more dynamical variables of the system exhibit two distinct states as the system evolves in time. In the "off" state the variables remain approximately constant in various time intervals. These periods are called laminar phases. The "on" states are characterized by irregular bursts of the variables away from their constant values.

The effect of on-off intermittency has been investigated in one-dimensional maps coupled to either random or chaotic signals [5,7], in a forced logistic map whose control parameter fluctuates either chaotically or stochastically [8], and in periodically forced coupled Duffing oscillators [9,10]. Like the other types, on-off intermittency is characterized by fundamental statistical properties with typical power-law scalings near the onset of intermittency: (i) for the mean laminar phase as a function of the coupling parameter with a critical exponent of −1 [7], and (ii) for the probability distribution of the laminar phase versus the laminar length with exponent −3/2 [7]. The on-off intermittency has also been detected experimentally in electronic circuits [11], in a gas discharge plasma [12], in a spin wave system [13], in nematic liquid crystals [14], and in a laser [15]. In the case of periodically driven systems, the same critical exponent of −1 for the mean laminar phase has been found in laser experiments as a function of both the amplitude and frequency of the parametric modulation near the onset of intermittency [15].

The possibility for controlling on-off intermittent dynamics was investigated first by Nagai, Hua, and Lai [10]. Their control method is based on the ideas of Ott, Grebogi, and Yorke (OGY) for controlling chaos [16]. Specifically, they devised an algorithm for stabilizing a trajectory in the vicinity of a desirable ("off") state by using arbitrarily small feedback perturbations to a system parameter. Their closed-loop control algorithm requires the knowledge of system equations. However, in many practical situations the detailed system equations are not available. For such a case, an open-loop control algorithm might be more realistic. Before the OGY method, Lima and Pettini [17] proposed a nonfeedback perturbative technique of stabilizing a chaotic system toward a periodic state. This technique was applied experimentally for eliminating chaotic oscillations in a bistable magnetoelastic system [18] and for stabilizing periodic orbits in a laser [19].

In this Brief Report we study the possibility of controlling on-off intermittency by harmonic modulation applied to an accessible system parameter. Our method to confine a trajectory in the "off" state is based on Lima and Pettini's idea of the open-loop control of chaos. Similarly to Nagai, Hua, and Lai [10], we assume that the desirable operational state of the system is the "off" state and the "on" state is undesirable. That is, we wish to avoid temporal bursts ("on" states) of dynamical variables. As distinct from the closed-loop control, the open-loop control is not restricted to small perturbations. The modulation amplitude may be arbitrarily large to achieve the control goal. However, the main advantage of this type of the control is that it does not require a prior knowledge of system equations. We also show in this work how the control works in the presence of external noise of different levels.

The analysis is carried out on an example of two coupled double-well Duffing oscillators with random driving. Along with many other complex systems, the coupled Duffing os-

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cillators exhibit coexistence of several attractors; some of them may be chaotic (intermittent), while the others are steady states. In such a situation, the control of intermittency may be manifested as annihilation of the intermittent attractors so that all trajectories are driven to the steady states. In our recent work [20] we showed that coexisting fixed points and limit cycles in multistable systems can be annihilated by harmonic parametric modulation. In this work we demonstrate how the annihilation effect is achieved with intermittent states in randomly driven Duffing oscillators.

The dynamics of two identical nonlinear oscillators with random driving can be governed by the equation,

$$\ddot{\mathbf{x}} + \gamma\dot{\mathbf{x}} - q\xi x = -\nabla V(\mathbf{x}), \tag{1}$$

where $\mathbf{x} \equiv (x,y)$, γ is a damping factor, ξ is uniformly distributed noise of level q in the unit interval [0,1], and $V(\mathbf{x})$ is a two-dimensional anharmonic potential function of coupled oscillators that for symmetric Duffing oscillators can be expressed as follows [10]:

$$V(x,y) = (1-x^2)^2 + (y^2-a^2)^2(x-d) + b(y^2-a^2)^4, \tag{2}$$

where a, d, and $b(>0)$ are parameters. We assume that one of the coupled subsystems (in the x direction) is randomly driven, i.e., noisy. The system Eqs. (1) and (2) can be written as four first-order differential equations in terms of the dynamical variables $x_1 = x$, $x_2 = \dot{x}$, $x_3 = y$, and $x_4 = \dot{y}$,

$$\dot{x}_1 = x_2, \tag{3}$$

$$\dot{x}_2 = -\gamma x_2 + 4x_1(1-x_1^2) - (x_3^2-a^2)^2 + q\xi x_1, \tag{4}$$

$$\dot{x}_3 = x_4, \tag{5}$$

$$\dot{x}_4 = -\gamma x_4 - 4x_3(x_3^2-a^2)(x_1-d) - 8bx_3(x_3^2-a^2)^3. \tag{6}$$

The system Eqs. (3)–(6) exhibits different dynamical regimes from regular states to on-off intermittency in a wide range of parameter values [9]. For simplicity we consider the case $\gamma = 0.04$, $a = 0.73$, $b = 0.008$, and $d = -1.8$. Due to the presence of two invariant subspaces at $x_3 = \pm a$ and $x_4 = 0$, there are two "off" states, i.e., the phenomenon referred to as *two-state on-off intermittency* [10]. The two "off" states arise from two wells in the potential $V(x,y)$ in the y direction. At relatively low noise ($q < 3$), the one-state and two-state intermittency regimes appear only as transients. The two-state on-off intermittent attractor is created at relatively high noise ($q \geqslant 3$) and coexists with two steady states corresponding to the potential wells. Our goal is to eliminate the intermittent attractors so that a trajectory initiated from a random initial condition stays in the vicinity of one of the potential wells in y, assuming that this potential well is the desirable state of the system.

We choose a to be the parameter to which the control modulation is applied in the following form:

$$a = a_0[1 - m\sin(2\pi f t)], \tag{7}$$

where m and f are the modulation depth and frequency and a_0 is the initial value of the parameter ($a_0 = 0.73$). A glimpse of the results is presented in Fig. 1 where we demonstrate the control effect on some of the coexisting attractors. The system, prior to the control, is in the chaotic state. When the control is switched on (at $t = 5000$), the intermittent attractors disappear, and the trajectory is attracted to one of the remaining steady states. Note that the external harmonic modulation creates a limit cycle around each fixed point so that the final state is a periodic orbit. When the modulation amplitude is applied but is not sufficiently large to eliminate an intermittent attractor, the system exhibits the coexistence of five attractors. In addition to two limit cycles in the vicinity of each potential well, two one-state and one two-state intermittency regimes coexist. In Figs. 1(b) and 1(c) we demonstrate how a sudden increase in m annihilates the intermittent states resulting in the periodic bistability.

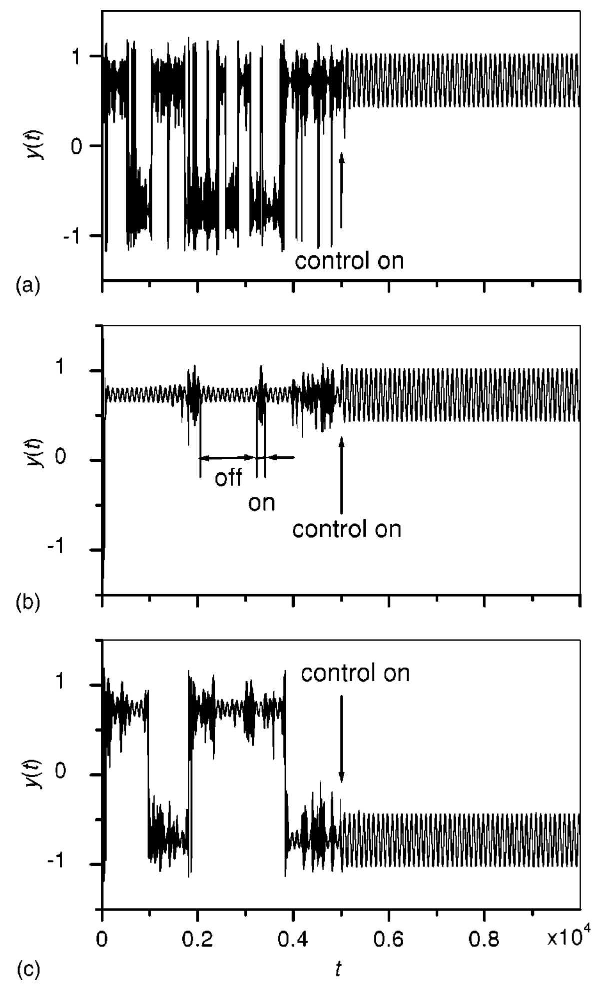

FIG. 1. A slow parametric modulation leads to the disappearance of intermittent attractors. The initial system states are (a) two-state on-off intermittency without modulation ($m = 0$), and (b) one-state and (c) two-state on-off intermittency with small modulation ($m = 0.1$). The arrows indicate the moments when the control with $m = 0.4$ and $f = 0.01$ is applied. The trajectory is attracted to the limit cycle in the vicinity of one of the potential wells. This demonstrates the flexibility of the control to select different desirable "off" states.

The required modulation amplitude for the control depends on both the noise level and the modulation frequency as shown in Fig. 2. In the presence of the parametric modu-

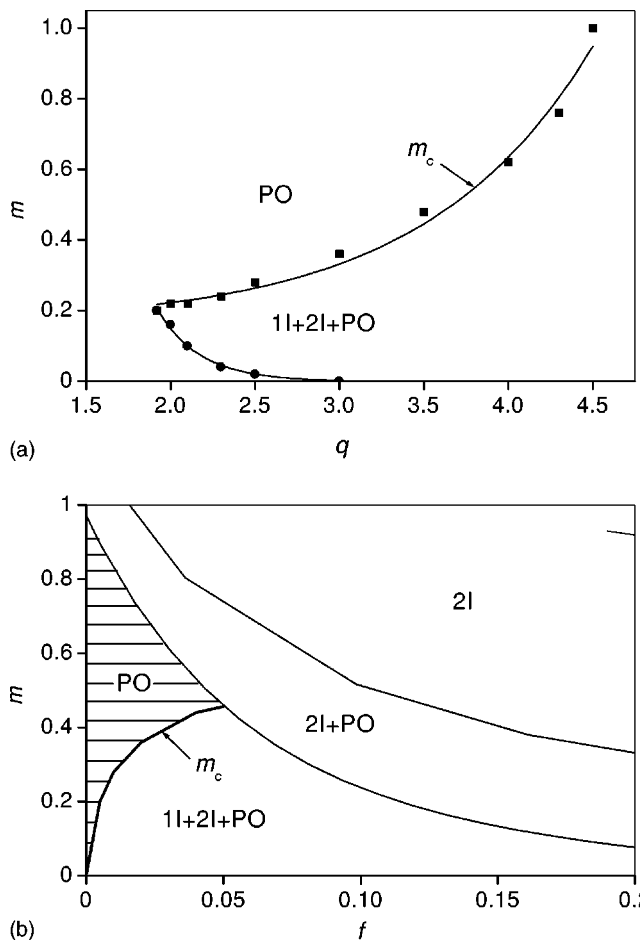

FIG. 2. Codimensional-2 bifurcation diagrams in space of (a) noise level and modulation depth at $f=0.01$ and (b) modulation frequency and depth at $q=3$. The boundaries between different dynamical regimes, one-state (1I) and two-state (2I) intermittency and periodical orbit (PO), are shown. The bifurcation lines for the onset of intermittency are indicated by the arrows. Only periodical regimes exist in the dashed region.

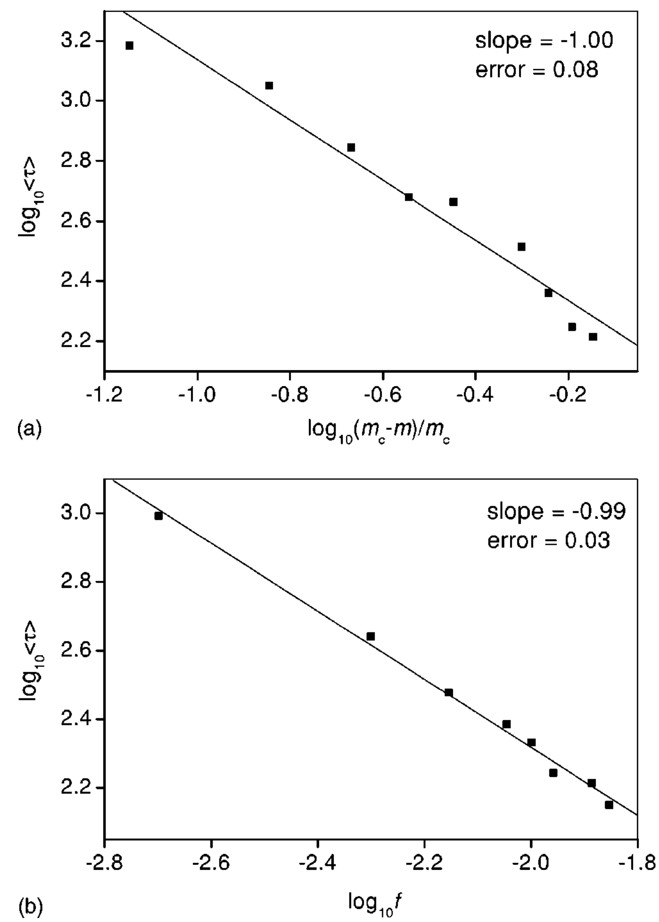

FIG. 3. Average laminar length (a) versus relative difference of modulation depth from its critical value at $f=0.01$ and (b) versus modulation frequency at $m=0.14$ on log-log scales. $q=2.5$. The fits of the data to straight lines are good, the slopes of which are -1.

lation Eq. (7), the intermittent attractors appear at a certain level of the noise ($q>1.9$ for $f=0.01$) [Fig. 2(a)]. To eliminate these attractors, the amplitude of the control modulation should be above some critical value m_c, i.e., above the corresponding bifurcation lines in Fig. 2. Note that, for relatively low noise ($1.9<q<3$), there are two critical values for the modulation amplitude, which correspond to the onset and offset of on-off intermittency. The two bifurcation lines in Fig. 2(a) are good fits of the data to the exponential growth and decay with critical exponents of 1 and 0.25, respectively. As seen from Fig. 2(b), the intermittent attractors can be destroyed only by slow parametric modulation ($f<0.05$) when $m>m_c$. In the regime of on-off intermittency, a typical trajectory spends a long time near one invariant subspace (laminar phase), and when the modulation is fast, the system has no time to respond to the control. In other words, the period of the modulation should be of the same order of magnitude as the characteristic time for which the trajectory spends near one invariant subspace before being repelled away. Of course, the duration of the laminar phase depends on the noise level. This suggests that the reason for the control effect is a resonant interaction of the modulation frequency with the frequency at which the trajectory was repelled away from one of the invariant subspaces.

Taking into account the above speculations, the mean duration of the laminar phase is one of the important characteristics both for achieving the control goal and for characterization of the observed intermittent behavior in general. In Fig. 3 we plot on a log-log scale the mean duration of the laminar phase, $\langle\tau\rangle$, as a function of both the relative difference of the modulation depth from its critical value $(m_c - m)/m_c$ [Fig. 3(a)] and the modulation frequency f. We find that in both cases these dependences obey the -1 scaling law that characterizes on-off intermittency. This result agrees well with other theoretical work where the control parameter was driven randomly [7] and with laser experiments where the parameter was modulated periodically [15].

In conclusion, the possibility of the open-loop control of a chaotic dynamical system that exhibits on-off intermittency has been demonstrated. We have shown that a trajectory can be stabilized in the vicinity of a desired state ("off" state) by slow harmonic modulation applied to an available system parameter. We have derived the conditions for the modulation amplitude and frequency to achieve the control goal in the presence of noise of different levels. A scaling law with a critical exponent of −1 for the mean duration of the laminar phase versus both the modulation amplitude and frequency has been found. The coincidence of this scaling relation with those of other work verifies the universal character of this scaling relation for different types of driving and different types of on-off intermittencies. The control can be realized easily in practice, because in the experimental situation the driving signal is well defined and hence the appropriate modulation parameters can be computed and applied to the system to eliminate intermittent attractors even without the knowledge of an adequate theoretical model.

This work has been supported through a grant from the Institute Mexico-USA of the University of California (UC-MEXUS) and Consejo Nacional de Ciencia y Tecnologia (CONACyT).

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