

Guiding optical flows by photonic crystal slabs made of dielectric cylinders

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We investigate the electromagnetic propagation in two-dimensional photonic crystals, formed by parallel dielectric cylinders embedded in a uniform medium. The frequency band structure is computed using the standard plane-wave expansion method, while the propagation and scattering of the electromagnetic waves are calculated by the multiple scattering theory. It is shown that within partial band gaps, the waves tend to bend away from the forbidden directions. Such a property may render novel applications in manipulating optical flows. In addition, the relevance with the imaging by flat photonic crystal slabs will also be discussed.

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I. INTRODUCTION

When propagating through periodically structured media such as photonic crystals (PCs), optical waves will be modulated with the periodicity. As a result, the dispersion of waves will no longer behave as in a free space, and so-called frequency band structures appear. Under certain conditions, waves may be prohibited from propagation in certain or all directions, corresponding to partial and complete band gaps, respectively. The photonic crystals revealing band gaps are called band gap materials.

Photonic crystals and band gap materials have a broad spectrum of applications, ranging from computing to digital communication and from laser cavities to optical transistors [1]. The possibilities are unlimited. In fact, applications have gone well beyond expectation, and are so far reaching that a fruitful new field called photonic crystals has come into existence. Most updated information about the research of photonic crystals and related materials can be found in the comprehensive webpage [2].

So far, most applications are associated with the properties of the complete band gaps of PCs. On one hand, the band gaps confine optical propagation within certain frequency regimes. On the other, when encountering the complete bands, optical waves can be guided into desired directions. For example, one of the main applications of PCs is to control optical flows, so that they can be used for things such as telecommunications. A comprehensive survey of photonic crystal research can be referred to Refs. [1–6]. To our knowledge, however, there have been very few attempts in the literature to explore possible usage of partial band gaps. In this paper, we wish to discuss a previously undiscussed phenomenon associated with partial band gaps, that is, deflection of optical waves. That is, the partial band gap can collimate wave propagation into certain directions. This property may allow for novel applications in manipulating optical flows.

The paper is organized as follows. The systems and the theory will be outlined in the following section. The results and discussion will be presented in Sec. III, followed by a short summary.

II. THE SYSTEMS AND FORMULATION

The systems considered here are two-dimensional photonic crystals made of arrays of parallel dielectric cylinders placed in a uniform medium, which we assume to be air. Such systems are common in both theoretical simulations or experimental measurements of two-dimensional PCs [1,2]. For brevity, we only consider the *E*-polarized waves (TM mode), that is, the electric field is kept parallel to the cylinders. The following parameters are used in the simulation.

(1) The dielectric constant of the cylinders is 14, and the cylinders are arranged to form a square lattice.

(2) The lattice constant is a and the radius of the cylinders is $0.3a$; in the computation, all lengths are scaled by the lattice constant.

(3) The unit for the angular frequency is $2\pi c/a$. After scaling, the systems become dimensionless; thus the features discussed here would be applicable to a wider range of situations.

While the frequency band structure in the systems can be calculated by the plane-wave expansion method [1], the propagation and scattering of electromagnetic (EM) waves in such systems can be studied by the standard multiple scattering theory. The theory originated from the self-consistent idea first discussed by Foldy [7], and then reached maturity through the significant efforts by Lax [8], Waterman and Truell [9], and particularly by Twersky [10]. The multiple scattering theory has been reformulated in various forms for two- and three-dimensional systems [11–14]. A review may be found in Ref. [15].

The essence of the theory is summarized as follows. In response to the incident wave from the source and the scattered waves from other scatterers, each scatterer will scatter waves repeatedly, and the scattered waves can be expressed in terms of a modal series of partial waves. When this scattered wave serves as an incident wave to other scatterers, a set of coupled equations can be formulated and computed rigorously. The total wave at any spatial point is the summation of the direct wave from the source and the scattered waves from all scatterers. The intensity of the wave is represented by the square of the wave field.

For the reader's convenience we present briefly the general multiple scattering theory. Consider that N straight cyl-

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inders of radius a^i located at \vec{r}_i with $i=1,2,\dots,N$ to form an array. A line source transmitting monochromatic waves is placed at \vec{r}_s . Here we take the standard approach with regard to the source. That is, the transmission from the source is calculated from the multiple scattering theory, and assume that the source is not affected by the surroundings. If some other sources such as a line of atoms are used, the reaction between the source and the backscattered waves should be taken into account.

The scattered wave from each cylinder is a response to the total incident wave composed of the direct wave from the source and the multiply scattered waves from other cylinders. The final wave reaching a receiver located at \vec{r}_r is the sum of direct wave from the source and the scattered waves from all the cylinders.

The scattered wave from the j th cylinder can be written as

$$p_s(\vec{r}, \vec{r}_j) = \sum_{n=-\infty}^{\infty} i\pi A_n^j H_n^{(1)}(k|\vec{r}-\vec{r}_j|) e^{in\phi_{\vec{r}-\vec{r}_j}}, \quad (1)$$

where k is the wave number in the medium, $H_n^{(1)}$ is the n th-order Hankel function of first kind, and $\phi_{\vec{r}-\vec{r}_j}$ is the azimuthal angle of the vector $\vec{r}-\vec{r}_j$ relative to the positive x axis. The total incident wave around the i th cylinder ($i=1,2,\dots,N; i \neq j$) is the summation of the direct incident wave from the source and the scattered waves from all other scatterers, and can be expressed as

$$p_m^i(\vec{r}) = \sum_{n=-\infty}^{\infty} B_n^i J_n(k|\vec{r}-\vec{r}_i|) e^{in\phi_{\vec{r}-\vec{r}_i}}. \quad (2)$$

In this paper, p stands for the electrical field in the TM mode and the magnetic field in the TE mode.

The coefficients A_n^i and B_n^i can be solved by expressing the scattered wave $p_s(\vec{r}, \vec{r}_j)$, for each $j \neq i$, in terms of the modes with respect to the i th scatterer by the addition theorem for Bessel function. Then the usual boundary conditions are matched at the surface of each scattering cylinder. This leads to

$$B_n^i = S_n^i + \sum_{j=1, j \neq i}^N C_n^{j,i}, \quad (3)$$

with

$$S_n^i = i\pi H_{-n}^{(1)}(k|\vec{r}_i|) e^{-in\phi_{\vec{r}_i}}, \quad (4)$$

and

$$C_n^{j,i} = \sum_{l=-\infty}^{\infty} i\pi A_l^j H_{l-n}^{(1)}(k|\vec{r}_i-\vec{r}_j|) e^{i(l-n)\phi_{\vec{r}_i-\vec{r}_j}}, \quad (5)$$

and

$$B_n^i = i\pi \tau_n^i A_n^i, \quad (6)$$

where τ_n^i are the transfer matrices relating the properties of the scatterers and the surrounding medium, and are given as

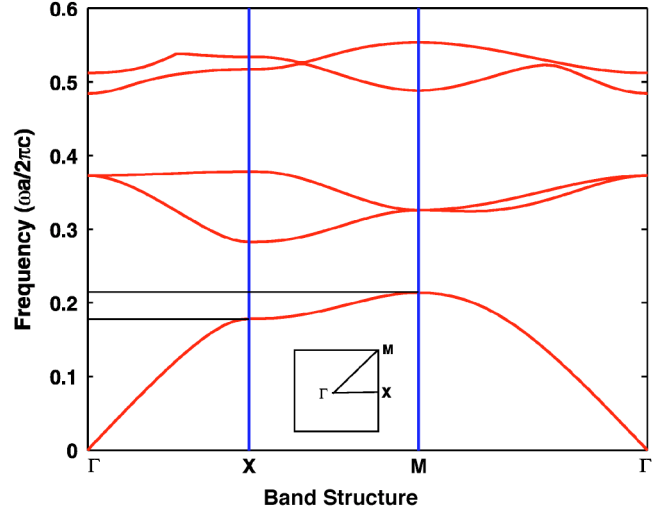


FIG. 1. (Color online) The band structure of a square lattice of dielectric cylinders. The lattice constant is a and the radius of the cylinders is $0.3a$. ΓM and ΓX denote the $[11,10]$ directions, respectively. A partial gap is between the two horizontal lines.

$$\tau_n^i = \frac{H_n^{(1)}(ka^i) J_n'(ka^i/h^i) - g^i h^i H_n^{(1)'}(ka^i) J_n(ka^i/h^i)}{g^i h^i J_n'(ka^i) J_n(ka^i/h^i) - J_n(ka^i) J_n'(ka^i/h^i)}, \quad (7)$$

where

$$h^i = \frac{1}{\sqrt{\epsilon^i}}, \quad \text{and} \quad g^i = \begin{cases} \epsilon^i & \text{for TE waves} \\ 1 & \text{for TM waves,} \end{cases}$$

in which ϵ^i is the dielectric constant ratio between the i th scatterer and the surrounding medium.

The coefficients A_n^i and B_n^i can then be inverted from Eq. (3). Once the coefficients A_n^i are determined, the transmitted wave at any spatial point is given by

$$p(\vec{r}) = p_0(\vec{r}) + \sum_{i=1}^N \sum_{n=-\infty}^{\infty} i\pi A_n^i H_n^{(1)}(k|\vec{r}-\vec{r}_i|) e^{in\phi_{\vec{r}-\vec{r}_i}}, \quad (8)$$

where p_0 is the field when no scatterers are present. The transmitted intensity field is defined as $|p|^2$.

III. RESULTS AND DISCUSSION

The frequency band structure is plotted in Fig. 1, and the qualitative features are similar to that obtained for a square array of alumina rods in air. A complete band gap is shown between frequencies of 0.22 and 0.28. Just below the complete gap, there is a regime, sandwiched by two horizontal lines, of partial band gap in which waves are not allowed to travel along the ΓX or $[10]$ direction. We will consider waves whose frequency is within this partial band gap. In particular, we choose the frequency to be 0.192.

First we consider the propagation of EM waves through two rectangular slabs of arrays of dielectric cylinders. Figure 2 shows the images of the fields. The left panel shows the real parts of the fields E_z , while the right panel presents the images of the intensity fields $|E_z|^2$. In (a1) and (a2), the slab measures 14×45 , and the slab is oriented such that the $[11]$

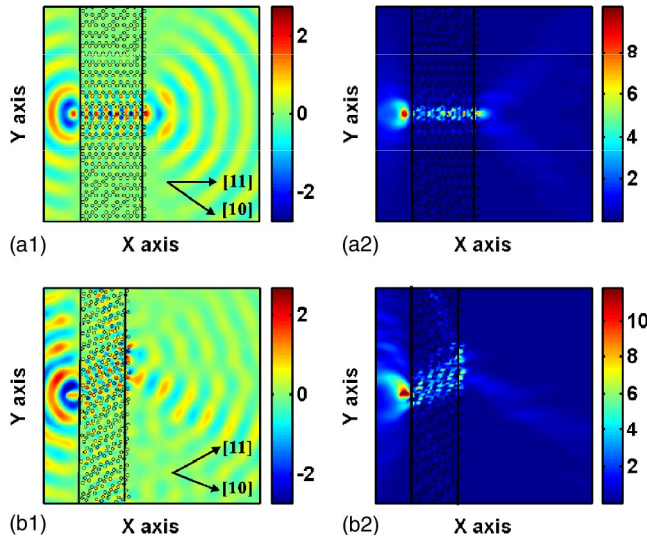


FIG. 2. (Color online) Imaging of the transmitted fields across two slabs of dielectric cylinders. The black circles denote the cylinders (for clarity, not all cylinders are plotted).

direction, i.e., the ΓM direction, is along the horizontal level. The size of the slab in (b1) and (b2) is 10×45 , and the [11] direction is tilted upwards and makes an angle of 22.5 deg with respect to the horizontal direction. The frequency is chosen at 0.192. A transmitting point source is placed at 2 lattice constants away from the left side of the slabs. The detailed geometrical information can be referred to in Fig. 2.

A few observations can be drawn from Fig. 2. First, there is a focused image across the slab in (a1) and (a2). Earlier, this focused image was attributed to the effect of negative refraction [6], inferred from the group velocity calculation. If this conjecture were valid, another focused image would be expected inside the slab as well. Our result does not support this conjecture. As seen from (a1) and (a2), there is no focused image inside the slab. Rather, the waves are mostly confined in a tunnel or self-collimated, and travel to the other side of the slab, then release to the free space. This is understandable, because the forbidden direction in (a1) and (a2) is along ΓX , which makes an angle of 45 deg from the ΓM direction that lies horizontally. The passing band in the ΓM direction thus acts as a transportation carrier that moves the source to the other side of the slab. The waves on the right hand side of the slab look as if they were radiated by an image that has been transported across the slab within a narrow guide. We note [16] that the present observation is in good agreement with the results in Refs. [17,18]. Second, the waves tend to bend towards the ΓM direction [18], as evidenced by Fig. 2 (b1) and (b2). Third, the decay of the transported intensity along the traveling path is not obvious, an indication of efficient guided propagation.

The results in Fig. 2 are promising. They show that in the presence of partial band gaps and when incident upon a slab of photonic crystals, waves tend to bend toward directions which are mostly away from forbidden directions. This would indicate that partial band gaps may be considered as a candidate for guiding wave flows. To verify this conjecture, we have further explored the guiding phenomenon associated with partial band gaps.

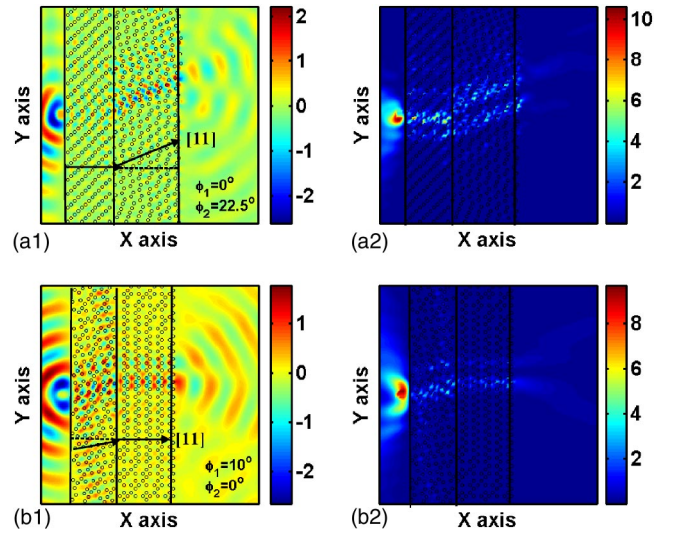


FIG. 3. (Color online) Imaging of the intensity fields across two consecutive slabs of arrays of dielectric cylinders in two arrangements. The left and right panels, respectively, plot the real part of the field and the intensity.

In Fig. 3, we show the EM wave propagation through stacks of photonic crystal slabs. Two setups are considered. In (a1) and (a2), two slabs of dielectric cylinders are stacked together. The first (left) slab is oriented such that the [11] direction is horizontal, while the second (right) slab is arranged to make the [11] direction tilted upward, making 22.5 deg with respect to the horizontal direction. The two slabs measure as 9×44 and 14×44 , respectively. In (b1) and (b2), two slabs are adjacently attached. The [11] direction is tilted upward by 10 deg for the first (left) slab, while it is along the horizontal direction in the second (right) slab. The sizes of the two slabs are 8×40 and 10×40 , respectively. In both situations (a) and (b), the point source is placed at a distance of 1.5 away from the left side of the stacks. The purpose here is to show how the light would travel when two adjacent slabs have different orientations.

Here it is clearly shown that the EM waves indeed always tend to travel along the [11] direction. In the case of (b1) and (b2), an image has been formed on the right hand side. Compared to the source, the image is uplifted by a distance about $8 \tan(\pi/18)$. In the cases considered here, the first slabs (left) serve as a collimating device, and then the collimated waves will be guided by subsequent photonic crystal slabs. This consideration can be extended to multiple consecutive slabs so that the wave flows can be guided into desired orientations, making possible alternative ways of controlling optical flows.

In Fig. 4, we consider two other situations of stacked photonic crystal slabs. The geometrical parameters are indicated in the figure. Again, the waves tend to move along the [11] direction. Here the amphoteric diffraction is observed. It draws analogy with the amphoteric refraction observed when waves propagate from an isotropic to an anisotropic medium [19].

The results from Figs. 2–4 clearly indicate that the partial band gaps can be indeed used as a guiding channel for opti-

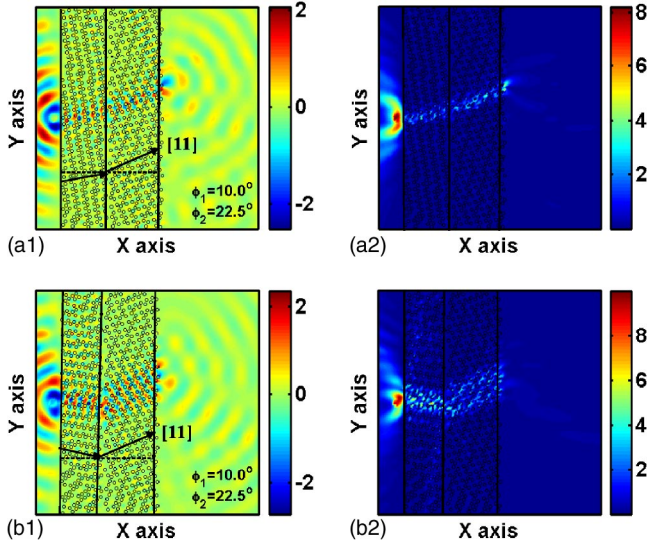


FIG. 4. (Color online) Imaging of the intensity fields across two consecutive slabs of arrays of dielectric cylinders in two arrangements. The left and right panels respectively plot the real part of the field and the intensity. Shown here is the amphoteric diffraction at the interfaces between two adjacent slabs: (a) positive and (b) negative. In both (a) and (b), the adjacent slabs measure as 10×50 and 12×50 .

cal flows. It can be also inferred that the guided transport is efficient. We have carried out further simulations against variations of frequencies, filling factors, and dielectric constants, the results are quantitatively the same for waves within partial band gaps. The observation presented here has also been confirmed by FDTD simulations. The controlled wave transport due to partial band gaps of PCs should be interpreted in terms of diffraction or scattering rather than

refraction; in fact, no refraction index can be determined for the phenomenon.

An immediate question may thus arise. Why do the waves of frequencies within the partial band gap tend to bend to particular directions? To answer this question, we have examined the properties of the energy flow of the eigenmodes which correspond to the frequency bands. While details will be published elsewhere, here we only outline our thoughts. The usual approach mainly relies on the curvatures of frequency bands to infer the energy flow. As documented in Ref. [20], an energy velocity is defined as

$$\vec{v}_e = \frac{\frac{1}{V} \int \vec{J}_{\vec{K}} d^3r}{\frac{1}{V} \int U_{\vec{K}} d^3r},$$

where $\vec{J}_{\vec{K}}$ and $U_{\vec{K}}$ are the energy flux and energy density of the eigenmodes, and the integration is performed in a unit cell. It can be shown that thus defined energy velocity equals the group velocity obtained as $\vec{v}_g = \nabla_{\vec{K}} \omega(\vec{K})$. Therefore it is common to calculate the group velocity to infer the energy velocity and subsequently the energy flows or refraction of waves. A few questions, however, may arise with regard to this approach. First, when the variation in the Bloch vector, i.e., $\delta\vec{K}$, is small, the changes in ω , $\vec{E}_{\vec{K}}$, and $\vec{H}_{\vec{K}}$ should also be small. Second, the variation operation should be exchangeable with the partial differential operations. When these two conditions fail, the energy velocity will become ill defined. Third, even if the two conditions hold, whether the net current flow through a unit cell really follows the direction of \vec{v}_e remains unclear. We note here that the average flux through a surface may be defined as $\langle \vec{J} \rangle = (\hat{n}/S) \int d\vec{S} \cdot \vec{J}$, where

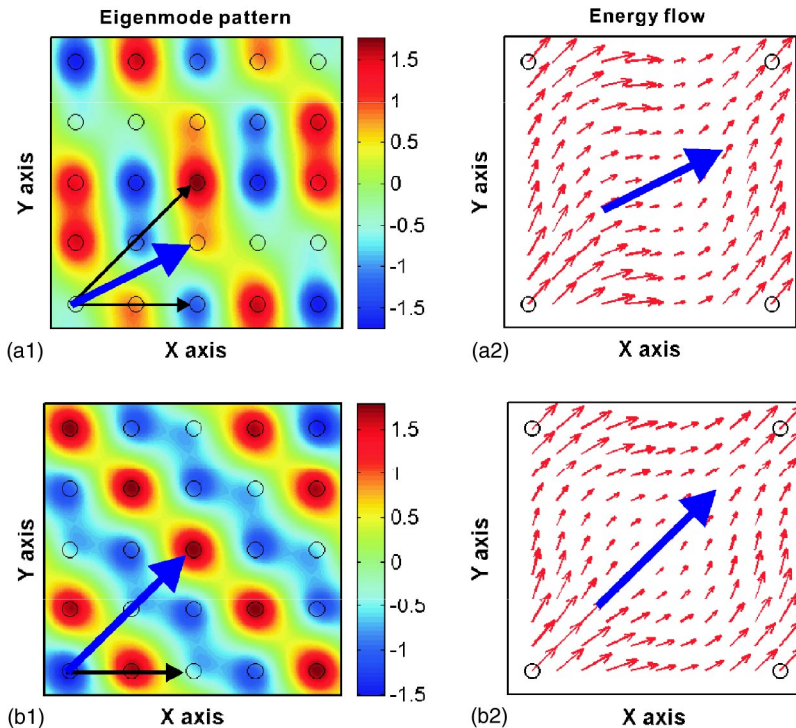


FIG. 5. (Color online) Left panel: the field pattern of eigenmodes. Right panel: the energy flow of the eigenmodes. The eigenmodes along two directions are considered: (a) $\vec{K} = (0.9\pi/a, 0.37\pi/a)$, i.e., along an angle of 22.5° exactly between ΓX and ΓM directions; the corresponding frequency is 0.185; (b) $\vec{K} = (0.7\pi/a, 0.7\pi/a)$, i.e., along ΓM ; the corresponding frequency is 0.192. The direction of the Bloch vectors are denoted by the blue arrows, while the red arrows denote the local energy flow including the direction and the magnitude. The circles refer to the cylinders. Both frequencies in (a) and (b) lie within the partial gap. Due to the periodicity, we only plot the energy flow within one unit cell. Note that although the features shown by (a) also hold for other Bloch vectors for which the corresponding frequencies lie within the partial gap regime, we only plot here for the case of 22.5° .

\hat{n} is the normal of the surface and the surface integration should not include the areas occupied by the cylinders. Another possible way in finding the actual current might be $\langle \vec{J} \rangle = (1/S) \int dS \vec{J}$. Again, the integration should not include the areas taken by the scatterers. Clearly, the volume averaged current within a unit cell, for which the integration includes the areas occupied by the scatterers, does not necessarily correspond to the actual current flow. We will publish verifications elsewhere.

To avoid possible ambiguities, here we consider the energy flow based upon its genuine definition. One advantage of this approach is that we are also able to examine the local properties of energy flows. By Bloch's theorem, the eigenmodes corresponding to the frequency bands of PCs can be expressed as $E_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{\vec{k}}(\vec{r})$, where \vec{k} is the Bloch vector, as the wave vector, and $u_{\vec{k}}(\vec{r})$ is a periodic function with the periodicity of the lattice. When expressing $E_{\vec{k}}(\vec{r})$ as $|E_{\vec{k}}(\vec{r})| e^{i\theta_{\vec{k}}(\vec{r})}$, the corresponding energy flow is derived as $\vec{J}_{\vec{k}}(\vec{r}) \propto |E_{\vec{k}}(\vec{r})|^2 \nabla \theta_{\vec{k}}(\vec{r})$; clearly, $\theta_{\vec{k}}$ combines the phase from the term $e^{i\vec{k}\cdot\vec{r}}$ and the phase from the function $u_{\vec{k}}(\vec{r})$. To explore the characteristics of the partial band gap, we have computed the eigenfield $E_{\vec{k}}(\vec{r})$ and also the energy flow of the eigenmodes. The results are shown in Fig. 5. Figure 5(a) shows that the energy eventually tends to flow into the direction of ΓM , i.e., the [11] direction, while the Bloch vector points to an angle of 22.5° that lies exactly between ΓX and ΓM .

Last, we would like to make a further comment on the effectiveness of the guided transport. For this, we consider the transmission versus the slab horizontal size. To be brief, we will consider the case in Fig. 1(a1). The results are shown in Fig. 6. The important feature is that the transmission does not decay as the sample size increases. Here we see that the transmission fluctuates around the value of one. The reason for the fluctuation is due to either constructive or destructive interference associated with the waves which are back and forth reflected inside the slab by the two interfaces, just like what would also happen when waves transmit through a slab of uniform medium. The results in Fig. 6 imply that the guided transport by partial gaps is efficient.

Although the above features are only investigated for the first partial band gap in this paper, we have found that they are also valid for other partial band gaps. For example, we have also considered the second partial band gap which is located between 0.283 and 0.325. All above features remain quantitatively valid. Within this second gap, however, the

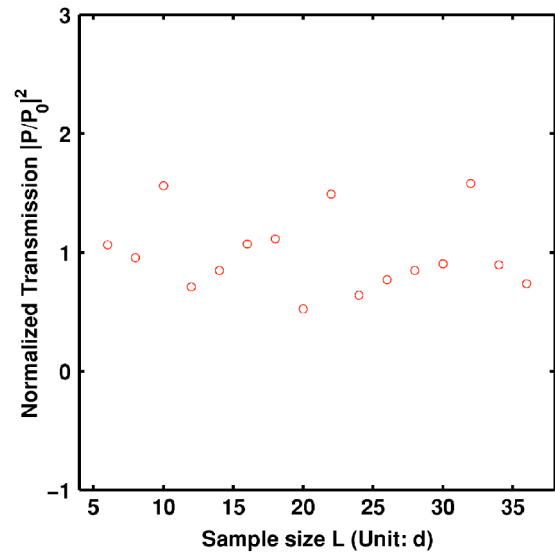


FIG. 6. (Color online) Normalized transmission versus the slab size. The normalization is to take out the trivial geometrical spreading effects. The size is scaled by the lattice constant. Here the slab height is 40 lattice constant.

waves are collimated to travel along the [10] direction rather than the [11] direction. In addition, we have also carried out simulations for various slab sizes, all the features are the same, thus excluding the boundaries as the possible cause.

IV. SUMMARY

We have considered EM wave propagation through slabs of photonic crystals which are made of arrays of dielectric cylinders. Properties of partial band gaps are investigated. It was shown that the partial band gaps may act as a guiding channel for wave propagation inside the photonic crystals. Such a feature may lead to novel applications in manipulating optical flows.

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