Soliton ratchets induced by excitation of internal modes

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Recently Flach *et al.* [Phys. Rev. Lett. **88**, 184101 (2002)] used a symmetry analysis to predict the appearance of directed energy current in homogeneously spatially extended systems coupled to a heat bath in the presence of an external ac field $E(t)$. Their symmetry analysis allowed them to make the right choice of $E(t)$ so as to obtain symmetry breaking which causes directed energy transport for systems with a nonzero topological charge. Their numerical simulations verified the existence of the directed energy current. They argued that the origin of their strong rectification in the underdamped limit is due to the excitation of internal modes and their interaction with the translational kink motion. The internal mode mechanism as a cause of current rectification was also proposed by Salerno and Zolotaryuk [Phys. Rev. E. **65**, 056603 (2002)]. We use a rigorous collective variable for nonlinear Klein-Gordon equations to prove that the rectification of the current is due to the excitation of an internal mode $\Gamma(t)$, which describes the oscillation of the slope of the kink, and due to a dressing of the bare kink by the ac driver. The internal mode $\Gamma(t)$ is excited by its interaction with the center of mass of the kink, $X(t)$, which is accelerated by $E(t)$. The external field $E(t)$ also causes the kink to be dressed. We derive the expressions for the dressing and numerically solve the equations of motion for $\Gamma(t)$, $X(t)$, and the momentum $P(t)$, which enable us to obtain the explicit expressions for the directed energy current and the ac driven kink profile. We then show that the directed energy current vanishes unless the slope $\Gamma(t)$ is a dynamical variable and the kink is dressed by the ac driver.

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I. INTRODUCTION

In a recent paper Flach *et al.* [1] studied the appearance of directed energy currents in homogeneous spatially extended systems described by nonlinear field equations coupled to a heat bath in the presence of an external ac field $E(t)$. As pointed out in Ref. [1], rectifying energy transform using fluctuations has been studied in connection with such problems as molecular motors in biological systems [3], electrical currents in superlattices [4–7], voltages in Josephson junction coupled systems [8–10], and other problems.

The authors of Ref. [1] showed by a symmetry analysis that the correct choices of $E(t)$ lead to directed energy transport for nonlinear Klein-Gordon systems with a nonzero topological charge. They used numerical simulations of the ac driven Klein-Gordon equation which confirmed their predictions which generalized recent rigorous theories of currents generated by broken time-space symmetries to the case of interacting many-particle systems [11,12]. They did this by replacing the fluctuations as a superposition of ac driving fields and uncorrelated white noise. They also showed the persistence of directed currents in the Hamiltonian limit of systems exposed to ac fields but decoupled from the heat bath. The authors of Ref. [1] then argued that the origin of the observed strong rectification in the underdamped limit is due to the nonadiabatic excitation of internal kink modes and their interaction with the translational kink motion.

In this paper we use a rigorous collective variable (CV) theory for nonlinear Klein-Gordon equations derived in Refs. [13,14] to prove that an external ac field causes the CV's for the center of mass $X(t)$ and the slope $\Gamma(t)$ to become time dependent and to interact with each other. The ac driver in addition to inducing time dependence in $X(t)$, the center of mass of the kink, and the slope $\Gamma(t)$, causes the kink to be dressed by phonons. The dressing changes the shape of the kink and increases the coupling of $\Gamma(t)$ to $X(t)$. We show that the nonvanishing of the dressing is a necessary condition for breaking time inversion symmetry of the energy current. However, the dressing of the kink alone in the absence of a time dependent collective variable $\Gamma(t)$ cannot cause current rectification. Consequently, we prove that the existence of the time dependence of $\Gamma(t)$ and the dressing $\chi(t)$ are necessary for time inversion symmetry breaking.

In Sec. II we derive the equations of motion for $X(t)$ and $\Gamma(t)$ including the terms due to the dressing of the kink by phonons. We present our results for the solutions $X(t)$ and $\Gamma(t)$, and for the generation of directed energy currents in Sec. III, and in Sec. IV and we discuss our results. The derivation of the dressing χ is given in the Appendix.

II. DERIVATION OF CV EQUATIONS OF MOTION

Before deriving the CV equations of motion used in this paper, we will make a few remarks about CV treatments of the Klein-Gordon equations. The first approach, which is derived in Refs. [13,14] and used in this paper, is to treat the center of mass $X(t)$ and the slope $\Gamma(t)$ as collective variables which satisfy coupled second-order differential equations, which also depend on the dressing of the kink. In this approach the equations of motion for $X(t)$ and $\Gamma(t)$ are not manifestly relativistic invariant. What has been proven is that when the solutions $X(t)$ and $\Gamma(t)$ are inserted in the kink $\phi[X(t),\Gamma(t)]$ ϕ satisfies the relativistic invariant nonlinear Klein-Gordon equation. An analogous well-known example of a nonmanifestly relativistic case is the use of the nonrelativistic Coulomb gauge which leads to the relativistic solution $E(t)$ and $B(t)$ of Maxwell's equations. The second approach would be to consider a single CV, $X(t)$. The equation of motion for $X(t)$ is fourth order in time because the Lagrangian contains the second derivative $\ddot{X}(t)$. The Lagrangian of a theory that contains a second derivative $\ddot{X}(t)$ leads to equations of motion that contain the fourth derivative $\ddddot{X}(t)$. The fourth-order equation for $X(t)$ has the same number of degrees of freedom as the equivalent CV theory for two variables which consists of two coupled second-order equations for $X(t)$ and $X(t)$.

We outline the derivation of the equations of motion for the collective variables $X(t)$ and $\Gamma(t)$ which are derived in detail in Refs. [13,14]. The damped nonlinear sine-Gordon (SG) equation for the field ϕ in the presence of an external potential $V(\phi)$ is

$$\phi_{,tt} - \phi_{,xx} + \sin\phi + \beta\phi_{,t} = -\frac{\partial V}{\partial \phi}, \tag{1}$$

where $\beta\phi_{,t}$ is the damping due to the heat bath, and where we are using dimensionless variables where the velocity of the phonons is $c=1$. We introduce the collective variables by writing the solution ϕ of Eq. (1) in the form

$$\phi(x,t) = \sigma[\xi(t)] + \chi[\xi(t)], \tag{2}$$

where $\xi \equiv \Gamma(t)[x-X(t)]$ and the single kink solution $\sigma[\xi(t)]$ is

$$\sigma[\xi(t)] = 4\tan^{-1}\exp\{\Gamma(t)[x - X(t)]\}, \tag{3}$$

and $\chi[\xi(t)]$ is the dressing of the kink by phonons due to the external potential $V(\phi)$ which for the applied ac field of this paper is given by

$$V(\phi) = (\epsilon_1\cos\omega t + \epsilon_2\cos[2\omega t + \theta])\phi(\xi(t)) \equiv f_1\phi(\xi(t)). \tag{4}$$

θ is an arbitrary phase, and ϵ_1 and ϵ_2 are perturbation parameters, i.e., we solve for ϕ to first order in ϵ_1 and ϵ_2. The CV's are the center of mass $X(t)$ and the slope of the kink evaluated at its center is $2\Gamma(t)$. In Ref. [2] a directed kink motion for the SG was obtained numerically for the first time using $V(\phi)$ in Eq. (4) for a wide range of momenta with an analytic approach for small momenta.

The equations of motion for X and Γ each contain many terms proportional to integrals of χ, its time derivatives and spatial derivatives. χ is a solution of the linearized ac driven SG equation and is proportional to ϵ_1 and ϵ_2. In the Appendix we solve for χ. The solution for χ is

$$\chi = \frac{4}{\pi} f(t)\,\mathrm{sech}^2\xi, \tag{5}$$

where

$$f(t) = (\epsilon_1/2)\cos\omega t\left\{\frac{1-\omega}{\beta^2 + (1-\omega)^2} + \frac{1+\omega}{\beta^2 + (1+\omega)^2}\right\} + (\epsilon_2/2)\cos(2\omega t + \theta)\left\{\frac{1-2\omega}{\beta^2 + (1-2\omega)^2} + \frac{1+2\omega}{\beta^2 + (1+2\omega)^2}\right\}.$$

Since χ is an even function of ξ many of the terms in Eqs. (2.4a) and (2.5a) of Refs. [13,14] for $\ddot{X}$ and $\ddot{\Gamma}$ that depend on integrals of χ vanish. The only terms which survive are

$$(1-b_X)M_X[\ddot{X} + \dot{X}(\dot{\Gamma}/\Gamma) + \beta\dot{X}] = 2\pi f_1 + \Gamma^2\langle\sigma'|\chi''\rangle(1-\dot{X}^2) - (\dot{\Gamma}/\Gamma)^2\langle\sigma'|\xi^2\chi''\rangle - 2(\dot{\Gamma}/\Gamma)\times(\dot{f}/f)\langle\sigma'|\xi\chi\rangle - (\ddot{\Gamma}/\Gamma)\times\langle\sigma'|\xi\chi\rangle, \tag{6}$$

where $M_X \equiv \Gamma\langle\sigma'|\sigma'\rangle = 8\Gamma$, $b_X \equiv (\Gamma/M_X)\langle\sigma''|\chi\rangle = 0$ and where $\langle f|g\rangle \equiv \int f^*(\xi)g(\xi)d\xi$. The corresponding equation for $\ddot{\Gamma}$ is

$$(1-b_\Gamma)M_\Gamma[\ddot{\Gamma} - 3\dot{\Gamma}^2/2\Gamma + (M_X/2\Gamma)(1-\dot{X}^2) + \beta\dot{\Gamma}] = (2\dot{X}\dot{\Gamma}/\Gamma^2)\langle\xi\sigma'|\chi''\xi\rangle + 2(\dot{X}/\Gamma)(\dot{f}/f)\langle\xi\sigma'|\chi'\rangle + (\ddot{X}/\Gamma)\times\langle\xi\sigma'|\chi'\rangle, \tag{7}$$

where $M_\Gamma \equiv \Gamma^{-3}\langle\xi\sigma'|\xi\sigma'\rangle = (2\pi^2/3\Gamma^3)$ and $b_\Gamma \equiv (\Gamma^3/M_\Gamma)^{-1}\langle\xi^2\sigma''|\chi\rangle = 0$.

We next eliminate the $\ddot{\Gamma}$ term in Eq. (6) and the $\ddot{X}$ term in Eq. (7) by using the zeroth order in ϵ_1, and ϵ_2 equations for $\ddot{X}$ and $\ddot{\Gamma}$. The elimination is justified because the corresponding terms are both multiplied by χ which is already first order in ϵ_1, and ϵ_2.

The zeroth-order expression for $\ddot{X}$ is $\ddot{X} = -\dot{X}(\dot{\Gamma}/\Gamma)$ and for $\ddot{\Gamma}$ is $\ddot{\Gamma} = 3\dot{\Gamma}^2/2\Gamma - M_X(2\Gamma M_\Gamma)^{-1}(1-\dot{X}^2)$. When we substitute for $\ddot{X}$ in Eq. (7), we obtain

$$\ddot{\Gamma} + \beta\dot{\Gamma} = (3\dot{\Gamma}^2/2\Gamma) + (6/\pi^2)\Gamma^3(1-\dot{X}^2) + (3/2\pi^2)\times[8\dot{\Gamma}\dot{X}(2\langle\xi\sigma'|\chi''\rangle - \langle\xi\sigma'|\chi'\rangle) + 8\dot{f}\Gamma^2\dot{X}\langle\xi\sigma'|\chi'\rangle], \tag{8}$$

and when we substitute for $\ddot{\Gamma}$ in Eq. (6) we obtain

$$M_X[\ddot{X} + \dot{X}(\dot{\Gamma}/\Gamma) + \beta\dot{X}] = 2\pi f_1 - 2\dot{f}(\dot{\Gamma}/\Gamma)\langle\sigma'\xi|\chi'\rangle + \Gamma^2(1-\dot{X}^2)\times[\langle\sigma'|\chi''\rangle - (6/\pi^2)\langle\sigma'|\xi\chi'\rangle] - (\dot{\Gamma}/\Gamma)^2(\langle\sigma'|\xi^2\chi'\rangle - 3/2\langle\sigma'\xi|\chi'\rangle). \tag{9}$$

The momentum P conjugate to X is $P = M_X\dot{X} = 8\Gamma\dot{X}$. Consequently we can write Eq. (8) for $\ddot{X}$ in terms of P, i.e.,

$$\frac{dP}{dt} + \beta P = 2\pi f_1 - 2\dot{f}(\dot{\Gamma}/\Gamma)\langle \sigma' \xi | \chi' \rangle + \left(\Gamma^2 - \frac{P^2}{64}\right)\left(\langle \sigma' | \chi'' \rangle - \frac{6}{\pi^2}\langle \sigma' | \xi \chi' \rangle\right) - \left(\frac{\dot{\Gamma}}{\Gamma}\right)^2 (\langle \sigma' | \xi^2 \chi'' \rangle - 3/2\langle \sigma' \xi | \chi' \rangle), \quad (10)$$

and Eq. (8) for $\ddot{\Gamma}$ in terms of P becomes

$$\ddot{\Gamma} - (3\dot{\Gamma}^2/2\Gamma) - (6/\pi^2)\Gamma[1 - \Gamma^2 + (P/8)^2] + \beta\dot{\Gamma} = (3/2\pi^2)[P\dot{\Gamma}(2\langle \xi\sigma' | \chi'' \rangle - \langle \xi\sigma' | \chi' \rangle) + \Gamma P(\dot{f}/f)\langle \xi\sigma' | \chi' \rangle]. \quad (11)$$

Finally after evaluating the integrals and replacing $\dot{X}^2$ by $P^2/64$ we obtain the final form of our equations of motion for $P(t)$ and $\Gamma(t)$:

$$\dot{P} + \beta P - 2\pi f_1 = f(t)\{0.47(8/\pi)(\dot{\Gamma}/\Gamma)^2 - (8/3)[\dot{f}(t)/f(t)](\dot{\Gamma}/\Gamma) - (8/\pi^2)[\Gamma^2 - (P/8)^2]\} \quad (12)$$

and

$$\ddot{\Gamma} + \beta\dot{\Gamma} - 3\dot{\Gamma}^2/2\Gamma - (6/\pi^2)\Gamma[1 - \Gamma^2 + (P/8)^2] = f(t)(2\pi^2)^{-1}\{(5/2 - \pi^2/16)P\dot{\Gamma} - [\dot{f}(t)/f(t)]P\Gamma\}. \quad (13)$$

Before solving the equations of motion for P and Γ it is worth making a few remarks about the properties of the coupled equations. In Eq. (12) the ac driver $f_1(t)$ directly drives $\dot{P}$ while in Eq. (13) for $\ddot{\Gamma}$ the ac driver $f_1(t)$ does not directly drive $\ddot{\Gamma}$ because the Γ mode $\partial\sigma/\partial\Gamma$ is orthogonal to the ac driver. However, both $\dot{P}$ and $\ddot{\Gamma}$ see the ac driver indirectly through the dressing χ which is proportional to $f(t)$ [Eq. (A7)], which also depends on the two frequencies ω and 2ω. As long as $\omega < 0.5$ the phonon radiation is small. In this paper we consider only frequencies which are much less than 1, which is the beginning of the lower band edge in the units of this paper. Consequently in this paper the emission of phonons is negligible. Since there are no modes of $X(t)$ and $\Gamma(t)$ in the band gap of the SG, there is no excitation of internal gap modes by the ac driver as there is, e.g., in ϕ^4 and the double sine-Gordon. The dressing χ which is proportional to ϵ changes the shape modes to $\sigma_{,X}+\chi_{,X}$ and to $\sigma_{,\Gamma}+\chi_{,\Gamma}$ in addition to changing the frequency of P and Γ directly.

Before discussing the results we discuss the symmetries of the coupled equations (12) and (13). The first symmetry is referred to as the shift symmetry of the driver which is $P \to -P$ and $t \to t+\tau/2$, provided $f_1(t) = -f_1(t+\tau/2)$ and $f(t) = -f(t+\tau/2)$ are always shift symmetric if and only if a Fourier expansion contains only odd terms. Thus $f_1(t)$ and $f(t)$ in Eqs. (12) and (13) always violate shift symmetry. A second symmetry is time inversion symmetry, i.e., $P \to -P$ when $t \to -t$ and $\beta=0$. Equations (12) and (13) satisfy time inversion symmetry when $\beta=0$ and $\theta=0, \pm n\pi$. When β is small, time inversion is approximately satisfied.

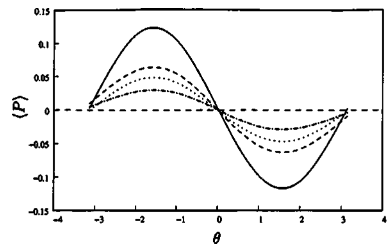

FIG. 1. This figure demonstrates symmetry breaking, i.e., the nonvanishing of $\langle P(t) \rangle$ as a function of θ for various values of the parameters ω, ϵ, and β. Solid curve, $\omega=0.1$, $\epsilon_1=\epsilon_2=0.03$, and $\beta=0$ (the curve is multiplied by 0.15); dashed curve $\omega=0.3$, $\epsilon_1=0.3$, $\epsilon_2=\epsilon_1/\sqrt{3}$, and $\beta=0.2$; dotted curve, $\omega=0.1$, $\epsilon_1=\epsilon_2=0.05$, and $\beta=0.12$; dash-dotted curve, $\omega=0.25$, $\epsilon_1=0.16$, $\epsilon_2=\epsilon_1/\sqrt{2}$, and $\beta=0.15$. $\langle P(t) \rangle$ has the units of momentum and θ is in radians.

III. RESULTS OF SIMULATIONS

In this section we present the computer solutions of Eqs. (12) and (13) for $\Gamma(t)$, $P(t)$, and for the time average of the energy current. In our units the energy current $J(t)$ is equal to $P(t)$ because

$$J(t) \equiv -\int_{-\infty}^{\infty} \sigma_{,t}(\xi)\sigma_{,x}(\xi)dx = 8\Gamma\dot{X} = P(t),$$

where $\sigma[\xi] = 4\tan^{-1}\exp(\Gamma(t)[x-X(t)])$. Consequently the time average of $J(t)$,$\langle J(t) \rangle$, is equivalent to the time average of $P(t)$,$\langle P(t) \rangle$.

In Fig. 1 we show the results for $\langle P(t) \rangle$ for a range of values ω, β, ϵ_1, and ϵ_2 which show clearly the directed energy current as a function of θ. We see that there is symmetry breaking for all the sets of parameter values. In Fig. 2 we show $\langle P(t) \rangle$ as a function of θ for fixed values of ω and $\epsilon_1 = \epsilon_2$, and various values of β. If $\beta \neq 0$, then time inversion symmetry is not valid. However as β goes to zero, time inversion symmetry is approximately restored at $\theta=0, \pm n\pi$. Thus we observe for small β, as β, decreases, exactly the same behavior as in Fig. 1 of Ref. [1], that is, the smaller the β the larger is the value of $\langle P(\theta=0) \rangle$, but the smaller is the value of θ at which $\langle P(\theta) \rangle = 0$. The values in Fig. 2 at $\theta=0$

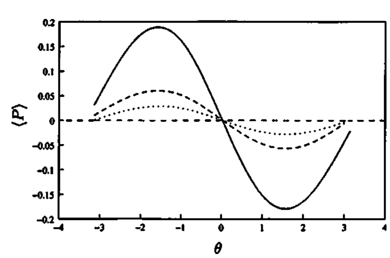

FIG. 2. $\langle P(t) \rangle$ as a function of θ for $\omega=0.1$, $\epsilon_1=\epsilon_2=0.03$ for various values of β show a monotonic decrease of the amplitude of $\langle P(t) \rangle$ as the damping β increases. Solid curve, $\beta=0.02$; dashed curve, $\beta=0.05$; dotted curve, $\beta=0.12$. In this simulation $\langle P(t) \rangle$ decreases as β increases but the values of $\langle P(t) \rangle$ are so small that they cannot be distinguished on the figure. $\langle P(t) \rangle$ has the units of momentum and θ is in radians.

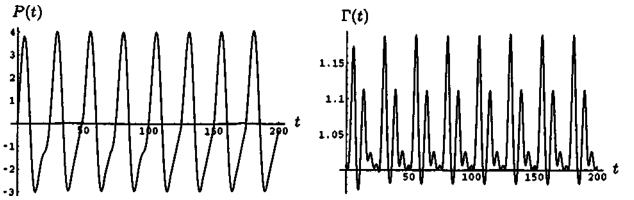

FIG. 3. The energy current $P(t)$ and the slope of the kink $\Gamma(t)$ for the parameters $\omega=0.25$, $\beta=0.15$, $\epsilon_1=0.16$, $\epsilon_2=\epsilon_1/\sqrt{2}$, and $\theta=1.61-\pi$. Both curves show the effect of two driving frequencies. The time average of $P(t)$ is nonzero and $\Gamma(t)$ has a multiple frequency oscillation about $\langle\Gamma(t)\rangle$ greater than Γ_0 with an amplitude change $\Delta\Gamma/\langle\Gamma\rangle\sim 20\%$. $P(t)$ has the units of momentum and $\Gamma(t)$ has the units of inverse length.

are $\langle P(t)\rangle=0.0008$ when $\beta=0.12$, $\langle P(t)\rangle=0.002$ when $\beta=0.05$, and $\langle P(t)\rangle=0.005$ when $\beta=0.02$.

In the three typical examples of $P(t)$ and $\Gamma(t)$ (Figs. 3–5) we have selected the shapes and magnitudes vary considerably. The shapes of the $P(t)$ and $\Gamma(t)$ curves show the effect of being driven by an ac driver with two frequencies which causes the curves to vary in amplitude and shape when we vary the parameters ω, ϵ_1, ϵ_2, and β. The changes in shape of $\Gamma(t)$ are often striking because $\Gamma(t)$ is a very nonlinear oscillator which has a complicated response to the ac driver and the dressing χ, whereas in lowest order the equation for $P(t)$ is linear. Generally the magnitudes of both $\Gamma(t)$ and $P(t)$ increase with increases in the strength of ϵ_1 and ϵ_2. The variable $\Gamma(t)$ oscillates about an average value of $\langle\Gamma(t)\rangle$ which is greater than $\Gamma_0=1$, the unperturbed kink value of Γ. It usually also takes instantaneous values less than 1. The relative change in slope, $\Delta\Gamma/\Gamma$, varies from a few percent to as much as 100% and is strongly dependent on the magnitude of ϵ_1 and ϵ_2. Large values of $\Delta\Gamma/\Gamma$ represent large distortions of the shape of the kink. When we compare $P(t)$ and $\Gamma(t)$ for the kink dressed by χ with the bare kink we find a strong dependence on the phase θ which leads to different shapes and amplitudes of $P(t)$ and $\Gamma(t)$ for different θ. For example, in Fig. 5 for $\omega=0.1$, $\beta=0.02$, $\epsilon_1=\epsilon_2=0.03$, and θ near $\theta=0,\pm n\pi$ the slope $\Gamma(t)$ of the dressed kink is a pattern of single peaks while for the bare kink with the same parameters $\Gamma(t)$ is a pattern of double kinks. On the other hand for θ appreciably different from $\theta=0,\pm n\pi$, e.g., $\theta=1.61-\pi$ the differences in shape of $P(t)$ between the bare and dressed kinks are relatively minor. At the same time although the

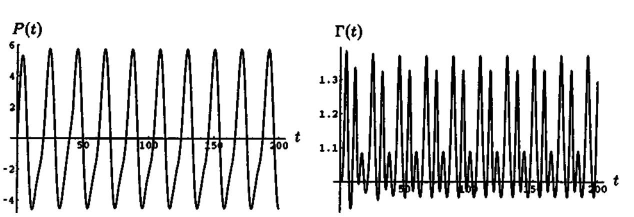

FIG. 4. $P(t)$ and $\Gamma(t)$ for the parameters $\omega=0.3$, $\beta=0.2$, $\epsilon_1=0.3$, $\epsilon_2=\epsilon_1/\sqrt{3}$, and $\theta=1.61-\pi$. $\langle P(t)\rangle$ is nonzero and the slope $\Gamma(t)$ has a multiple frequency oscillation about $\langle\Gamma(t)\rangle>\Gamma_0$ with an amplitude change $\Delta\Gamma/\langle\Gamma\rangle\sim 50\%$. $P(t)$ has the units of momentum and $\Gamma(t)$ has the units of inverse length.

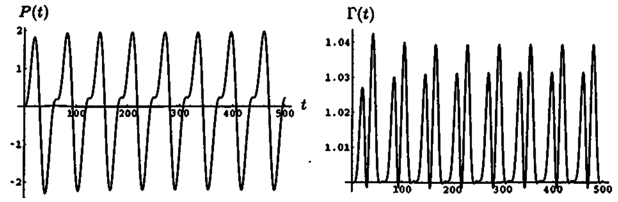

FIG. 5. $P(t)$ and $\Gamma(t)$ for the parameters $\omega=0.1$, $\beta=0.02$, $\epsilon_2=\epsilon_1=0.03$, and $\theta=\pi$. $\langle P(t)\rangle=0$ as is required by $\theta=\pi$. $\Gamma(t)$ has a two-frequency oscillation about $\Gamma(t)>\Gamma_0$ with only an amplitude change of $\Delta\Gamma/\langle\Gamma\rangle\sim 4\%$. The relatively weak response of $P(t)$ and $\Gamma(t)$ is due to the smallness of the driver ϵ. $P(t)$ has the units of momentum and $\Gamma(t)$ has the units of inverse length.

shapes of $\Gamma(t)$ are qualitatively similar the bare kinks have appreciably reduced amplitudes of $\Gamma(t)$.

IV. DISCUSSION

The ac driver causes the center of mass X and the slope Γ to become time dependent and the kink to be dressed by phonons given by the expression $\chi(t)=(4/\pi)f(t)\mathrm{sech}^2\,\xi(t)$. The dressing χ which is not a CV internal mode can be observed as a modulation of the structure of the kink σ. We proved that the existence of a directed energy current arises from the existence of the internal degree of freedom $\Gamma(t)$ combined with the dressing $\chi(t)$. We found that the directed energy current vanished when Γ was set equal to Γ_0. When we set $\chi=0$ in Eq. (10) for $\dot{P}$ the right-hand side vanishes and we obtain

$$\dot{P}+\beta P=2\pi f_1.$$

The infinite time average of this equation vanishes when we use the fact that the thermal average of the initial value of P vanishes. Thus there is no directed current in the SG unless the slope depends on t and the kink is dressed by χ.

We observe in the computation of $\langle P(t)\rangle$ that the kink sees the heat bath only through the damping term $-\beta\langle P(t)\rangle$, i.e., $\langle P(t)\rangle$ does not see the fluctuations of the heat bath. The reason is that when we represent the bath as a generalized Fokker-Planck equation and calculate $\langle P(t)\rangle$ the damping term contributes $-\beta\langle P(t)\rangle$ because the damping is represented by $\beta\partial/\partial P$. However the fluctuation term is proportional to a second derivative $\partial^2/\partial P\partial P$ and thus gives a vanishing contribution to $\langle P(t)\rangle$. Note a fluctuation such as $\langle P^2\rangle$ or $\langle P(t)P\rangle$ would see both the damping term and the bath fluctuations.

In conclusion, we have proven that the symmetry breaking that leads to a directed energy current in the ac driven SG is generated by the existence of the time dependence of the slope $\Gamma(t)$ and by the dressing $\chi(t)$. In Ref. [15], Salerno and Quintero showed that a double SG showed ratchet behavior. In Ref. [16], Marchesoni obtained a directed kink transport by the sin ϕ potential for $\epsilon_2=0$. Costantini *et al.* [17] observed ratchet behavior in ac driven asymmetric kinks.

ACKNOWLEDGMENT

We would like to thank Sergej Flach for useful discussions on symmetry breaking.

APPENDIX

We calculate the dressing χ generated by the ac driven $f_1(t)$ in the linear approximation, i.e., the lowest order in ϵ_1, ϵ_2 and by using the Green's function of the linearized SG equation in the presence of a soliton Eq. (C5) of Ref. [14]. The formal solution for χ is

$$\chi(\xi) = 2\ \mathrm{Re}\int_{-\infty}^{\infty} dk[\Omega(k)]^{-1}\psi_k(\xi)\int_{-\infty}^{\infty} d\xi'\,\psi_k^*(\xi') \times \int_0^{\infty} dt'\ \sin\Omega(k)(t-t')e^{-\beta(t-t')}f_1(t'), \tag{A1}$$

where $\Omega(k)\equiv(1+k^2)^{1/2}$ and the eigenfunctions of the linearized SG are

$$\psi_k(\xi) = (2\pi)^{-1/2}e^{ik\xi}[ik-\tanh\,\xi]. \tag{A2}$$

The ξ' integral is

$$\int_{-\infty}^{\infty} d\xi'\,\psi_k(\xi') = -ik\int_{-\infty}^{\infty} d\xi'\cos k\xi' - \int_{-\infty}^{\infty} d\xi'\sin k\xi'\tanh\,\xi'. \tag{A3}$$

The first integral is an irrelevant constant which we can neglect. Integrating the second integral by parts we obtain

$$-(i/k)\int_{-\infty}^{\infty}\cos k\xi'\,\mathrm{sech}^2\,\xi'\,d\xi' = -(i/k)F(k), \tag{A4}$$

where $F(k)\equiv\pi k[2\ \sinh(\pi k/2)]^{-1}$ which decays rapidly with large k. The k integration in Eq. (A1) is

$$\begin{aligned}\mathrm{Re}(-i)\int_{-\infty}^{\infty} dk\ e^{ik\xi}[ik-\tanh\,\xi]k^{-1}F(k) &= \int_{-\infty}^{\infty}\cos k\xi F(k)dk \\ &\quad -(\pi/2)\tanh\,\xi\int_{-\infty}^{\infty} dk\ \sin k\xi[\sinh(\pi k/2)]^{-1} \\ &= (2/\pi)(\mathrm{sech}^2\xi-\tanh^2\xi) \\ &= \frac{4}{\pi}\mathrm{sech}^2\xi,\end{aligned} \tag{A5}$$

where we have dropped the irrelevant constant $(2/\pi)$. The time integral in Eq. (A1) is $f(t)$ where

$$\begin{aligned}f(t) &\equiv \int_0^{\infty} dt'\sin\Omega(k)(t-t')e^{-\beta(t-t')}f_1(t') \\ &= (\epsilon_1/2)\cos\,\omega t\left\{\frac{\Omega(k)-\omega}{\beta^2+[\Omega(k)-\omega]^2}+\frac{\Omega(k)+\omega}{\beta^2+[\Omega(k)+\omega]^2}\right\} \\ &\quad +(\epsilon_2/2)\cos(2\omega t+\theta)\left\{\frac{\Omega(k)-2\omega}{\beta^2+[\Omega(k)-2\omega]^2}+\frac{\Omega(k)+2\omega}{\beta^2+[\Omega(k)+2\omega]^2}\right\}.\end{aligned} \tag{A6}$$

In this paper we only consider values of ω which are appreciably less than 1. While $\Omega(k)=(1+k^2)^{1/2}$ where in our units the lower band edge has the value 1. Consequently with ω's in this paper as in Ref. [1], there is essentially no radiation of SG phonons generated by the ac driver but only a dressing of the soliton that is localized on the soliton. The presence of the SG phonons would not qualitatively alter the symmetry breaking but for the ac driver frequencies used in this paper the SG phonons would not be observable because they would occur only in very high orders of perturbation theory. Since $F(k)$ decreases rapidly with increasing k and $\omega \ll 1$ we can treat $f(t)$ as effectively independent of k and equal to

$$\begin{aligned}f(t) &= (\epsilon_1/2)\cos\,\omega t\left\{\frac{1-\omega}{\beta^2+(1-\omega)^2}+\frac{1+\omega}{\beta^2+(1+\omega)^2}\right\} \\ &\quad +(\epsilon_2/2)\cos(2\omega t+\theta)\left\{\frac{1-2\omega}{\beta^2+(1-2\omega)^2}+\frac{1+2\omega}{\beta^2+(1+2\omega)^2}\right\}.\end{aligned} \tag{A7}$$

Finally we have

$$\chi = (4/\pi)f(t)\mathrm{sech}^2\xi, \tag{A8}$$

with $f(t)$ given by Eq. (A7).

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