Ion temperature gradient instability in a dusty plasma

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An analysis of the temperature-gradient-driven (η_i) instability of drift waves in dusty plasma is presented. Various limits that allow for the coupling of the drift wave with the dynamics of dust grains are discussed. In particular, the cases of tiny (magnetized) and relatively heavy (unmagnetized) grains are studied. It is shown that in both limits the behavior of the η_i mode is considerably affected by the dust dynamics. The growth rate turns out to be higher in the presence of dust, and the instability threshold is lower, resulting in a more unstable plasma.

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I. INTRODUCTION

There exist a number of instabilities that are associated with drift waves in inhomogeneous magnetized plasmas. Some of these instabilities are dissipative, such as the one caused by electron-ion collisions, which results in a complex dispersion equation for the drift wave. Others are due to geometric effects (and are consequently termed as reactive instabilities). A typical example is the instability driven by the gradient of the ion temperature, known as the η_i instability. Here $\eta_i = d\ \ln T_i / d\ \ln n_i$, and it represents the ratio of the temperature and density gradients. The problem of the η_i instability can be treated in the framework of a plain slab geometry. It then yields an unstable ion sound perturbation propagating along the magnetic field lines, which is modified due to the ion temperature gradient. More complex is the case which includes the curvature of magnetic field lines (the so-called toroidal η_i instability). Details on the standard theory of the η_i mode can be found in Refs. [1–3]. The linearly unstable mode grows in time so that at some point the nonlinearities become essential. That has been studied recently in Refs. [4–6]. In Ref. [6] a dispersion equation describing parametric generation of zonal flow has been derived with the complete inclusion of linear as well as nonlinear effects of the ion polarization drifts.

Dust in a plasma can modify standard linear and nonlinear plasma modes (typically when the heavy dust grains are taken as stationary), or induce completely new ones when the dust dynamics is taken into account. The former case implies that dust enters into the plasma equations only through the equilibrium quasineutrality condition; however, even in such a simple case a new linear mode can be deduced [7], which in the nonlinear limit modifies standard plasma modes resulting in different localized strongly nonlinear solutions. The latter, due to the large difference in mass, implies dust modes with frequencies that are usually much smaller compared to the standard plasma frequencies (e.g., gyrofrequencies and plasma frequencies for light plasma species). Thus, the temporal and spatial scales of ion and heavy dust fluids can be very different in general and the coupling of dust and ion modes is usually rather weak. However, realistic dusty plasmas include grains of various size and in principle one has to deal with the grain size (mass) distribution, rather than taking a fixed value for the grain mass as is standardly done due to the complexity of the problem. In the present study we shall demonstrate the effect of dust grains on the η_i instability mentioned above. In other words, we shall investigate some limits in which the dust and ion dynamics are coupled, resulting in some distinct plasma features.

II. THE MODEL

We take a nonuniform dusty plasma embedded in a magnetic field, with density gradients for every plasma species in the direction that is perpendicular to the magnetic field lines. The equilibrium ion temperature is taken also with a gradient in the perpendicular direction. To be general we include the magnetic field curvature and gradient in the equations for the ion dynamics.

If the magnetic field is assumed to have curvature, in the steady state the following relation should then hold:

$$\nabla\left(p_0 + \frac{1}{2\mu_0}B_0^2\right) = \frac{1}{\mu_0}(\vec{B}_0 \cdot \nabla)\vec{B}_0. \tag{1}$$

The momentum equation for hot ions gives the perpendicular velocity,

$$\vec{v}_{i\perp 1} = \vec{v}_E + \vec{v}_{*i}, \tag{2}$$

where $\vec{v}_E = \vec{E}_{\perp 1} \times \vec{e}_z$ and $\vec{v}_{*i} = -\nabla p_{i1} \times \vec{e}_z/(eB_0 n_{i0})$. In writing the above equation we use the standard drift approach, $\partial/\partial t \ll \Omega_j$, where $j=e,i$, and $\vec{E}_1 = -\nabla\phi_1$, so that the polarization drift is neglected. The parallel component of the ion equation of motion yields

$$v_{iz1} = \frac{k_z}{m_i\omega}\left(e\phi_1 + T_{i1} + T_{i0}\frac{n_{i1}}{n_{i0}}\right). \tag{3}$$

In Eq. (3) and in the further text the subscript 0 denotes equilibrium quantities while the subscript 1 denotes perturbations. The magnetic field has both a gradient and a curvature, therefore $\nabla \cdot \vec{v}_E \neq 0$, and we have $\nabla \cdot (n_i \vec{v}_{*i})$

$=(\vec{v}_{Di}\cdot\nabla p_{i1})/T_{i0}$, and $\nabla\cdot(n_i\vec{v}_E)=q_i(\vec{v}_{Di}\cdot\nabla\phi_1)/T_{i0}$, where we have the sum of the curvature and ∇B drift given by $\vec{v}_{Di}=T_{i0}(\vec{e}_z\times\nabla\ln B_0+\vec{e}_z\times\vec{\kappa})/m_i\Omega_i$, and $\vec{\kappa}=(\vec{e}_z\cdot\nabla)\vec{e}_z$ is the local curvature of the field lines. The ion continuity equation

$$\frac{\partial n_i}{\partial t}+\nabla_\perp\cdot(n_i\vec{v}_{i\perp})+\partial_z(n_i v_{iz})=0 \tag{4}$$

then becomes

$$\left(\omega-\omega_{Di}-\frac{c_s^2k_z^2}{\tau\omega}\right)\frac{n_{i1}}{n_{i0}}-\tau(\omega_{Di}+\omega_{*i})\frac{e\phi_1}{T_{e0}}-\frac{T_{i1}}{T_{i0}}\omega_{Di}-\frac{c_s^2k_z^2}{\omega}\left(\frac{e\phi_1}{T_{e0}}+\frac{T_{i1}}{T_{i0}}\right)=0. \tag{5}$$

Here, the following notations are used: $c_s^2=T_{e0}/m_i$, $\omega_{Di}=\vec{v}_{Di}\cdot\vec{k}_\perp$, $\omega_{*i}=T_{i0}\vec{\kappa}_{ni}\cdot(\vec{e}_z\times\vec{k})\nabla_\perp\ln n_{i0}/(eB_0)$, and $\tau=T_{e0}/T_{i0}$.

The adiabatic ion temperature perturbation is dominantly due to the convective term, therefore we have

$$\frac{T_{i1}}{T_{i0}}=-\eta_i\frac{\omega_{*i}}{\omega}\frac{e\phi_1}{T_{e0}}. \tag{6}$$

Using Eq. (6), the ion density perturbation can be expressed in terms of the potential perturbation as

$$\left(\omega-\omega_{Di}-\frac{c_s^2k_z^2}{\omega\tau}\right)\frac{n_{i1}}{n_{i0}}=\left[\tau(\omega_{*i}+\omega_{Di})-\frac{\eta_i\omega_{*i}\omega_{Di}}{\omega}+\frac{c_s^2k_z^2}{\omega}\left(1-\frac{\eta_i\omega_{*i}}{\omega}\right)\right]\frac{e\phi_1}{T_{e0}}. \tag{7}$$

In the absence of dust one can replace $\tau\omega_{*i}$ with ω_{*e} which yields the appropriate expression from Ref. [1].

Negatively charged dust can take part in the collective interaction in the plasma, and the appropriate linearized equation of motion for a cold dust fluid can be written in the form

$$m_d n_{d0}\frac{\partial\vec{v}_{d1}}{\partial t}=-n_{d0}q_d(\nabla\phi_1-\vec{v}_{d1}\times\vec{B}_0). \tag{8}$$

Realistic space and laboratory dusty plasma systems include dust grains of various size, ranging from very tiny particles of the size of a few angstroms (like in the case of polycyclic aromatic hydrocarbons, observed in astrophysical dusty plasmas [8], which typically consist of a few tens of atoms only) to microns and much-above-micron sized grains. Therefore, various limits in the sense of dust grain size are possible. In principle, the charge on dust grains fluctuates even in the absence of perturbations. However, for very low frequency processes the charging and discharging of grains is an "adiabatic" process that is insignificant for the wave behavior [9]. Namely, in the theory of unmagnetized dusty plasma it is shown that there appears a phase difference between the charge variation and the wave potential which is such that the charge fluctuation on grains is of no importance for the frequencies that are much smaller or much higher than the characteristic attachment frequency of electrons. On the other hand, analytical estimates for some astrophysical conditions show [8] that the average grain charge number Z_d can be given by $Z_d\approx-1/[1+(\tau_0/\tau_1)^{1/2}]+\psi\tau_1$, where $\tau_1=aT/e^2\leqslant0.2$, T is the plasma temperature, a is the grain radius, the quantity τ_0 is the reduced temperature for which the ion collision rate with a negatively charged grain with $Z_d=-1$ equals the electron collision rate with a neutral grain, ψ is the solution to a transcedental Spitzer's equation [10], and its values for an electron-proton and a heavy-ion plasma are -2.504 and -3.799, respectively. As example, for $\tau_1\approx100$, we have $Z_d\approx300$. The appropriate response of charged grains on collective electrostatic perturbations in a plasma will consequently depend on the grain size and charge.

III. MAGNETIZED GRAINS

For very tiny but relatively highly charged grains the ratio Ω_i/Ω_d can become of the order of unity, or not much larger than 1, like when m_d/m_i goes up to 10^3–10^4, and Z_d takes values from 10^3 to 10^4. In that case the standard drift approximation $\omega\ll\Omega_{i,d}$ is valid, and it is appropriate to include the effects of the Lorentz force on the grain dynamics, so that the perpendicular grain velocity is given by

$$\vec{v}_{d\perp1}=-\frac{1}{B_0}\nabla\phi_1\times\vec{e}_z. \tag{9}$$

Using the dust continuity equation we can express the dust density fluctuations by

$$\frac{n_{d1}}{n_{d0}}=\left(\frac{\omega_{*d}}{\omega}-\frac{c_d^2k_z^2}{\omega^2}\right)\frac{e\phi_1}{T_{e0}}. \tag{10}$$

Here $\omega_{*d}=T_{e0}\nabla\ln n_{d0}(\vec{e}_z\times\vec{k}_\perp)/(eB_0)$, $c_d^2=Z_dT_{e0}/m_d$

Further, using the quasineutrality condition

$$n_{i1}=n_{e1}+Z_dn_{d1}, \tag{11}$$

one obtains the following dispersion relation that couples the dynamics of grains with the drift wave:

$$\left[\frac{n_{e0}}{n_{i0}}\omega+Z_d\frac{n_{d0}}{n_{i0}}\left(\omega_{*d}-\frac{c_d^2k_z^2}{\omega}\right)\right](\tau\omega^2-\tau\omega_{Di}\omega-c_s^2k_z^2)=\tau^2\omega_{*i}\omega^2+\tau(\tau\omega_{Di}\omega-\eta_i\omega_{*i}\omega_{Di})\omega+\tau c_s^2k_z^2(\omega-\eta_i\omega_{*i}). \tag{12}$$

The electron dynamics is represented by the Boltzmann distribution in the parallel direction.

The effects of dust can easily be seen in the simple case when the magnetic field is homogeneous. In that case Eq. (12) reduces to

$$\left[\frac{n_{e0}}{n_{i0}}\omega+Z_d\frac{n_{d0}}{n_{i0}}\left(\omega_{*d}-\frac{c_d^2k_z^2}{\omega}\right)\right](\tau\omega^2-c_s^2k_z^2)=\tau[\tau\omega_{*i}\omega^2+c_s^2k_z^2(\omega-\eta_i\omega_{*i})]. \tag{13}$$

For static dust grains from Eq. (13) one readily gets a modified (due to the term n_{i0}/n_{e0}) dispersion equation for the η_i mode in slab geometry,

$$\omega^3 - \omega_{*i}\frac{n_{i0}}{n_{e0}}\omega^2 - c_s k_z^2\left(\frac{n_{i0}}{n_{e0}} + \frac{1}{\tau}\right)\omega + \eta_i\omega_{*i}\frac{n_{i0}}{n_{e0}}c_s k_z^2 = 0.$$

The modification is only due to the term n_{i0}/n_{e0}, and the corresponding stability conditions which follow from this dispersion equation are well known from the literature [1]. However, we note that the mode increment is generally higher in the presence of static dust grains.

With the dust dynamics included, we can rewrite Eq. (13) conveniently in the form

$$\omega^4 + h_1\omega^3 + h_2\omega^2 + h_3\omega + h_4 = 0, \tag{14}$$

where

$$h_1 = Z_d\frac{n_{d0}}{n_{e0}}\omega_{*d} - \frac{n_{i0}}{n_{e0}}\tau\omega_{*i}, \quad h_2 = -c_s^2k_z^2\left(\frac{1}{\tau} + \frac{n_{i0}}{n_{e0}} + \frac{Z_d n_{d0}}{n_{e0}}\frac{c_d^2}{c_s^2}\right),$$

$$h_3 = c_s^2k_z^2\left(\frac{n_{i0}}{n_{e0}}\eta_i\omega_{*i} - \frac{Z_d}{\tau}\frac{n_{d0}}{n_{e0}}\omega_{*d}\right),$$

$$h_4 = \frac{Z_d}{\tau}\frac{n_{d0}}{n_{e0}}c_d^2c_s^2k_z^4.$$

Equation (14) can be solved numerically yielding four modes, i.e., two drift modes and two sound-type modes (modified due to the η_i term). For numerical purposes we normalize the frequency ω to the ion gyro frequency Ω_i, and further rewrite the coefficients $h_{1,2,3,4}$ in a more suitable form as

$$h_1 = (1-s)\frac{nr^2}{l}, \quad h_2 = -r^2\kappa_z^2\left(n + \frac{1}{\tau} + (n-1)\frac{Z_d m_i}{m_d}\right),$$

$$h_3 = -\frac{r^4\kappa_z^2 n}{\tau l}(\eta_i - s), \quad h_4 = r^4\kappa_z^4\frac{n-1}{\tau}\frac{Z_d m_i}{m_d}. \tag{15}$$

Here, we have introduced the dimensionless parameters defined as $r=\rho_s k_y$, $n=n_{i0}/n_{e0}$, $l=k_y L_{ni}$, $\kappa_z=k_z/k_y$, $s=l_{nd}(n-1)/n$, $l_{nd}=L_{ni}/L_{nd}$, and the equilibrium density gradients are taken as negative. Clearly, $n=1$ (which also yields $s=0$) corresponds to the case without dust.

To demonstrate the effects of dust dynamics on the η_i instability we adopt $\tau=0.3$, $r=0.2$, $l=60$, $Z_d m_i/m_d=0.1$, $l_{nd}=1.5$, and solve Eq. (14) by varying the parameters κ_z, η_i, for $n=1$ (no dust), and $n=3$, searching for the threshold $\omega_i=0$. The locus of the pairs (κ_z, η_i) is presented by the lines 1 and 3 in Fig. 1, where line 1 corresponds to the case without dust and line 3 to the case $n=3$. The unstable region for each case is located above the corresponding line. The dashed line will be discussed later. Consequently, the presence of dust destabilizes the mode in the sense that the threshold value for η_i decreases for a fixed value of k_z. For example, it is seen that for $k_z=0.04$ the threshold value for η_i without dust (i.e., for $n=1$) is ≈ 13, while for $n=3$ it becomes ≈ 8.2, i.e., it is about 37% smaller than the former value.

In Fig. 2 we fix $\eta_i \approx 9.8$ (which in line 1 in Fig. 1 corresponds to $\kappa_z \approx 0.03$) and solve Eq. (14) in terms of κ_z. This yields the frequencies and increments/decrements of the modes. The imaginary parts of the frequencies (in units of Ω_i, and multiplied by 10^{-3}) are presented in Fig. 2 for the cases $n=1$ (no dust), $n=1.5$, and $n=3$. Clearly, the dust changes the stability of the η_i mode in two ways: first, it increases the growth rate (note that for $\kappa_z=0.025$ the increment in the presence of dust, $n=3$, is about 2.4 times higher then the corresponding increment without dust), and, second, it extends the range of the values of κ_z for which the mode is unstable (it is seen that the threshold value of κ_z in the presence of dust is shifted by about 60 %). The limit of very small values of k_z has no much physical sense since in that case the parallel ion dynamics becomes unimportant, and in the same time the conditions and assumptions used to describe electron dynamics by the Boltzmann distribution are violated, so that the complete model becomes invalid.

Further, for a given value $\kappa_z=0.025$ we solve Eq. (14) in terms of η_i. The imaginary part of the frequency (in units of Ω_i, and multiplied by 10^{-3}) is presented in Fig. 3, where again we have three curves for $n=1,1.5,3$. It is seen, first, that the mode increment in the presence of dust becomes

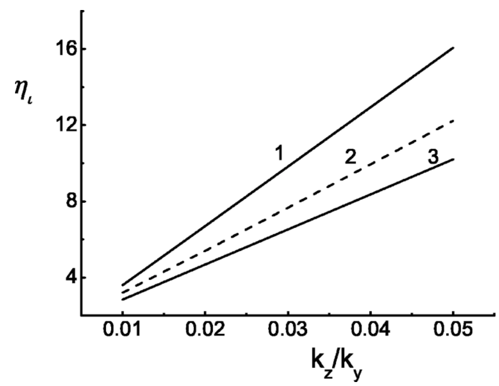

FIG. 1. The locus of pairs $\kappa_z \equiv k_z/k_y$ yielding $\omega_i=0$ in Eq. (14). The line 1 corresponds to the case without dust, while the line 3 is for $n_{i0}/n_{e0}=3$, both for the magnetized dust. The dashed line (number 2) is for $n=3$ and for unmagnetized grains. The parameters are given in the text.

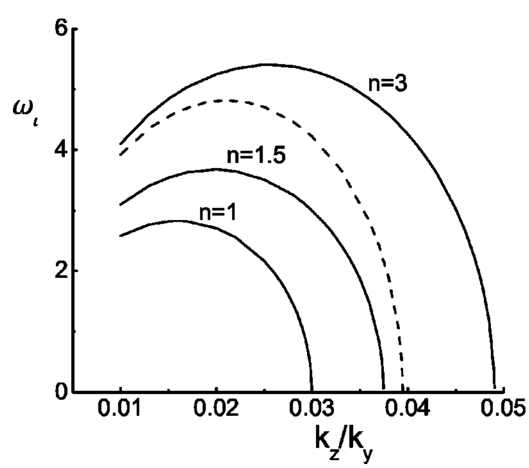

FIG. 2. The increment of the η_i mode for magnetized dust (in units of Ω_i and multiplied by 10^{-3}) in terms of k_z/k_y for $n=1,1.5,3$. Here $\eta_i \approx 9.8$, and other parameters are given in the text. The dashed curve is for the unmagnetized dust, for $n=3$ and $\delta=1$.

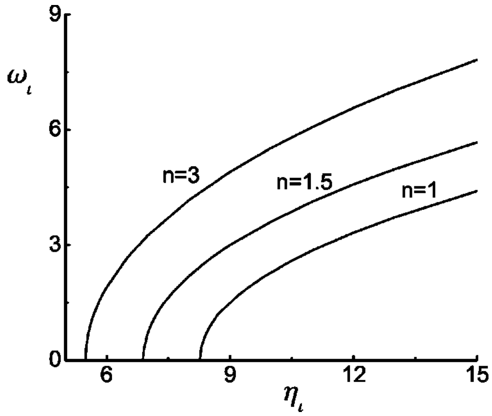

FIG. 3. The increment (in units of Ω_i and multiplied by 10^{-3}) in terms of η_i for a fixed $k_z/k_y=0.025$ for $n=1,1.5,3$. The corresponding instability thresholds are around the values $\eta_i \approx 8.26$, 6.80, and 5.48, respectively.

higher, and, second, that the threshold value of η_i is shifted from ≈ 8.26 (for $n=1$) to ≈ 5.48 (for $n=3$). In other words the plasma becomes more unstable in the presence of dust.

IV. UNMAGNETIZED GRAINS

For relatively heavy grains the effects of the Lorentz force is negligible (this is equivalent to the case $\omega \gg \Omega_d$), and it is appropriate to keep the first term only in the right-hand side of Eq. (8). Instead of Eq. (10) we then have

$$\frac{n_{d1}}{n_{d0}} = -\frac{c_d^2 k_y^2}{\omega^2}\frac{e\phi_1}{T_{e0}}, \tag{16}$$

and instead of Eq. (13) we have

$$\left(\frac{n_{e0}}{n_{i0}}\omega^2 - c_d^2 k_y^2 \frac{Z_d n_{d0}}{n_{i0}}\right)(\tau\omega^2 - c_s^2 k_z^2) = \tau\omega[\tau\omega_{*i}\omega^2 + c_s^2 k_z^2(\omega - \eta_i\omega_{*i})]. \tag{17}$$

Neglecting the parallel ion dynamics one obtains a dispersion equation for the modes propagating strictly perpendicular, in the form $\omega^2 - \omega_{*e}\omega n_{i0}/n_{e0} - c_d^2 k_y^2 Z_d n_{d0}/n_{e0} = 0$. This dispersion equation describes coupled drift and dust sound modes; in the absence of the ion density gradient we have two sound modes propagating in opposite directions. It is worth noting that here the sound response of ions (along the magnetic field lines) and the dust (along the wave vector) become comparable, i.e., $c_s^2 k_z^2/(c_d^2 k_y^2) = m_d k_z^2/(m_i k_y^2 Z_d)$ becomes of the order of unity for $m_d \sim 10^8 - 10^{10}$, $Z_d \sim 10^2 - 10^4$, and $k_z/k_y \sim 10^{-2} - 10^{-3}$.

The dispersion equation (17), normalized to Ω_i, can be rewritten as

$$\omega^4 + \beta_1\omega^3 + \beta_2\omega^2 + \beta_3\omega + \beta_4 = 0. \tag{18}$$

Here, using the dimensionless parameters introduced earlier we have

$$\beta_1 = \frac{r^2 n}{l}, \quad \beta_2 = -r^2\kappa_z^2\left(\frac{1}{\tau} + n + \delta\right), \quad \beta_3 = -\frac{\eta_i r^4 \kappa_z^2 n}{\tau l},$$

$$\beta_4 = (n-1)\frac{r^4\kappa_z^2}{\tau}\frac{Z_d m_i}{m_d}, \quad \delta = \frac{n-1}{\kappa_z^2}\frac{Z_d m_i}{m_d}.$$

It is seen that the parameters δ and β_4 vanish in the absence of dust. Also, we notice that $\delta=0.2$ for $Z_d m_i/m_d = 10^{-5}$, $\kappa_z = 10^{-2}$, and $n=3$. Thus, the case $n=1$ corresponds to the absence of grains, and the threshold for the η_i instability is, as earlier, described by the locus of points represented by the line 1 in Fig. 1. For a comparison with the previous case, as an illustration, Eq. (18) is solved for $n=3$ and $\delta=1$. The locus of the threshold for the η_i instability is presented by the dashed line (line 2) in Fig. 1. Hence, the instability is again influenced by the grains although the dust fluid dynamics is of a very different nature in the present case. Namely, for the drift wave propagating in y direction the ions perform oscillations typical for a drift wave, i.e., in the direction of the density gradient (due to the Lorentz force), while the motion of grains is in the direction of the perturbed electric field, i.e., in the direction of the wave. The appropriate wave increment for an unstable mode can be calculated in a straightforward manner. As an example we have presented it in Fig. 2 by the dashed curve. Here $\eta_i = 9.8214$, $n=3$, and $\delta=1$. Its behavior is qualitatively similar to the curves representing the magnetized dust case. The mode is again destabilized.

V. CONCLUSIONS

To conclude, we have investigated the effects of the dust on the η_i instability by including the dynamics of dust grains in two different limits, i.e., for tiny and magnetized grains, and for relatively heavy grains which consequently do not feel effects of the Lorentz force in the time and space scales typical for a drift wave. The results obtained in this study clearly show that the stability conditions are significantly changed due to the presence of dust. In both cases, for light and heavy grains the mode growth rates become higher in the presence of dust, while the threshold for the η_i instability is reduced. Therefore, plasma is generally destabilized due to the effects of dust. Physically, this is due to the smaller amount of free electrons in the system; namely, a considerable part of the negative charge (usually attributed to light particles—electrons) is now bound to the heavy grains which, on the other hand, cannot replace highly mobile electrons in their generally stabilizing role in an ideal plasma, so the effects observed here are expected. Consequently, similar to the previously mentioned studies of a plasma without dust [4–6], the nonlinear development of the η_i instability in a plasma comprising dust grains would be worth studying. Some other aspects of the nonlinear phenomena in dusty plasmas have been studied intensively in the recent past [7,11], yet the nonlinear phenomena related to the η_i instability in dusty plasmas remain an open issue.

As it is said in the text, the charge on grains, which in the present study for clarity is taken as constant, should fluctuate and it could affect the mode behavior. Various aspects of the charge fluctuation, and its effects on modes in unmagnetized plasmas, have been studied in our earlier works [12,13]. The eventual interplay of the charge fluctuation and the η_i mode, with the dust dynamics included, would be interesting to study. However, this is not a simple task as an appropriate model for the fluctuation of charge on grains in a magnetized plasma seems to be not available in the existing literature. We believe that the model and the qualitative results presented in this work could be applied to realistic space and laboratory dusty plasmas, and to plasmas with impurities.

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