PHYSICAL REVIEW E **69**, 055601(R) (2004)

Prediction of simultaneously large and opposite generalized Goos-Hänchen shifts for TE and TM light beams in an asymmetric double-prism configuration

Chun-Fang Li[1,2,*] and Qi Wang[1]

[1]*Department of Physics, Shanghai University, 99 Shangda Road, Shanghai 200436, People's Republic of China*

[2]*State Key Laboratory of Transient Optics Technology, Xi'an Institute of Optics and Precision Mechanics, Academia Sinica, 322 West Youyi Road, Xi'an 710068, People's Republic of China*

(Received 23 September 2003; published 12 May 2004)

It is predicted that large and opposite generalized Goos-Hänchen (GGH) shifts may occur simultaneously for TE and TM light beams upon reflection from an asymmetric double-prism configuration when the angle of incidence is below but near the critical angle for total reflection, which may lead to interesting applications in optical devices and integrated optics. Numerical simulations show that the magnitude of the GGH shift can be of the order of beam's width.

DOI: 10.1103/PhysRevE.69.055601 PACS number(s): 42.25.Gy, 42.25.Ja, 78.20.Bh

It is well established that a light beam that is totally reflected from a dielectric interface undergoes a lateral shift from the position predicted by geometrical optics. This phenomenon is known as the Goos-Hänchen (GH) effect [1,2] and was theoretically explained first by Artmann [3]. The GH shift depends on the polarization state of the beam [4–7] and its magnitude for incidence angles close to the critical angle for total reflection is about the order of the wavelength. The smallness of the shift for optical wavelengths has impeded its direct measurement in a single-reflection experiment [7,8]. Since the investigation of the GH shift has been extended to frustrated total internal reflection [5,9–12] and partial reflection [13–19] and to other areas of physics [4], such as acoustics [20], nonlinear optics [21,22], surface physics [23], and quantum mechanics, attention has been paid to the mechanism for enlarging its magnitude [13,15,18,24–32].

It was predicted that the GH shift can be enhanced by resonance by an order or more for TE polarization in spatially dispersive semiconductors [30] or for TM polarization in cesium vapor [31]. In this paper we report that large and opposite GH shifts may occur simultaneously for TE and TM light beams upon reflection from an asymmetric double-prism configuration when the angle of incidence is below but near the critical angle for total refection, which may have interesting applications in optical devices and integrated optics. These large GH shifts are in connection with transmission resonances in much the same way as in a total internal reflection configuration [32]. Numerical simulations show that the magnitude of the GH shift is about the order of beam's width at transmission resonances.

Historically, the phenomenon of the GH shift [1,2] involves the evanescent wave in an optically thinner medium. But the beam shift discussed in this paper has nothing to do with the evanescent wave. So we term it as generalized GH (GGH) shift for the rest of the paper, since it keeps the main features of the GH shift, that is to say, it is due to the finite width of the light beam and is different from the prediction of geometrical optics.

The asymmetric double-prism configuration considered here is shown in Fig. 1, where two prisms of refractive indices n_u and n_s are placed adjacently with a thin dielectric layer of refractive index n_l and thickness a being formed in between them, so that $n_u, n_s > n_l$. Our discussions are in two dimensions. A light beam of wavelength λ and angular frequency ω is incident from lower left at incidence angle θ_0 that is assumed to be less than the critical angle, $\theta_c = \sin^{-1}(n_l/n_u)$, for total reflection. Let $\psi_{in}(\vec{x}) = A(k_y)\exp(i\vec{k}_u \cdot \vec{x})$ be the electric (or magnetic) field of the Fourier angular spectrum of the incident TE (or TM) beam, where time dependence $\exp(-i\omega t)$ is implied and suppressed, $\vec{k}_u \equiv (k_{ux}, k_y) = (k_u \cos\theta_u, k_u \sin\theta_u)$, $k_u = n_u k_0$, $k_0 = 2\pi/\lambda$ is the wave number in the vacuum, and θ_u is the incidence angle of the plane-wave component under consideration, $A(k_y)$ is the amplitude angular-spectrum distribution. Combining Maxwell's equations and boundary conditions yields the electric (or magnetic) fields of corresponding Fourier angular spectrum $\psi_r(\vec{x}) = r(k_y)A(k_y)\exp[i(-k_{ux}x + k_y y)]$ for reflected beam, and $\psi_t(\vec{x}) = t(k_y)A(k_y)\exp[ik_{sx}(x-a) + ik_y y]$ for transmitted beam, where $r(k_y)$ is the amplitude reflection coefficient $r(k_y) = g_1 \exp(-i\phi_1)/[g_0 \exp(-i\phi_0)]$, $t(k_y)$ is the amplitude transmission coefficient $t(k_y) = 1/[g_0 \exp(-i\phi_0)]$, g_0 and ϕ_0 are defined by

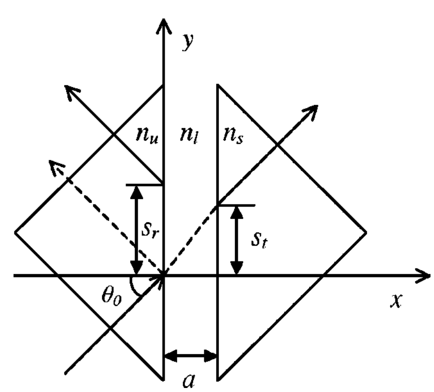

FIG. 1. Schematic diagram of the GGH shift in asymmetric double-prism configuration.

*Electronic address: cfli@mail.shu.edu.cn

$$g_0 \exp(-i\phi_0) \equiv \frac{1}{2}\left(1+\frac{\eta_s}{\eta_u}\right)\cos k_{lx}a - \frac{i}{2}\left(\frac{\eta_s}{\eta_l}+\frac{\eta_l}{\eta_u}\right)\sin k_{lx}a, \tag{1}$$

g_1 and ϕ_1 are defined by

$$g_1 \exp(-i\phi_1) \equiv \frac{1}{2}\left(1-\frac{\eta_s}{\eta_u}\right)\cos k_{lx}a - \frac{i}{2}\left(\frac{\eta_s}{\eta_l}-\frac{\eta_l}{\eta_u}\right)\sin k_{lx}a, \tag{2}$$

$$\eta_j = \begin{cases} n_j \cos\theta_j, & \text{for TE,} \\ \cos\theta_j/n_j, & \text{for TM,} \quad j=u,l,s, \end{cases}$$

θ_j is determined by Snell's law, $n_j \sin\theta_j = n_u \sin\theta_u$, $k_{jx} = n_j k_0 \cos\theta_j$.

It is noted that $\phi_0-\phi_1$ is the phase of reflection coefficient, and ϕ_0 itself is the phase of transmission coefficient. The GGH shift of reflected beam from the position predicted by geometrical optics as is shown in Fig. 1 is, according to stationary-phase theory [3,5,10], $s_r=-d(\phi_0-\phi_1)/dk_y|_{\theta_u=\theta_0}$. The GGH shift of transmitted beam which is defined [18] as the lateral displacement of the peak of the transmitted beam at the second layer/prism interface from the peak of the incident beam at the first prism/layer interface is given by, according to stationary-phase theory,

$$s_t = -\left.\frac{d\phi_0}{dk_y}\right|_{\theta_u=\theta_0} = \frac{a}{4g_0^2}\left\{\left(1+\frac{\eta_s}{\eta_u}\right)\left[\left(\frac{\eta_s}{\eta_l}+\frac{\eta_l}{\eta_u}\right)-\left(\frac{\eta_s}{\eta_l}-\frac{\eta_l}{\eta_u}\right)\frac{\sin 2k_{lx}a}{2k_{lx}a}\right]\tan\theta_l + \frac{\eta_s}{\eta_u}\left[\left(\frac{\eta_s}{\eta_l}-\frac{\eta_l}{\eta_s}\right)\frac{n_l\cos\theta_l}{n_u\cos\theta_u}\tan\theta_u + \left(\frac{\eta_u}{\eta_l}-\frac{\eta_l}{\eta_u}\right)\frac{n_l\cos\theta_l}{n_s\cos\theta_s}\tan\theta_s\right]\frac{\sin 2k_{lx}a}{2k_{lx}a}\right\}\Bigg|_{\theta_u=\theta_0}. \tag{3}$$

Substituting ϕ_1 defined in Eq. (2) and noticing Eq. (3), we finally get $s_r=s_t+s_0$ for the GGH shift of reflected beam, where

$$s_0 = \frac{d\phi_1}{dk_y} = -\frac{a}{4g_1^2}\left\{\left(1-\frac{\eta_s}{\eta_u}\right)\left[\left(\frac{\eta_s}{\eta_l}-\frac{\eta_l}{\eta_u}\right)-\left(\frac{\eta_s}{\eta_l}+\frac{\eta_l}{\eta_u}\right)\frac{\sin 2k_{lx}a}{2k_{lx}a}\right]\tan\theta_l - \frac{\eta_s}{\eta_u}\left[\left(\frac{\eta_s}{\eta_l}-\frac{\eta_l}{\eta_s}\right)\frac{n_l\cos\theta_l}{n_u\cos\theta_u}\tan\theta_u - \left(\frac{\eta_u}{\eta_l}-\frac{\eta_l}{\eta_u}\right)\frac{n_l\cos\theta_l}{n_s\cos\theta_s}\tan\theta_s\right]\frac{\sin 2k_{lx}a}{2k_{lx}a}\right\}\Bigg|_{\theta_u=\theta_0}. \tag{4}$$

Since the power reflectance R satisfies $R=g_1^2/g_0^2 \leq 1$, we see that $g_1^2 \leq g_0^2$. In fact, we have from Eq. (2) that

$$g_1^2 = \frac{1}{4}\left(1-\frac{\eta_s}{\eta_u}\right)^2\cos^2 k_{lx}a + \frac{1}{4}\left(\frac{\eta_s}{\eta_l}-\frac{\eta_l}{\eta_u}\right)^2\sin^2 k_{lx}a. \tag{5}$$

g_1^2 can be very small and even be equal to zero under certain conditions. So let us look carefully at s_0, which will dominate s_r when g_1^2 is very small.

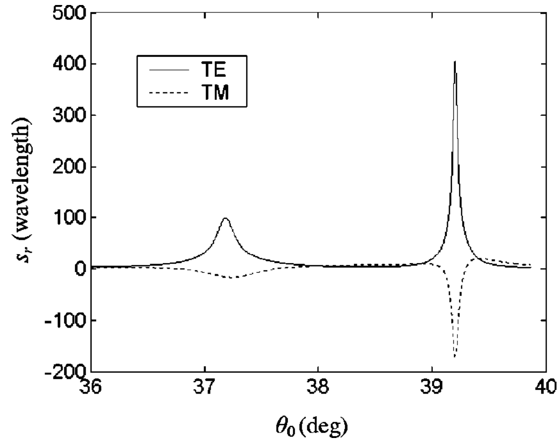

FIG. 2. Dependence of theoretical GGH shifts for reflected TE and TM beams on the angle of incidence, where $n_u=1.56$, $n_l=1$, $n_s=2.22$, $a=3\lambda$, the TE beam is shown by solid curve, and the TM beam is shown by dotted curve.

First of all, when the incidence angle of the beam is near the critical angle θ_c in the asymmetric configuration $(1-\eta_s/\eta_u \neq 0)$, s_0 will be dominated by its first part,

$$s_0 \approx -\frac{a}{4g_1^2}\left(1-\frac{\eta_s}{\eta_u}\right)\left[\left(\frac{\eta_s}{\eta_l}-\frac{\eta_l}{\eta_u}\right)-\left(\frac{\eta_s}{\eta_l}+\frac{\eta_l}{\eta_u}\right)\frac{\sin 2k_{lx}a}{2k_{lx}a}\right]\tan\theta_l\Bigg|_{\theta_u\approx\theta_c}. \tag{6}$$

Second, because $\eta_l \to 0$ near the critical angle, g_1^2 reaches its minima $(1/4)(1-\eta_s/\eta_u)^2$ at $k_{lx}a=m\pi (m=1,2,3,\ldots)$. This means that the maxima of s_0 near the critical angle are

$$s_{0max} \approx -\frac{\eta_s/\eta_l-\eta_l/\eta_u}{1-\eta_s/\eta_u}a\tan\theta_l\Bigg|_{\theta_u\approx\theta_c},$$

which is very large in that $\eta_l \to 0$ and $\theta_l \to \pi/2$ near the critical angle. At last, it is noted that near the critical angle, $\eta_s/\eta_l-\eta_l/\eta_u$ is positive, but the sign of $1-\eta_s/\eta_u$ depends on the polarization of the beam for a definite double-prism structure, that is, for definite n_u and n_s. For instance, if $n_u > n_s$, $1-\eta_s/\eta_u > 0$ for TE polarization and $1-\eta_s/\eta_u < 0$ for TM polarization (it should be kept in mind that the incidence angle of the beam is assumed to be always below the critical angle). On the other hand, if $n_u < n_s$, then $1-\eta_s/\eta_u < 0$ for TE polarization and $1-\eta_s/\eta_u > 0$ for TM polarization. Thus it is clear that the reflection GGH shift of TE beam is opposite to that of TM beam near the critical angle. In Fig. 2 is shown the dependence of the GGH shift s_r on the incidence angle of the beam, where $n_u=1.56$, $n_l=1$ (the critical angle $\theta_c=\sin^{-1}(1/1.56)\approx 39.87°$), $n_s=2.22$, the thickness of the layer $a=3\lambda$, the TE beam is shown by solid curve, and the TM beam is shown by dotted curve. All the physical quantities that have length dimension, such as the GGH shift, the thickness of the thin layer and the width of the beam are in units of wavelength in this paper. It is shown that the GGH shift is very large and is positive for TE beam and negative for TM beam at $\theta_0 = 39.2°$.

When the incidence angle is near the critical angle, the reflectance is minimal at $k_{lx}a = m\pi$ as readers easily verify. This minimum is equal to $R_{min} = (1 - \eta_s/\eta_u)^2/(1+\eta_s/\eta_u)^2$. So the power transmission reaches its maximum $T_{max} = 1 - R_{min} = (4\eta_s/\eta_u)/(1+\eta_s/\eta_u)^2$ at $k_{lx}a = m\pi$. This shows that the GGH shift of reflected beam is greatly enhanced by transmission resonance in much the same way as in a total internal reflection configuration [32].

Now that the peaks of the GGH shift of reflected beam are determined by $k_{lx}a = m\pi$, the angular distance $\Delta\theta_0$ between two adjacent peaks for a given a is determined by $|\Delta k_{lx}|a = \pi$, which gives $\Delta\theta_0 = \pi/k_{ux}a \tan\theta_l$. In order to retain the profile of the beam in reflection, it is required that $\Delta\theta_0$ be much larger than the divergence of the beam, $\delta\theta \sim \lambda/n_u\pi w_0$, where w_0 is the width of the beam. As a result, for a beam of given width, the thickness of the layer between the two prisms is required to satisfy

$$a \ll a_0 \equiv \pi w_0/2 \cos\theta_u \tan\theta_l \sim \frac{\pi w_0}{2}\frac{\cos\theta_l}{\cos\theta_u}$$

near the critical angle. This means that the restrictions adopted by Steinberg and Chiao in Ref. [10] [e.g., in the discussions around their equations (34)–(39)] are sufficient, but not necessary, to ensure that the stationary-phase method is valid.

To show the validity of the above stationary-phase analysis, we have made numerical simulations in which the incident beam is assumed to be of Gaussian profile,

$$\psi_{in}(\vec{x})|_{x=0} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A(k_y)\exp(ik_y y)dk_y, \qquad (7)$$

where the amplitude angular-spectrum distribution is Gaussian, $A(k_y) = w_y \exp[-(w_y^2/2)(k_y - k_{y0})^2]$, $k_{y0} = k_u \sin\theta_0$, $w_y = w_0 \sec\theta_0$. Consequently, the field of reflected beam has the following form,

$$\psi_r(\vec{x}) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} r(k_y)A(k_y)\exp(-ik_{ux}x + ik_y y)dk_y. \qquad (8)$$

The integral from $-\infty$ to $+\infty$ in Eq. (7) guarantees that the field of the incident beam has a perfect Gaussian profile with respect to y. But for a real incident beam, the incidence angles of its angular-spectrum components extend from $-\pi/2$ to $\pi/2$. So the integral in Eq. (8) in numerical simulations is performed from $-k_u$ to k_u,

$$\psi_r^N(\vec{x}) = \frac{1}{\sqrt{2\pi}}\int_{-k_u}^{k_u} r(k_y)A(k_y)\exp(-ik_{ux}x + ik_y y)dk_y.$$

The numerically calculated GGH shift s_r^N of reflected beam is defined by $|\psi_r^N(x=0, s_r^N)| = \max|\psi_r^N(x=0,y)|$.

In Fig. 3 we draw the dependence of numerically calculated GGH shifts for reflected TE beams on the thickness of the layer in comparison with the result of theoretical analysis (solid curve), where two different widths of the beam are involved, $w_0 = 117\lambda$ (corresponding to beam's divergence $\delta\theta = 0.1°$ and being shown by dashed curve), and $w_0 = 1170\lambda$ (corresponding to beam's divergence $\delta\theta = 0.01°$ and being shown by dotted curve), the angle of incidence is chosen to be $\theta_0 = 39.2°$, all other parameters are as in Fig. 2. It should be pointed out that the discrepancy between theoretical and numerical results is due to the distortion of the reflected beam, especially when the width of the beam is narrow. So the wider the incident beam is, the lesser the reflected beam is distorted, and the closer to the theoretical result the numerical result is. The peak of the numerical shift at $a = 3\lambda$ is about 194.1λ for $w_0 = 117\lambda$. And it is about 383.3λ for $w_0 = 1170\lambda$, which is almost equal to the theoretical result 403.4λ. This shows that the magnitude of the GGH shift at transmission resonance is over 100 times of the wavelength and is of the order of beam's width.

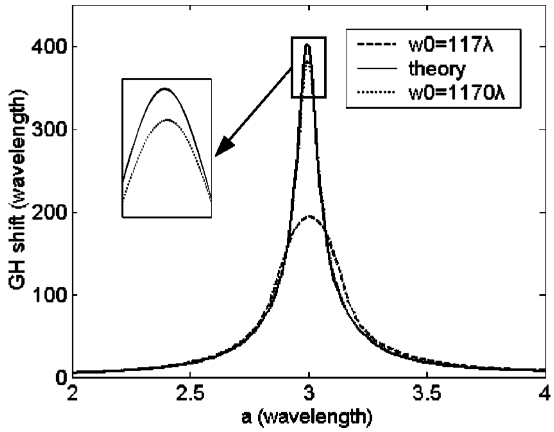

FIG. 3. Dependence of numerically calculated GGH shifts for reflected TE beams on the thickness of the layer, where $w_0 = 117\lambda$ is shown by dashed curve, $w_0 = 1170\lambda$ is shown by dotted curve, $\theta_0 = 39.2°$, and all other parameters are as given in Fig. 2. For comparison, the result of theoretical analysis is also shown by solid curve.

According to Eq. (3), the GGH shift of transmitted beam in the sense of above definition can be much larger than what is predicted by geometrical optics when the incidence angle is near the critical angle. Since near the critical angle, g_0 is minimal at $k_{lx}a = m\pi$, the large GGH shift of transmitted beam occurs at $k_{lx}a = m\pi$. Numerical simulations show that the GGH shift of transmitted beam can also be of the order of beam's width.

When the refractive indices of the two prisms are the same, s_0 will vanish according to Eq. (4), so that the GGH shift of reflected beam will be equal to that of transmitted beam. This shows that the properties of the reflection GGH shift do result from the asymmetry of the configuration.

It is worthwhile to point out that in the above discussions where the angle of incidence is near the critical angle, the minima of g_1^2 at which the GGH shift of reflected beam reaches its maxima is equal to $(1/4)(1 - \eta_s/\eta_u)^2$. Therefore the corresponding reflectance $R = g_1^2/g_0^2$ is not equal to zero. Readers may notice that apart from this case, there are other cases in which the power reflectance can be equal to zero and the corresponding GGH shift of reflected beam seems to tend to infinity. These happen when $1 - \eta_s/\eta_u = 0$ or $\eta_s/\eta_l - \eta_l/\eta_u = 0$. In the present asymmetric double-prism configuration where $n_u, n_s > n_l$, these two equations have solutions only for TM polarization. The solution to the former equation is nothing but the Brewster angle,

$$\theta_u = \tan^{-1}(n_s/n_u), \tag{9}$$

for the interface between media of refractive indices of n_u and n_s. The solution to the latter equation which can be rewritten as

$$\sin 2\theta_u \sin 2\theta_s = \sin^2 2\theta_l \tag{10}$$

determines another angle of incidence. If $n_u > n_l > n_s$ (or $n_u < n_l < n_s$), the latter equation also has solution for TE polarization, which can be rewritten as

$$\tan \theta_u \tan \theta_s = \tan^2 \theta_l. \tag{11}$$

Though at an angle of incidence that satisfies any of Eqs. (9)–(11) for appropriate polarization, the GGH shift of reflected beam has resonant peaks which tend to infinity, these peaks are physically meaningless, because they are located at the zero points of power reflectance. At those points, the reflected beam is very weak and distorted so severely that it cannot be described in terms of a shifted beam [15]. The resonant peaks in those situations result from the discontinuity of ϕ_1 with respect to k_y at the zero points of $g_1 \exp(-i\phi_1)$. After all, the phase of complex number zero is mathematically undefined. In order to make use of such resonant peaks in the GGH shift, additional mechanisms are needed, such as the weak absorption [28,29]. Discussions of these problems will be presented elsewhere.

This work was supported in part by the National Natural Science Foundation of China (Grant No. 60377025), the Science Foundation of Shanghai Municipal Commission of Education (Grant No. 01SG46), the Science Foundation of Shanghai Municipal Commission of Science and Technology (Grant No. 03QMH1405), and the Shanghai Leading Academic Discipline Program.

[1] F. Goos and H. Hänchen, Ann. Phys. (Leipzig) **1**, 333 (1947).
[2] F. Goos and H. Hänchen, Ann. Phys. (Leipzig) **5**, 251 (1949).
[3] K. Artmann, Ann. Phys. (Leipzig) **2**, 87 (1948).
[4] For a comprehensive reference until 1971, see H. K. V. Lotsch, Optik (Stuttgart) **32**, 116 (1970); **32**, 189 (1970); **32**, 299 (1971); **32**, 553 (1971).
[5] J. J. Cowan and B. Anicin, J. Opt. Soc. Am. **67**, 1307 (1977).
[6] L. Dutriaux, Ph. Balcou, F. Bretenaker, and A. Le Floch, Europhys. Lett. **33**, 359 (1996).
[7] H. Gilles, S. Girard, and J. Hamel, Opt. Lett. **27**, 1421 (2002).
[8] F. Bretenaker, A. Le Floch, and L. Dutriaux, Phys. Rev. Lett. **68**, 931 (1992).
[9] A. K. Ghatak, M. R. Shenoy, I. C. Goyal, and K. Thyagarajan, Opt. Lett. **56**, 313 (1986).
[10] A. M. Steinberg and R. Y. Chiao, Phys. Rev. A **49**, 3283 (1994).
[11] Ph. Balcou and L. Dutriaux, Phys. Rev. Lett. **78**, 851 (1997).
[12] A. Haibel, G. Nimtz, and A. A. Stahlhofen, Phys. Rev. E **63**, 047601 (2001).
[13] T. Tamir and H. L. Bertoni, J. Opt. Soc. Am. **61**, 1397 (1971).
[14] L. A. A. Read, Man Wong, and G. E. Reesor, J. Opt. Soc. Am. **68**, 319 (1978).
[15] C. W. Hsue and T. Tamir, J. Opt. Soc. Am. A **2**, 978 (1985).
[16] R. P. Riesz and R. Simon, J. Opt. Soc. Am. A **2**, 1809 (1985).
[17] M. A. Porras, Opt. Commun. **135**, 369 (1997).
[18] J. Broe and O. Keller, J. Opt. Soc. Am. A **19**, 1212 (2002).
[19] C.-F. Li, Phys. Rev. Lett. **91**, 133903 (2003).
[20] R. Briers, O. Leroy, and G. Shkerdin, J. Acoust. Soc. Am. **108**, 1622 (2000).
[21] O. Emile, T. Galstyan, A. Le Floch, and F. Bretenaker, Phys. Rev. Lett. **75**, 1511 (1995).
[22] B. M. Jost, A.-A. R. Al-Rashed, and B. E. A. Saleh, Phys. Rev. Lett. **81**, 2233 (1998).
[23] N. J. Harrick, Phys. Rev. Lett. **4**, 224 (1960).
[24] J. E. Midwinter and F. Zernike, Appl. Phys. Lett. **16**, 198 (1970).
[25] O. C. de Beauregard, C. Imbert, and Y. Levy, Phys. Rev. D **15**, 3553 (1977).
[26] V. Shah and T. Tamir, J. Opt. Soc. Am. **73**, 37 (1983).
[27] P. Mazur and B. Djafari-Rouhani, Phys. Rev. B **30**, 6759 (1984).
[28] W. J. Wild and C. L. Giles, Phys. Rev. A **25**, 2099 (1982).
[29] H. M. Lai and S. W. Chan, Opt. Lett. **27**, 680 (2002).
[30] J. L. Birman, D. N. Pattanayak, and A. Puri, Phys. Rev. Lett. **50**, 1664 (1983).
[31] E. Pfleghaar, A. Marseille, and A. Weis, Phys. Rev. Lett. **70**, 2281 (1993).
[32] R. Kaiser, Y. Levy, J. Fleming, S. Muniz, and V. S. Bagnato, Pure Appl. Opt. **5**, 891 (1996).