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Suppression of the pulsed regimes appearing in free-electron lasers using feedback control of an unstable stationary state

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We show that the pulsed regimes observed in free-electron lasers (FELs) can be suppressed using feedback control. By applying tiny parameter perturbations, the feedback allows to keep the systems onto a stationary state that is naturally existing in phase space, but is usually inaccessible because of its unstable nature. We test this method numerically on a master equation derived from the classical iterative model. Then we present the experimental results obtained on the super-ACO FEL. This method is in principle directly applicable to the other free-electron lasers, whose instabilities have a dynamical (deterministic) origin.

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Emission of coherent light using stimulated emission by the electron bunches of an accelerator provides an efficient way to produce tunable wavelengths (from far infrared to x rays) in the so-called free-electron lasers (FELs). The laser emission occurs as a train of picosecond/femtosecond pulses (the *micropulses*) that follows the train of electron bunches.

However in FEL oscillators (with an optical cavity as in conventional lasers), instabilities can occur: The emitted train of desired micropulses has then an envelope presenting slow full-scale fluctuations (typically in the subkilohertz range) that can be problematic for practical use. It has been shown that these nonstationary regimes have a *deterministic dynamical origin*, which leads typically to unwanted limit cycles, either with linear accelerators (LINACs) [1,2] or storage rings [3–6]. A typical example is represented in Fig. 1.

Proving that these regimes have a deterministic origin (historically in storage ring FELs [7], and also in LINACs FELs [1]) revealed a fundamental limit: Even perfect suppression of all noise sources in the accelerator and in the laser cavity would not suffice to suppress the pulsed regimes. This constraint leads to consequences on the optimization of FEL operation, which can be severe (Fig. 2).

However we will see that the deterministic nature of the instability simultaneously opens a powerful way for stabilizing FELs. Indeed, in such a dynamical system presenting a pulsed behavior, there often exists one or several stationary states that are usually not observed because they are unstable. The simple existence of one such state theoretically allows us to control the system. It is indeed almost always possible to stabilize one of these stationary states, using a suitable feedback loop between a measured value and a parameter [10,11].

In this paper, we demonstrate the possibility of controlling FEL oscillators using these concepts on the example of a storage-ring free-electron laser (SRFEL). We prove that stabilization is possible using electronic feedback, taking advantage of the existence of an unstable stationary state. This principle is not restricted to storage ring FELs and is applicable to any other FEL subjected to instabilities with deterministic origins.

Let us emphasize that the fact that the target state *already exists* in the uncontrolled system is of fundamental importance here. This implies that the needed modifications of the control parameter by the feedback are theoretically very small (they would be arbitrary small in the absence of noise [11,12]). This is of paramount importance here, where huge parameter modifications of an electron accelerator would be a nontrivial task.

In the following, we demonstrate the principle of the control on a storage ring free-electron laser. First, in a preliminary step, we will transform the map model [13] of SRFELs into a partial derivative equation that will be our reference equation for testing the control scheme. Then we describe the control scheme and demonstrate numerically its efficiency. Finally, we present the experimental results on the stabilization of the super-ACO FEL.

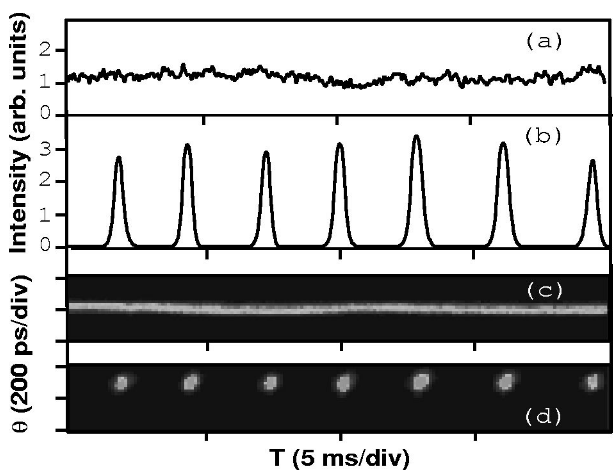

FIG. 1. Typical instabilities observed of a FEL oscillator (in the UV FEL of super-ACO). (a) and (b) represent the evolutions of the picosecond train envelope, respectively, in a stable and an unstable situation. (c) and (d) observation of the associated evolution of the picosecond pulses using a double sweep streak camera. In these images, successive vertical cuts can be viewed schematically [8] as the picosecond pulse profiles (vs the fast time θ) at the successive round trips in the cavity (associated with the slow time T).

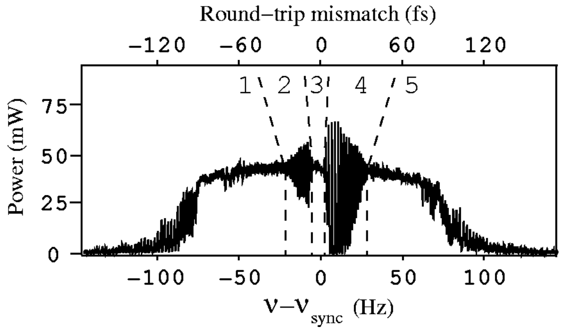

FIG. 2. Output power vs the mismatch between the photon round-trip time and the electron passage time, for the FEL of super-ACO. This mismatch is controlled by the ring RF frequency ν ($\nu_{\rm sync}\approx$100 MHz corresponds to perfect synchronism). The best micropulse properties (duration and time-bandwidth product) are obtained in zone 3, i.e., precisely in the small domain between the two unstable regions. This behavior is observed in the other FELs [3–5]. In the recent FEL of Elettra [9], zone 3 is even not systematically observed. The signals of Fig. 1 correspond to zones 3 and 4.

The dynamical instabilities reported in the SRFEL are well reproduced by an iterative model that involves the longitudinal profiles of the laser intensity $y_n(\tau)$ and of the gain provided by the electron bunch $G_n(\tau)$, at each cavity round trip n [13]:

$$y_{n+1}(\tau)=(1-\epsilon)[1+\epsilon G_n(\tau+\delta\tau_n)]y_n(\tau)+\epsilon\eta G_n(\tau+\delta\tau_n), \tag{1a}$$

with

$$G_n(\tau)=G_i\frac{\sigma_0}{\sigma_n}\exp\left(-\frac{\sigma_n^2-\sigma_0^2}{2\sigma_0^2}\right)\exp\left(-\frac{\tau^2}{2\sigma_\tau^2}\right) \tag{1b}$$

and

$$\sigma_{n+1}^2=\sigma_n^2+\frac{2\tau_R}{\tau_s}[\sigma_0^2-\sigma_n^2+(\sigma_e^2-\sigma_0^2)I_n]. \tag{1c}$$

In these equations $\delta\tau_n$ accounts for the delay between the electron bunch and the laser pulse at each round trip:

$$\delta\tau_n=\delta\tau_{n-1}+\tau_R^2\Delta\omega_n/2\pi, \tag{2}$$

with $\Delta\omega_n$ the instantaneous difference between the electron bunch passage frequency and the laser round-trip frequency, and τ_R the cavity round-trip time. τ_s is the synchrotron damping time. G_i is the single pass gain at perfect tuning in units of the round-trip cavity losses ϵ. $\epsilon\eta G_n$ is the amount of spontaneous emission per pass injected in the optical cavity. σ_τ is the temporal width of the electron bunch. σ_n is the width of the electron energy distribution at round trip n. σ_0 and σ_e are, respectively, the energy distribution widths without laser, and at equilibrium. σ_e is linked to the other parameters by the relation $G_i(\sigma_0/\sigma_e)\exp[-(\sigma_e^2-\sigma_0^2)/2\sigma_0^2]=1$. I_n is the micropulse energy: $I_n=\int_{-\tau_R/2}^{+\tau_R/2}y_n(\tau)d\tau$.

The study will be simplified by rewriting (1) in the form of a partial differential equation, as is usually done in the case of pulsed (mode-locked) lasers [14,15]. This is possible because the solution for $y_n(\tau)$ is typically a pulse whose shape varies slowly with the round-trip number n, thanks to the small values of the loss ϵ, gain ϵG_i, and detuning $\Delta\omega_n$. As in Ref. [15], we replace the discrete time n by a continuous time T that we express in units of the cavity photon lifetime: $T=\epsilon n=\epsilon t/\tau_R$, where t is the time. We also define the new dimensionless fast time synchronous with the electron bunch passage $\theta=(\tau+\delta\tau_n)/\tau_R$ [16]. We then perform the substitutions:

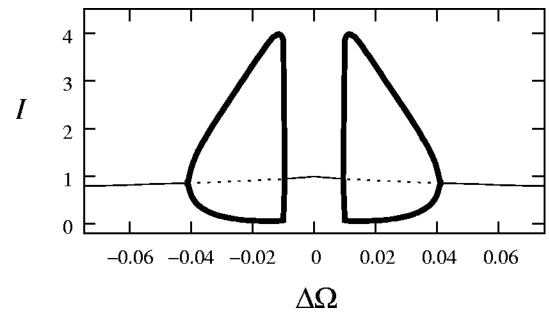

FIG. 3. Bifurcation diagram of the FEL model (4). The thin lines represent stationary states (solid and dashed lines are associated with stable and unstable states, respectively), the thick solid line represents the periodic regime. For this latter, the maxima and minima of the power $I(t)$ is represented. The key point is the existence of the unstable steady state in the parameter region where the regime is periodic. This potentially allows to stabilize the laser with arbitrary small perturbations of a control parameter.

$$\Delta\omega_n\rightarrow 2\pi\epsilon\Delta\Omega(T)/\tau_R, \tag{3a}$$

$$G_n(\tau)\rightarrow G(\theta,T), \tag{3b}$$

$$y_n(\tau)=y_n(\tau_R\theta-\delta\tau_n)\rightarrow Y(\theta,T), \tag{3c}$$

$$I_n\rightarrow I(T). \tag{3d}$$

Noting that we have the chain rule $y_{n+1}(\tau)=Y(\theta,T)+\epsilon\partial_T Y(\theta,T)-\epsilon\Delta\Omega\partial_\theta Y(\theta,T)+O(\epsilon^2)$, we obtain at first order in ϵ the following master equation that will be our reference model in the following:

$$\partial_T Y-\Delta\Omega\partial_\theta Y=-Y+G[Y+\eta], \tag{4a}$$

$$\frac{d}{dT}\sigma^2=\gamma[\sigma_0^2-\sigma^2+(\sigma_e^2-\sigma_0^2)I], \tag{4b}$$

with $\gamma=(2\tau_R/\epsilon\tau_s)(\gamma\ll 1)$, and $G(\theta,T)$ being defined by Eq. (1b) where the discrete index n is replaced by the continuous time T and τ by θ. In this form, our laser appear as a spatio-temporal system for which the relevant "space" and time are, respectively, θ and T, and the effect of detuning appears as an advection term $\Delta\Omega\partial_\theta y$. This form presents the advantage of allowing faster numerical integration, in particular, in the cases of large ratios between the synchrotron damping time and the photon lifetime.

For controlling the laser, the first step consists of examining the bifurcation diagram of Eq. (4). A diagram corresponding to typical parameter values of the super-ACO FEL is presented in Fig. 3. As experimentally, a stable stationary state exists in the vicinity of perfect tuning ($\Delta\Omega$ near zero),

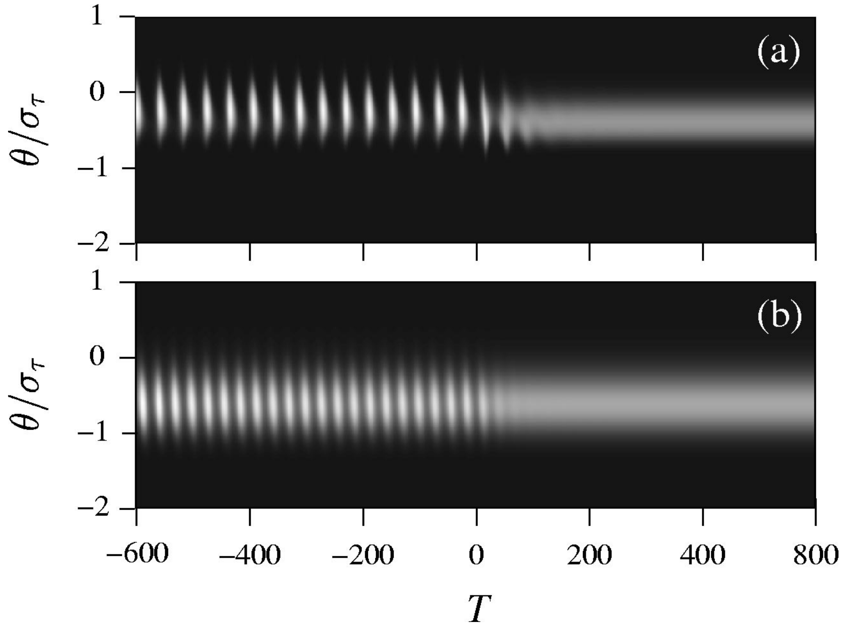

FIG. 4. Stabilization of the unstable steady state in the model (4) and (5). $Y(\theta,T)$ are represented as a gray scale, white being associated to largest values. As in the experimental streak recordings (Fig. 1), the figures can be viewed as the succession of the pulse profiles (vertical cuts) in the cavity at each round trip (horizontal coordinate). The feedback is off ($\beta=0$) at $T<0$ and on ($\beta=0.1$) at $T\geqslant 0$. Parameters are $G_i=5$, $\sigma_e^2=3.09\sigma_0^2$, $\sigma_\tau=1$ (by choice of the fast time unit), $\gamma=0.0056$, $\eta=10^{-3}$, $\gamma_{\Delta\Omega}=0.1$. (a) $\Delta\Omega_0=0.01$, (b) $\Delta\Omega_0=0.04$. This corresponds to the physical values: $\epsilon=0.5\%$, $\tau_R=120$ ns, $\tau_s=8.5$ ms.

and for the case of large detuning (with far less interesting pulse properties). Between these regions, the dynamics present the self-pulsing behavior. However, as a key point, the associated limit cycle coexists together with an unstable stationary state. This implies that it is in principle possible to control the laser by changing the stability of the latter.

To achieve stabilization, the feedback can be *a priori* applied on several parameters. We concentrate here on parameters of the RF cavities, which present the advantage of not requiring the introduction of intracavity elements. More precisely, we test a feedback on the RF frequency, a parameter that can be varied on super-ACO with response time scales of the order of microseconds. Guided by previous work on laser control [17,18], we test a feedback proportional to the derivative of the intensity, measured with a detector that has a bandpass smaller than the pulse round-trip frequency (in the MHz range), and much larger than the macropulse frequency (typically hundreds of Hertz). The detuning $\Delta\Omega$ in Eq. (4) becomes a dynamical variable:

$$\frac{d\Delta\Omega}{dt}=\gamma_{\Delta\Omega}[\Delta\Omega_0-\Delta\Omega+\beta I_T], \tag{5}$$

with $\Delta\Omega_0$ the prescribed detuning and $\gamma_{\Delta\Omega}$ the response time of the electron bunch revolution frequency to a variation of the RF frequency.

We have tested this feedback using various parameters that correspond to present SRFELs, and found each time a range for the feedback gain β allowing stabilization of the steady state. Examples of transients leading to control are presented in Fig. 4. In addition to the feasibility test, this calculation reveals that the pulse position is in general notably different in the periodic regime, and in the unstable stationary state. Moreover, we observe that this effect decreases at high values of $|\Delta\Omega|$.

We have tested experimentally this scheme on the super-ACO FEL [13]. The ring operated with two positron bunches at a current in the 30–70 mA range, and with two phase-locked rf cavities (at 100 and 500 MHz). The laser was tuned near 350 nm, and in these conditions the optical mode is TEM_{00}. The feedback input variable $I(T)$ is provided by a photomultiplier device whose bandpass exceeds the macropulse repetition frequency (in the 300–500 Hz range), but does not allow to resolve the repetition of the picosecond pulses (in the MHz range). The signal is phase shifted using an analog derivator, and amplified. The rf frequency ν applied to the storage ring is the sum of a prescribed value ν_0 (near 100 MHz) and the output of the feedback loop:

$$\nu=\nu_0+\beta_{\rm expt}\frac{dP}{dt}, \tag{6}$$

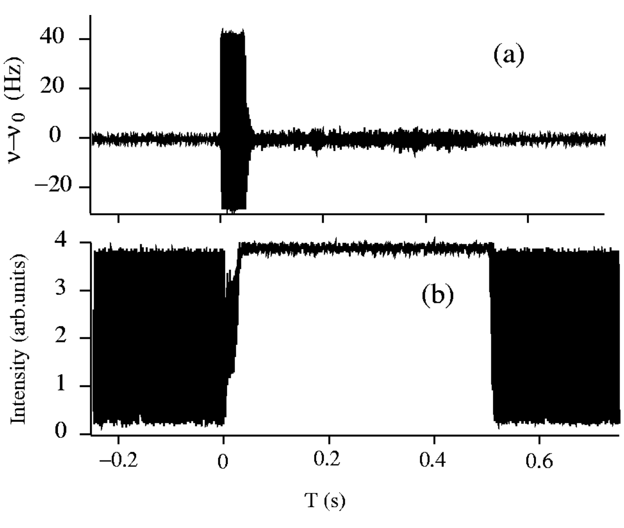

FIG. 5. Typical transient observed experimentally when the feedback is switched on (at $T=0$) and then off (at $T=0.5$ s). (a) Control signal applied on the RF frequency, (b) output power (the train of picosecond pulses is not resolved by the detector). The black zones are oscillations (the macropulses) at frequencies of the order of 300–500 Hz, as in Fig. 1(b). Note that once the stabilization has occured, the feedback corrections tend to zero.

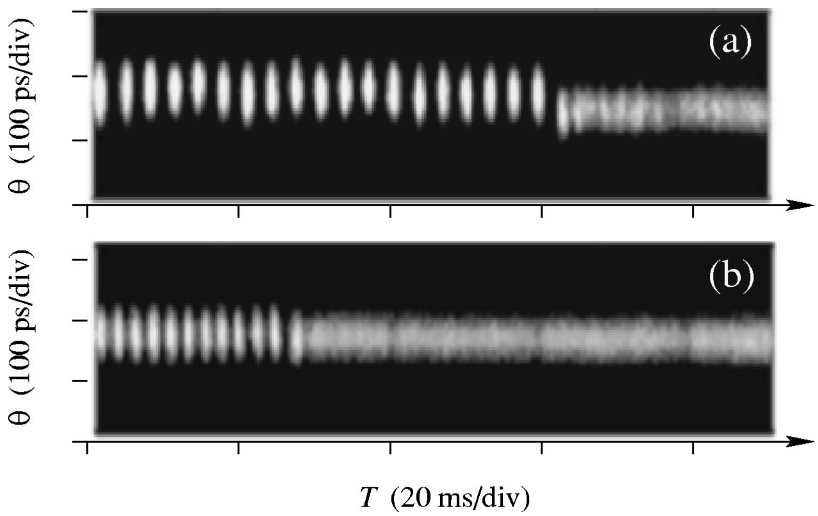

FIG. 6. Picosecond pulse train evolution monitored with a streak camera, during the transient following the application of the control. (a) and (b) correspond to low and high detunings, respectively.

where P is the output power of the laser. In these conditions, we have been able to control the laser, by choosing empirically the adequate feedback gain (β_{expt} is of the order of $1-5\times10^{-4}$ mW^{-1}). Figure 5 represents typical transients observed when the control is switched on and off, with the FEL operating in zone 4. For observing the micropulse evolution during the transient, we have also performed streak camera recordings (Fig. 6). It appears that the difference of the pulse position with and without feedback are different and depend on the rf detuning, in a similar way as in the model (4). We have observed shifts ranging from 0 to 90 ps.

Finally, let us note that the principle of the control can be realized with other feedback schemes that are worth to be tested (e.g., with different frequency responses or with action on other control parameters). For instance, we have been able to stabilize the laser by applying the feedback loop on the RF amplitude of the 500 MHz cavity. Further systematic tests could contribute to improve the feedback scheme and reduce the eventual noise.

In conclusion, we have demonstrated that the deterministic nature of the dynamics of a range of FELs (FEL oscillators) allows one to obtain suppression of their instabilities, using a feedback strategy. The principle is to stabilize a stationary state that naturally exists (in an unstable form), using feedback control. Let us emphasize that the needed modifications of the control parameters are *small* (they would tend to zero in the absence of noise), because we are stabilizing a stationary state that already exists in the uncontrolled system (in an unstable form). In addition to the interest for FEL applications (e.g., in pump-probe experiments), we believe that the access to the system steady state can potentially allow testing (and possibly refining) further the existing FEL models. This method can be extended *a priori* directly not only to other storage ring free-electron lasers, but also to LINAC-based FEL oscillators.

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