Importance of beam-beam tune spread to collective beam-beam instability in hadron colliders

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In hadron colliders, electron-beam compensation of beam-beam tune spread has been explored for a reduction of beam-beam effects. In this paper, effects of the tune-spread compensation on beam-beam instabilities were studied with a self-consistent beam-beam simulation in model lattices of Tevatron and Large Hodron Collider. It was found that the reduction of the tune spread with the electron-beam compensation could induce a coherent beam-beam instability. The merit of the compensation with different degrees of tune-spread reduction was evaluated based on beam-size growth. When two beams have a same betatron tune, the compensation could do more harm than good to the beams when only beam-beam effects are considered. If a tune split between two beams is large enough, the compensation with a small reduction of the tune spread could benefit beams as Landau damping suppresses the coherent beam-beam instability. The result indicates that nonlinear (nonintegrable) beam-beam effects could dominate beam dynamics and a reduction of beam-beam tune spread by introducing additional beam-beam interactions and reducing Landau damping may not improve the stability of beams.

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I. INTRODUCTION

In storage-ring colliders, beam-beam interactions are one of the major factors that reduce beam lifetime and limit luminosity. Beam-dynamics experiments in SPS (Super Proton Synchrotron, CERN, Geneva) [1] and Tevatron (Fermilab, Chicago) [2,3] showed that in the presence of high-order resonances of beam-beam interactions or nonlinear fields in lattice, a large beam tune spread due to head-on beam-beam interactions could result in a significant emittance growth and beam-particle loss. For LHC (Large Hadron Collider) being constructed at CERN and Tevatron, efforts are being made to reduce beam-beam effects in order to achieve or exceed the designed luminosity. Electron-beam compensation is one of the schemes being developed for Tevatron RUN II to reduce bunch-to-bunch tune variation due to PACMAN effect of beam-beam interactions [4–6]. It has been explored that the electron-beam compensation scheme could also be used for a reduction of beam-beam tune spread (amplitude dependence of tunes) due to head-on beam-beam interactions and, therefore, to compensate nonlinear beam-beam effects [7]. In this scheme of the nonlinear compensation of beam-beam tune spread with electron beams, high-intensity low-energy electron (e) beams will collide with antiproton ($\overline{p}$) beam at certain locations in the ring other than nominal interaction points (IP) for proton (p) and $\overline{p}$ beam. The tune spread of the $\overline{p}$ beam due to the collisions between the p and $\overline{p}$ beam will then be compensated by the collisions between the e and $\overline{p}$ beams. Previous studies [7] based on non-self-consistent treatments (strong-weak model) of beam-beam interactions with either single-resonance analysis or numerical simulation have shown that the use of electron beams can effectively reduce beam-beam tune spread and possibly improve beam dynamics of the $\overline{p}$ beam. The effect of nonlinear perturbations due to the original beam-beam interactions between the p and $\overline{p}$ beam and the additional beam-beam interactions between the $\overline{p}$ and e beams have however not been carefully studied.

To have a better understanding of the electron-beam compensation of beam-beam tune spread, let us take a glance at formal Hamiltonian for the transverse motion of the $\overline{p}$ beam. Neglecting nonlinearities in lattice (nonlinear magnetic field errors), the Hamiltonian can be written as

$$\begin{aligned} H &= \vec{\nu}\cdot\vec{I} + H_{\overline{p}p}(\vec{I},\vec{\phi},t) + H_{\overline{p}e}(\vec{I},\vec{\phi},t) \\ &= \vec{\nu}\cdot\vec{I} + t\langle H_{\overline{p}p}\rangle + \langle H_{\overline{p}e}\rangle + \{H_{\overline{p}p}\} + \{H_{\overline{p}e}\}, \end{aligned} \quad (1)$$

where $\vec{\nu}$ is the betatron tune and $(\vec{I},\vec{\phi})$ the action-angle variables for the transverse motion of the $\overline{p}$ beam. $H_{\overline{p}p}$ is the perturbative Hamiltonian for beam-beam interactions between the $\overline{p}$ and p beam at the nominal IPs and $H_{\overline{p}e}$ the perturbative Hamiltonian for beam-beam interactions between the $\overline{p}$ and e beam for the electron-beam compensation. In the second line of Eq. (1), $\langle H_{\overline{p}p}\rangle$ and $\langle H_{\overline{p}e}\rangle$ are the average of $H_{\overline{p}p}$ and $H_{\overline{p}e}$ over $\vec{\phi}$ and t, respectively, and are the first-order phase-independent (integrable) perturbations of beam-beam interactions that lead to the lowest-order beam-beam tune spread. Because of the opposite charge of p and e, $\langle H_{\overline{p}p}\rangle$ and $\langle H_{\overline{p}e}\rangle$ have an opposite sign and cancel each other if the e beam has the same intensity and charge distribution as that of the p beam. In the use of the electron-beam compensation, the degree of the cancellation between $\langle H_{\overline{p}p}\rangle$ and $\langle H_{\overline{p}e}\rangle$ can be varied by adjusting the intensity and charge distribution of the e beam. $\{H_{\overline{p}p}\} = H_{\overline{p}p} - \langle H_{\overline{p}p}\rangle$ and $\langle\{H_{\overline{p}e}\rangle\} = H_{\overline{p}e} - \langle H_{\overline{p}e}\rangle$ are the nonintegrable (nonlinear) phase-dependent (oscillating) parts of the perturbative Hamiltonians that could lead to nonlinear resonance effects and beam-beam instability [8]. Note that integrable ($\langle H\rangle$) and nonintegrable ($\{H\}$) perturbation are usually referred to linear and nonlinear perturbation, respectively, in nonlinear dynamics although both $\langle H\rangle$ and $\{H\}$ are nonlinear functions

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of phase-space variables. In this paper, we also follow this terminology. For a weakly perturbed (near-integrable) Hamiltonian system, it is commonly believed even though it is not always true that the phase-independent perturbations are the dominant effect when a system is away from major resonances. The electron-beam compensation of beam-beam tune spread is therefore based on the assumption that $\langle H_{\overline{p}p}\rangle$ is the dominant term of $H_{\overline{p}p}$ and a smaller beam-beam tune spread would always improve beam performance because of less crossings of resonances. When the nonlinear phase-dependent perturbations of beam-beam interactions are important, however, the beam-beam interactions could lead to onset of chaotic coherent beam-beam instability that could result in a spontaneous chaotic coherent beam oscillation and an enhanced beam-size growth [8]. In that situation, having a large tune spread could benefit the beam stability since the existence of the tune spread is a necessary condition for Landau damping that could suppress the coherent beam-beam instability [9–11]. Moreover, the existence of a large tune spread reduces the possibility of trapping particles inside bad resonances. The compensation of beam-beam tune spread may therefore damage the beam stability by reducing the Landau damping when coherent beam-beam effects are dominant. Moreover, the compensation of the beam-beam tune spread with electron beams unavoidably introduces an additional nonlinear phase-dependent perturbation of $\{H_{\overline{p}e}\}$ to the $\overline{p}$ beam that could further enhance the coherent beam-beam effect. The question is then how important the nonlinear phase-dependent perturbations $\{H_{\overline{p}p}\}$ and $\{H_{\overline{p}e}\}$ are when the beam tune spread is reduced as desired by using the electron-beam compensation. On the other hand, in pursuing a better understanding of beam-beam effects in storage-ring colliders, the electron-beam compensation of beam-beam tune spread is an effective means, not only for simulation study but also for future beam-dynamics experiments, to probe the importance of the phase-independent and phase-dependent perturbations of beam-beam interactions.

It should be noted that the coherent (collective) beam-beam instability could occur in both cases of strong-strong (symmetrical or nearly symmetrical) and strong-weak (very un-symmetrical) beam-beam interactions when nonlinear beam-beam perturbations dominate beam dynamics. Recent simulation studies and beam-dynamics experiments of beam-beam effects in HERA (Hadron Electron Ring Accelerator at DESY, Hamburg, Germany) showed that the collective beam-beam instability could occur in the HERA Upgrade. It was predicated by a self-consistent beam-beam simulation [12] and observed in beam-dynamics experiments [13] that when the beam-beam parameter of the positron beam exceeds a threshold that corresponds to an overlap of the positron beam with the fourth-order resonance, the onset of the collective beam-beam instability results in a significant emittance growth of the proton beam. In the HERA Upgrade, the beam-beam parameter of the positron beam is over 20 and 100 times larger than that of the proton beam in the horizontal and vertical directions, respectively, and the two rings have a very different working point. Traditionally, the beam-beam effect in such a situation is considered as a typical strong-weak or very unsymmetrical case. For the strong-weak case of beam-beam interactions, it is commonly believed that the collective beam-beam effect is not important. The simulation study together with the HERA experiments showed that in the nonlinear regime of beam-beam interactions the traditional boundary between the strong-strong and strong-weak case of beam-beam interactions is no longer valid and beam-beam effects have to be studied self-consistently .

To understand collective beam-beam instabilities, many efforts have been made to formulate the beam-beam problem self-consistently by using methods of approximations or perturbation expansions. For lepton storage-ring colliders, two different types of theoretical models have been shown to be successful [14–16]. In the first type of models nonlinear maps for the moments of beams are obtained by a truncation of a moment expansion for beam-particle distributions, while in the second the instabilities of equilibrium distributions of beams are analyzed with the linearized Vlasov equation. In both models, steady states of coherent beam oscillations were obtained and the instability of the coherent oscillations was studied. For high-energy electron beams, because of the radiation damping, the time scale for a beam to reach the equilibrium distribution is much less than the storage time. Consequently, the study of beam dynamics can be focused on the behavior of the distribution near its steady states. Moreover, a fast damping of high-order fluctuations permits the truncation of the moment expansion at fairly low orders. For lepton storage-ring colliders, therefore, methods of perturbation are usually effective in the study of beam-beam effects. For high-energy hadron beams, on the other hand, the damping time scale is usually larger than the storage time so that motions of beam particles are determined by Hamiltonian dynamics. In the presence of nonlinear perturbations due to either beam-beam interactions or nonlinear field errors in lattice, the particle distribution may not reach any steady state within a fraction of the storage time. In the near-linear (near-integrable) regime of beam-beam interactions in which the phase-dependent perturbations of beam-beam interactions are not important, the beam distributions change very little due to beam-beam interactions in the time scale of interest. In this case, quasistationary states of the Vlasov equation could be considered and methods of perturbation could be employed to study beam-beam effects. In the nonlinear (non-integrable) regime of beam-beam interactions in which the phase-dependent perturbations of beam-beam interactions are dominant, no stationary distribution for the nonlinear Vlasov equation that is relevant to beams in accelerators has been found theoretically or observed experimentally. Note that the Gaussian distribution is not or not even close to an equilibrium distribution when the beam-beam parameter is large. Computer simulations have shown that the beam distribution could deviate significantly from its initial Gaussian distribution due to the formation of beam halo [8,12]. Consequently, the truncation of the moment expansions or the linear stability analysis of equilibrium distributions of the nonlinear Vlasov equation is no longer valid in this case. Moreover, it has been well recognized in the field of nonlinear dynamics that in the nonintegrable regime of a Hamiltonian system, the use of perturbation expansions such as various canonical perturbation methods usually distorts the dynamics of the system and may result in incorrect conclusions.

The difference in characteristics of particle distributions of high-energy lepton and hadron beams in storage-ring colliders can also be understood in the aspect of statistical physics [17,18]. In statistical physics, a dynamical system could attain a thermodynamical equilibrium through interactions with its environment (heatbath) or through internal stochastic processes as either external or internal stochastic interactions introduce fluctuations and dissipation into motions of particles. In the case of high-energy lepton beams, leptons interact with the synchrotron radiation, which results in quantum fluctuations and synchrotron damping of leptons. The phase-space distribution of leptons can therefore attain equilibrium in a time scale that is much smaller than the storage time. In the case of high-energy hadron beams, however, the interaction between hadrons and the synchrotron radiation is much weaker and other external stochastic interactions are usually negligible in the time scale of interest here. A high-energy hadron beam is more like an isolated thermodynamical system and cannot attain equilibrium in the time scale of interest through interactions with a heat bath. The intrabeam scattering, on the other hand, could redistribute energy among different degrees of freedom and exchange energy among ''hot'' particles in beam tails and ''cold'' particles in beam core. Due to a very low density of a typical high-energy hadron beam (several orders of magnitude smaller) as compared with a gas under normal conditions or fusion plasma, however, the relaxation time due to the intrabeam scattering is usually even longer than that due to the synchrotron radiation. Chaotic dynamics of beam particles could also lead to the ergodicity of particle motions and result in a thermodynamical equilibrium. It could, however, only happen when most beam particles are in fully developed chaotic regions [19], which never be a case in normal operation conditions of particle accelerators.

In order to understand the beam-beam effect of hadron beams, one has therefore to study transient states of the nonlinear Vlasov equation. For transient states in the nonlinear regime of beam-beam interactions, only validated method currently available for a theoretical understanding of beam-beam effects is self-consistent numerical simulation. To understand the importance of the phase-independent and phase-dependent perturbations of beam-beam interactions, we therefore conducted a self-consistent beam simulation with both lattice models of Tevatron and LHC. In this paper the emphasis is given to the results of Tevatron model, meanwhile, to make the study more general the results of LHC model is presented as well but with less detail. The paper is organized as follows. In Sec. II, the simulation models are briefly discussed. The simulation results are presented in Sec. III. Sec. IV contains a summary remark.

II. SIMULATION AND LATTICE MODELS

Two test lattices used in this study were Tevatron RUN IIB in Fermilab [20] and LHC in CERN [21]. Only linear lattices were used since multipole field errors in the lattice normally do not change the characteristic of the beam-beam instability [8]. In the case of the Tevatron, p and $\bar{p}$ beams are collided only at one interaction point D0 in the simulation. The value of the β function at the IP is $\beta^*=0.35$ m. The electron-beam compensation is located at F0 where β function is about 70 m in both horizontal and vertical planes. As the intensity of the p beam is much larger than that of the $\bar{p}$ beam, the $\bar{p}$ beam is perturbed more severely than the p beam due to the collision between the p and $\bar{p}$ beams. The electron-beam compensation of the beam-beam tune spread was thus only applied on the $\bar{p}$ beam as proposed for the Tevatron RUN IIB [4]. In the case of the LHC, two p beams are collided at two high-luminosity interaction points IP1 and IP5 ($\beta^*=0.5$ m) in the simulation. Since the p beams have equal intensity, the electron-beam compensation of the beam-beam tune spread was applied on both beams in this study, in which each p beam was collided with an e beam at either IP2 or IP8 where β function is about 250 m in both horizontal and vertical planes.

Our self-consistent (strong-strong) beam-beam simulation code has been fully tested and presented in detail in our previous paper [8]. The reliability of the code has also been verified by a comparison between the simulation and beam-dynamics experiments at the HERA recently [12]. In this code, each beam is represented by a large number of macroparticles with given initial distributions in transverse phase space. In this study, the initial phase-space distributions of two counter-rotating beams are chosen to be round Gaussian beams in the normalized transverse phase space with standard deviation σ_0 and truncated at $\pm 4\sigma_0$. $\sigma_0=\sigma^*/\sqrt{\beta^*}$, where σ^* is the nominal transverse beam size at IP. Without beam-beam interactions, the initial beam distribution used in the simulation matches exactly with the lattice. During the tracking, beam-beam kicks in four-dimensional transverse phase space are calculated at each IP by using the particle-in-cell method. This task consists of three major steps [8]: (a) The beam charge distributions at each crossing of IP are obtained by assigning macroparticles to the grid points of an uniform mesh in two-dimensional transverse configuration space for each beam using the four-point cloud-in-cell technique. (b) The beam-beam kicks are calculated at the grid points using the precalculated Green's functions for the beam-beam kicks. (c) The kicks are then interpolated to the position of every macroparticle. In order to ensure the convergence of the simulation parameters and to avoid any artificial result due to those numerical approximations, the size of mesh, the grid constant (the length between nearest neighboring grid points), and the number of macroparticles have to be carefully tested [8]. In this study, we found that a uniform mesh extending to $6\sigma_0$ in all directions of the normalized configuration space with a grid constant of $0.2\sigma_0$ is good enough. To have a reliable beam-beam simulation for hadron beams, on the other hand, the number of macroparticles has to be large enough, typically $>10^5$ [8]. In this study, we use 5×10^5 macroparticles for each beam. Tracking of particle motion has been done in four-dimensional transverse phase space without synchrotron oscillations and momentum deviations. The beam dynamics has been studied with up to 10^6-turns tracking that corresponds to about 21 and 90 sec run for the case of the Tevatron and the LHC, respectively.

For the simulation of the electron-beam compensation, we considered the idea case that the electron beam has the same type of charge distribution of the initial p beam. The distribution of the electron beam is therefore chosen to be the Gaussian distribution with a standard deviation that changes with the root-mean-square (rms) beam size of the p beam during the tracking. The momentum kick exerted by the electron beam was simply calculated with the standard formula for the beam-beam interaction of a Gaussian beam [22].

III. SIMULATION RESULTS

A. Model lattice for tevatron RUN IIB

To probe the importance of the beam-beam tune spread to beam-beam effects, the electron-beam compensation with different intensity of the e beam were studied, i.e., $\xi_{\bar{p}c} = -\lambda \xi_{\bar{p}p}$, where $\xi_{\bar{p}e}$ and $\xi_{\bar{p}p}$ are the beam-beam parameters of the $\bar{p}$ beam for $\bar{p}$-e and $\bar{p}$-p collisions, respectively, and $\lambda \leqslant 1$ represents the degree of the compensation. Two different working points were used in this study. $(\nu_x, \nu_y) = (20.582, 20.574)$ is the nominal betatron tunes for Tevatron and $(\nu_x, \nu_y) = (20.740, 20.730)$ is close to the fourth-order resonance. In addition, the situation in which the p and $\bar{p}$ beams have a slight tune split was also studied.

1. Symmetrical rings with nominal betatron tune

The study of the electron-beam compensation of tune spread at the nominal working point of Tevatron is to understand how the electron beam would affect dynamics of the $\bar{p}$ beam in a good working point that is far away from major resonances. In this case, both p and $\bar{p}$ beams have the same betatron tunes and the intensity of the $\bar{p}$ beam is one fifth of that of the p beam, i.e., $\xi_{\bar{p}p} = -0.01$ and $\xi_{p\bar{p}} = -0.002$, where $\xi_{p\bar{p}}$ is the beam-beam parameter of the p beam for the p-$\bar{p}$ collision. In the nominal condition of Tevatron, the ratio of emittance between the $\bar{p}$ and p beams is $\epsilon_{\bar{p}}/\epsilon_p = 3/4$ and that results in a mismatched (in beam size) collision at the IP. The beam experiments in HERA and SPS have shown that the beam-beam interaction due to a mismatched collisions gives rise to a shorter beam lifetime than that in the case of matched collisions [23,1]. In Fig. 1, the evolution of rms beam size of the $\bar{p}$ beam was calculated for both the cases of matched and mismatched collision of the p and $\bar{p}$ beams in Tevatron without the electron-beam compensation. It confirms that the beam-size growth rate in the mismatched case is larger than that in the matched case in Tevatron. In order to be close to the realistic Tevatron situation, however, the mismatched p-$\bar{p}$ collision with $\epsilon_{\bar{p}}/\epsilon_p = 3/4$ was used in the following study of the electron-beam compensation of beam-beam tune spread in Tevatron. Note that the rms beam size (σ) plotted in this paper is the average of horizontal and vertical rms beam sizes.

To exam the reduction of beam tune spread after the electron-beam compensation, the initial tune spread of the $\bar{p}$ beam was calculated with 420 test particles during the first 2000-turns tracking. In all cases we studied ($\lambda = 0.25$, 0.5,

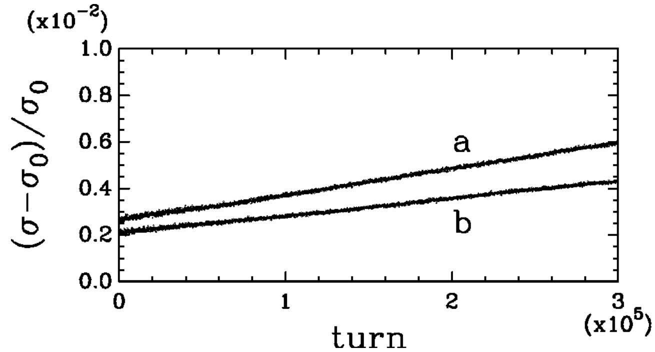

FIG. 1. Evolution of rms beam size of the $\bar{p}$ beam without the compensation of the beam-beam tune spread. $(\nu_x, \nu_y) = (20.582, 20.574)$, $\xi_{\bar{p}p} = -0.01$, and $\xi_{p\bar{p}} = -0.002$. σ is the average of the horizontal and vertical beam sizes and σ_0 the initial beam size. (a) $\epsilon_{\bar{p}} = 0.75\epsilon_p$ and (b) $\epsilon_{\bar{p}} = \epsilon_p$. $\epsilon_{\bar{p}}$ and ϵ_p are the emittance of the $\bar{p}$ and p beams, respectively.

and 1.0), the tune spread is shrunk as expected after the electron-beam compensation. Figure 2 is an example of the initial tune spread of the $\bar{p}$ beam without or with the electron-beam compensation ($\lambda = 1.0$). A small remaining tune spread of $\Delta\nu \sim 0.002$ after the electron-beam compensation [see Fig. 2(b)] is the high-order contributions from both p-$\bar{p}$ and e-$\bar{p}$ collisions that are, in general, not compensated each other and become observable when the lowest-order tune spread is eliminated. In Fig. 3, the evolution of rms beam size of the $\bar{p}$ beam was plotted for the cases of $\lambda = 0.0$, 0.25, and 1.0. Without the electron-beam compensation, the size of the $\bar{p}$ beam increases less than 0.6% in 3×10^5 turns (see curve a of Fig. 3). With the electron-beam compensation, however, the beam-size growth becomes more severe even though the tune spread is reduced. Moreover, the rate of the beam-size growth (the tangent of curves in Fig. 3) increases with the intensity of the e beam. Since the original tune spread of the $\bar{p}$ beam does not lead to crossings of any major resonance, the reduction of the tune spread with the electron-beam compensation does not improve the beam dynamics but introduce more nonlinearity [$\{H_{\bar{p}e}\}$ in Eq. (1)] to the beam that is responsible for the increased beam-size growth. Since there is no obvious single dominant resonance that is responsible to the beam-size growth, the enhanced beam-size growth after the use of the electron-beam compensation is not a single resonance effect. It should also be noted that the initial beam-size blowup (see Fig. 3) is due to the nonlinear beam filamentation in phase space resulting from the nonlinear beam-beam perturbation and is not an emittance smear due to a linear mismatch between the beam and the linear ring including the linear beam-beam tune shift. When the nonlinear beam-beam perturbations of $\{H_{\bar{p}e}\}$ and $\{H_{\bar{p}p}\}$ are important, the initial particle distributions that are Gaussian in the normalized phase space are far away from a constant of motion for the Hamiltonian with beam-beam interactions. That results in a very subtle, since the beam-beam perturbation is still weak relatively, but rapid change in the distributions as well as the beam-beam interactions within a very short period of time (the first 1000 turns). The beam sizes therefore increase quickly at the be-

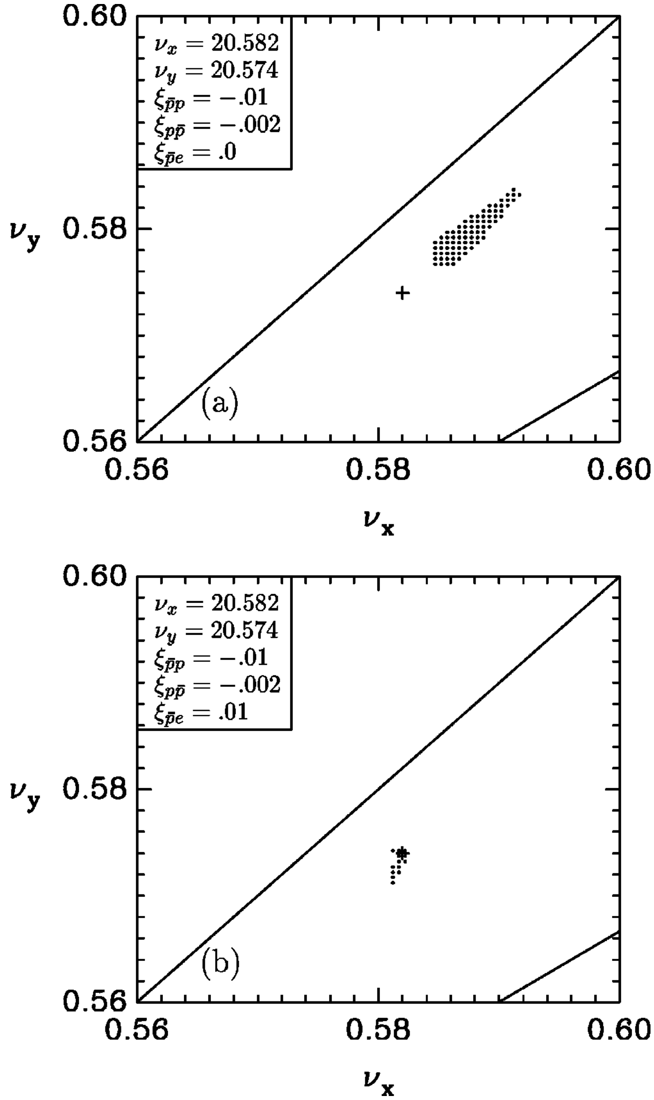

FIG. 2. Tune spread of the $\bar{p}$ beam during the first 2000 turns when $(\nu_x, \nu_y) = (20.582, 20.574)$, $\xi_{\bar{p}p} = -0.01$, and $\xi_{p\bar{p}} = -0.002$. (a) Without the tune-spread compensation and (b) with the compensation of $\lambda = 1.0$. Solid lines are the even-order resonances up to the tenth order and + indicates the lattice bare tune.

ginning. As such the "nonlinear mismatch" increases with the beam-beam perturbation in general, the initial beam-size blowup increases with the intensity of the e beam (see Fig. 3). A similar study was also conducted for the case of $\xi_{p\bar{p}} = -0.01$ and $\xi_{\bar{p}p} = -0.005$, i.e., the ratio of the intensity between the $\bar{p}$ and p beams is 1/2. The result obtained is consistent with that presented here.

In Ref. [7], a study based on a single-resonance analysis and simulation with a weak-strong (non-self-consistent) model of beam-beam interactions suggested that the electron-beam compensation of beam-beam tune spread with 50% strength ($\lambda = 0.5$) would improve the $\bar{p}$ beam. The discrepancy between the results here and that of Ref. [7] suggests that the non-self-consistent treatment of beam-beam interactions is not valid in the case when the nonlinear phase-dependent perturbations $\{H_{\bar{p}e}\}$ and $\{H_{\bar{p}p}\}$ are dominant and

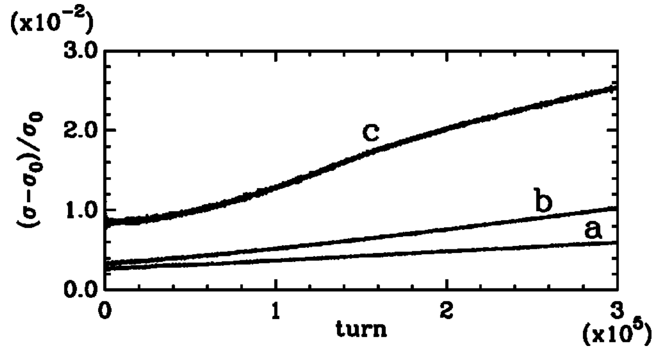

FIG. 3. Evolution of rms beam size of the $\bar{p}$ beam for the cases of Fig. 2. σ is the average of the horizontal and vertical beam sizes and σ_0 the initial beam size. (a) Without the tune-spread compensation, (b) with the compensation of $\lambda = 0.25$, and (c) with the compensation of $\lambda = 1.0$.

the collective beam-beam effect, in which particle distributions of both beams are involved with time, becomes important.

2. Unsymmetrical rings

In Tevatron, the p and $\bar{p}$ beams could have slightly different betatron tunes because they circulate along different closed orbits and thus experience slightly different magnetic fields. On the other hand, the Landau damping in an unsymmetrical system, due to unsymmetrical rings or/and unsymmetrical beams, could suppress the coherent beam-beam instability that could be the reason of the enhanced beam-size growth. Because of the differences in the emittance and intensity between the two colliding beams, the Landau damping should exist in Tevatron even with symmetrical rings. A small difference in the betatron tunes of the two beams could, however, further enhance the Landau damping. The simulation was therefore conducted also for the case of unsymmetrical rings. In order to be compared with the symmetrical case, the betatron tunes for the $\bar{p}$ beam were still kept at the nominal working point $(\nu_x, \nu_y) = (20.582, 20.574)$, while for the p beam $(\nu_x, \nu_y) = (20.582 + \delta\nu, 20.574 + \delta\nu)$, where $\delta\nu$ is the tune split between the two beams. The simulation studies in Refs. [24,25] have suggested that the Landau damping due to a tune split could have significant effect on the coherent beam-beam effects only when the tune split is close to or larger than the linear beam-beam tune shifts. Two cases of $\delta\nu = 0.001$ and 0.005 were therefore studied. When $\delta\nu = 0.001$, the tune split is smaller than the linear beam-beam tune shifts of both beams, while $\delta\nu = 0.005$ corresponds to the case of $\delta\nu = \xi_{p\bar{p}}$.

When $\delta\nu = 0.001$, the result is roughly the same as that in the symmetrical rings and the e beam does more harm than good to the $\bar{p}$ beam in all cases of λ. Figure 4 plots the beam-size growth of the $\bar{p}$ beam for $\delta\nu = 0.005$. Without the electron-beam compensation, the beam size increases about 3% in 10^6 turns. With the full-strength compensation ($\lambda = 1.0$), the collision between the e and $\bar{p}$ beam again deteriorates the performance of the $\bar{p}$ beam (curve c in Fig. 4). When $\lambda = 0.5$, on the other hand, the electron-beam compensation improves beam dynamics slightly. The increase of the

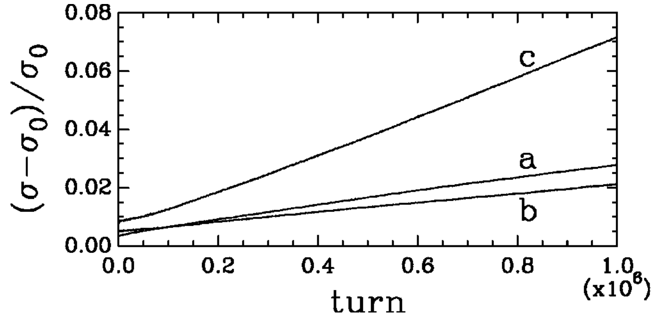

FIG. 4. Evolution of rms beam size of the $\bar{p}$ beam in unsymmetrical rings. The betatron tune for the $\bar{p}$ and p beams are $(\nu_x,\nu_y)=(20.582,20.574)$ and $(\nu_x,\nu_y)=(20.587,20.579)$, respectively. $\xi_{\bar{p}p}=-0.01$ and $\xi_{p\bar{p}}=-0.002$. σ is the average of the horizontal and vertical beam sizes and σ_0 the initial beam size. (a) Without the tune-spread compensation, (b) with the compensation of $\lambda=0.5$, and (c) with the compensation of $\lambda=1.0$.

$\bar{p}$ beams size during 10^6 turns reduces to about 2.5% (curve b in Fig. 4). Moreover, the rate of the beam-size growth is significantly reduced after the compensation. Note that the difference between the cases of symmetrical and unsymmetrical rings is only a small change in betatron tunes of the p beam that can only affect the dynamics of the $\bar{p}$ beam through collective beam-beam effects, i.e., changes of particle distributions due to the nonlinear phase-dependent beam-beam perturbation of $\{H_{\bar{p}e}\}$ and $\{H_{\bar{p}p}\}$. The change of the characteristic of the beam-size growth at $\delta\nu\sim0.005$ indicates that the enhanced beam-size growth after the electron-beam compensation is a collective (coherent) beam-beam effect. In this system, there are two competing forces affecting the coherent beam-beam effect. The nonlinear phase-dependent beam-beam perturbation is the source of coherent beam-beam instabilities, while the Landau damping tends to stabilize the beams. A recent study on the beam-beam effects in the HERA Upgrade has shown that when the nonlinear beam-beam perturbation is dominant the coherent beam-beam instability could occur in a very unsymmetrical system in which the Landau damping is supposed to be significant [12]. On the other hand, the existence of the tune spread is a necessary condition for the Landau damping. With the 100% compensation of the tune spread, the lowest-order tune spread is eliminated and therefore the system becomes very unstable because of a much weakened Landau damping. In the case of $\delta\nu=0.001$, $\delta\nu<\xi_{p\bar{p}}<\xi_{\bar{p}p}$. The increase of the Landau damping due to such a small tune split has little effect on the coherent beam-beam effect and the nonlinear beam-beam perturbations of $\{H_{\bar{p}e}\}$ and $\{H_{\bar{p}p}\}$ are dominant. When $\delta\nu\geqslant\xi_{p\bar{p}}$, on the other hand, a stronger Landau damping due to a larger tune split could suppress the coherent beam-beam effect if the perturbation of $\{H_{\bar{p}e}\}$ is not too strong. A less-than-100% compensation of the tune spread with the e beam could therefore benefit the $\bar{p}$ beams if the p and $\bar{p}$ beams have an appropriate tune split and if the nonlinear beam-beam perturbations of $\{H_{\bar{p}p}\}$ and $\{H_{\bar{p}e}\}$ are not dominant.

3. Effect of major resonances

If the original tune spread of the $\bar{p}$ beam leads to crossings of major resonances due to either a bad working point

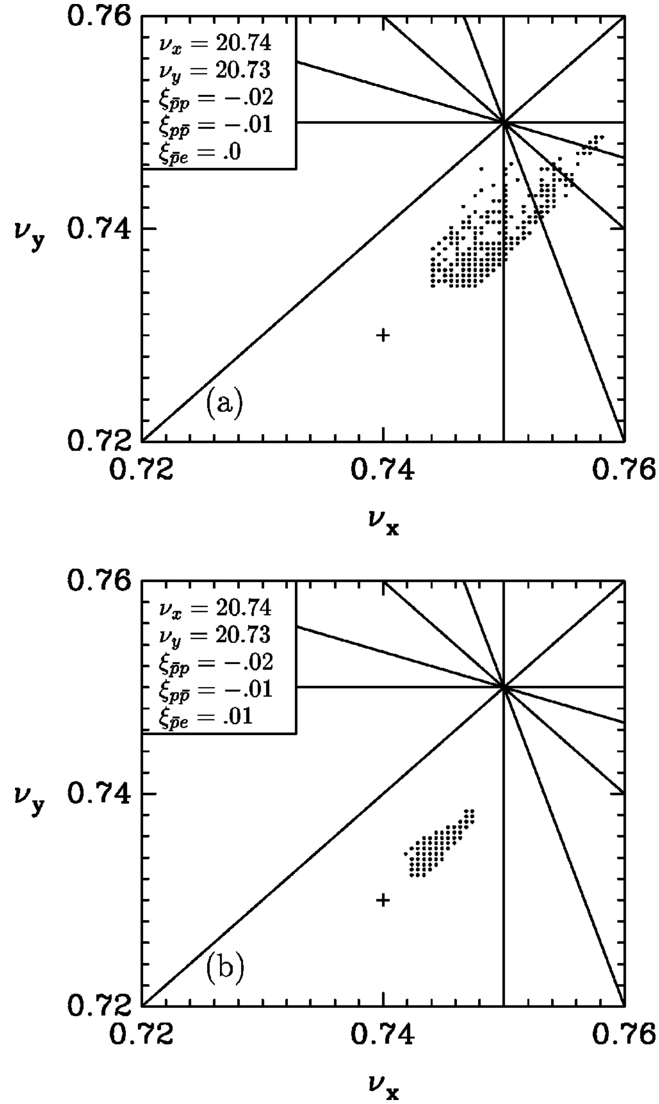

FIG. 5. Tune spread of the $\bar{p}$ beam during the first 2000 turns when $(\nu_x,\nu_x)=(20.74,20.73)$, $\xi_{\bar{p}p}=-0.02$, and $\xi_{p\bar{p}}=-0.01$. (a) Without the tune-spread compensation, the beam core crosses the fourth-order resonances and (b) with the compensation of $\lambda=0.5$. Solid lines are the even-order resonances up to the tenth order and + indicates the lattice bare tune.

or/and a large beam-beam tune shift, a reduction of the tune spread with the electron-beam compensation could move the beam away from the resonances and improve the beam dynamics. In order to explore this possible benefit of the tune-spread compensation, we studied the case that the working point of both the p and $\bar{p}$ beams is at $(\nu_x,\nu_y)=(20.740,20.730)$ that is close to the fourth-order resonance. To have a significant resonance crossing, a larger beam-beam parameter is also used for the p-$\bar{p}$ collision, i.e., $\xi_{\bar{p}p}=-0.02$ and $\xi_{p\bar{p}}=-0.01$.

In Figs. 5 and 6, the initial tune spread and the beam-size growth of the $\bar{p}$ beam were plotted with or without the electron-beam compensation. Without the electron-beam compensation, the core of the $\bar{p}$ beam crosses the fourth-order resonance [see Fig. 5(a)] and that results in a 60%

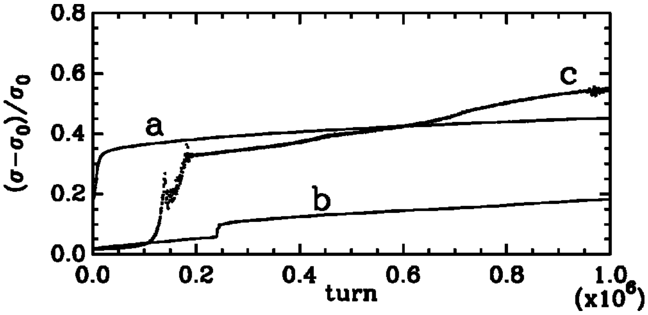

FIG. 6. Evolution of rms beam size of the $\bar{p}$ beam for the cases of Fig. 5. σ is the average of the horizontal and vertical beam sizes and σ_0 the initial beam size. (a) Without the tune-spread compensation, (b) with the compensation of $\lambda=0.5$, and (c) with the compensation of $\lambda=1.0$.

increase in beam size during 10^6 turns tracking (see curve *a* in Fig. 6). Note that the most of this beam-size growth is due to the nonlinear beam filamentation in phase space during the initial 20 000 turns because of a severe deformation of the phase-space area nearby the resonance. With the electron-beam compensation of $\lambda \geqslant 0.5$, the tune spread of the $\bar{p}$ beam is shrunk as expected and the crossing of the fourth-order resonance is eliminated as shown in Fig. 5(b). Consequently, the initial blowup of the $\bar{p}$ beam is suppressed (see curve *b* and *c* in Fig. 6). The tune-spread compensation, therefore, effectively reduces the incoherent beam-beam effect. In the case of $\lambda=1.0$, however, the nonlinear phase-dependent perturbations of p-$\bar{p}$ and e-$\bar{p}$ collisions induce a chaotic coherent beam-beam instability [8] afterwards as the beam-size jumps suddenly during 10^5 to 2×10^5 turn (see curve *c* in Fig. 6). After the onset of the coherent beam-beam instability, the phase-space area nearby origin becomes unstable for the beam centroid and the initially centered beams develop a spontaneous unstable coherent oscillation in the vertical direction as shown in Fig. 7(d). The dynamics of the unstable coherent oscillation has characteristics of chaotic transport in phase space and the growth of the beam size is significantly enhanced by this chaotic coherent oscillation. The beam-size growth rate (the slop of curves in Fig. 6) after the sudden jump in the beam size is therefore substantially larger than that in the case without the compensation. Note that in the case without the electron-beam compensation, no spontaneous coherent beam oscillation was observed after the initial nonlinear beam filamentation [see Figs. 7(a) and 7(b)]. With the full-strength compensation, therefore, the nonlinear phase-dependent beam-beam perturbations of $\{H_{\bar{p}e}\}$ and $\{H_{\bar{p}p}\}$ dominate the beam-beam interaction and make significant damage to the $\bar{p}$ beam even though the $\bar{p}$ beam benefits initially from a reduction of the incoherent beam-beam effect through the elimination of the resonance crossing. In the case of $\lambda=0.5$, the nonlinear beam-beam perturbation from the *e* beam is weaker than that of $\lambda=1.0$, while the Landau damping is stronger because of a larger tune spread of the $\bar{p}$ beam than that in the case of $\lambda=1.0$. Consequently, a weaker spontaneous coherent oscillation in horizontal direction due to a weaker coherent beam-beam instability dissipates quickly [see Fig. 7(c)]. The onset of the coherent beam-beam instability also induces a small jump in the beam size, but unlike

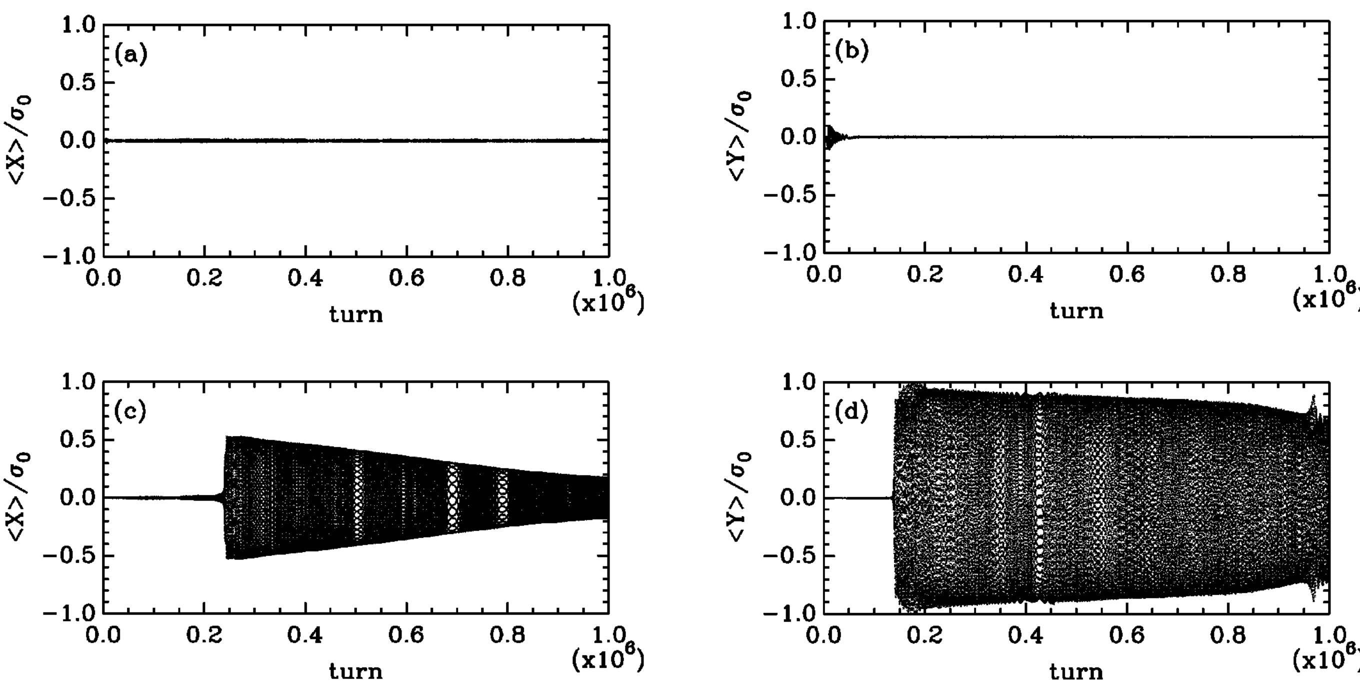

FIG. 7. Beam-centroid motion of the $\bar{p}$ beam for the cases of Fig. 6. $\langle X\rangle$ and $\langle Y\rangle$ are the normalized horizontal and vertical coordinates averaged over each bunch of particles. σ_0 is the initial beam size. Both the p and $\bar{p}$ beams are centered in phase space initially. (a) The beam-centroid motion in horizontal plane without the tune-spread compensation, (b) the beam-centroid motion in vertical plane without the tune-spread compensation, (c) the spontaneous coherent oscillation in horizontal plane with the tune-spread compensation of $\lambda=0.5$, and (d) the spontaneous unstable coherent oscillation in vertical plane with the tune-spread compensation of $\lambda=1.0$.

the case of $\lambda=1.0$, the beam-size growth rate after the jump is still the same as that in the case without the electron-beam compensation because of the suppression of the coherent beam-beam instability by the Landau damping and because of a weaker nonlinear perturbation of $\{H_{\bar{p}e}\}$. When the beam is close to major resonances, the electron-beam compensation could therefore improve the beam dynamics if the strength of the *e* beam is carefully chosen in such a way that the damping mechanism can suppress the coherent beam-beam instability so that the damage effects of the nonlinear phase-dependent beam-beam perturbations are insignificant or can be outweighed by the benefit of the tune-spread compensation.

4. Importance of the difference between particle distributions of the *p* and *e* beams

When the phase advances between the $\bar{p}$-p and $\bar{p}$-e collision points are multiples of 2π, the beam-beam interactions of the $\bar{p}$-p and the $\bar{p}$-e collisions cancel each other completely if there is no any other nonlinearity between the collision points and if the charge distribution of the *e* beam is the same as that of the *p* beam. As the particle distribution of the *p* beam is constantly changing with time due to nonlinear perturbations including beam-beam interactions, in reality, it is impossible to generate an *e* beam that has a microscopically exact charge distribution of the *p* beam. For a slight difference in the distribution of the *e* and *p* beams, such 2π-cancellation of beam-beam interactions leaves small, but usually high-order, nonlinear beam-beam perturbations. If those remains of high-order, nonlinear beam-beam perturbations were not important, the ideal situation of the electron-beam compensation of beam-beam effects would be of the 2π cancellation [4]. Such that the 2π-cancellation of beam-beam interactions has been attempted in the DCI (Dispositif de Collisions dans l'Igloo) storage ring at the Laboratoire de l'Accélérateur Linéaire (Orsay, France), where two pairs of electron and positron beams were brought into a collision at a single interaction point (four-beams cancellation). In that case, all four beams had particle distributions that were very close to a Gaussian distribution and the differences in the distributions were small. The cancellation of the most of beam-beam interactions was achieved and the beam-beam effect was expected to be much weaker. The beam experiment [26] and numerical simulation [27] however showed otherwise. The beam intensities in the DCI with the four beams were severely limited by coherent beam-beam effects, which indicated that the High-order nonlinear beam-beam perturbations could be very damaging when low-order beam-beam perturbations are removed. Recently, this four-beams compensation scheme was studied again for the KEK Super B-factory [28]. Similar to the case of DCI, it was found that the coherent beam-beam instability could occur after a reduction of the beam-beam tune spread with the four-beam compensation in the Super B-factory.

To test the 2π cancellation of beam-beam interactions in Tevatron, a simulation was conducted with both the $\bar{p}$-p and $\bar{p}$-e collisions at a single interaction point (D0). The betatron tune used here for both the p and $\bar{p}$ beams is the nominal Tevatron working point of $(\nu_x,\nu_y)=(20.582,20.574)$. The initial distribution of the *p* beam is a Gaussian distribution. Due to the beam-beam interaction between the $\bar{p}$ and p beams, the distribution of the *p* beam deviates from the Gaussian but the change in distribution is small since the beam-beam perturbation on the *p* beam is weak. Similar to the previous cases, the distribution of the *e* beam is also a Gaussian distribution with the standard deviation changing with the rms beam size of the *p* beam. Figure 8 plots the $\bar{p}$ beam size growth for $\lambda=0.75$ and 1.0. It shows that the electron-beam compensation with 2π cancellation of beam-beam interactions could damage the stability of the beam. In the case of the 100% cancellation ($\lambda=1.0$), the tune spared together with all Gaussian-type beam-beam perturbations of the $\bar{p}$ beam are canceled. The remaining high-order non-Gaussian-type beam-beam perturbations induce a chaotic coherent beam-beam instability as the initially centered beams develop a spontaneous unstable beam-centroid oscillation (see Fig. 9). Consequently, the beam size blows up quickly (see curve *c* in Fig. 8). This result is consistent with the DCI result. The lack of the Landau damping due to the elimination of the tune spread and the addition of the high-order nonlinear perturbations due to the $\bar{p}$-e collision could be the reasons of the onset of the instability.

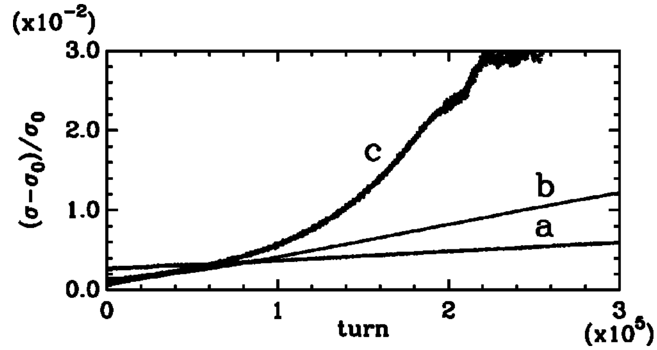

FIG. 8. Same as shown in Fig. 3 but with both the $\bar{p}$-p and $\bar{p}$-e collisions being at a single IP (D0). (a) Without the tune-spread compensation, (b) with the compensation of $\lambda=0.75$, and (c) with the compensation of $\lambda=1.0$.

B. Model lattice for the LHC

To test the generality of the above results obtained with Tevatron, the effect of the electron-beam compensation of the beam-beam tune spread was also studied on a model lattice of LHC. The betatron tune used here is the LHC nominal working point, i.e., the fractional part of the betatron tune is $(\nu_x,\nu_y)=(0.31,0.32)$. Since the two *p* beams have the same intensity, the compensation was used on both the beams in the simulation. Each *p* beam collided with an electron beam at locations of either IP2 or IP8 of LHC [21]. Because two interaction points (IP1 and IP5) of the p-p collisions were included in the simulation, the 100% degree of the compensation corresponds to $\xi_{pe}=-2\xi_{pp}$, where ξ_{pe} and ξ_{pp} are the beam-beam parameter for each p-e and p-p collisions, respectively. In order to explore, relatively easily, nonlinear beam-beam effects, the beam-beam parameter used here is $\xi_{pp}=0.01$. Note that the designed parameter for LHC is $\xi_{pp}=0.0034$.

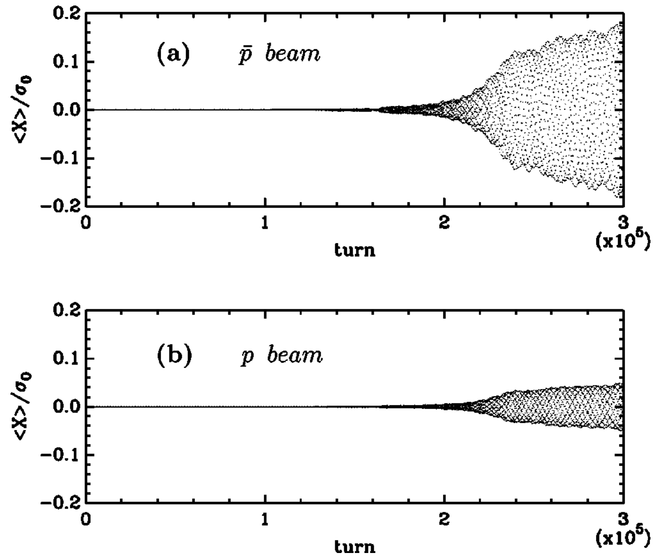

FIG. 9. Spontaneous unstable coherent oscillation in horizontal plane for case (e) in Fig. 8. (a) The $\overline{p}$ beam and b) the p beam. Both the beams are centered in phase space initially. $\langle X\rangle$ is the normalized horizontal coordinate averaged over each bunch of particles. σ_0 is the initial beam size.

Figure 10 plots the evolution of the rms beam size of one p beam with or without the compensation of the beam-beam tune spread. Since the betatron tune of the lattice is away from major resonances, without the electron-beam compensation the original tune spread of the beam-beam interactions between p beams does not lead to any significant beam-beam effect that is associated with low-order resonances. Moreover, the threshold for the coherent beam-beam instability is $\xi_c \simeq 0.03$ in this case when two p-p interaction points are considered [8]. Without the electron-beam compensation the p beams are therefore stable and very little beam-size growth was observed (see curve a in Fig. 10). With both the full-strength ($\lambda=1.0$) and half-strength ($\lambda=0.5$) electron-beam compensation, the tune spread of the p beams is reduced as those similar to the case of Tevatron. The p beams, however,

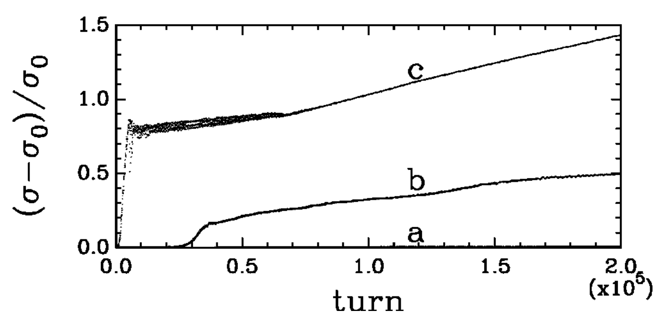

FIG. 10. Evolution of rms beam size of the p beam in LHC when the fractional part of the betatron tune is $(\nu_x,\nu_y)=(0.31,0.32)$ and $\xi_{pp}=0.01$. σ is the average of the horizontal and vertical beam sizes and σ_0 the initial beam size. (a) Without the tune-spread compensation. There is little beam-size growth so that the curve overlaps with x axis, (b) with the compensation of $\lambda=0.5$ used on both colliding beams, and (c) With the compensation of $\lambda=1.0$ used on both colliding beams.

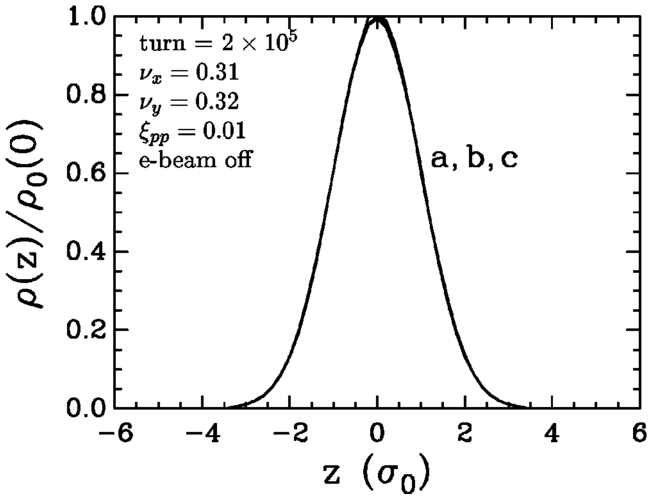

FIG. 11. Projection of the distribution of p beam in LHC without the tune-spread compensation. $(\nu_x,\nu_y)=(0.31,0.32)$ and $\xi_{pp}=0.01$. (a) Initial Gaussian distribution, (b) $z=X$ for the projection to the horizontal normalized coordinate at the 2×10^5th turn, (c) $z=Y$ for the projection to the vertical normalized coordinate at the 2×10^5th turn. $\rho_0(0)$ is the maximum of the initial Gaussian distribution and σ_0 the initial beam size. Note that all three curves overlap in this case.

became very unstable instead as a spontaneous chaotic coherent oscillation was excited and, consequently, the emittance of p beams blows up quickly (curves b and c in Fig. 10). The onset of the coherent beam-beam instability after a reduction of the beam-beam tune spread confirms again that the nonlinear phase-dependent beam-beam perturbations dominate the beam-beam effect and the additional nonlinear perturbations from the $\overline{p}$-e collision could significantly damage the beam stability even though the system is away from any major resonance.

To investigate the mechanism of the enhanced beam-size growth after the onset of the coherent beam-beam instability, we also studied the dynamics of the particle distribution of the p beams in phase space during the tracking. Because a large number of macroparticles was used in the tracking, we were able to reconstruct a smooth particle distribution with very little noise. In Figs. 11 and 12, projections of the distribution at the 2×10^5th turn were plotted for the cases with or without the electron-beam compensation. For a comparison, the initial distribution of the Gaussian beam was also plotted. Without the electron-beam compensation, the p beams are stable and the particle distribution of the p beams is maintained as a Gaussian distribution (Fig. 11). With the electron-beam compensation, the center of the distribution oscillates around the origin and the shape of the distribution deviates from the Gaussian distribution significantly due to the coherent beam-beam instability (Fig. 12). In this case, the density of protons in the beam core drops about 40% and 20% in the horizontal and vertical plane after 2×10^5 turns, respectively, and most of those particles originally in the beam cores escape to the intermediate zone (1σ-4σ, where σ is the rms beam size) of the distribution via the chaotic transport in phase space [29]. The beam-size blowup is therefore mainly

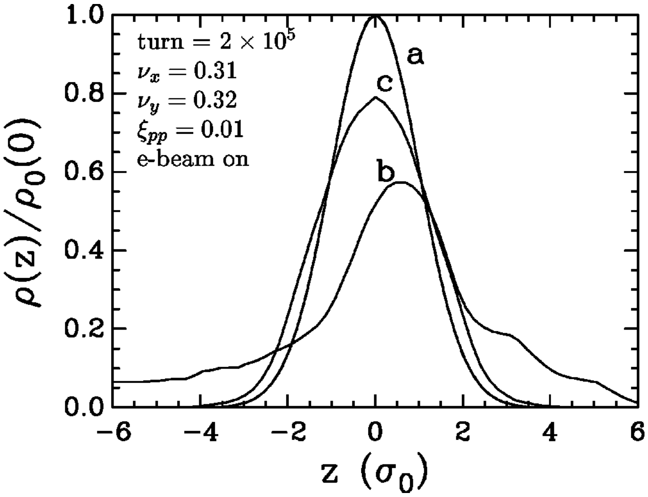

FIG. 12. The same as in Fig. 11, but with the tune-spread compensation of $\lambda = 1.0$.

due to the formation of beam halo after the onset of the coherent beam-beam instability.

IV. SUMMARY

The onset of the collective beam-beam instability due to the tune-spread compensation with electron beams indicates that the nonlinear phase-dependent perturbations of beam-beam interactions could dominate beam-beam instabilities of high-intensity hadron beams. Although the tune-spread compensation could reduce incoherent beam-beam effects, the associated reduction of the Landau damping and increase of the nonlinear phase-dependent beam-beam perturbations significantly limit possible benefits of the compensation. At both nominal work points of Tevatron and LHC that are away from major resonances, the use of the tune-spread compensation with electron beams can damage the beam stability and result in a significantly increased beam-size growth. When the working point is close to major resonances, on the other hand, the beam-beam interactions of hadron beams could lead to crossings of the resonances and result in a beam blowup due to the nonlinear beam filamentation. In this case, the tune-spread compensation can effectively reduce the incoherent beam-beam effect by moving the beams away from the resonances. The nonlinear phase-dependent perturbation from the *e* beam could, however, still damage the stability of the beams by exciting a coherent beam-beam instability. In the case of unsymmetrical rings, the effect of the Landau damping to the coherent beam-beam instability could be significant. A less-than-100% compensation of the tune spread could improve the beam stability if the strength of the *e* beam is carefully chosen in such a way that the Landau damping can suppress the coherent beam-beam instability. For Tevatron RUN II, it was found that a compensation with 50%-or-less reduction of the tune spread could benefit the $\bar{p}$ beam if the difference in the betatron tune between the two colliding beams is close to or larger than the beam-beam parameter. With consideration of other nonlinearities in the lattice such as field errors and long-range beam-beam interactions, there could be cases that the beam dynamics is dominated by a few major resonances and other limitations prevent a change of a better working point. The electron-beam compensation of the beam-beam tune spread could then be used to avoid those resonance effects if the damage effects of the nonlinear phase-dependent beam-beam perturbations from the *e* beams can be outweighed by the benefit of the tune-spread compensation.

The beneficial effect of the beam-beam tune spread on beam stability has also been observed in HERA recently [13]. In a simulation study of beam-beam effects in HERA, it was found that at some working points the coherent beam-beam instability is easier to be induced in the case of one IP than that in the case of two IPs. Note that the incoherent beam-beam tune shift as well as the beam-beam tune spread in the case of two IPs is about twice as large as that in the case of one IP. Recently, a beam experiment in HERA was conducted to compare the difference in emittance growth between the case of one IP and the case of two IPs. In both cases, the betatron tunes of both beams were kept the same and the positron beam crossed the fourth-order resonance. In the experiment, a significant emittance growth of both the proton and positron beams was observed in the case of one IP but not in the case of two IPs. The simulation study showed that the emittance growth in the case of one IP is due to the onset of the chaotic coherent beam-beam instability and in this case a large number of positrons were trapped inside the fourth-order resonance. The fourth-order resonance in HERA is therefore more harmful in the case of one IP even though the positron beam crosses more fourth-order resonance lines in the case of two IPs. These numerical and experimental observations in HERA further confirmed that a large beam-beam tune spread may benefit the stability of beams.

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