PHYSICAL REVIEW E **69**, 016410 (2004)

Vertical oscillations of paramagnetic particles in complex plasmas

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(Received 19 September 2003; published 30 January 2004)

Vertical vibrations of a single magnetized dust grain and a one-dimensional string of magnetized particles in discharge plasmas are treated taking into account the magnetic force associated with gradients of an external magnetic field. For a single particle a novel type of oscillation associated with these gradients is found. Such vibrations can be stable or unstable depending on the distribution of the magnetic field inside the particle cloud. In a one-dimensional particle string the magnetic force causes a new low-frequency oscillatory mode which is characterized by inverse optical-like dispersion when the wavelength far exceeds the intergrain distance. The study of vertical vibrations of magnetized grains provides a tool for determining complex plasma parameters.

DOI: 10.1103/PhysRevE.69.016410 PACS number(s): 52.40.Hf, 52.25.Vy, 52.35.Fp

I. INTRODUCTION

In the last decade, complex plasmas have attracted great interest as a new developing field in plasma physics and because of the many interactions with other research areas, such as strong coupling phenomena, colloid physics, environmental research, astrophysics, plasma processing technologies, etc. In fundamental physics the most surprising discovery was the observation of crystal-like structures which spontaneously form when charged microparticles are trapped in a sheath electric field [1–7]. Such plasma crystal structures can support longitudinal and transverse vibrational modes, which have been extensively studied in the last years [4–14]. However, ground based complex plasma experiments deal with either a monolayer or a cloud of a few layers (quasi-two-dimensional systems). Three-dimensional structures can be obtained either by experiments under microgravity conditions using, very small (submicron) particles or when some other nonelectric force compensates gravitation. The idea of gravity compensation by a nonelectric force looks very promising from a standpoint of recent experimental research. In particular, experiments involving the thermophoretic force lifting the particles above the sheath region, which is characterized by strong plasma inhomogeneities and ion flows [15], give an opportunity to study, e.g., void formation even under gravity conditions [16,17]. At the same time, the nonlinear temperature distribution produces specific vibrational modes [18], which can provide a tool for determining the complex plasma parameters.

Recently, multilayer complex plasma structures were observed in an external magnetic field [19]. Measurements were carried out in a capacitively coupled rf discharge with plastic microspheres (Dynobeads M-450) of radius $a=4.5\ \mu\text{m}$ and material density 1.5 g/cm^3. The grains contained 20% Fe_2O_3 and Fe_3O_4 they were superparamagnetic with a magnetic permeability $\mu=4$. A magnetic field coaxial with the chamber, was created by magnetic coils, located above the discharge chamber. The distance between the lower electrode and the bottom edge of the coils was 28 cm. The coils were powered with a current up to 1.5 kA, yielding a magnetic field up to 0.12 T in the middle of the chamber. The field gradient pointed up, exerting a levitating force on paramagnetic particles. For further technical details we refer to the recent paper [19].

When grains were injected into the discharge in the absence of the magnetic field, they charged up and levitated forming a few-layer structure in the plasma sheath of the lower electrode [Fig. 1(a)]. As soon as the magnetic field was applied, the main particle cloud rose slightly higher than in the absence of the magnetic field [Fig. 1(b)]. By further increasing the strength of the magnetic field it was possible to lift the microparticles into the central part of the discharge chamber or even to the upper sheath of the discharge [Fig. 1(c)]. Hence, contrary to the electrostatic levitation in the sheath electric field, the levitation height in the magnetic experiments can be controlled by variations of the electric current in the magnetic coils. This means that magnetic forces open new opportunities for controlling complex plasmas.

Among the novel features demonstrated by the magnetized particles was dust agglomeration: some particles coalesce into chains, oriented vertically, parallel to the field lines of the external magnetic field. This phenomenon is well studied both in theory [20] and in the laboratory [19]. In this paper we will focus on the grain levitation in an external magnetic field and vertical vibrations of the particles. Up to now, oscillatory modes of magnetized particles in a gas discharge plasma considered only in homogeneous magnetic fields [21], which implies that a magnetic force levitation was not discussed as well as the role of inhomogeneous magnetic field on the particle dynamics.

The paper is structured as follows. In Sec. II we first discuss levitation of the magnetized particles and consider the vertical oscillations of a single grain in a discharge plasma, before turning in Sec. III to the existence of new collective modes in a string of paramagnetic particles. Finally, our conclusions are given in Sec. IV.

II. VERTICAL OSCILLATIONS OF A SINGLE GRAIN

The static magnetic field is considered to be vertical, parallel to the z axis. The equation describing the vertical motion of particles is

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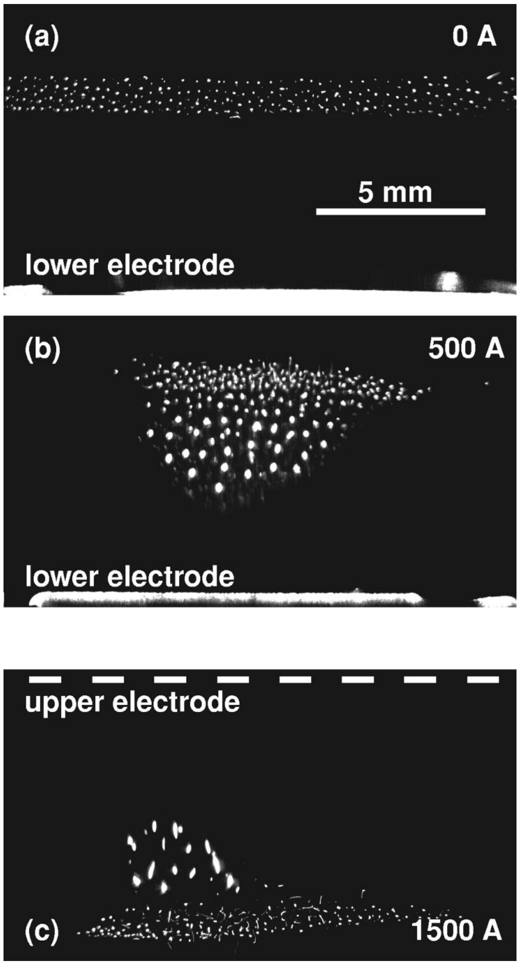

FIG. 1. Particles levitated in the plasma. The current in the magnetic coils is indicated in the right corner. (a) Without the magnetic field. Particles form a multilayer cloud in the lower plasma sheath (lower viewing area). (b) Magnetic field of 0.04 T. Some particles agglomerate and levitate in the lower sheath below the main cloud (lower viewing area). The main cloud is compressed and slightly shifted upwards. (c) Magnetic field of 0.12 T. The cloud is levitated in the upper sheath (upper viewing area). The larger agglomerated particles levitate above the main cloud. Gravity can be compensated by the magnetic force. Reprinted from Ref. [19].

$$\ddot{z}+2\gamma\dot{z}=\frac{F}{M}-g, \tag{1}$$

where F is the electromagnetic force acting on the particle of mass M, g denotes the gravitational acceleration, and γ is the damping rate due to neutral gas friction [22].

In the experiments which do not invoke a magnetic field, the particles are trapped in the vertical direction z, by the balance of gravitational and electric forces. Since to a good approximation, the sheath electric field $E(z)$ increases almost linearly with z, there is always a position where gravitation can be compensated by the electric field,

$$Mg=Q_0E_0. \tag{2}$$

Here $Q_0=Q(z=0)$ is the particle charge, and $E_0=E(z=0)$ is the electric field at the point $z=0$ corresponding to the equilibrium position. Relation (2) implies that the particles with different ratios Q_0/M will be suspended at different positions: the grains of larger Q_0/M (smaller sizes) are trapped by the electric force at larger heights, while the larger grains require stronger electric fields and therefore can only levitate in the vicinity of the electrode, where the electric field is strong enough.

If the particle is perturbed from its equilibrium position and assuming sufficiently small oscillation amplitude, one can expand

$$E(z)\simeq E_0+E_0'z, \tag{3}$$

(the prime denotes the derivative $\partial E/\partial z$ at the equilibrium $z=0$). In the plasma sheath, the equilibrium particle charge is a function of the distance from the electrode [23], but we assume that the particle charge does not change crucially over the scales of the small vertical oscillations and use the same series expansion for $Q(z)$,

$$Q(z)\simeq Q_0+Q_0'z. \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (1) gives in the linear order

$$\ddot{z}+2\gamma\dot{z}-\frac{Q_0E_0'+Q_0'E_0}{M}z=0 \tag{5}$$

from which the resonance frequency follows as

$$\Omega_E^2=-\frac{(QE)_0'}{M}. \tag{6}$$

The particle charge is practically independent on the vertical position only in the bulk plasma and in the presheath region, but its value increases rapidly in the sheath, achieving a maximum and then starts to decrease in the vicinity of the lower electrode [23]. Normally, the particle are trapped near the sheath edge, where $(QE)_0'\simeq Q_0E_0'<0$, so that the vertical oscillations are stable. Recently, an excitation of the vertical vibrations and the measurement of the particle's resonance frequency Eq. (6) have been used for measuring its charge and the spatial distribution of the electric field in the plasma sheath [6,24,25]. Note that we do not include into present analysis the "delayed" charging effect and related to this specific instability of the vertical vibrations predicted by Nitter *et al.* [26] and observed in a discharge plasma at low gas pressure [27–29].

Consider now the case when the grain levitation occurs mainly due to the magnetic force. A spherical particle of radius a and magnetic permeability μ in an external magnetic field B parallel to the z axis, gets a magnetic moment [30]

$$m=\frac{(\mu-1)}{(\mu+2)}a^3B. \tag{7}$$

Such a magnetized grain is subjected to a magnetic force, associated with the vertical magnetic field gradient

$$F_m = -\partial(mB)/\partial z = -2\alpha B \partial B/\partial z, \tag{8}$$

where the coefficient α is defined through $\alpha = (\mu - 1)a^3/(\mu + 2)$.

Expanding the magnetic force around the equilibrium position $z=0$, letting $F_m \simeq F_0 + F_0' z$, yields

$$F_0 = -2\alpha B_0 B_0', \tag{9}$$

$$F_0' = -2\alpha(B_0'^2 + B_0 B_0''). \tag{10}$$

Once again subscript "0" denotes the quantities at the equilibrium position $z=0$. Contrary to the condition of electrostatic levitation Eq. (2) , both sides of the force balance $Mg = -2\alpha B_0 B_0'$ are now proportional to a^3, and grains of various sizes are suspended at the same height, which can be completely specified by the external magnetic field gradients. Substituting Eqs. (9) and (10) into Eq. (1) and using the balance equation gives in the linear approximation

$$\ddot{z} + 2\gamma \dot{z} = -\frac{2\alpha}{M}(B_0'^2 + B_0 B_0'')z. \tag{11}$$

The resonance frequency of vertical oscillations is now

$$\Omega_m^2 = \frac{2\alpha(B_0'^2 + B_0 B_0'')}{M} = \frac{2\alpha(B'B)_0'}{M}. \tag{12}$$

Since the particle mass can be written as $M = 4\pi\rho a^3/3$ (ρ is the material density), Eq. (12) reduces to

$$\Omega_m^2 = \frac{6(\mu-1)(B'B)_0'}{4\pi(\mu+2)\rho}. \tag{13}$$

It follows immediately that all magnetized grains have the same resonance frequency independent on size (13), but specified by the structure of the external magnetic field and the magnetic properties of the grain material. As a result, the value Ω_m^2 can be positive or negative depending on B_0, B_0', and B_0'' at the levitation height. When the electric current in the coils is fixed, these values are mainly determined by the position of the discharge chamber relative to the magnetic coils. Calculated field profiles for the experiments with paramagnetic particles (Fig. 2) show that when the distance between the lower electrode and the magnetic coils is small enough (less than 0.1 m), B_0'' becomes strongly negative leading to $\Omega_m^2 < 0$ and thus to a vertical oscillation instability. Nevertheless the main conclusion is that by an appropriate choice of the magnetic field regime, the squared frequency Ω_m^2 can be always made positive to stabilize particle vibrations.

To obtain a numerical estimate of the frequency Ω_m, we consider typical plasma parameters in the experiments and a magnetic field gradients sufficient for gravity compensation [19]. Particle levitation was observed for average magnetic fields $B_0 \sim 0.1$–0.15 T and field gradients of $|B_0'|$ ~ 0.75–1.13 T/m. We now need only to specify B_0'' to calculate the frequency of vertical oscillations (13). An estimate of B_0'' can be obtained using a model presentation of the field profile (Fig. 2). Taking $B_0'' \sim 5$–7.5 T/m^2, corresponding to the position of the discharge chamber used in the experiments (a distance ~ 0.25–0.28 m), this yields a frequency $\Omega_m \sim 50$–70 s^{-1}. These values are similar to the frequencies of vertical vibrations in a sheath electric field (6) and can be easily measured.

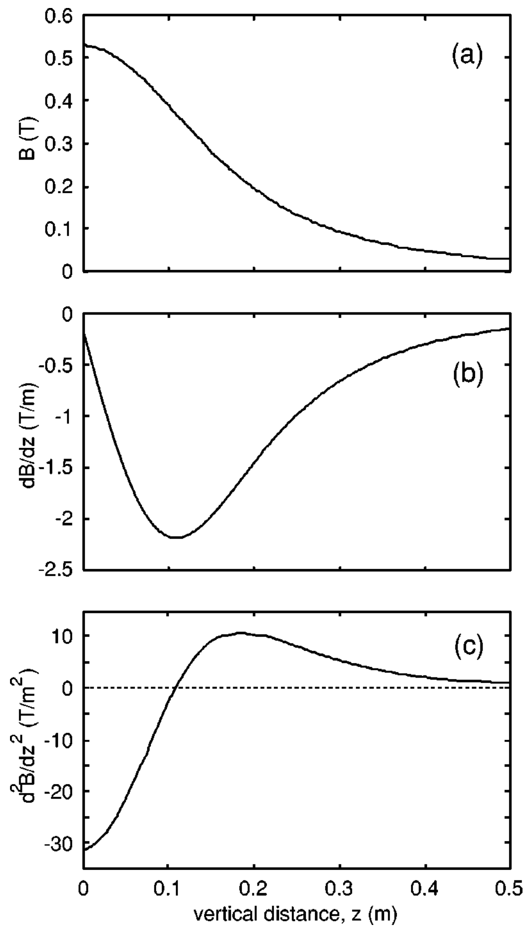

FIG. 2. Magnetic field on the axis of the magnetic coils at the current of 1 kA. (a) Magnetic induction, (b) first and (c) second spatial derivatives. The field is produced by a 17 cm thick coil with 30 cm bore. The current can be varied from 0 to 1.6 kA.

Finally, when both the magnetic and electric forces contribute to balancing gravitation, the frequency of vertical oscillations can be written as a combinations of Eqs. (6) and (13),

$$\Omega_v^2 = \Omega_m^2 + \Omega_E^2 = \frac{6(\mu-1)(B'B)_0'}{4\pi(\mu+2)\rho} - \frac{(QE)_0'}{M}. \tag{14}$$

This describes stable or unstable vertical variations depending on the input made by Ω_m^2 and Ω_E^2. While the electrostatic squared frequency $\Omega_E^2 \sim (QE)_0'$ is either positive or negative within the sheath region [23], the magnetic term Ω_m^2 can be made positive (by varying the magnetic field) and exceeding $|\Omega_E^2|$ and thus stabilize the vibrations which would be unstable in the absence of the magnetic field.

The question now is what complex plasma parameters can be recovered from measurements of the resonance frequencies (13) or (14). If the frequencies of the vertical vibrations have been accurately measured and since values of B_0, B_0', and B_0'' are determined numerically, expression (13) immediately gives the value of the magnetic permeability μ and thus from Eq. (7) the magnetic moment. If the levitation is due to the combination of the electrostatic and magnetic forces, then using Eq. (14) one can determine the $(QE)_0'$ profile at given μ, B_0, B_0', and B_0''. Varying the current in the magnetic coils (and also B_0, B_0', and B_0'') it is possible to calculate the values of $(QE)_0'$ even in the close vicinity of the electrode where $(QE)_0' > 0$, and the oscillations are unstable in the absence of the magnetic field. There is another opportunity when the magnetic force (8) lifts up the particle in the presheath region, where the charge is nearly constant and $\Omega_E^2 \simeq Q_0 E_0'$. Then measuring Eq. (14) one can find either the equilibrium charge Q_0 at given μ, B_0, B_0', and B_0'' or the value of the equilibrium gradient E_0' if Q_0 is known. Such measurements could be of importance to understand how the particle charge or the sheath structure is influenced by an external magnetic field. Hence the observations of the vertical oscillation of a single particle in an inhomogeneous magnetic field look very promising for complex plasma diagnostics.

III. VIBRATIONAL MODES IN A HORIZONTAL STRING OF PARAMAGNETIC GRAINS

In order to describe the vertical modes in a complex plasma with magnetized grains, we consider the simplest model of one-dimensional horizontal string (oriented along the x axis) [9,12], where the spherical dust grains have charge Q, mass M, and magnetic moment $\mathbf{m}$ (7), parallel to the external magnetic field. The grains are separated by the average distance Δ. The electrostatic potential of each particle is assumed to be the screened Coulomb potential. The energy of the electrostatic and magnetic interactions between the nth and mth grains of the string can be written as

$$U_{n,m} = \frac{Q_n Q_m}{|\mathbf{r}_{n,m}|} \exp\left(-\frac{|\mathbf{r}_{n,m}|}{\lambda_D} \right) - \frac{\mu_0}{4\pi} \left[\frac{\mathbf{m}_n \mathbf{m}_m}{|\mathbf{r}_{n,m}|^3} - \frac{3(\mathbf{m}_n \cdot \mathbf{r}_{n,m})(\mathbf{m}_m \cdot \mathbf{r}_{n,m})}{|\mathbf{r}_{n,m}|^5} \right], \quad (15)$$

where λ_D is the screening length of the plasma and $|\mathbf{r}_{n,m}|$ is the distance between the grains. The second term in the right-hand side Eq. (15) accounts for interactions of two magnetic dipoles [32]. The corresponding force acting on the nth particle can then be presented as $\mathbf{F}_{n,m} = -\partial U_{n,m}/\partial \mathbf{r}_n$.

Usually in complex plasma structures, the particle separation Δ, exceeds the screening length λ_D, viz. $\Delta/\lambda_D \sim 1.5$–$2.5$. The dipole interactions are also short ranged, so that we need only to consider the nearest neighbor particle interactions. The equation of motion for the nth particle in the string is

$$\ddot{\mathbf{r}}_n + 2\gamma \dot{\mathbf{r}}_n = M^{-1}[\mathbf{F}_{n,n+1} + \mathbf{F}_{n,n-1} + \mathbf{F}_{n,ext}]. \quad (16)$$

Here $\mathbf{F}_{n,n\pm1} = \mathbf{F}(\mathbf{r}_{n\pm1} - \mathbf{r}_n)$ are the particle interaction forces and $\mathbf{F}_{n,ext} = -\partial U_{ext}/\partial \mathbf{r}_n$ is the force acting on the nth grain in the external magnetic and electric fields. The external potential U_{ext} can be approximated by a parabolic potential in z direction, just as in the case of a sheath electric confinement

$$U_{ext} = \frac{1}{2} M (\Omega_m^2 + \Omega_E^2) z^2, \quad (17)$$

with Ω_m^2 and Ω_E^2 being the vertical resonance frequencies defined by Eq. (14).

Using Eqs. (15)–(17) and considering only small vertical oscillations $z \ll \Delta$ around the equilibrium position $z = 0$ gives the linear equation of motion

$$\ddot{z}_n + 2\gamma \dot{z}_n = -(\Omega_E^2 + \Omega_m^2) z_n + \left(\Omega_\perp^2 + 9\frac{m_0^2}{\Delta^5} \right) \times (2z_n - z_{n+1} - z_{n-1}) = 0, \quad (18)$$

where z_n is the vertical displacement of the nth particle and m_0 stands for the equilibrium magnetic moment of the grains, $m_0 = (\mu - 1) a^3 B_0 / (\mu + 2)$. The quantity $\Omega_\perp$ denotes the frequency of the transverse dust-lattice waves [31]

$$\Omega_\perp^2 = \frac{Q^2}{M\Delta^3} \exp\left(-\frac{\Delta}{\lambda_D} \right) \left(1 + \frac{\Delta}{\lambda_D} \right). \quad (19)$$

To derive Eq. (18) we have neglected the small corrections $[\sim (a/\Delta)^3 \ll 1]$ associated with variations of the magnetic moments over the scales of the particle displacement around the equilibrium position.

Assuming now that z_n varies as $\sim \exp(-i\omega t + ikn\Delta)$, Eq. (18) gives a dispersion relation describing transverse dust-lattice waves in the external electric and magnetic fields,

$$\omega^2 + 2i\gamma\omega = \Omega_E^2 + \Omega_m^2 - 4\left(\Omega_\perp^2 + 9\frac{m_0^2}{\Delta^5} \right) \sin^2 \frac{k\Delta}{2}. \quad (20)$$

If $\Omega_E^2 + \Omega_m^2 > 0$, then Eq. (20) is similar to the dispersion dependence for the vertical vibrational mode found in the sheath region [12]: the wave is characterized by an optical-like inverse dispersion (the maximum frequency is achieved at $k = 0$ and then the frequency decreases with growing wave number k). The main difference is in the increase of the total resonance frequency of vertical vibrations $\Omega_v^2 = \Omega_E^2 + \Omega_m^2$ for positive Ω_m^2 or its decrease if $\Omega_m^2 < 0$. It is even possible for Ω_v^2 to be negative ($\Omega_E^2 < |\Omega_m^2|$) and then the vertical mode becomes unstable due to magnetic interactions. An opposite situation is also possible, when the magnetic term Ω_m^2 exceeds the negative Ω_E^2 and thus plays the stabilizing role.

When particle levitation is mainly due to the magnetic force (8) outside of the sheath region, $\Omega_E^2 \rightarrow 0$, and then only $\Omega_\perp^2$ remains from the electrostatic interactions of dust charges in the dispersion relation Eq. (20) describing a vertical dust-lattice mode, namely,

$$\omega^2 + 2i\gamma\omega = \Omega_m^2 - 4\left(\Omega_\perp^2 + 9\frac{m_0^2}{\Delta^5}\right)\sin^2\frac{k\Delta}{2}. \tag{21}$$

Finally note that just as in the case of vertical vibrations of a single particle, the measurements of the wave dispersion $\omega(k)$ corresponding to Eqs. (20) or (21) could give an opportunity to estimate a few important complex plasma parameters: an equilibrium charge in the presheath or bulk plasmas (via determining $\Omega_\perp$), magnetic moment of the grain (magnetic permeability), or even the vertical profile of $(QE)'$ (by means of measurement Ω_E at different magnetic fields).

IV. CONCLUSIONS

We have demonstrated the appearance of a novel type of vertical vibrations of a single magnetized particles in discharge plasmas when an external magnetic field is applied. Such vibrations can be stable or unstable depending on the distribution of the magnetic field inside the particle cloud. Moreover, it is shown that the vertical oscillations of a one-dimensional string of grains supported by the magnetic force give rise to a new low-frequency mode which is characterized by the inverse optical-like dispersion when the wavelength far exceeds the intergrain distance. The identifications of a vertical mode can provide a tool for determining the electric field profile, particle charge or magnetic moment of the grains. The characteristics of the mode are specified by the gradients of the external magnetic field and thus can be effectively controlled in experimental conditions. This opens new possibilities for the investigation of the particle behavior at the kinetic level as well as for stimulating phase transitions in the system, and for the study of self-organized structures in the experiments.

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