Amplitude death induced by dynamic coupling

Keiji Konishi*
Department of Complex Systems, Future University—Hakodate, 116-2 Kamedanakano, Hakodate, Hokkaido 041-8655, Japan
(Received 8 April 2003; published 23 December 2003)

The present paper shows that dynamic coupling induces amplitude death in coupled identical oscillators. For a simple limit-cycle oscillator, our theoretical analysis provides the necessary and sufficient condition for amplitude death. Furthermore, we guarantee that amplitude death never occurs, if each oscillator satisfies the odd number property that is known in the field of delayed-feedback control of chaos.

DOI: 10.1103/PhysRevE.68.067202 PACS number(s): 05.45.Xt, 05.45.Ra

There have been various investigations on amplitude death in coupled oscillators [1–3]; this phenomenon is a coupling-induced stabilization of the origin in the oscillators. For two coupled oscillators, Aronson, Ermentrout, and Kopell have investigated this phenomenon in detail [4]. From these results, we can see that amplitude death never occurs in coupled identical oscillators.

Reddy, Sen, and Johnston reported that a time-delay coupling, which is frequently observed in laser and biological systems, induces an amplitude death in coupled identical oscillators [5]. Their result has created considerable interest in recent years [6,7]. The theoretical analysis on time-delay-induced amplitude death has been shown in Ref. [8]; furthermore, this phenomenon was experimentally observed in electronic circuits [9], living oscillators [10], and thermo-optical oscillators [11]. In addition, the time-delay-induced stabilization of coupled identical discrete-time systems has been investigated [12].

Kuntsevich and Pisarchik showed amplitude death in a dual-wavelength class-B laser with modulated losses [13]. This laser is a nonautonomous system, since the losses in a channel are modulated by an external sinusoidal force. In studies on amplitude death of autonomous systems [1–4], the coupling signal is proportional to the difference between the oscillators' states. The proportionality factor is a constant value; hence, it can be considered as *static* coupling. In other words, static coupling without delay does not induce amplitude death in coupled identical oscillators.

The present paper proposes a *dynamic* coupling that has not only the proportionality factor but also its own dynamics; however, the dynamic-coupled systems are classified into the autonomous systems. The motivations of our proposal are as follows: it is a rough approximation of the time-delayed coupling for low-frequency oscillators and/or short-time delay [14]; RC-ladder coupling [15], which is an approximation of RC wire delay connections in VLSI chips [16], can be considered as a kind of the dynamic coupling. From these motivations, the dynamic coupling is reasonable from a practical viewpoint. We shall show the dynamic-coupling-induced amplitude death, and provide the stability analysis.

Let us consider two identical limit-cycle oscillators,

$$\begin{bmatrix}\dot{x}_{\alpha1}\\ \dot{x}_{\alpha2}\end{bmatrix}=\begin{bmatrix}x_{\alpha1}(1-x_{\alpha1}^2-x_{\alpha2}^2)-\omega x_{\alpha2}\\ x_{\alpha2}(1-x_{\alpha1}^2-x_{\alpha2}^2)+\omega x_{\alpha1}\end{bmatrix}+\begin{bmatrix}u_\alpha\\ 0\end{bmatrix}, \qquad (1)$$

$$\begin{bmatrix}\dot{x}_{\beta1}\\ \dot{x}_{\beta2}\end{bmatrix}=\begin{bmatrix}x_{\beta1}(1-x_{\beta1}^2-x_{\beta2}^2)-\omega x_{\beta2}\\ x_{\beta2}(1-x_{\beta1}^2-x_{\beta2}^2)+\omega x_{\beta1}\end{bmatrix}+\begin{bmatrix}u_\beta\\ 0\end{bmatrix}, \qquad (2)$$

where $x_{\alpha i,\beta i}\in\boldsymbol{R}(i=1,2)$ and $u_{\alpha,\beta}\in\boldsymbol{R}$ are the system variables and the coupling signals, respectively. The parameter $\omega>0$ is the natural frequency. It is well accepted that the oscillator without coupling (i.e., $u_\beta\equiv u_\alpha\equiv 0$) has been considered as a typical model of limit cycle. This is because the oscillator shows a stable limit cycle with unit amplitude. Several investigations of the static-coupled oscillators have been reported in Refs. [3–5,8]. The main purpose of this paper is to propose the following dynamic coupling:

$$\dot{z}_\alpha=-z_\alpha+x_{\beta1}, \quad u_\alpha=k(z_\alpha-x_{\alpha1}), \qquad (3)$$

$$\dot{z}_\beta=-z_\beta+x_{\alpha1}, \quad u_\beta=k(z_\beta-x_{\beta1}), \qquad (4)$$

where $z_{\alpha,\beta}\in\boldsymbol{R}$ are the additional variables in the dynamic coupling. $k\in\boldsymbol{R}$ corresponds to the coupling strength. It should be noted that Eqs. (3) and (4) include the other oscillator variables $x_{\beta1}$ and $x_{\alpha1}$, respectively. The steady states of subsystems (1) and (2) without coupling ($k=0$) are

$$[x_{\alpha1}\,x_{\alpha2}]^T=[0\;0]^T, \quad [x_{\beta1}\,x_{\beta2}]^T=[0\;0]^T,$$

which never change even by dynamic coupling; hence, coupling influences only the state stability.

The parameter and the coupling strength are set at $\omega=10$ and $k=4.0$. Figure 1 shows the numerical simulation of the coupled limit-cycle oscillators. Each oscillator without coupling ($k=0$) behaves periodically until $t=150$; then, the dynamic coupling is achieved at $t=150$. It can be seen that the oscillations vanish after the coupling. This phenomenon is the amplitude death induced by dynamic coupling. The bifurcation diagram ($\omega=4$) is shown in Fig. 2, where the coupling strength k is used as a bifurcation parameter. We can observe amplitude death in the wide range of k. The variable $x_{\alpha1}$ presents oscillatory behavior in $k\in[0,2.3]$; the amplitude death occurs in $k\in[2.3,8.5]$, where all variables converge on the origin. The stable fixed point, which differs from the origin, appears for $k\geqslant 8.5$. Now, an important ques-

*Electronic address: kkonishi@fun.ac.jp

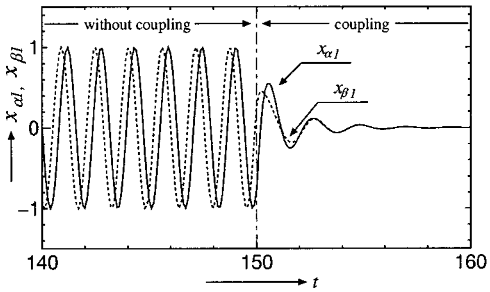

FIG. 1. Behavior of the coupled limit-cycle oscillators just before and after the dynamic coupling ($\omega=10,k=4.0$). Two isolated oscillators are coupled at $t=150$.

tion remains about the analytical derivation of the coupling strength range. This paper shall answer it on the basis of linear stability analysis.

We consider two identical m-dimensional subsystems $\Sigma_{\alpha,\beta}$,

$$\Sigma_\alpha:\begin{cases}\dot{\boldsymbol{x}}_\alpha=\boldsymbol{F}(\boldsymbol{x}_\alpha)+\boldsymbol{B}\boldsymbol{u}_\alpha\\ \boldsymbol{y}_\alpha=\boldsymbol{C}\boldsymbol{x}_\alpha,\end{cases}\quad \Sigma_\beta:\begin{cases}\dot{\boldsymbol{x}}_\beta=\boldsymbol{F}(\boldsymbol{x}_\beta)+\boldsymbol{B}\boldsymbol{u}_\beta\\ \boldsymbol{y}_\beta=\boldsymbol{C}\boldsymbol{x}_\beta,\end{cases}$$

where $\boldsymbol{x}_{\alpha,\beta}\in\boldsymbol{R}^m$ are the system variables, $\boldsymbol{u}_{\alpha,\beta}\in\boldsymbol{R}^l$ and $\boldsymbol{y}_{\alpha,\beta}\in\boldsymbol{R}^p$ are the input and output signals. $\boldsymbol{F}:\boldsymbol{R}^m\rightarrow\boldsymbol{R}^m$ denotes the nonlinear function that has an unstable steady state $\boldsymbol{0}$ [i.e., $\boldsymbol{F}(\boldsymbol{0})=\boldsymbol{0}$]. $\boldsymbol{B}\in\boldsymbol{R}^{m\times l}$ and $\boldsymbol{C}\in\boldsymbol{R}^{q\times m}$ are the input and output matrices. These subsystems $\Sigma_{\alpha,\beta}$ are coupled by

$$\Pi_\alpha:\begin{cases}\dot{\boldsymbol{z}}_\alpha=-\boldsymbol{z}_\alpha+\boldsymbol{y}_\beta\\ \boldsymbol{u}_\alpha=\boldsymbol{K}(\boldsymbol{z}_\alpha-\boldsymbol{y}_\alpha),\end{cases}\quad \Pi_\beta:\begin{cases}\dot{\boldsymbol{z}}_\beta=-\boldsymbol{z}_\beta+\boldsymbol{y}_\alpha\\ \boldsymbol{u}_\beta=\boldsymbol{K}(\boldsymbol{z}_\beta-\boldsymbol{y}_\beta),\end{cases}$$

where $\boldsymbol{z}_{\alpha,\beta}\in\boldsymbol{R}^p$ are the additional variables for dynamic coupling. $\boldsymbol{K}\in\boldsymbol{R}^{l\times p}$ implies the coupling strength. Figure 3 illustrates the structure of the coupled systems. The steady state of the coupled systems is described by

$$[\boldsymbol{x}_\alpha\,\boldsymbol{z}_\alpha\,\boldsymbol{x}_\beta\,\boldsymbol{z}_\beta]^T=[\boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0}]^T. \tag{5}$$

The amplitude death induced by dynamic coupling can be considered as a stabilization of steady state (5).

In order to analyze local stability of Eq. (5), we linearize subsystems $\Sigma_{\alpha,\beta}$ around the steady state; the linearized subsystems are as follows:

$$\Delta\Sigma_\alpha:\begin{cases}\dot{\boldsymbol{x}}_\alpha=\boldsymbol{A}\boldsymbol{x}_\alpha+\boldsymbol{B}\boldsymbol{u}_\alpha\\ \boldsymbol{y}_\alpha=\boldsymbol{C}\boldsymbol{x}_\alpha,\end{cases}\quad \Delta\Sigma_\beta:\begin{cases}\dot{\boldsymbol{x}}_\beta=\boldsymbol{A}\boldsymbol{x}_\beta+\boldsymbol{B}\boldsymbol{u}_\beta\\ \boldsymbol{y}_\beta=\boldsymbol{C}\boldsymbol{x}_\beta,\end{cases}$$

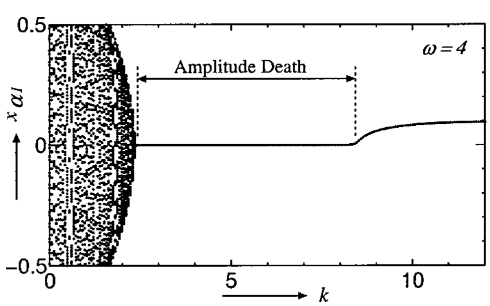

FIG. 2. Bifurcation diagram of the coupled limit-cycle oscillators for $k\in[0,12]$ ($\omega=4.0$).

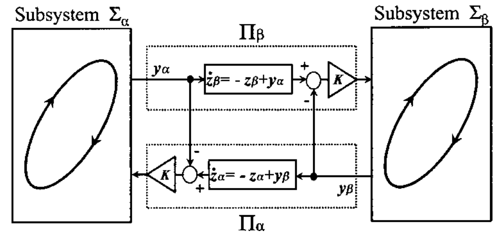

FIG. 3. Dynamic-coupled nonlinear systems. $\Sigma_{\alpha,\beta}$, subsystems and $\Pi_{\alpha,\beta}$, dynamic coupling.

where

$$\boldsymbol{A}:=\left.\frac{\partial\boldsymbol{F}(\boldsymbol{x})}{\partial\boldsymbol{x}}\right|_{\boldsymbol{x}=\boldsymbol{0}}. \tag{6}$$

These subsystems are coupled by $\Pi_{\alpha,\beta}$. $\boldsymbol{A}$ is a Jacobi matrix of the nonlinear function at the steady state $\boldsymbol{0}$, and has at least one eigenvalue in the open right half complex plane (i.e., $\boldsymbol{A}$ is an unstable matrix).

The local stability of steady state (5) in the coupled system is the same as the stability of linear subsystems $\Delta\Sigma_{\alpha,\beta}$ coupled by $\Pi_{\alpha,\beta}$. The coupled linear systems can be given by

$$\begin{bmatrix}\dot{\boldsymbol{x}}_\alpha\\ \dot{\boldsymbol{z}}_\alpha\\ \dot{\boldsymbol{x}}_\beta\\ \dot{\boldsymbol{z}}_\beta\end{bmatrix}=\begin{bmatrix}\boldsymbol{A}-\boldsymbol{BKC} & \boldsymbol{BK} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & -\boldsymbol{I}_p & \boldsymbol{C} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}-\boldsymbol{BKC} & \boldsymbol{BK}\\ \boldsymbol{C} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{I}_p\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_\alpha\\ \boldsymbol{z}_\alpha\\ \boldsymbol{x}_\beta\\ \boldsymbol{z}_\beta\end{bmatrix}, \tag{7}$$

where $\boldsymbol{I}_p$ is the p-dimensional identity matrix. The stability of linear system (7) depends only on the characteristic function $f(\lambda)=f_1(\lambda)f_2(\lambda)$, where

$$f_1(\lambda):=\det\begin{bmatrix}\lambda\boldsymbol{I}_m-\boldsymbol{A}+\boldsymbol{BKC} & -\boldsymbol{BK}\\ -\boldsymbol{C} & (\lambda+1)\boldsymbol{I}_p\end{bmatrix},$$

$$f_2(\lambda):=\det\begin{bmatrix}\lambda\boldsymbol{I}_m-\boldsymbol{A}+\boldsymbol{BKC} & -\boldsymbol{BK}\\ \boldsymbol{C} & (\lambda+1)\boldsymbol{I}_p\end{bmatrix}.$$

Linear system (7) is stable if and only if all roots $\lambda_i[i=1,2,\ldots,2(m+p)]$ of $f(\lambda)=0$ are in the open left half complex plane. These roots can be obtained by solving $f_1(\lambda)=0$ and $f_2(\lambda)=0$.

If $\lim_{\lambda\rightarrow\infty}f_1(\lambda)=\infty$ and $f_1(0)<0$ are satisfied, at least one root of $f_1(\lambda)=0$ is in the open right half complex plane. It is obvious that the first condition $\lim_{\lambda\rightarrow\infty}f_1(\lambda)=\infty$ always holds, and the second condition can be described by

$$f_1(0)=\det[-\boldsymbol{A}]=\sum_{q=1}^{m}(-\sigma_q),$$

where $\sigma_q(q=1,2,\ldots,m)$ are the eigenvalues of $\boldsymbol{A}$. Hence, if $\boldsymbol{A}$ has an odd number of real positive eigenvalues (*odd number property*), then we have $f_1(0)<0$. These arguments can be summarized as follows: If the Jacobi matrix $\boldsymbol{A}$ of oscillators satisfies the odd number property, the dynamic coupling never induces amplitude death at the origin in coupled identical systems. The odd number property has been known in the field of delayed-feedback control of chaos [17]. A similar stability analysis can be found in Refs. [18–20].

We shall provide two numerical examples to confirm our theoretical results. First, let us look again at limit-cycle oscillators (1) and (2) coupled by dynamic coupling (3) and (4). The above linear stability analysis can be applied to these coupled oscillators. The Jacobi matrix described in Eq. (6),

$$\boldsymbol{A}=\begin{bmatrix} 1 & -\omega \\ \omega & 1 \end{bmatrix},$$

has the eigenvalues $\sigma_{1,2}=1\pm i\omega$; hence, $\boldsymbol{A}$ is an unstable matrix, and does not satisfy the odd number property. The other parameters are

$$\boldsymbol{B}=[1\ 0]^T, \quad \boldsymbol{C}=[1\ 0], \quad \boldsymbol{K}=k.$$

From linear stability analysis, we have

$$f_1(\lambda)=\lambda^3+(k-1)\lambda^2+(k-1+\omega^2)\lambda+1+\omega^2-2k,$$

$$f_2(\lambda)=\lambda^3+(k-1)\lambda^2+(\omega^2-1-k)\lambda+1+\omega^2.$$

The steady state of the coupled oscillators are locally stable if and only if all the roots of the characteristic equations, $f_1(\lambda)=0$ and $f_2(\lambda)=0$, are in the open left half complex plane. These roots are not so simple; therefore, we apply the Routh stability criterion to the characteristic equations. This criterion has been used to check the stability of characteristic equations in the field of control theory [14]. These characteristic equations are stable if and only if k and ω satisfy the following inequalities: (a) $1+\omega^2-2k>0$, (b) $1-\omega^2-k<0$, (c) $1-\omega^2+k<0$, (d) $k>1$, (e) $k^2+\omega^2k-2\omega^2>0$, and (f) $k^2-\omega^2k+2\omega^2<0$. From these inequalities, we obtain the amplitude death region shown in Fig. 4. The coupling strength range can be described as

$$\tfrac{1}{2}(\omega^2-\omega\sqrt{\omega^2-8})<k<\tfrac{1}{2}(\omega^2+\omega\sqrt{\omega^2-8})$$

for $2\sqrt{2}<\omega\leqslant\sqrt{4+\sqrt{17}}$, and

$$\tfrac{1}{2}(\omega^2-\omega\sqrt{\omega^2-8})<k<\tfrac{1}{2}(1+\omega^2)$$

for $\sqrt{4+\sqrt{17}}\leqslant\omega$. This theoretical result allows us to obtain the coupling strength range $2.343<k<8.500$ for $\omega=4$. It must be noted that the range agrees well with the numerical result shown in Fig. 2.

Second, we consider the two identical Rössler systems,

$$\dot{\xi}_{\alpha1}=-\xi_{\alpha2}-\xi_{\alpha3}, \qquad \dot{\xi}_{\beta1}=-\xi_{\beta2}-\xi_{\beta3},$$

$$\dot{\xi}_{\alpha2}=\xi_{\alpha1}+\gamma_1\xi_{\alpha2}+u_\alpha, \qquad \dot{\xi}_{\beta2}=\xi_{\beta1}+\gamma_1\xi_{\beta2}+u_\beta,$$

$$\dot{\xi}_{\alpha3}=\gamma_2+\xi_{\alpha3}(\xi_{\alpha1}-\gamma_3), \quad \dot{\xi}_{\beta3}=\gamma_2+\xi_{\beta3}(\xi_{\beta1}-\gamma_3),$$

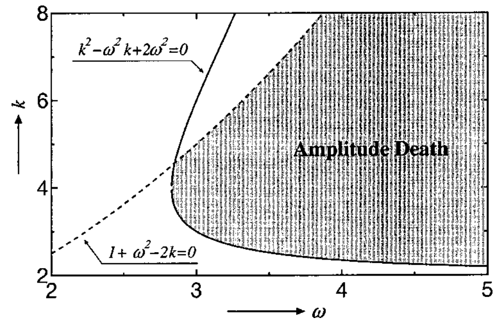

FIG. 4. Amplitude death region for (ω,k) space.

coupled by

$$\dot{w}_\alpha=-w_\alpha+\xi_{\beta2}, \quad u_\alpha=k(w_\alpha-\xi_{\alpha2}),$$

$$\dot{w}_\beta=-w_\beta+\xi_{\alpha2}, \quad u_\beta=k(w_\beta-\xi_{\beta2}).$$

$\xi_{\alpha i,\beta i}\in\boldsymbol{R}(i=1,2,3)$ are the system variables and $\gamma_{1,2,3}\in\boldsymbol{R}$ are the parameters. Each isolated subsystem has the steady state

$$\boldsymbol{\xi}_{\alpha,\beta}:=[\xi_{\alpha1,\beta1}\ \xi_{\alpha2,\beta2}\ \xi_{\alpha3,\beta3}]^T=[\xi_{f1}\ \xi_{f2}\ \xi_{f3}]^T, \tag{8}$$

where

$$\xi_{f1}=\frac{\gamma_3-\sqrt{\gamma_3^2-4\gamma_1\gamma_2}}{2}, \quad \xi_{f2}=\frac{-\gamma_3+\sqrt{\gamma_3^2-4\gamma_1\gamma_2}}{2\gamma_1},$$

$$\xi_{f3}=\frac{\gamma_3-\sqrt{\gamma_3^2-4\gamma_1\gamma_2}}{2\gamma_1}.$$

This steady state can be shifted to the origin via a change of variables,

$$x_{\alpha i,\beta i}:=\xi_{\alpha i,\beta i}-\xi_{fi} \quad (i=1,2,3), \quad z_{\alpha,\beta}:=w_{\alpha,\beta}-\xi_{f2},$$

then we obtain the subsystems $\Sigma_{\alpha,\beta}$ coupled by $\Pi_{\alpha,\beta}$. Since the nonlinear function is

$$\boldsymbol{F}(\boldsymbol{x})=\begin{bmatrix} -(x_2+\xi_{f2})-(x_3+\xi_{f3}) \\ x_1+\xi_{f1}+\gamma_1(x_2+\xi_{f2}) \\ \gamma_2+(x_3+\xi_{f3})(x_1+\xi_{f1}-\gamma_3) \end{bmatrix},$$

we obtain the Jacobi matrix

$$\boldsymbol{A}=\begin{bmatrix} 0 & -1 & -1 \\ 1 & \gamma_1 & 0 \\ \xi_{f3} & 0 & \xi_{f1}-\gamma_3 \end{bmatrix}.$$

The input and output matrices and the coupling gain are given by

$$\boldsymbol{B}=[0\ 1\ 0]^T, \quad \boldsymbol{C}=[0\ 1\ 0], \quad \boldsymbol{K}=k.$$

The parameters are fixed at the well-known values: $\gamma_1=0.398$, $\gamma_2=2.0$, and $\gamma_3=4.0$, where each isolated subsystem $(k=0)$ behaves chaotically. The eigenvalues of $\boldsymbol{A}$ are

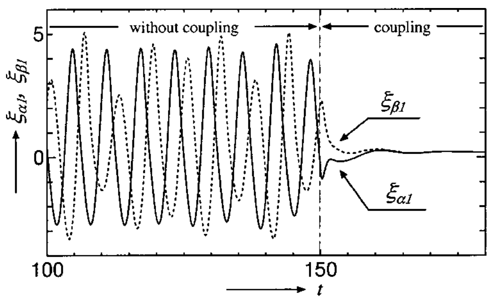

FIG. 5. Behavior of the coupled Rössler systems just before and after the dynamic coupling ($k=3.0$). Two isolated systems are coupled at $t=150$.

$\sigma_1=-3.655$ and $\sigma_{2,3}=0.131\pm i0.981$; hence, we notice that $\boldsymbol{A}$ is an unstable matrix and does not satisfy the odd number property. From linear stability analysis, we notice that amplitude death could occur at steady state. Figure 5 shows the behavior of the coupled systems just before and after dynamic coupling ($k=3$). They behave chaotically before the coupling and converge on steady state (8) after that. Unlike the limit-cycle oscillators, it is not so easy to derive the coupling strength range in which amplitude death occurs. However, a numerical analysis supports us in estimating k as shown in Fig. 6. The bifurcation diagram for $k\in[0,10]$ is indicated in Fig. 6(a). Figure 6(b) presents the maximum real part of roots of $f_1(\lambda)=0$ and $f_2(\lambda)=0$ [i.e., $\lambda_{\max}:=\max_{i\in[1,8]}\mathrm{Re}(\lambda_i)$]. From our stability analysis, we know that steady state (8) is stable if and only if this value is negative. It can be stated that amplitude death occurs in a coupling strength range where the maximum real part of the roots is negative. The above numerical estimation of $\lambda_{\max}$ is a simple calculation of eigenvalues of system matrix (7); hence, the stability can be easily analyzed even for high-dimensional oscillators. On the contrary, for the time-delayed coupled-induced amplitude death, a laborious task of graphical method, Nyquist plot [14], is needed to determine the stability.

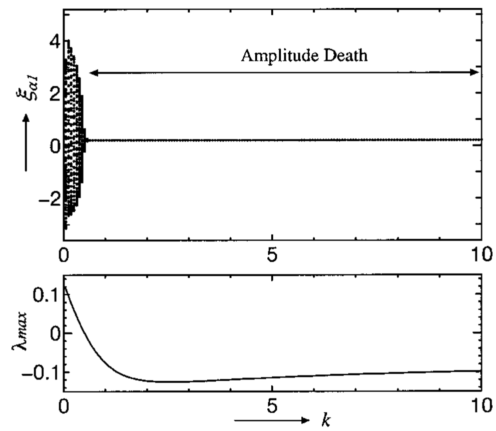

FIG. 6. Bifurcation diagram of the coupled Rössler systems for $k\in[0,10]$.

In conclusion, this paper introduced the dynamic coupling that induces amplitude death in coupled identical oscillators. We have analyzed the amplitude death, and obtained the sufficient condition under which it never occurs. Furthermore, we observed amplitude death in coupled limit-cycle oscillators and in coupled Rössler systems.

This research was supported by the Grants-in-Aid for Young Scientists (B) from the Ministry of Education, Culture, Sports, Science and Technology of Japan (Grant No. 15760326) and by the Special Research Funds of Future University—Hakodate.

[1] Y. Yamaguchi and H. Shimizu, Physica D **11**, 212 (1984).
[2] K. Bar-Eli, Physica D **14**, 242 (1984).
[3] R.E. Mirollo and S.H. Strogatz, J. Stat. Phys. **60**, 245 (1990).
[4] D.G. Aronson, G.B. Ermentrout, and N. Kopell, Physica D **41**, 403 (1990).
[5] D.V. Ramana Reddy, A. Sen, and G.L. Johnston, Phys. Rev. Lett. **80**, 5109 (1998).
[6] S.H. Strogatz, Nature (London) **394**, 316 (1998).
[7] See http://focus.aps.org/story/v6/st15
[8] D.V. Ramana Reddy, A. Sen, and G.L. Johnston, Physica D **129**, 15 (1999).
[9] D.V. Ramana Reddy, A. Sen, and G.L. Johnston, Phys. Rev. Lett. **85**, 3381 (2000).
[10] A. Takamatsu, T. Fujii, and I. Endo, Phys. Rev. Lett. **85**, 2026 (2000).
[11] R. Herrero, M. Figueras, J. Rius, F. Pi, and G. Orriols, Phys. Rev. Lett. **84**, 5312 (2000).
[12] K. Konishi, Phys. Rev. E **67**, 017201 (2003).
[13] B.F. Kuntsevich and A.N. Pisarchik, Phys. Rev. E **64**, 046221 (2001).
[14] B.C. Kuo, *Automatic Control Systems* (Wiley, New York, 2003).
[15] J.P. Fishburn and C.A. Schevon, IEEE Trans. Circuits Syst., I: Fundam. Theory Appl. **42**, 1020 (1995).
[16] X. Zhang, IEEE Circuits Devices Mag. **12**, 12 (1996).
[17] K. Pyragas, Phys. Lett. A **170**, 421 (1992).
[18] T. Ushio, IEEE Trans. Circuits Syst., I: Fundam. Theory Appl. **43**, 815 (1996).
[19] K. Konishi, M. Ishii, and H. Kokame, IEEE Trans. Circuits Syst., I: Fundam. Theory Appl. **46**, 1285 (1999).
[20] H. Nakajima and Y. Ueda, Physica D **111**, 143 (1998).