Superluminal propagation of light pulses: A result of interference

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The propagation of pulses through dispersive media was investigated by solving Maxwell's equations without any approximation. We show that the superluminal propagation of pulses through anomalous dispersive media is a result of the interference of different frequency components composed of the pulse. The coherence of the pulse plays an important role for the superluminal propagation. With the decrease of the coherence of the pulse, the propagation changes from superluminal to subluminal. We have shown that the anomalous dispersion (the real part of the susceptibility) not the amplification (the imaginary part of the susceptibility) plays the essential role in the superluminal propagation. Although the superluminality always exists as long as the spectrum of the coherent pulse is within the anomalously dispersive region, both the energy propagation velocity and the frontal velocity never exceed the light speed in the vacuum. The output pulse through the medium is not the original pulse; instead it carries the information of the original pulse and the information of the prepared medium.

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I. INTRODUCTION

Superluminal propagation is a phenomenon that the group velocity of an optical pulse in a medium is greater than the light speed in vacuum [1,2]. This phenomenon has been discussed theoretically and experimentally in many different media [3–10]. However, the mechanism about the superluminal propagation remains controversial.

What is the mechanism of the superluminal group velocity of pulses through the medium (the advance of the pulse peak)? There are two different viewpoints. One of them is that the front and the back of the pulse undergo different gain or attenuation [11–15]. Some statements are as follows. "The pre-excited active medium plays the role of a nonlinear amplifier, amplifying the front of the laser pulse and absorbing energy at the rear of the pulse in such a manner as to maintain its shape but at the same time increase the overall pulse speed [11]." "We show that in a gain medium, the lowest-order effect is that the pulse propagates with velocity c and undergoes differential gain, i.e., a distortion in which the front of the pulse is amplified more than the back [12]." "The leading edge of the Gaussian pulse entering the medium induces the gain (or accumulation) effect that makes the peak of the pulse appear at the exit of the medium earlier than it appears through the vacuum. At a later time, when the peak of the pulse reaches the medium, the absorption (or dissipation) effect becomes more important, and so, a part of the pulse energy would be dissipated inside the medium [13]." "We demonstrate that a well designed linear medium with two resonances can be used to reshape a pulse so that the front is amplified and the back is attenuated [15]."

Another viewpoint is that it is due to the interference between the different frequency components of the pulse, which undertake different phase shifts after passing through a medium of anomalous dispersion [16–23]. "It can be understood by the classical theory of wave propagation in an anomalous dispersion region where interference between different frequency components produces this rather counterintuitive effect [6]." "In this experiment, each of the different frequency components making up the pulse experiences a slightly different dispersion in the medium. The relative phases between them are therefore changed and the pulse shape is shifted to bring the pulse wave packet (or group velocity) forward in time [16]." "Therefore, in a sufficiently strong anomalous dispersion medium, the redder incident ray will have a shorter wavelength and hence becomes a bluer ray, while an incident bluer ray will have a longer wavelength to become a redder ray. This results in an unusual situation where the phases of the different frequency components of a pulse become aligned at the exit surface of the medium earlier than even in the case of the same pulse propagating through the same distance in a vacuum [17]."

Recently, we have shown the effect of coherence of light on superluminal propagation and the important role of the interference between different frequency components for the superluminal propagation [24]. In this paper, we will investigate the nature of the superluminal propagation, and use partially coherent pulses to solve this controvery. From the viewpoint that the interference between different frequency components results in the superluminality, the superluminality depends on the coherence of the pulse. If we can change the interference which can be realized by varying the coherence of the pulses, while keeping the intensity profile of the pulse the same, the superluminal propagation will be changed (even disappears). In the following, we will investigate the dependence of the group velocities on the coherence of the pulses, discuss the effect of coherence on the superluminal propagation, and demonstrate that the coherence plays a key role in the superluminal phenomena. In Sec. II, we will introduce temporal partially coherent pulses. In Sec. III, we will discuss the propagation of temporal partially coherent pulses in a gain medium. In Sec. IV, we will point out that the key role for superluminal propagation of pulses in a dispersive medium is the real part of the susceptibility and is not the imaginary part of the susceptibility. In Sec. V, the group velocity, the energy velocity, and the frontal velocity of the pulses are again investigated in detail; we show the differences among these velocities. Our final conclusion can be found in Sec. VI.

II. TEMPORAL PARTIALLY COHERENT PULSES

The interference can be varied by changing the coherence of light. In order to investigate the effects of coherence and interference on the superluminal propagation, we introduce a kind of temporal partially coherent pulses. It is known that any light field from real source is not fully coherent [25]. For stationary fields, the theory of coherence has been studied for a long time [25,26]. Recently, the theory of coherence for nonstationary fields is established [27–32]. The correlation function of a pulse in space-time domain is the key quantity for discussing partially coherent pulses.

The correlation function of a fully coherent plane-wave pulse [25] is defined by $\Gamma(z_1,t_1;z_2,t_2)=E^*(z_1,t_1)E(z_2,t_2)$. The reflection in current case is negligible. Decomposing the electric field into Fourier components, $E(z,t)=\int E(0,\omega)e^{ik(\omega)z}e^{-i\omega t}d\omega$, where $k(\omega)$ is a complex wave vector, we can write the correlation function for a fully coherent pulse as

$$\Gamma(z_1,t_1;z_2,t_2)=\int\int W(0,\omega_1;0,\omega_2)\times e^{ik(\omega_2)z_2-ik^*(\omega_1)z_1}e^{i(\omega_1 t_1-\omega_2 t_2)}d\omega_1 d\omega_2, \tag{1}$$

where

$$W(0,\omega_1;0,\omega_2)=E^*(0,\omega_1)E(0,\omega_2)=\int\int\Gamma(0,t_1;0,t_2)e^{-i(\omega_1 t_1-\omega_2 t_2)}dt_1 dt_2, \tag{2}$$

with $\Gamma(0,t_1;0,t_2)$ being the initial correlation function of the pulse at $z=0$. By using Eqs. (1) and (2), we can obtain the evolution of the correlation function.

For a partially coherent pulse the correlation function is defined [25] by $\Gamma(z_1,t_1;z_2,t_2)=\langle E^*(z_1,t_1)E(z_2,t_2)\rangle$, where "$\langle\cdots\rangle$" represents the statistical ensemble average (the phases of all components of the light field is partially random, and the average is taken on all these components). The evolution of the correlation function for partially coherent pulses is still controlled by Eqs. (1) and (2). In the above and the following discussion, we assume that the medium is stationary. For the fully coherent plane-wave pulses, we have

$$\Gamma(0,t_1;0,t_2)=[I(0,t_1)I(0,t_2)]^{1/2}\exp[i\omega_0(t_1-t_2)]. \tag{3}$$

For the partial coherent pulses, the temporal correlation usually depends only on the time difference, and we assume that the initial correlation function is Gaussian,

$$\Gamma(0,t_1;0,t_2)=[I(0,t_1)I(0,t_2)]^{1/2}\exp\left[-\frac{(t_1-t_2)^2}{4\sigma_{L0}^2}\right]\times\exp[i\omega_0(t_1-t_2)], \tag{4}$$

where σ_{L0} is the correlation time width, which measures the correlation between two different space-time points. Note that the initial intensity of light field $I(0,t_i)=\Gamma(0,t_i;0,t_i)$ $(i=1,2)$ is not dependent on σ_{L0}. That is to say, the space-time intensity profile of the pulse is the same for any value of σ_{L0}. We call the pulse defined by Eq. (4) as a kind of Schell-Model plane-wave pulse [30].

It is obvious from Eq. (4) that the completely coherent plane-wave light pulse is obtained at the limit $\sigma_{L0}\to\infty$. In the opposite limit $\sigma_{L0}\to 0$, all the space-time points become uncorrelated. Therefore, when the parameter σ_{L0} varies from zero to infinity, Eq. (4) represents a class of temporal partially coherent pulses with the same space-time intensity profile but with different coherence. In the following, we will consider two special partially temporal coherent pulses.

III. PROPAGATION OF TEMPORAL PARTIALLY COHERENT PULSES IN A GAIN MEDIUM

We consider the propagation of a partially coherent pulse in a gain medium from $z=0$ to L surrounded by the vacuum. The susceptibility of the gain medium is assumed as a double Lorentz oscillator, which describes a three-level system with two closely placed Raman gain peaks [6],

$$\chi(\omega)=\frac{M}{\omega-\omega_0-\Delta+i\gamma}+\frac{M}{\omega-\omega_0+\Delta+i\gamma}, \tag{5}$$

where M is related to the gain coefficient, and γ is the spectral width of two gain lines. The parameters used in this section are $\omega_0/2\pi=3.5\times10^{14}$ Hz, $M/2\pi=2.262$ Hz, $\gamma/2\pi=0.46\times10^6$ Hz, $\Delta/2\pi=1.35\times10^6$ Hz, which are fit to the experimental data reported in Ref. [6]. The relation between complex wave number and the susceptibility is $k(\omega)=\omega n(\omega)/c$ with $n(\omega)=\sqrt{1+\chi(\omega)}$ $[\mu(\omega)=1]$.

A. Gaussian pulses

First we consider the propagation of partially coherent Gaussian pulses whose initial intensity is

$$I(0,t)=\exp\left(-\frac{t^2}{\sigma_{\tau 0}^2}\right), \tag{6}$$

where $\sigma_{\tau 0}=1.2\times10^{-6}$ s is the pulse width. So the initial correlation function of the partially coherent Gaussian pulse is

$$\Gamma(0,t_1;0,t_2)=\exp\left(-\frac{t_1^2}{2\sigma_{\tau 0}^2}\right)\exp\left(-\frac{t_2^2}{2\sigma_{\tau 0}^2}\right)\times\exp\left[-\frac{(t_1-t_2)^2}{4\sigma_{L0}^2}\right]\exp[i\omega_0(t_1-t_2)]. \tag{7}$$

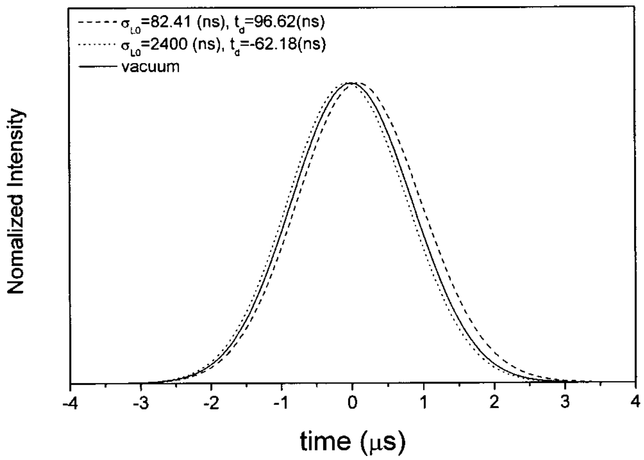

FIG. 1. The temporal evolution of the pulse envelops at z_L. Solid line, the pulse through the vacuum; dot line, the pulse through the medium for $\sigma_{L0}=2400$ ns; dashed line, the pulse through the medium for $\sigma_{L0}=82.41$ ns [24].

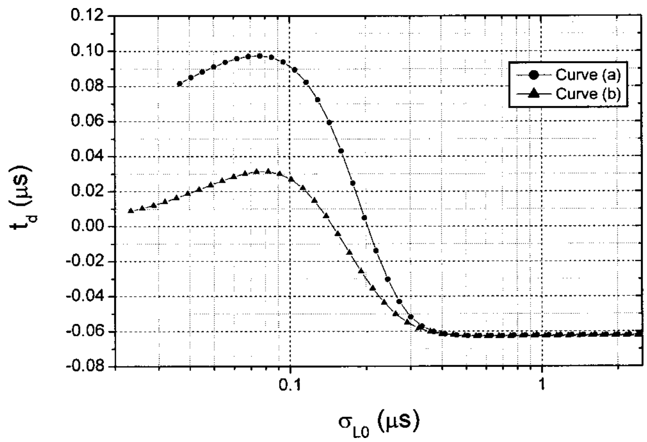

FIG. 2. Peak delay t_d as a function of the correlation time σ_{L0}. Curve a for the partially coherent GSMP, the relevant parameters are the same as in Fig. 1; curve b for the partially coherent quasirectangle pulse, $\tau_0=20\times 10^{-6}$ s and $\delta_0/2\pi=0.5\times 10^6$ Hz, other parameters are the same as Fig. 1.

Such a pulse is called temporal partially coherent Gaussian Schell-model pulse (GSMP) [30,24,31].

Let the partially coherent Gaussian pulse enter the medium at $z=0$ and exit the medium at $z=L$. Substituting Eq. (7) into Eq. (2), we have the generalized spectral density

$$W(0,\omega_1;0,\omega_2)=\frac{1}{2\pi}\sqrt{\frac{1}{1+(\sigma_{\tau 0}/\sigma_{L0})^2}}\times\exp\left[-\frac{(\omega_1-\omega_0)^2+(\omega_2-\omega_0)^2}{2\left(\frac{1}{\sigma_{\tau 0}^2}+\frac{1}{\sigma_{L0}^2}\right)}-\frac{(\omega_1-\omega_2)^2}{\frac{4(\sigma_{\tau 0}^2+\sigma_{L0}^2)}{\sigma_{\tau 0}^4}}\right]. \tag{8}$$

The shape and the width of the generalized spectrum of such a pulse depend on both $\sigma_{\tau 0}$ and σ_{L0}. When $\sigma_{L0}\gg\sigma_{\tau 0}$, the light pulse is essentially fully temporal correlated (fully coherent), and the width of the generalized spectrum is determined by the temporal width $\sigma_{\tau 0}$. When $\sigma_{L0}\ll\sigma_{\tau 0}$, the light pulse is globally temporal uncorrelated (incoherent), and the generalized spectral width determined by the correlated time width σ_{L0} becomes very broad. The intensity of the pulse is determined by $I(0,t)=\Gamma(0,t;0,t)$, which is independent of σ_{L0}, that is to say, the initial intensity profile is independent of σ_{L0}. Substituting Eq. (8) into Eq. (1), we can get the pulse evolution through the medium. In Fig. 1, we plot the intensity profile $I(z,t)=\Gamma(z,t;z,t)$ after passing through the medium for different σ_{L0}. When σ_{L0} is large, the pulse is essentially coherent and its propagation is superluminal. For $\sigma_{L0}=2400$ ns (dot line), the peak delay is $t_d=t_m-t_v=-62.2$ ns, where t_m and t_v are the times that the peaks appear for the pulses passed through the medium and through the vacuum, respectively. When σ_{L0} becomes small, the pulse becomes partially coherent and the peak delay increases. When σ_{L0} is sufficiently small (see the dashed line), the pulse propagation is no longer superluminal, but subluminal. The dashed line in the figure is for $\sigma_{L0}=82.4$ ns, where we have a peak delay of $t_d=96.6$ ns (subluminal propagation). In Fig. 2 we plot the peak delay t_d as a function of the correlation time width σ_{L0} (see curve a). It is very clear that the time decay increases as the pulse changes from almost fully coherent to almost incoherent. The transition from superluminal to subluminal propagation happens near $\sigma_{L0}\approx 200$ ns. As the correlation time σ_{L0} becomes smaller than 75 ns, the time delay decreases again. At extremely small σ_{L0}, the time delay tends to a constant, which is determined from the average refractive index of the medium. In our case, the average refractive index is $\overline{n}=1$, which means the time delay tends to zero in the limit of $\sigma_{L0}\rightarrow 0$. From the above discussion it is clear that the dispersion is result of the interference between different frequency components.

B. Quasirectangle pulses

We consider the propagation of a temporal partially coherent quasirectangle pulse. For a fully quasirectangle coherent pulse, its spectrum [24] is

$$E(0,\omega)=\begin{cases}\dfrac{\sin[\tau_0(\omega-\omega_0)]}{\tau_0(\omega-\omega_0)} & \text{when}|\omega-\omega_0|\leq\delta_0\\ 0 \quad \text{when}|\omega-\omega_0|>\delta_0. & \end{cases} \tag{9}$$

The intensity of the quasirectangle pulse at initial position $z=0$ is

$$I(0,t)=\left|\int E(\omega)\exp(-i\omega t)d\omega\right|^2 \tag{10}$$

(see Fig. 3). In the following we choose $\tau_0=20\times 10^{-6}$ s and $\delta_0/2\pi=0.5\times 10^6$ Hz, and other parameters are the same as

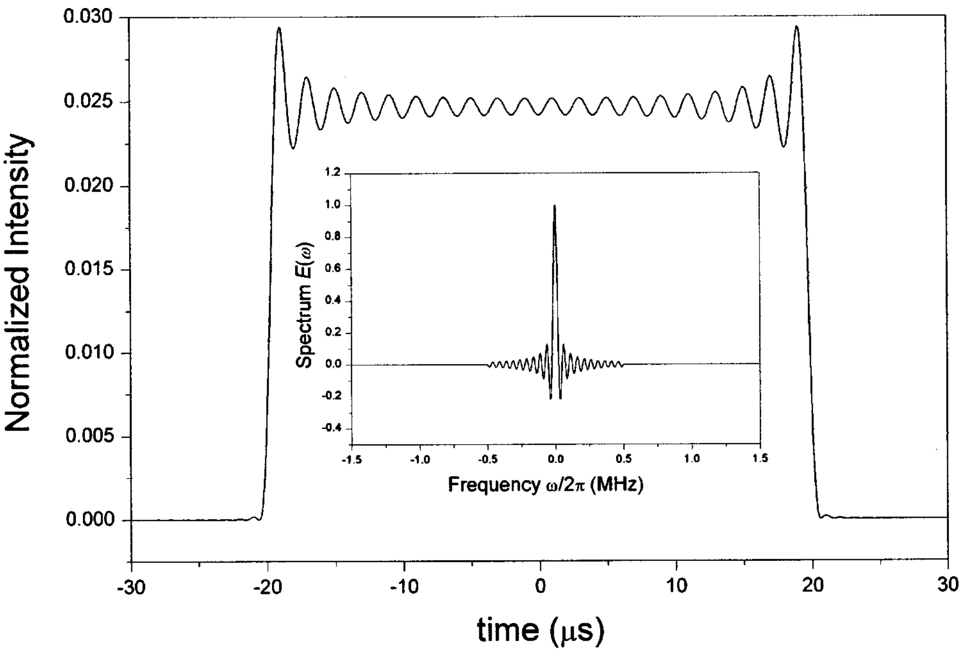

FIG. 3. The quasirectangle pulse, here $\tau_0 = 20\times10^{-6}$ s and $\delta_0/2\pi = 0.5\times10^6$ Hz. Its spectrum is shown in the inset.

the above. Substituting Eq. (10) into Eq. (4), we can obtain the initial shape of the temporal partially coherent quasirectangle light pulse of any σ_{L0}, as shown in Fig. 3. Using Eqs. (1), (2), and (4), we can get the evolution of partially coherent quasirectangle pulse, and the intensity of the pulse in the medium at any position. The intensity of the quasirectangle pulse has a lot of peaks. The arrival of the first peak (sharply ascending to highest value for the initial pulse) can be considered as the arrival of the pulse (or the signal carried by the pulse) [20]. We study the time difference of the arrival of the first peak of the pulse after propagating through the medium and the vacuum. In Fig. 2, curve *b* is the peak delay time versus σ_{L0} for the quasirectangle pulse. When the coherence decreases (σ_{L0} decreases), the propagation changes from superluminal to subliminal. The behavior is similar to that of curve *a*.

When σ_{L0} becomes smaller from a large value (almost infinity), the peak delay time first increases to a maximum and then decreases to zero as σ_{L0} goes to zero. As well known, the spectrum width $\Delta\omega$ of a coherent pulse is related to its duration ΔT as $\Delta\omega \approx 1/\Delta T$. For a partially coherent pulse, we do not have its spectrum. However, we can use $W(\omega,\omega)$ as an equivalent spectrum [30], which depends on the correlation time width σ_{L0} and its duration ΔT. For the partially coherent Gaussian pulse, we can obtain from Eq. (8) the equivalent power spectrum width $\Delta\omega = \sqrt{(1/\sigma_{\tau 0})^2 + (1/\sigma_{L0})^2}$. The equivalent spectrum width increases as the correlation time decreases. When the correlation time decreases so that the equivalent power spectrum of the pulse covers the whole normal dispersion region besides the central anomalous dispersion region, the peak delay time reaches a maximum value. When the correlation time decreases continuously so that the equivalent power spectrum width reaches the flat dispersion region outside the normal dispersion region, the peak delay time decreases again.

IV. DISCUSSION OF THE ROLE OF THE REAL AND IMAGINARY PARTS OF THE SUSCEPTIBILITY

In the above section, we have shown that the coherence of the light pulses plays a very important role in superluminal propagations, and superluminal propagation is a wave interference phenomenon. Reducing the coherence, the superluminal will disappear. Meanwhile, we note that the superluminal propagation is always accompanied by amplification (due to the gain of the medium). Now we examine what the role of the amplification is (i.e., what is the role of the imaginary part of the susceptibility?). In the following, we will show that the gain (or absorption) of the medium is not important for superluminal propagation, although it cannot be removed for a practical medium.

A. Fully coherent Gaussian pulses

In order to examine the effect of the gain on pulse propagation, we consider three situations: (1) Only keep the imaginary part of the susceptibility (pure amplification or absorption); (2) only keep the real part of the susceptibility (pure dispersion); (3) keep both parts (real medium). In Fig. 4(a), we plot the profiles of the pulse at the exit end for the three situations together with the profile after passing through the vacuum. The susceptibility is still given by Eq. (5) with the same parameters as in the above section. Comparing curve *b* (for the pure amplification) and curve *d* (for the vacuum), we find their peak positions are the same and the amplification for the front and tail is symmetrical [see Fig. 4(b)]. Curve *a* (for the pure dispersion) and curve *c* (for the real medium) have the same peak advancement. Curve *b* (for the pure amplification) does not have the peak advancement. Therefore, the amplification is not an essential factor for the advancement of the peak. In Ref. [33], the authors divided the time delay (defined by the time "center of mass" not by the peak time delay) into two parts: "the net group delay" and "the

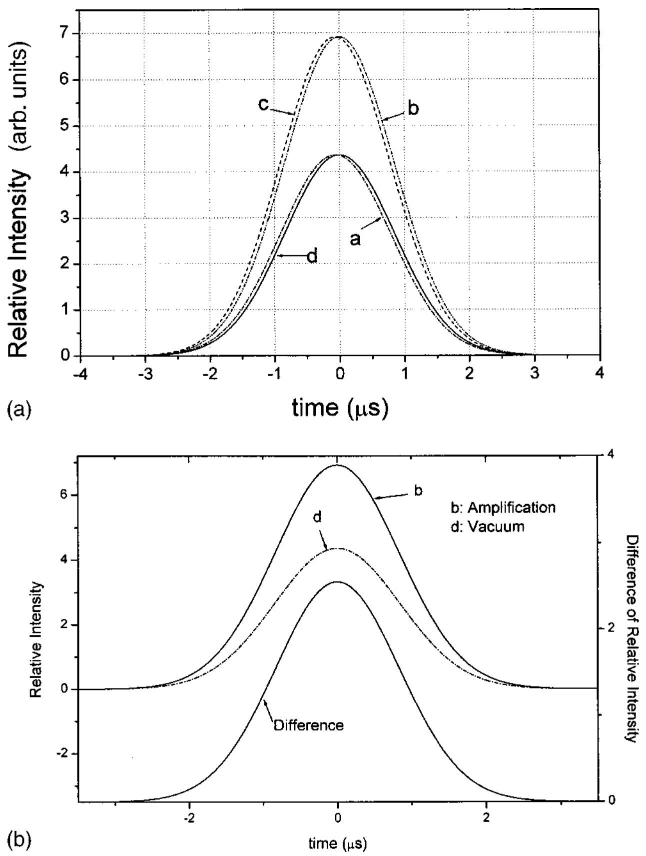

FIG. 4. A coherent Gaussian pulse through the medium with gain. (a) The profiles of the pulse at the exit end for the three situations together with the profile after passing through the vacuum. Curve *a* for the pure dispersion, curve *b* for the pure amplification, curve *c* for the real medium, and curve *d* for the vacuum. (b) The difference between the pulses through the pure amplification and the vacuum.

reshaping delay.'' In current case, due to the symmetry of the spectrum of the pulse around the central frequency of the gain medium, the amplification of each frequency component in the medium is also symmetrical. Consequently the symmetry of the pulse profile would be kept, if we only consider the imaginary part of the susceptibility. Therefore, the reshaping delay induced by the amplification (or attenuation) of the medium is always zero for the Gaussian pulses. The effect of amplification induced by the imaginary part of the susceptibility only makes the pulse compressed or broadened. The real part of the susceptibility (i.e., the anomalous dispersion) plays the essential role in the advancement of pulses through such a gain medium.

In Fig. 5, we compare the shape of the pulse after passing through the medium with the initial shape. The pulse passing through the medium is rescaled so that it contains the same energy as the initial pulse. We align the two pulses' centers to examine the shape difference between the two pulses. We find that the pulse is compressed slightly (distorted), because the amplification at different part of the pulse is different, although it is still symmetrical with respect to the center of the Gaussian pulse.

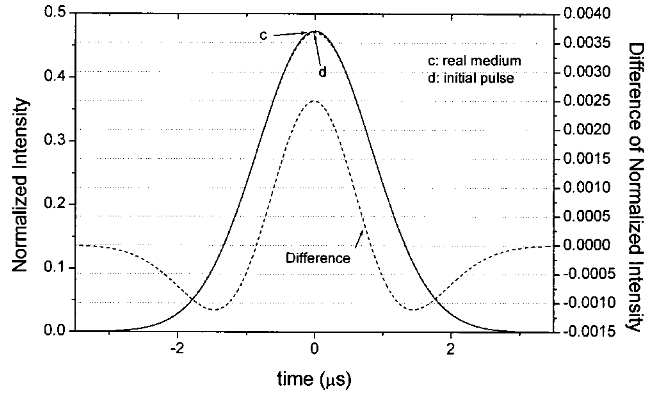

FIG. 5. Comparison between the shapes of the pulses after passing through the medium with the initial shape. The shape of pulses is normalized by the areas of the pulse being equal to 1.

B. Fully coherent quasirectangle pulses

In Fig. 6, we show the three situations as discussed above for the fully coherent quasirectangle pulse. In order to show the effects clearly, we choose $M/2\pi = 5.26$ Hz , and other

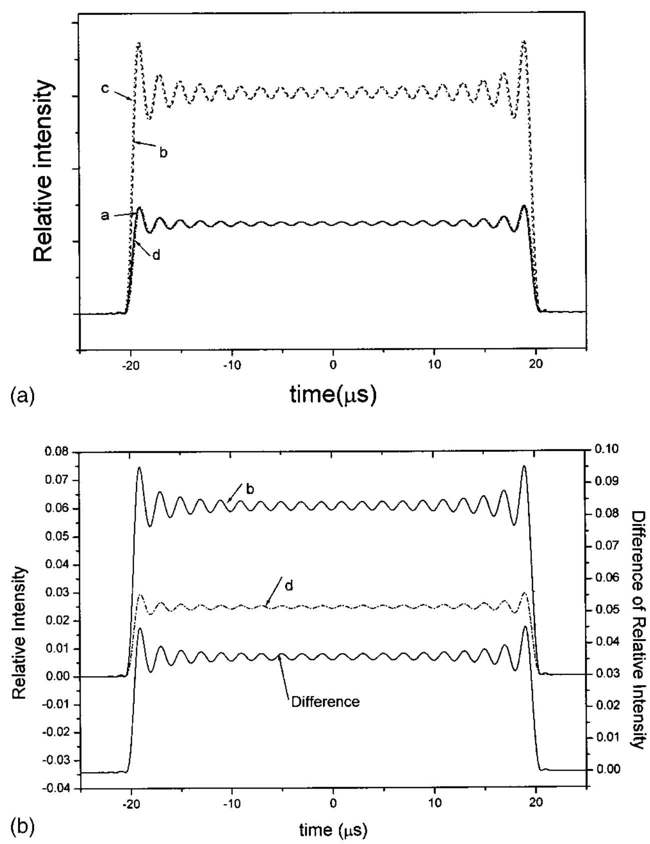

FIG. 6. A coherent quasirectangle pulse through the medium with gain, where $M/2\pi = 4.524$ Hz and others parameters are the same as before. (a) The profiles of the pulse at the exit end for the three situations together with the profile after passing through the vacuum: Curve *a* for the pure dispersion, curve *b* for the pure amplification, curve *c* for the real medium, and curve *d* for the vacuum. (b) The difference between the pulses through the pure amplification and the vacuum.

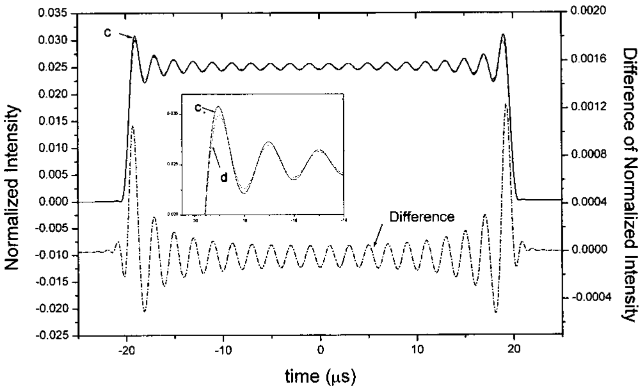

FIG. 7. Comparision between the shapes of the quasirectangle pulses after passing through the medium with the initial shape. The shape of pulses is nomalized by the areas of the pulse being equal to 1.

parameters are the same as before. In the figure, curves *a*, *b*, *c*, and *d* are the cases of the pure dispersion, the pure amplification, the real medium and the vacuum, respectively. Carefully comparing curve *b* (for the pure amplification) and curve *d* (for the vacuum), we find that those peaks in the front half of the pulse in curve *b* are shifted forwards, and the back half peaks in curve *b* are shifted backwards. The amount of forward shift and backward shift is symmetrical [see Fig. 6(b)], and consequently curve *b* is still symmetrical and slightly broadened. The maximum amount of 22.5 ns shift is for the first peak and the last peak. This effect is similar to the reshaping delay [33]. Comparing curve *a* (for the pure dispersion) and curve *d*, we find that all the peaks of curve *a* are shifted forwards by the same amount (127.5 ns), which are much larger than the shift (22.5 ns) induced by the pure amplification. From curve *c* (for real medium), we find that all peaks are advanced, and the first peak has the maximum advancement (about 150 ns) and the last peak has the minimum advancement (about 97.5 ns). Therefore, the amplification (or absorption) is not an essential factor for the advancement of the peaks. In Fig. 7, we compare the shape of the pulse after passing through the medium with the initial shape. Here we also align the two pulses' centers. It is clear that the shape is changed. Each peak is compressed.

Here we emphasize that, the shape distortion of pulses can be divided into two parts: the symmetrical distortion and the nonsymmetrical distortion. The former is due to the amplification (or attenuation) (i.e., the imaginary part of the susceptibility). The symmetrical distortion could not lead to the advancement of the peaks. The nonsymmetrical distortion,

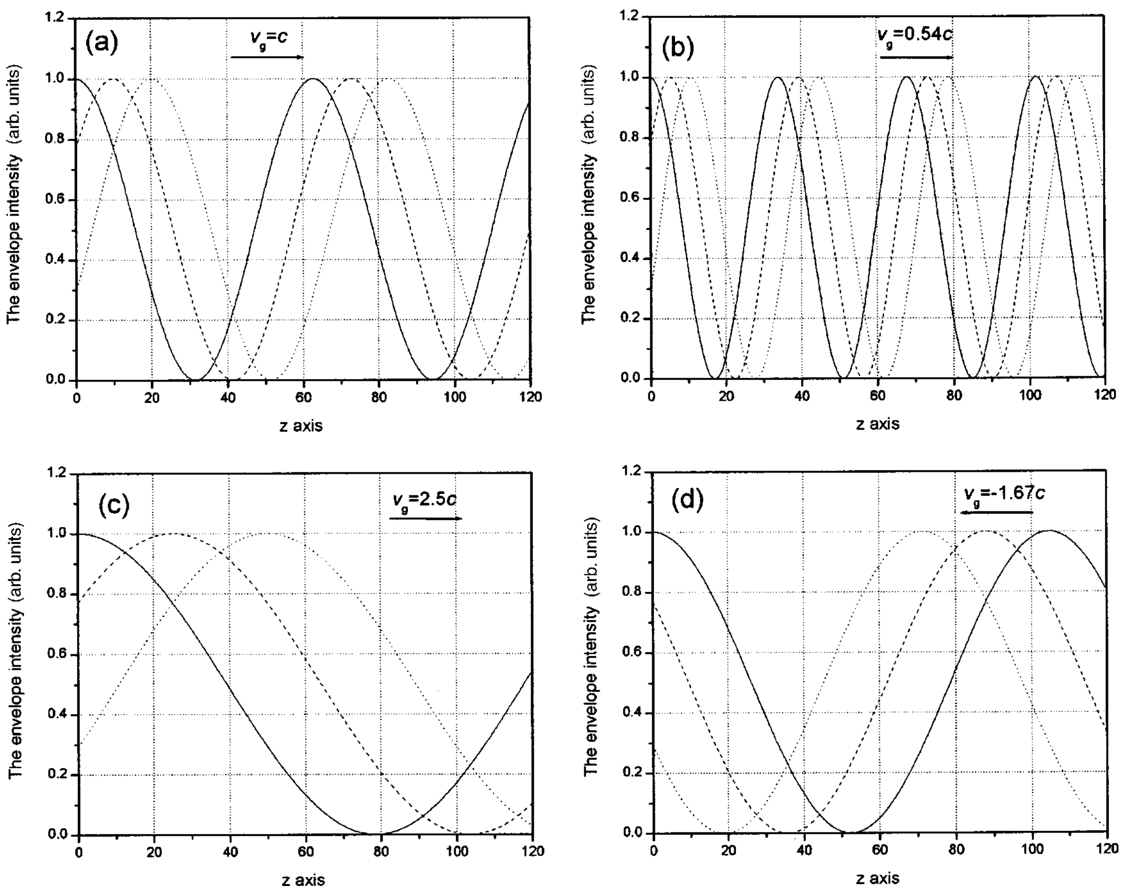

FIG. 8. The evolution of the envelope intensity under different dispersions. Here $\omega_2 = 0.9\omega_1$. The solid line shows the waveform at $t = 0$ s, and the dash line shows the waveform at $t = 10$ s, and the dotted line shows the waveform at $t = 20$ s. (a) In the vacuum ($n_1 = n_2 = 1$); (b) in a normal dispersive medium, $n_1 = 1.4$ and $n_2 = 1.35$; (c) in a weak anomalous dispersive medium, $n_1 = 1.3$ and $n_2 = 1.4$; (d) in a strong anomalous dispersive medium, $n_1 = 1.2$ and $n_2 = 1.4$. Note that the *z* axis is in the scale of 3×10^8 m.

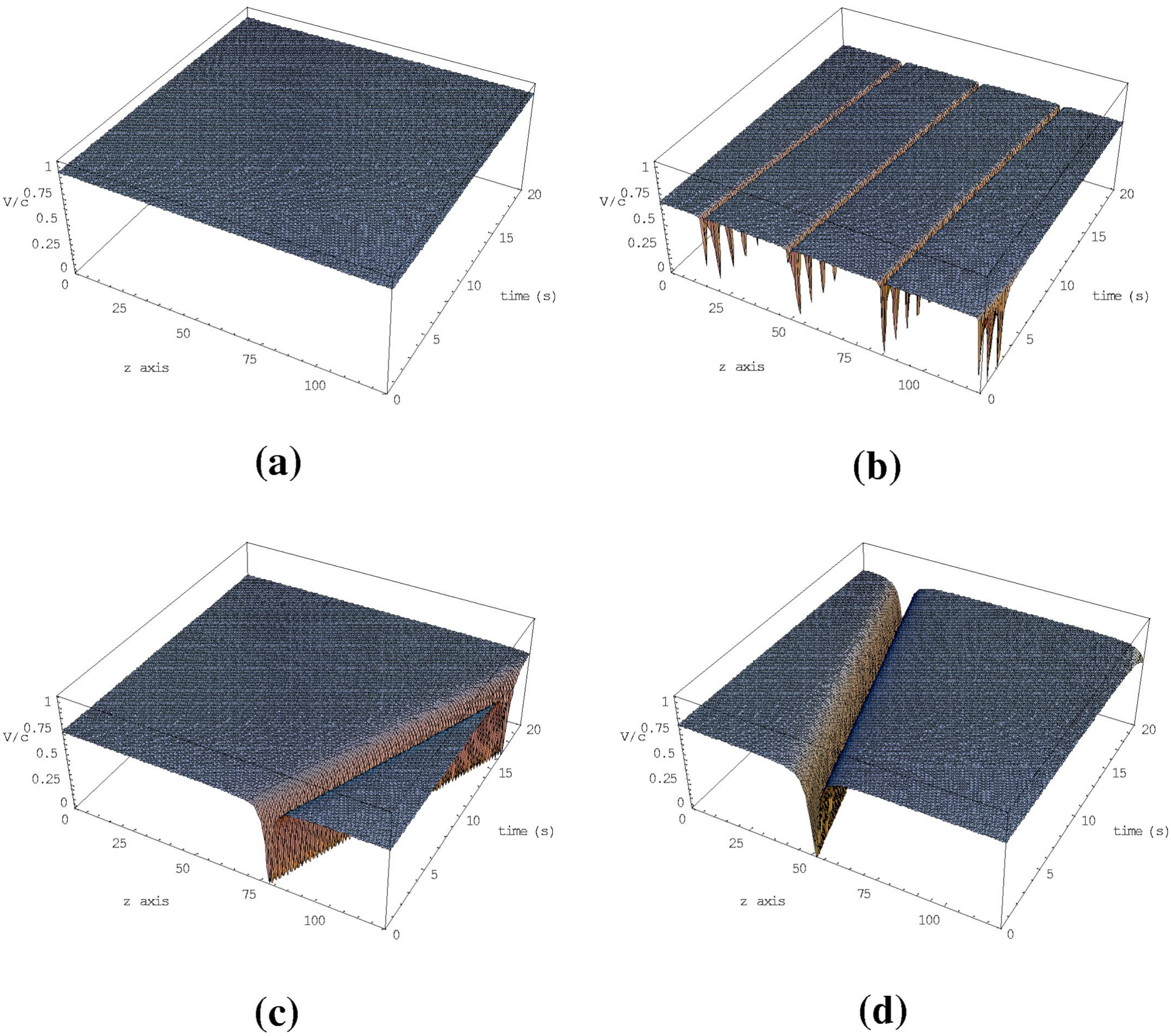

FIG. 9. The energy velocity of a two-frequency-component plane wave propagtion under different dispersions. (a) In the vacuum ($n_1 = n_2 = 1$); (b) in a normal dispersive medium, $n_1 = 1.4$ and $n_2 = 1.35$; (c) in a weak anomalous dispersive medium, $n_1 = 1.3$ and $n_2 = 1.4$; (d) in a strong anomalous dispersive medium, $n_1 = 1.2$ and $n_2 = 1.4$. Note that the z axis is in the scale of 3×10^8 m.

which is essential for the superluminal propagation, is due to high order dependence of the wave vector on the frequency (the real part of the susceptibility).

V. GROUP, ENERGY, AND FRONTAL VELOCITIES OF PULSES

In order to better understand the group velocity (the envelope evolution) and energy velocity of a pulse, we first consider the simplest example of the superposition of two monochromatic plane waves passing through different dispersion materials (assuming zero gain or absorption for both waves). Then we will discuss the group velocity of a pulse as a function of frequency in different dispersion media. We would emphasize that in our calculation, all orders of the dispersion effect are taken into account since the dispersion relation of $k(\omega)$ has not been expanded into the Taylor series.

Two monochromatic plane waves propagate in z direction with frequencies ω_1, ω_2, and wave numbers k_1, k_2 in a dispersion medium. The electric and magnetic fields of the two waves are

$$\vec{E}_i(z,t) = \vec{E}_0 \exp[i(k_i z - \omega_i t)],$$

$$\vec{H}_i(z,t) = \sqrt{\frac{\varepsilon_i}{\mu_i}} \vec{H}_0 \exp[i(k_i z - \omega_i t)], \tag{11}$$

where ε_i and μ_i ($i = 1,2$) are the corresponding permittivities and permeabilities at frequencies ω_1 and ω_2. The total electric field and intensity $I(z,t)$ are as follows:

$$\vec{E}(z,t)=\vec{E}_1(z,t)+\vec{E}_2(z,t)$$
$$=\vec{E}_0\{\exp[i(k_1z-\omega_1t)]+\exp[i(k_2z-\omega_2t)]\}$$
$$=2\vec{E}_0\cos\left[\frac{1}{2}(\omega_1-\omega_2)t-\frac{1}{2}(k_1-k_2)z\right]$$
$$\times\cos\left[\frac{1}{2}(\omega_1+\omega_2)t-\frac{1}{2}(k_1+k_2)z\right], \tag{12}$$

$$I(z,t)=\langle\vec{E}^*(z,t)\vec{E}(z,t)\rangle$$
$$=|E_0|^2\{2+2\cos[(k_1-k_2)z-(\omega_1-\omega_2)t]\}, \tag{13}$$

where $\langle\cdots\rangle$ represents a time average [much longer than $1/(\omega_1+\omega_2)$ and much shorter than $1/|\omega_1-\omega_2|$]. The group velocity describes the evolution of the peak of the amplitude envelope or of the peak of the intensity envelope,

$$v_g=\frac{\delta\omega}{\delta k}=\frac{\omega_1-\omega_2}{k_1-k_2}=\frac{c}{n_1}\left(\frac{\omega_1-\omega_2}{\omega_1-\frac{n_2}{n_1}\omega_2}\right), \tag{14}$$

where $n_i=ck_i/\omega_i$ $(i=1,2)$ are the refraction indices.

In Fig. 8 we plot the evolutions of the intensity envelope under different dispersion conditions. We assume that the medium is nonmagnetic, i.e., $\mu_1=\mu_2=1$. The directions of v_g are indicated by the arrows. In Figs. 8(a)–(c), we can see that the group velocities are equal to, less than, and larger than the light speed in the vacuum. Furthermore, in Fig. 8 (d), the group velocity becomes negative, i.e., the peak of the intensity envelope is propagating backward instead of forward. Since the two monochromatic waves are propagating forward, and the energy velocity of the compound wave must be forward. Therefore, the group velocity is not the energy velocity of the compound wave.

The energy velocity is defined by [34,26]

$$\vec{v}_e=\frac{\vec{S}}{w_e+w_m}, \tag{15}$$

where $\vec{S}$ is the Poynting vector and w_e and w_m are the electric energy density and the magnetic energy density. In the case of two monochromatic waves, the Poynting vector has the form of the compound wave after the average:

$$\vec{S}=\frac{c}{4\pi}[\vec{E}\times\vec{H}^*]$$
$$=\frac{c}{4\pi}\left(\sqrt{\frac{\varepsilon_1}{\mu_1}}+\sqrt{\frac{\varepsilon_2}{\mu_2}}\right)$$
$$\times E_0H_0\{1+\cos[(k_1-k_2)z-(\omega_1-\omega_2)t]\}\vec{k}, \tag{16}$$

where $\vec{k}$ is a unit vector and indicates the direction of the energy flux of each composite wave. After the average the energy densities in this case are

$$w_e=\frac{1}{8\pi}[\vec{E}\cdot\vec{D}^*]$$
$$=\frac{1}{8\pi}[(\vec{E}_1+\vec{E}_2)\cdot(\varepsilon_1\vec{E}_1^*+\varepsilon_2\vec{E}_2^*)]$$
$$=\frac{1}{8\pi}(\varepsilon_1+\varepsilon_2)E_0^2\{1+\cos[(k_1-k_2)z-(\omega_1-\omega_2)t]\}, \tag{17a}$$

$$w_m=\frac{1}{8\pi}[\vec{B}\cdot\vec{H}^*]$$
$$=\frac{1}{8\pi}[(\mu_1\vec{H}_1+\mu_2\vec{H}_2)\cdot(\vec{H}_1^*+\vec{H}_2^*)]$$
$$=\frac{1}{8\pi}\left\{(\varepsilon_1+\varepsilon_2)H_0^2+\sqrt{\varepsilon_1\varepsilon_2}\left(\sqrt{\frac{\mu_1}{\mu_2}}+\sqrt{\frac{\mu_2}{\mu_1}}\right)\right.$$
$$\left.\times H_0^2\cos[(k_1-k_2)z-(\omega_1-\omega_2)t]\right\}. \tag{17b}$$

Substituting Eqs. (16), (17a), and (17b) into Eq. (15) we have

$$\vec{v}_e=\frac{2c\left(\sqrt{\frac{\varepsilon_1}{\mu_1}}+\sqrt{\frac{\varepsilon_2}{\mu_2}}\right)}{(\varepsilon_1+\varepsilon_2)+\frac{(\varepsilon_1+\varepsilon_2)+\sqrt{\varepsilon_1\varepsilon_2}\left(\sqrt{\frac{\mu_1}{\mu_2}}+\sqrt{\frac{\mu_2}{\mu_1}}\right)\cos[(k_1-k_2)z-(\omega_1-\omega_2)t]}{1+\cos[(k_1-k_2)z-(\omega_1-\omega_2)t]}}\vec{k}. \tag{18}$$

In Fig. 9, we plot the energy velocity as a function of time t and position z. The energy velocity $\vec{v}_e$ of the compound wave is not a constant. It varies with the time and position due to the interference of the two plane waves. From Eq. (18), we can prove that the energy velocity is always positive and always equal to or less than the light speed in the vacuum on the condition that the permittivities ε_i of the medium are larger than unit. That is, to say the group velocity is not the energy velocity of the wave. They are the same only in the case of the vacuum (or the nondispersion media). They are always different in the dispersion medium even for a normal dispersion one.

In the above case, it is too simple and ideal but much more explicit. Now, we still consider the case discussed in

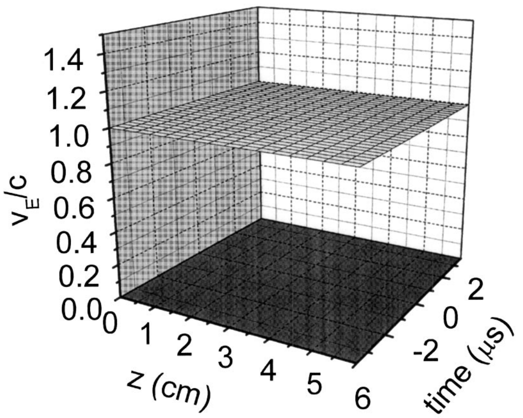

FIG. 10. The energy velocity of a completely coherent Gaussian pulse through the two-gain medium. The relative parameters are the same as in Fig. 1.

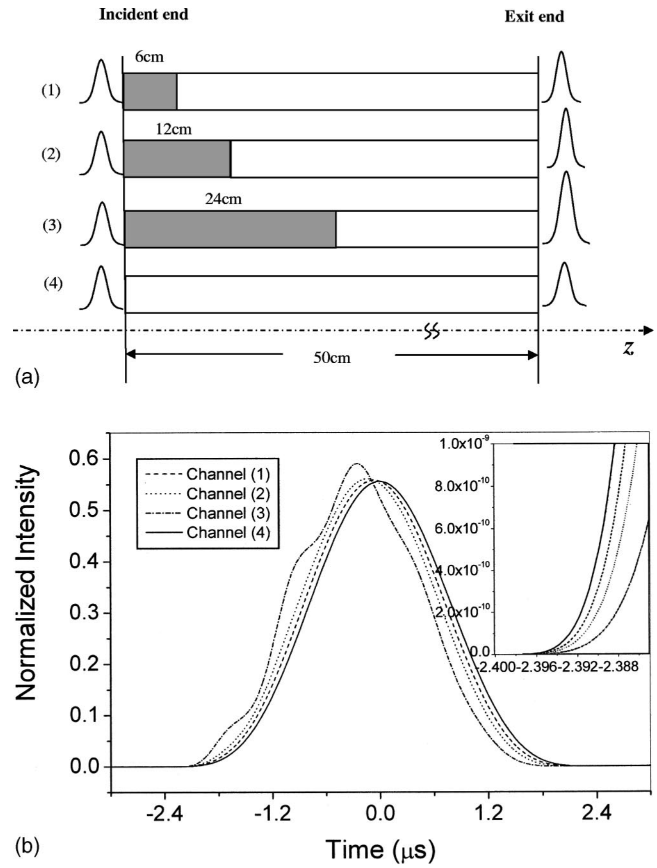

FIG. 11. The evolution of such pulse through four different channels. (a) is the schematic diagrams of these channels: (1) with a 6 cm medium, (2) with a 12 cm medium, (3) with a 24 cm medium, and (4) with a 50 cm free space, and (b) shows the normalized intensity profiles of such pulse at the exit end when it passed through these different channels, and the inset is the amplification of the front leading of the pulse. The other parameters of the medium are the same as in Fig. 1.

Sec. III. The medium can be seen as a single-layer photonic crystal. By using the method developed in Ref. [23], for a coherent pulse, we obtain the energy velocity from Eq. (15). Here we would like to emphasize that both the transmitted and reflected electromagnetic fields are included in the calculation. (The result is almost the same if the reflected field is neglected.) In Fig. 10, we plot the energy velocity for the fully coherent Gaussian pulse propagating in the anomalous dispersion medium with a gain as discussed in Sec. III. The energy velocity is approximately equal to *c*. No superluminality for the energy velocity of the wave propagation occurs in such a medium. Obviously, the group velocity is also not the energy velocity of the pulse (wave).

Although the intensity envelope is proportional to the energy density, the energy velocity is not equal to the group velocity. The energy density in the medium and at the exit end comes from two contributions: One from the incoming electromagnetic field and another from the energy preserved in the medium [36]. The energy velocity determined by Eq. (15) is the propagating velocity of the electromagnetic field energy of the wave only. Hence the group and energy velocities are different.

Now we turn to consider the velocity of pulse's front [2,14,15,34,35]. Here we consider the initial incident light pulse has the following form:

$$E(0,t)=\begin{cases}1+\cos\left(\dfrac{\pi t}{2\tau_0}\right) & \text{when}-2\tau_0\leqslant t\leqslant 2\tau_0\\ 0 & \text{otherwise.}\end{cases}\tag{19}$$

The pulse has well-defined start and end ($\tau_0=1.2\ \mu$s), and there is no singularity in the pulse envelop. The coherent pulse passes through the mxedium considered in Sec. III. In order to make our calculation convinced, we take into account both the transmitted wave and reflected wave in the medium by using the method of Ref. [23], which can include all orders of the dispersion. Figure 11 shows the evolution of such pulse through four different channels with different length of the anomalously dispersive medium. Figure 11(a) shows the schematic diagrams of four channels: (1) 6-cm-long medium +44-cm-long vacuum; (2) 12-cm-long medium + 38-cm-long vacuum; (3) 24-cm-long medium +26-cm-long vacuum; (4) 50-cm-long vacuum, and Fig. 11(b) shows the normalized intensity profiles of such pulse at the exit end when it passes through these four different channels, while the inset figure is the amplification of the front leading of the pulse. From these curves, we obtain the time delays of the pulse peak through each channel are: (1) $t_d=63$ ns, (2) $t_d=132$ ns, (3) $t_d=249$ ns, and (4) $t_d=0$, respectively. From the insert figure, we can find that the front leading ends of the pulses propagate with the same velocity *c*, although the peaks of the pulses propagate in the medium with a negative group velocity due to anomalous dispersion. The peak in the third channel emerges first at the exit end. However, the shape of the pulse from the third channel is greatly distorted. The time advancement of the peak can never be larger than the time interval between the start point and the peak of the original pulse (here it is 2.4 μs). Therefore, the distance of such pulse with a front start point passing the medium is limited if we want to preserve the pulse shape (nearly unchanged) after passing through the medium.

We have shown that the front leading end is always propagating at the speed of light *c* (no superluminality); the propagation of the pulse's peak is superluminal. We would like to point out that, if the pulse is partially coherent, the front end also propagates at the speed of light *c*, and the superluminal propagation of the peak gradually becomes subluminal as the coherence of the pulse decreases.

VI. CONCLUSION

In the linear media with dispersion and gain (or absorption), each Fourier component of the pulse propagates in a different way. Each Fourier component obtains a phase shift when it propagates and is amplified (in gain media) or attenuated (in absorptive media). However, these phase shifts are not the same, and the amplifications (or attenuation) for different Fourier components are also not identical. Although each Fourier component propagates in a velocity not faster than *c*, the resuperposition of all frequency components at the end of the medium will produce a new pulse which will appear in advance (for anomalous dispersion) or delay (for normal dispersion) compared with the pulse propagating in vacuum through the same distance. When the pulse loses the coherent properties, as the partially coherent pulses discussed above (or pulses containing strong noise), the superluminality disappears. As demonstrated above, the superluminality always exists as long as the spectrum of the coherent pulse is within the anomalously dispersive region whatever the shape of the pulse is, and the evolution of such coherent pulses are always transmitted superluminally. We show that the amplification is not the essential reason for the advancement of the pulse peak, and the amplification is responsible for pulse compression or broadening. The main reason for the advanced peak is the real part of the susceptibility, which leads to the different phase shifts for different frequency components through the medium. Although the peak of the pulses appear in advance, both the energy and frontal velocity of the pulse never exceed the light speed in the vacuum.

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