Continuum traffic model with the consideration of two delay time scales

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This paper presents a continuum traffic model. The derivation of this model is based upon the assumption that the stream velocity u reaches the equilibrium velocity u_e within the relaxation time T, while the equilibrium velocity u_e is adjusted to be attained through the driver's reaction time t_r. It is also assumed that the former delay time scale is greater than the latter. A motion equation with nonconstant propagation velocity of a disturbance in traffic flow is derived that can reflect the anisotropy of disturbance propagation in real traffic, unlike some other higher-order continuum models. It indicates that in our model the undesirable "wrong-way travel" phenomenon and gaslike behavior have been eliminated. The formation and diffusion of traffic shock can be better simulated.

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I. INTRODUCTION

Recently, traffic problems have attracted considerable attention from scientists because of the increasing requirements of traffic construction and management, and also due to a variety of interesting nonlinear phenomena occurring in traffic systems. The theory of traffic flow is a subject using mathematics and physics to describe the characteristics of traffic, in which mathematical models are established and then solved. The first paper about traffic flow can be traced back to the year of 1933, when Kinzer proposed and discussed the applicability of the Poisson distribution to traffic flow [1]. After the Second World War, the theory developed rapidly. In particular, in the 1950s, Lighthill and Whitham, and Richards put forward the kinematics traffic model (called the LWR model later) by introducing the hypothesis of a continuous medium and the continuity equation from fluid dynamics [2,3], and since then traffic flow research has drawn much attention from fluid dynamicists. In 1971, Payne proposed a high-order continuum model that includes the effects of the driver's reaction and acceleration by considering the limiting case of the car-following model [4] and introducing the "motion equation" [5]. Later on, he applied his high-order model to compile the computer simulation program FREFLO [6]. In recent decades, a variety of traffic models, including follow-the-leader models, continuum models, gas kinetic models, cellular automaton models, etc., have been presented and some empirical observations reported (for reviews, see Refs. [7–13]).

The continuum models of traffic flow aim to describe, through a system of partial differential equations, the evolution of traffic states such as the flow rate $q(x,t)$, the vehicle density $\rho(x,t)$, and the travel velocity or mean velocity $u(x,t)$ over space x and time t. A number of continuum traffic flow models have been proposed over these decades. Among them, the LWR theory [2,3] is the earliest and most fundamental. The model is a continuity equation describing the conservation of vehicle number:

$$\frac{\partial\rho(x,t)}{\partial t}+\frac{\partial q(x,t)}{\partial x}=0. \tag{1}$$

Because the equilibrium relation between the velocity and density is introduced in the LWR model, it is impossible to describe correctly the traffic flows in nonequilibrium states, such as phantom traffic jams or stop-and-go waves [7,14,15] and forward propagation of disturbances in heavy traffic [16]. Thus, there were no equilibrium curves $(\rho,u_e(\rho))$ in the fundamental diagram obtained by various early observations when traffic was not in equilibrium. Moreover, acceleration and deceleration flows follow distinctively different paths in the phase plane (ρ,u), and these paths usually form one or more hysteresis loops [17]. To solve these problems, Payne complemented the continuity equation by a dynamic velocity equation [5], derived from Newell's car-following model [18] by means of the Taylor expansion. He claimed that the velocity of traffic flow will reach equilibrium with the reaction time T, and the velocity at the location (x,t) is determined by the traffic density at location $(x+\Delta x,t)$ due to drivers' anticipation of traffic conditions ahead. His dynamic velocity equation is

$$\frac{du}{dt}=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{1}{T}[u-u_e(\rho)]-\frac{v}{\rho T}\frac{\partial\rho}{\partial x}, \tag{2}$$

where the first term on the right side, called the relaxation term, describes the adaptation of the average velocity $u(x,t)$ to the density-dependent equilibrium velocity $u_e(\rho)$. This adaptation is exponential in time with the driver reaction time T. The second term, called the anticipation term, reflects the reaction of identical drivers to the traffic situation in their surroundings, where $v=(1/2)du_e(\rho)/d\rho$ is called the anticipation exponent. Payne's model does not require the velocity to satisfy the equilibrium relation and allows some departure from it. So it can depict real traffic rather well in some respects. However, Daganzo [19] and del Castillo *et al.* [20] pointed out that the two equations (1) and (2) violate the following principles: A fluid particle responds to stimuli

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from the front and from behind, but a car is an anisotropic particle that mostly responds to frontal stimuli.

From Eqs. (1) and (2), one can derive the two characteristic velocities $\lambda_1 = u - \sqrt{v/T}$ and $\lambda_2 = u + \sqrt{v/T}$, which determine how traffic disturbances are propagated in a traffic stream. Note that the second characteristic velocity λ_2 is larger than the velocity u of the traffic flow. This means that waves associated with the second characteristic velocity always reach vehicles from behind, either slowing traffic down or speeding traffic up. This evidently violates the anisotropy of traffic flow and does not accord with real traffic. The main reason is that these models tried to mimic the gas dynamic equations and led to so-called gaslike behavior. Aw and Rascle proposed a model to suppress the gaslike behavior by replacing the space derivative of the density with a convective derivative [21]. The model is a system of hyperbolic equations with no diffusion and no relaxation. Zhang [22] derived a macroscopic equation with anisotropy and no relaxation from Pipes's car-following model [4]. Jiang *et al.* [23] also presented a macroscopic equation with anisotropy derived from a car-following model by adding the relative velocity. But the propagation velocity of disturbance in this model is constant independent of the density. In fact, the propagation velocity of disturbance in a Payne-Whitham-like model should depend on the density [24,25]. In this paper, we attempt to improve these anisotropic continuum traffic models. The paper is organized as follows. In Sec. II, we establish a continuum model with anisotropy by introducing the drivers' reaction time t_r and the vehicle relaxation time T to the forward anticipation velocity; thus the propagation velocity of a small disturbance results that is related to the density. We then analyze the characteristics of our model in Sec. II. A numerical simulation is described in Sec. IV, which validates the correctness of the theoretical analysis. All of the results indicate that gaslike behavior does not exist in our model, and the nonequilibrium phase transition and nonlinear dynamical phenomena are correctly revealed.

II. MODELING AND ANALYSIS

Payne's assumption of forward anticipation [5] is important and reasonable, but there exists the drawback that the velocity at location (x,t) was assumed to be determined by the velocity-density relation at location $(x+\Delta x,t)$ in front, and to be reached instantaneously. According to the car-following theory, the adjustment of the state of traffic flow is performed in a certain vehicle relaxation time T. The reaction time t_r of the drivers is not included, which differs from the relaxation time varying with headway [26,27]. The relaxation time includes mechanical delay of the vehicles as well as the drivers' reaction time t_r, about 1 s [28]. When the traffic is in a near-jamming state, the vehicle relaxation time can approach the driver reaction time. According to the data given by the National Safety Council of the United States, the average reaction time of drivers is 0.75 s. Hence, we determine the effect of reaction time on traffic. We consider that the forward anticipation of velocity u is accomplished in the relaxation time T, while this process is completed by an adjustment with driver reaction time t_r to the anticipated one u_e. The stream velocity u in the relaxation time T just reaches the anticipated value u_e corresponding to the density in time $t+t_r$ at the location $x+ut_r$. Thus, we deduce the following relation:

$$u(x+uT,\ \ t+T) = u_e(\rho(x+ut_r, t+t_r)). \tag{3}$$

Using the Taylor series expansion for Eq. (3) and neglecting higher-order terms,

$$u(x,t) + \frac{\partial u}{\partial t}T + uT\frac{\partial u}{\partial x} + O(\tau^2) = u_e(\rho) + \frac{du_e}{d\rho}\left(t_r\frac{\partial \rho}{\partial t} + ut_r\frac{\partial \rho}{\partial x}\right) + O(t_r^2). \tag{4}$$

Equation (4) can be rearranged as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{u_e(\rho)-u}{T} + \frac{t_r}{T}\frac{du_e}{d\rho}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x}\right). \tag{5}$$

Applying Eq. (1), we have

$$\frac{\partial u}{\partial t} + \left(u + \rho\frac{t_r}{T(\rho)}\frac{du_e}{d\rho}\right)\frac{\partial u}{\partial x} = \frac{u_e(\rho)-u}{T(\rho)}, \tag{6}$$

where $c = -\rho[t_r/T(\rho)]du_e/d\rho \geqslant 0$ is the "sonic speed" in traffic flow, at which small disturbances propagate relative to a moving traffic stream. $T(\rho)$ is the relaxation time, which is a nonlinear function of the density ρ and can be expressed as [29]

$$T(\rho) = t_r\left[1 + \frac{E}{1+(\rho/\rho_m)}\right], \tag{7}$$

where ρ_m is the critical density, t_r is the constant reaction time, θ is a parameter $(\theta>0)$, and E is a constant $(E>0)$ that denotes the difference of reaction times between congested and uncongested traffic situations. As $\rho \to 0$, $T(\rho) \to (E+1)t_r$; while as $\rho > \rho_m$, $T(\rho) \to t_r$. This means that at high density levels the relaxation time is smaller, and at low density levels the relaxation time is larger. Moreover, del Castillo and Bentez [30] have investigated the general velocity-density model and attempted to provide a general characterization of velocity-flow relationships. Two were obtained. One is the exponential curve given by

$$u_e = u_f\left\{1 - \exp\left[\frac{c_{\mathrm{jam}}}{u_f}\left(1 - \frac{\rho_{\mathrm{jam}}}{\rho}\right)\right]\right\}. \tag{8a}$$

The other, called the maximum sensitivity curve, is expressed by

$$u_e = u_f\left(1 - \exp\left\{1 - \exp\left[\frac{c_{\mathrm{jam}}}{u_f}\left(\frac{\rho_{\mathrm{jam}}}{\rho} - 1\right)\right]\right\}\right), \tag{8b}$$

where u_f is the free-flow velocity; c_{jam} is the propagation velocity of a disturbance under the jam density ρ_{jam}. Both relationships satisfy the properties of traffic flow, where the free-flow velocity is the limit of the desired velocity, vehicles are stopped at the jam density, and the velocity decreases with increasing density. Thus we have established a continuum model with the propagation velocity of small disturbances depending on the traffic density, which is consistent with other high-order Payne-Whitham-like models. It is comprised of two partial differential equations as follows:

$$\frac{\partial \rho(x,t)}{\partial t}+\frac{\partial q(x,t)}{\partial x}=0, \tag{9a}$$

$$\frac{\partial u}{\partial t}+(u-c)\frac{\partial u}{\partial x}=\frac{u_e(\rho)-u}{T(\rho)}, \tag{9b}$$

with

$$c=-\rho\frac{t_r}{T(\rho)}\frac{du_e}{d\rho}\geqslant 0.$$

If the difference between the relaxation and reaction terms is not considered, that is, $T(\rho)=t_r$, Eq. (9b) will be reduced to the anisotropic higher-order model developed by Zhang [22], but the "sonic speed" in our model is directly derived without any assumptions. So this model might have a universal meaning. We can analyze the characteristics of Eqs. (9a) and (9b), and obtain the characteristic velocity and corresponding eigenvectors

$$\lambda_1=u, \quad \lambda_2=u-c, \tag{10a}$$

$$r_1=\begin{pmatrix}1\\ u_e'(\rho)\end{pmatrix}, \quad r_2=\begin{pmatrix}1\\0\end{pmatrix}. \tag{10b}$$

The model of Eqs. (9a) and (9b) is therefore strictly hyperbolic. Note that the motion equation in the model does not explicitly depend on the density gradient. Because $c\geqslant 0$, the characteristic velocity is no greater than the traffic velocity. We can validate the anisotropy of traffic flow using the following example constructed by Daganzo [19]: We try to find the traffic evolution, if at the initial instant

$$u=0, \quad \rho=\rho_{\max}H(x), \quad \forall \ x\leqslant A, \quad t=0 \quad (A>0),$$

$$u=0, \quad x=A, \quad t>0, \tag{11}$$

where $H(x)$ is the Heaviside unit step function, and $\rho_{\max}$ is the maximum density. Under these initial conditions, the correct solution should be that nothing would happen, namely, vehicles do not move and $du/dt=0$. Substituting $du/dt=0$ in Eq. (9b), we have

$$u(x,t)=u_e\geqslant 0. \tag{12}$$

This means the vehicles always move forward and there is no backward-travel problem in our model.

III. QUALITATIVE PROPERTIES OF THE MODEL

Before the numerical simulations are carried out, we will analyze the disturbance propagation and linear stability in our model.

A. Disturbance propagation

To study how a small disturbance propagates, we assume an equilibrium solution (ρ_0,u_0) with a small disturbance $(\xi(x,t),w(x,t))$, where $(\xi(x,t),w(x,t))$ are sufficiently smooth functions of x and t. Substituting the small disturbance into Eq. (9b), using the Taylor series expansions and neglecting higher-order terms of $\xi(x,t)$, $w(x,t)$, we get

$$\begin{pmatrix}\xi\\ w\end{pmatrix}_t+\begin{pmatrix}u_0 & \rho_0\\ 0 & u_0-c_0\end{pmatrix}\begin{pmatrix}\xi\\ w\end{pmatrix}_x=\begin{pmatrix}0\\ \dfrac{u_e'(\rho_0)\xi-w}{T(\rho)}\end{pmatrix}, \tag{13}$$

where $c_0=-\rho_0u_e'(\rho_0)t_r/T(\rho)$. By eliminating w from Eq. (13) we can obtain

$$\tau[\partial_t+(u_0-c_0)\partial_x](\partial_t+u_0\partial_x)\xi=\partial_t\xi+C(\rho_0)\partial_x\xi, \tag{14}$$

where $C(\rho_0)=(\rho_0u_e'(\rho_0)t_r/T(\rho)+u_0)$ and $(\partial_t+u_0\partial_x)\xi=\partial_t\xi+u_0\partial_x\xi$ is the wave operator, where ∂_t and ∂_x denote partial derivatives with respect to time and space. Equation (14) implies that any small disturbance in the model propagates in the form of two waves: the slower wave travels at a velocity u_0-c_0 and the faster wave travels at the velocity u_0 of the traffic stream that carries it. The propagating velocities of the two waves mean that the disturbances are carried downstream by the vehicles that generated them and, on the other hand, propagate upstream through a line of vehicles that are behind the disturbance-generating vehicles. Neither wave travels faster than the traffic that carries it. The model is therefore anisotropic, and we shall call it the anisotropic nonequilibrium traffic model.

B. Linear stability analysis

We conduct a linear stability analysis of Eqs. (9a) and (9b) governing the development of the disturbances $(\xi_0(x,t),w_0(x,t))$ of the quantities (ρ,u) from a certain equilibrium value (ρ_0,u_0). Introducing a special form of the small disturbances,

$$\xi(x,t)=\xi_0\exp(\omega t-ikx), \quad w(x,t)=w_0\exp(\omega t-ikx),$$

in the linearized equations, we obtain

$$\xi_0(\omega-u_0ik)-\rho_0w_0ik=0,$$

$$w_0\left(\omega-(u_0-c_0)ik+\frac{1}{T(\rho)}\right)=\frac{u_e'(\rho_0)}{T(\rho)}\xi_0. \tag{15}$$

Eliminating ξ from Eq. (15) yields

$$(\omega-u_0ik)^2+c_0(\omega-u_0ik)ik+\frac{(\omega-u_0ik)}{T(\rho)}=\frac{\rho_0}{T(\rho)}u_e'(\rho_0)ik. \tag{16}$$

When $\mathrm{Re}(\omega)\geqslant 0$, the traffic flow is in a stable state. We have

$$u_0-c_0\geqslant u_0+\rho_0u_e'(\rho_0). \tag{17}$$

Thus, the condition of stability is

$$c_0 \leqslant -\rho_0 u_e'(\rho_0) \Rightarrow t_r \leqslant T. \tag{18}$$

The criterion of stability indicates that the flow is stable when drivers can respond to the changes ahead within the reaction time; otherwise the flow is unstable, which is consistent with one of Kühne *et al.*'s models at $\mu=0$ [31,32]. Unstable traffic will lead to the occurrence of a traffic flow pattern like stop-start waves or a spontaneous traffic jam.

IV. NUMERICAL COMPUTATION

We use the finite difference method to discretize Eqs. (9a) and (9b). For the discretization of the conservation equation (9a), we use the difference format adapted to the physical meaning of traffic flow and apply the first-order upwind scheme to the motion equation (9b) [33,34]. Thus, we have

$$\rho_i^{j+1} = \rho_i^j + \frac{\Delta t}{\Delta x}\rho_i^j(u_i^j - u_{i+1}^j) + \frac{\Delta t}{\Delta x}u_i^j(\rho_{i-1}^j - \rho_i^j). \tag{19}$$

(I) when traffic is heavy $[u_i^j < c_0(\rho_i^j)]$,

$$u_i^{j+1} = u_i^j + \frac{\Delta t}{\Delta x}[c(\rho_i^j) - u_i^j](u_{i+1}^j - u_i^j) + \frac{\Delta t}{T(\rho)}[u_e(\rho_i^j) - u_i^j]. \tag{20}$$

(II) when traffic is light $[u_i^j \geqslant c_0(\rho_i^j)]$,

$$u_i^{j+1} = u_i^j + \frac{\Delta t}{\Delta x}[c(\rho_i^j) - u_i^j](u_i^j - u_{i-1}^j) + \frac{\Delta t}{T(\rho)}[u_e(\rho_i^j) - u_i^j], \tag{21}$$

where the index i represents the road section and the index j represents time. To investigate congestion and dissipation of the traffic flow, we use two Riemann initial conditions. We first assume that

$$\rho_u = 0.04 \ \text{veh/m}, \quad \rho_d = 0.18 \ \text{veh/m}, \tag{22a}$$

$$\rho_u = 0.18 \ \text{veh/m}, \quad \rho_d = 0.04 \ \text{veh/m}, \tag{22b}$$

where ρ_u and ρ_d are, respectively, the upstream and downstream densities. Equation (22a) corresponds to the appearance of shock waves when free-flow traffic meets stopped vehicles, while Eq. (22b) corresponds to the rarefaction wave as a queue dissolves. Two initial conditions are

$$u_u = u_e(\rho_u), \quad u_d = u_e(\rho_d). \tag{23}$$

Free boundary conditions are applied here [25]. Consider a section of freeway 20 km long, divide it into 100 cells, and take the time interval as 1 s. According to real observation and the parameter identification process, the values of the parameters are chosen as follows:

$$u_f = 30 \ \text{m/s}, \quad \rho_{\text{jam}} = 0.2, \ \text{veh/m}, \quad c_{\text{jam}} = 6.0 \ \text{m/s},$$

$$\rho_m = 0.168 \ \text{veh/m}, \quad E = 0.5, \quad \theta = 1.5,$$

$$\text{vehicle relaxation time} \ T_0 = 7 \ \text{s} \ [35],$$

$$\text{driver reaction time} \ t_r = 0.75 \ \text{s} \ [28].$$

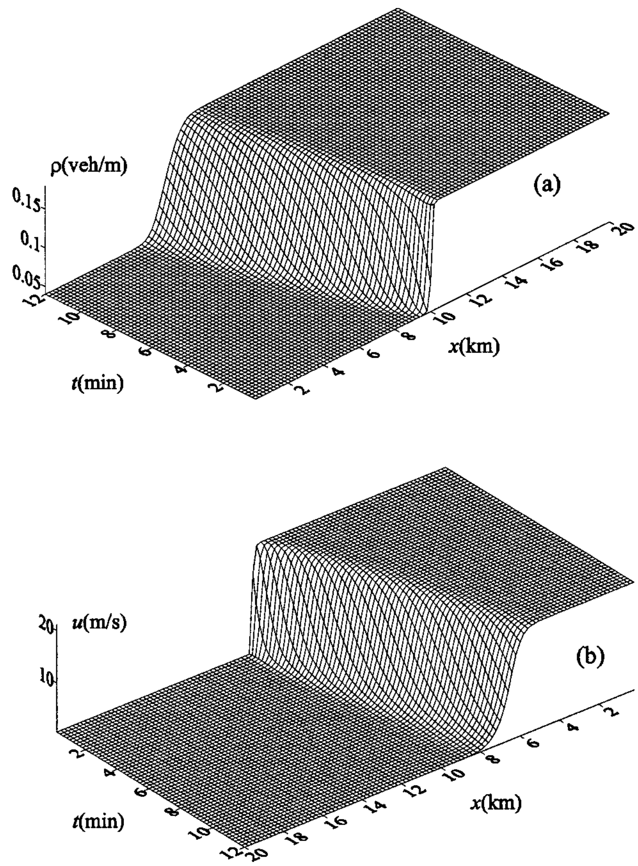

FIG. 1. Shock waves under the Riemann initial conditions of Eq. (22a): temporal evolution of (a) density $\rho(x,t)$ and (b) velocity $u(x,t)$.

The computational results under the two Riemann initial conditions (22a) and (22b) are shown in Figs. 1(a) and 1(b) and Figs. 2(a) and 2(b), respectively.

From Figs. 1 and 2, we can see that different Riemann initial conditions lead to different fronts between the congested and free-flow traffic. Figure 1 shows how the backward-moving shock wave front evolves under the condition (22a). This means that traffic becomes more congested, which we often see in rush hours. Figure 2 shows how the rarefaction wave front evolves under the condition (22b). It is a queue in the process of dissolution, which is consistent with our daily experiences in real traffic. The variations of the propagation velocity $c(\rho)$ of a disturbance with the density and its temporal evolution processes under the two Riemann initial conditions (22a) and (22b) are shown, respectively, in Figs. 3(a) and 3(b). From Fig. 3(a), we can clearly understand that the propagation velocity of a disturbance is density dependent and decreases with an increase of the density. Figure 3(b) shows that the temporal evolution of the propagation velocity $c(\rho)$ of a disturbance with different values of the driver reaction time t_r in the 50th cell. These curves indicate that the driver reaction time t_r has a remarkable effect on the propagation velocity of the disturbance. The lower curves labeled (2) correspond to the Riemann initial conditions of (22a), which shows that the change in the propagation velocity of a disturbance in the 50th cell is very drastic when a shock wave occurs. The

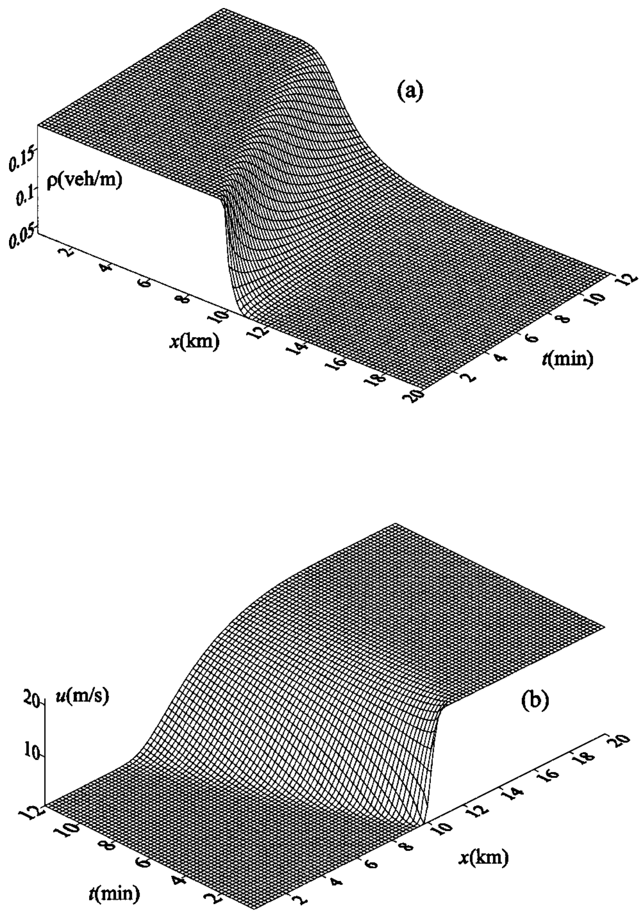

FIG. 2. Rarefaction waves under the Riemann initial conditions of Eq. (22b): temporal evolution of density $\rho(x,t)$ and (b) velocity $u(x,t)$.

upper curves labeled (1) correspond to Eq. (22b), indicating that the propagation velocity of a disturbance in the 50th cell approaches a high value when a queue is dissolving. All the results illustrate that the propagation velocity of a disturbance is very large during queue dissolution, but is reduced to a smaller value in the case of congestion.

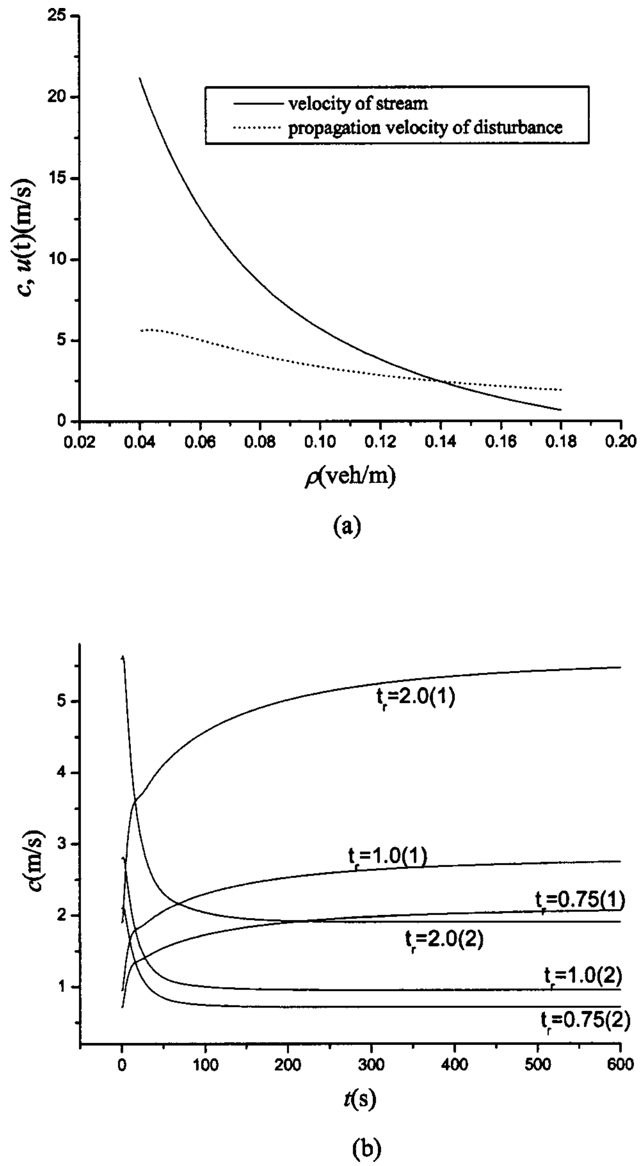

FIG. 3. The variation of propagation velocity $c(\rho)$ of a disturbance of vehicle density and its temporal evolution under the two Riemann initial conditions of Eqs. (22a) and (22b), respectively labeled as (1) and (2).

V. CONCLUSION

In this paper, we assume that the stream velocity u reaches the equilibrium velocity u_e within the driver relaxation time while the equilibrium velocity u_e is attained through adjustment of the driver reaction time in this dynamic process. We derive the motion equation with a nonconstant propagation velocity of disturbance in the traffic flow. The model has anisotropy, which is confirmed by the formation and diffusion of a traffic shock wave in numerical simulations and validated by using the model. The model reflects that the propagation velocity of a disturbance is very large in the free-flow state, but small in congested traffic, consistent with real situations. Moreover, the model has an analogy with Zhang's, Aw and Rascle's, and Jiang and Wu's models. It can describe certain traffic phenomena that evade the LWR model, such as vehicle clustering and forward propagation of disturbances in dense traffic [22] and modeling stop-and-go traffic [23]. When we carry out traffic measures on the ground and overhead traffic in Shanghai, we also find differences between the reaction time t_r and the relaxation time. The relaxation time T is about 7 s while the driver reaction time t_r is about 1 s [35]. When the vehicle relaxation time T approaches the driver reaction time t_r at a red light at an intersection, small disturbances caused by some drivers late for this change in dense traffic will be propagated fast through the vehicle stream and enlarged with time, and finally lead to a traffic jam, which corresponds to the propagation velocity of disturbance $c(\rho)$ in Fig. 3(b) under the two Riemann initial conditions of Eq. (22a). In contrast, when the traffic stream dissipates, the disturbances decrease with time. All the results show that the relationships between the relaxation time T and the driver reaction time t_r are very important for traffic flow. Hence, we plan to explore traffic bottleneck effects in our future work, together with numerical approximations, experimental validation, and extension of the model to describe traffic on real roads.

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