Convection in a very compressible fluid: Comparison of simulations with experiments

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The time profile $\Delta T(t)$ of the temperature difference, measured across a very compressible fluid layer of supercritical ^{3}He after the start of a heat flow, shows a damped oscillatory behavior before steady-state convection is reached. The results for $\Delta T(t)$ obtained from numerical simulations and from laboratory experiments are compared over a temperature range where the compressibility varies by a factor of ≈ 40. First the steady-state convective heat current j^{conv} as a function of the Rayleigh number Ra is presented, and the agreement is found to be good. Second, the shape of the time profile and two characteristic times in the transient part of $\Delta T(t)$ from simulations and experiments are compared, namely (1) t_{osc}, the oscillatory period, and (2) t_p, the time of the first peak after starting the heat flow. These times, scaled by the diffusive time τ_D versus Ra, are presented. The agreement is good for t_{osc}/τ_D, where the results collapse on a single curve showing a power-law behavior. The simulation hence confirms the universal scaling behavior found experimentally. However for t_p/τ_D, where the experimental data also collapse on a single curve, the simulation results show systematic departures from such a behavior. A possible reason for some of the disagreements, both in the time profile and in t_p, is discussed.

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I. INTRODUCTION

In a Rayleigh-Bénard (RB) cell, the start of a constant heat flow across the fluid layer produces an initial rise in the temperature difference across this layer, $\Delta T(t)$, with a transient profile determined by the fluid convection dynamics, and which then tends to a steady-state value, labeled ΔT. Recently such heat flow experiments were carried out [1,2] on a very compressible fluid in its convective state, supercritical ^{3}He, along the critical isochore $\langle\rho\rangle=\rho_c$, where the critical temperature is $T_c=3.318$ K. The fluid layer height in the experiments was $L=0.106$ cm and the diameter of the cylindrical cell was 5.7 cm. For this large aspect ratio, the predicted critical Rayleigh number is $\mathrm{Ra}_c=1708$, as was confirmed by the experiments [1]. Over the reduced temperature range $0.009\leqslant\epsilon\equiv(T-T_c)/T_c\leqslant 0.2$ where the experiments were analyzed, the isothermal compressibility increases by a factor of ≈ 40 as T_c is approached. The substantial change in the fluid properties along ρ_c is reflected in large changes of the transient profile, where damped oscillations were observed after the first peak of $\Delta T(t)$ for $\epsilon\geqslant 0.009$.

This paper compares the results of experimental data, and of simulations extending those of Refs. [3,4], for the time profile $\Delta T(t)$ in the regime where the fluid approaches steady-state convection. Reference is also made of recently published simulation results by Amiroudine and Zappoli [5]. In the simulations done in Refs. [3,4], two new terms are added in the heat conduction equation; the first takes into account the adiabatic heating taking place throughout the cell (the "piston effect"), and the second accounts for the adiabatic temperature gradient effect within plumes leading to the Schwarzschild criterion of the convection onset in compressible fluids [6]. In the work of Ref. [5] the Navier-Stokes equation together with an enthalpy equation were used, without a specific term to simulate the piston effect, since the latter is implicitly taken into account in the work of the pressure forces term in the enthalpy equation. The simulations were made in two dimensions, where the fluid was contained in a cell with the same height L as in the experiment. The aspect ratio was 4 in Refs. [3,4] and 2 in Ref. [5] with the periodic boundary condition in the horizontal direction in order to reproduce the conditions of the experimental cell with a large aspect ratio. The simulations in Ref. [4] extended until $[\mathrm{Ra}-\mathrm{Ra}_c]\approx 4\times 10^6$ for $\epsilon=0.05$ and until $\approx 1.7\times 10^5$ for other values of ϵ. Simulation results in this paper, except those of $\epsilon=0.05$ in Ref. [3], are obtained using the scheme in Ref. [4].

The outline for the remainder of this paper is as follows: First, a general discussion of the profiles $\Delta T(t)$ will be made, with presentation of some examples and an appraisal of the degree of agreement between experiments and simulations. Second, a comparison of the steady-state results from experiments and simulations will be presented, expressed in terms of the convection heat current j^{conv} versus the reduced Rayleigh number $\mathrm{ra}^*\equiv[\mathrm{Ra}-\mathrm{Ra}_c]/\mathrm{Ra}_c$. Third, a comparison of the transients from experiments and simulations will be made by the examination of two "characteristic times," the time t_p of the first peak after the start of the heat current, and t_{osc}, the oscillation period in the damped oscillatory decay. Both will be presented in a scaled form, again versus $[\mathrm{Ra}-\mathrm{Ra}_c]$. This quantitative comparison illustrates well the agreements and deviations between simulations and experiments, which will be discussed in the Conclusions.

II. GENERAL OBSERVATIONS ON THE PROFILES ΔT VERSUS TIME

We briefly emphasize, as was done in Refs. [1,2], that the experiments were done under conditions where the stratifi-

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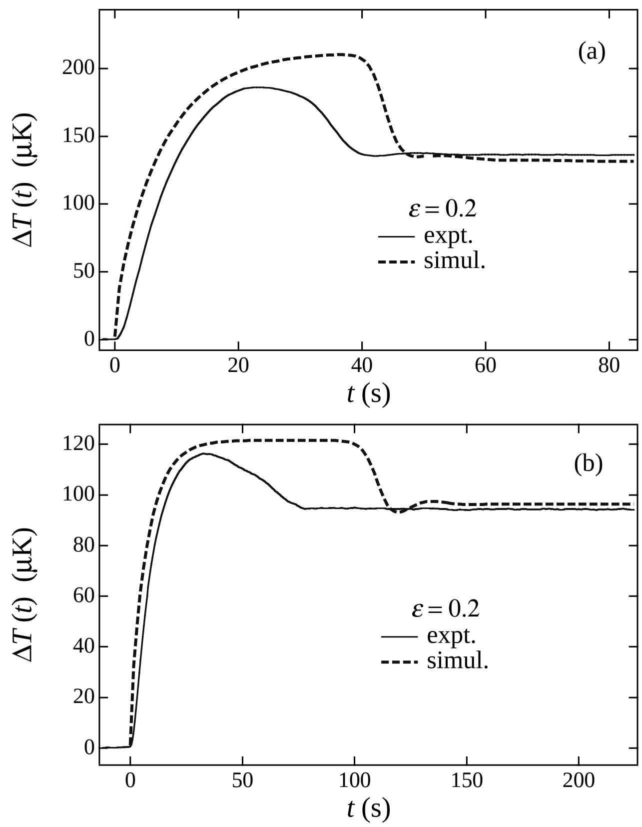

FIG. 1. Plots of the profile $\Delta T(t)$ vs time for two values of the heat flow q (in 10^{-7} W/cm^2) at ϵ=0.2 and comparison between experiments and simulations (a) $q=3.89$ (expt), 3.67 (simul) and (b) $q=2.16$ (expt), 2.10 (simul).

cation from gravity was small and where the temperature changes across the fluid were kept small enough that the changes in the fluid properties across the fluid layer were smaller than a few percent. Thus the conditions for the approximations in a Boussinesq-like fluid in the momentum equation were maintained. In Fig. 9 in Ref. [1] a representative evolution is shown of the observed profile $\Delta T(t)$ at ϵ=0.05 as a function of the heat current q.

Both experiments and simulations show over a certain range of heat flow q and of temperature a damped oscillatory profile $\Delta T(t)$ as shown in Figs. 1 and 2 of Ref. [3], Fig. 1 of Ref. [4], both at ϵ=0.05 and Fig. 2 of Ref. [5] at ϵ=0.01, which are not reproduced here. Considering that the experimental data have not been corrected for the time lag introduced by the temperature recording instrumentation with a time constant of τ=1.3 s, the qualitative agreement is quite satisfactory. However, as ϵ increases and the compressibility decreases, the disagreement between the transient regime of experiments and simulations becomes sizable. This can be seen in Fig. 1 at ϵ=0.2 for two values of the heat flow $q \approx 3.7\times 10^{-7}$ and 2.1×10^{-7} W/cm^2. By contrast, the steady-state value for ΔT reached in both the experiments and in simulations for the same value of q remains nearly the same. In the experimental trace for the lower value of q, no damped oscillations are seen, but rather a nonexponential decay of the overshoot. This is the regime labeled "truncated oscillations," described in Fig. 2 of Ref. [2]. (See also remark [7].)

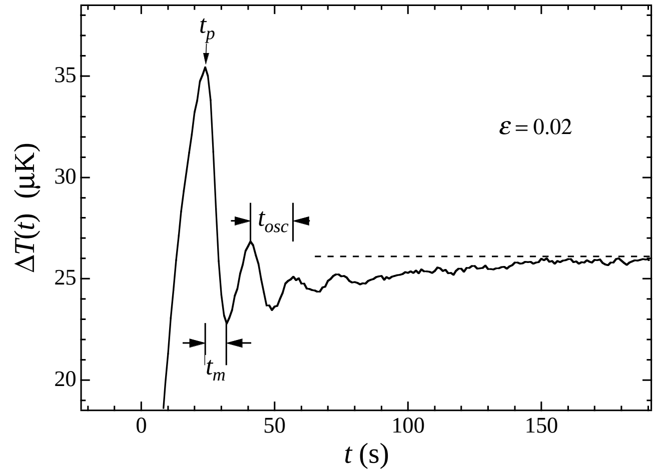

FIG. 2. A representative profile $\Delta T(t)$ after starting the heat current, with the definition of the times t_p, t_m, and t_{osc} and the slow exponential relaxation to the steady-state value of ΔT (dashed line) with a time constant τ_{tail}. The recording is for ϵ=0.02 with the heat flow $q=1.69\times 10^{-7}$ W/cm^2.

Simulations, however, always show damped oscillations in the convective state.

In Fig. 2, we present an enlarged segment near the peak of $\Delta T(t)$ for a representative trace at ϵ=0.02, which exhibits and defines various characteristic times in the transient. We will consider two of these times in this paper, namely (1) the peak t_p of the first peak after the start of the heat current and (2) the period t_{osc} of the damped oscillations. The profile tends asymptotically from below to the steady-state value ΔT with a relaxation time τ_{tail}, obtained by fitting this transient tail with a simple exponential. We note that this transient tail has not been detected in the simulations either in Ref. [4] or in Ref. [5]. The transient shape, principally at short times where the changes in $\Delta T(t)$ are rapid, is affected by the instrumentation time constant, and also the modulated minimum shown in Fig. 2 makes an analysis of the oscillation amplitude decay rate uncertain. However, inspection of the many recorded experimental traces shows that for a given ϵ (or compressibility), the rates for both the oscillations and their amplitude decay increase with the heat current. For values large enough of q at a given ϵ, the first peak and the oscillations become attenuated and averaged out. (See Fig. 4 of Ref. [2].) The likely cause for this observation is that t_{osc} becomes comparable or smaller than the instrumentation time constant. An averaging effect of the oscillations due to a negative interference between nonsynchronous plumes that results from the large lateral dimension of the experimental cell was suggested in Ref. [4]. This suggestion might be very relevant in the regime $\epsilon<0.009$ where no oscillations could be observed, even when their expected period was well above that of the instrumental time constant (see Ref. [2], Sec. III).

III. COMPARISON OF STEADY-STATE RESULTS IN EXPERIMENTS AND SIMULATIONS

In Ref. [1] the steady-state experimental results were presented in terms of the dimensionless convective heat current

j^{conv} versus ra*. Here j^{conv} is the ratio of the convective portion of the heat current to that conducted through the fluid at the transition to convection, which leads to the relation [8]

$$j^{\mathrm{conv}} \equiv (\mathrm{Nu}-1)(\mathrm{ra}^*+1), \tag{1}$$

where Nu is the Nusselt number. Along the critical isochore of a fluid, where the compressibility diverges as T_c is approached, Gitterman and Steinberg [9] have shown that for the fluid onset of mechanical instability there is a crossover from the Rayleigh to Schwarzschild (or "adiabatic temperature gradient") criterion as T_c is approached. Then it has been shown (see Ref. [10] and references therein) that at the onset of convection, the temperature drop across the fluid layer is given by

$$\Delta T_{\mathrm{onset}} = \Delta T_{\mathrm{R}} + \Delta T_{\mathrm{ad}}, \tag{2}$$

where $\Delta T_{\mathrm{R}} = \mathrm{Ra}_c \nu D_T/\alpha_P g L^3$, the familiar Rayleigh term, and $\Delta T_{\mathrm{ad}} = LgT\alpha_P/C_P$, the adiabatic temperature difference contribution (for ΔT_{R} and ΔT_{ad} see, for instance, Tritton's monograph [11]). Here ν is the kinematic viscosity, D_T the thermal diffusivity, α_P the isobaric thermal expansion coefficient, g the gravity acceleration, L the height of the fluid layer, and C_P the specific heat at constant pressure.

As has been described in some detail in Refs. [1,2], Nu and Ra have to be corrected for the contribution from the adiabatic temperature gradient. One then obtains

$$\mathrm{Ra}_{\mathrm{corr}} = \frac{\mathrm{Ra}(\Delta T - \Delta T_{\mathrm{ad}})}{\Delta T} \quad \text{and} \quad \mathrm{Nu}_{\mathrm{corr}} = \frac{(\Delta T_{\mathrm{diff}} - \Delta T_{\mathrm{ad}})}{(\Delta T - \Delta T_{\mathrm{ad}})} \tag{3}$$

and hence

$$\mathrm{ra}^*_{\mathrm{corr}} \equiv (\mathrm{Ra}_{\mathrm{corr}}/\mathrm{Ra}_c - 1)$$

and

$$j^{\mathrm{conv}}_{\mathrm{corr}} \equiv (\mathrm{Nu}_{\mathrm{corr}}-1)(\mathrm{ra}^*_{\mathrm{corr}}+1). \tag{4}$$

Here ΔT_{diff} is the temperature drop across the fluid in the diffusive regime for the same heat current producing the observed ΔT. Because both $j^{\mathrm{conv}}_{\mathrm{corr}}$ and $\mathrm{ra}^*_{\mathrm{corr}}$ vary over about five orders of magnitude for the range covered by the data, a more sensitive way is to present the ratio $j^{\mathrm{conv}}_{\mathrm{corr}}/\mathrm{ra}^*_{\mathrm{corr}}$ versus $\mathrm{ra}^*_{\mathrm{corr}}$. Furukawa and Onuki [4] theoretically justified the validity of the scaling relations in terms of these corrected quantities. This is done in Fig. 3(a) where the points recorded close to the transition to convection and showing rounding instead of a sharp convection onset have been omitted. Furthermore the data for $\epsilon < 0.009$, where no damped oscillations were obtained in the transients and for which no simulations were carried out, have not been used in this figure. As can be seen, within the scatter all the data points nearly collapse on a common curve. For $\mathrm{ra}^*_{\mathrm{corr}} < 1$, the data extrapolate to a horizontal line with an amplitude of 1.3 ± 0.1. This asymptotic result $j^{\mathrm{conv}}_{\mathrm{corr}} = 1.3\mathrm{ra}^*_{\mathrm{corr}}$, which represents data slightly above the onset of convection, has been discussed in Ref. [1] where it was concluded that the amplitude is consistent with a straight roll convection prediction [12]. In Fig. 3(b), the experimental data are replaced by a solid curve representing its average, and the results from simulations are shown by the symbols at various values of ϵ. In the intermediate range of $\mathrm{ra}^*_{\mathrm{corr}}$ these data points collapse on a curve slightly above the experimental average, and also within their scatter they tend to the same limiting amplitude of 1.3, as do the experiments. Therefore, they also imply straight roll convection, a picture which is plausible in the two-dimensional (2D) simulation which represents a cross section of rolls in a geometry with parallel vertical periodic boundaries.

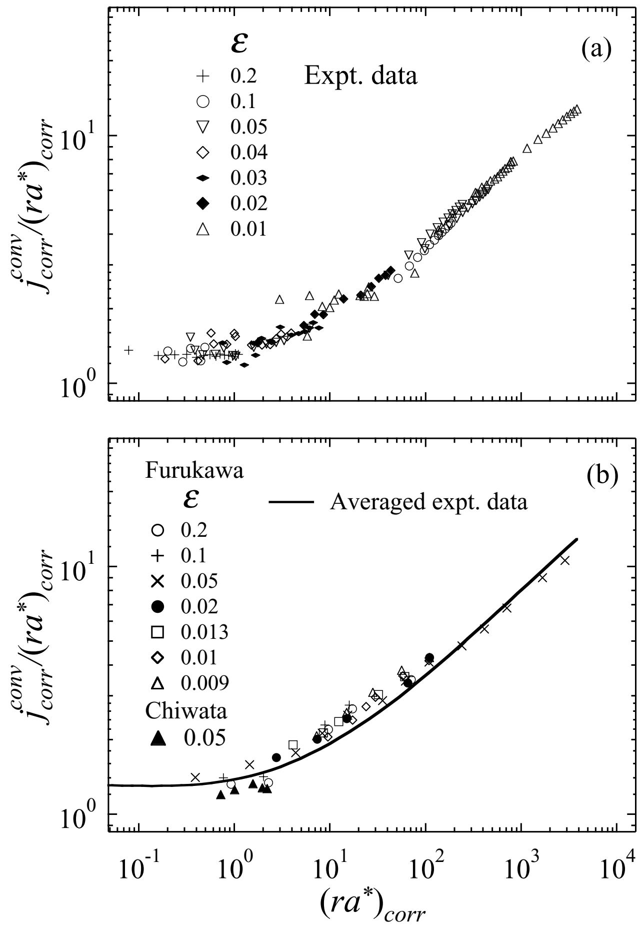

FIG. 3. The convective heat current $j^{\mathrm{conv}}_{\mathrm{corr}}$ divided by the reduced Rayleigh number $\mathrm{ra}^*_{\mathrm{corr}} = [\mathrm{Ra}_{\mathrm{corr}} - \mathrm{Ra}_c]/\mathrm{Ra}_c$ vs $\mathrm{ra}^*_{\mathrm{corr}}$, both corrected for the adiabatic temperature gradient contribution. (a) Experimental data at various values of ϵ. (b) Data from simulations at different values of ϵ shown by symbols. The solid line is the average of the experimental data in (a).

IV. TRANSIENT CHARACTERISTIC TIMES IN EXPERIMENTS AND SIMULATIONS

A. Background information

The mechanism responsible for the damped oscillations for a highly compressible fluid at constant average density has been discussed most recently by Furukawa and Onuki [4] and by Amiroudine and Zappoli [5]. From their simulations,

TABLE I. The ratio $C_P/C_V \equiv \gamma$ for ^{3}He along its critical isochore for several values of the reduced temperature ϵ, the ratio $B(\gamma)$ of the diffusive times, calculated from Fig. 2 of Ref. [13] (Behringer *et al.*) and the diffusive time τ_D.

ϵ	$C_P/C_V \equiv \gamma$	$B(\gamma) \equiv \tau_D/\tau_D$	τ_D(s)
0.01	119	1.01	265
0.02	57.7	1.03	134
0.03	38.1	1.06	92.0
0.04	28.5	1.08	69.9
0.05	22.8	1.10	57.0
0.07	16.5	1.13	42.2
0.10	11.8	1.19	31.8
0.20	6.48	1.35	19.8

these authors presented a detailed analysis of the time evolution of the temperature in various locations in the fluid layer. We refer to their description on how maxima and minima of $\Delta T(t)$ are produced by the "piston effect" that leads to the vertical flow of successive "warm" and "cold" fluid masses. The snapshots of the simulations by Chiwata and Onuki [3] had already pictured the formation of plumes during these processes, and the evidence of warm and cold vertical flows.

In the analysis of the experimental data [2], two characteristic times describing the remarkable oscillatory behavior, t_m and $t_{\rm osc}$, were discussed, which are shown in Fig. 2 for a representative $\Delta T(t)$ profile. The experimental results for the time t_p have not been analyzed before and are presented here. In order to keep the analysis of the results tractable, we will not include t_m in this discussion. In the experimental data analysis, the relaxation time $\tau_{\rm tail}$ to the steady-state convection was also presented in Ref. [2]. All the data of $t_{\rm osc}$ and $\tau_{\rm tail}$ at the various reduced temperatures ϵ, scaled by the diffusion time $t_D \equiv L^2/4D_T$, and plotted versus the Rayleigh number difference $[{\rm Ra}_{\rm corr} - {\rm Ra}_c]$ were found to collapse within a scatter of $\pm 15\%$ on two respective curves, an unexpected result.

We now discuss the choice of the scaling time in the data for $t_{\rm osc}$ and t_p versus the Rayleigh number. As mentioned above, the diffusion time t_D was used as the scale for $t_{\rm osc}$ in the plot versus $[{\rm Ra}_{\rm corr} - {\rm Ra}_c]$. This time t_D was first used in the Navier-Stokes equation leading to the simulations of Ref. [3] under the conditions of constant average density and valid for $C_P/C_V \equiv \gamma \gg 1$. In the absence of convection, t_D can be related to the relaxation time $\tau_0 = L^2/\pi^2 D_T$ of the lowest diffusion mode in a RB—or in a standard thermal conductivity cell [13]. However, over the experimental and simulation range where $0.2 \geqslant \epsilon \geqslant 0.009$, the condition $\gamma \gg 1$ is only progressively realized as ϵ decreases to 0.01. In Ref. [13] expressions for the solution of τ_0 as a function of γ were derived. As $\gamma \to 1$, $\tau_0 \to 4L^2/\pi^2 D_T$, and this last value is the same as for a fluid relaxing at constant pressure. In general $\tau_0(\gamma) = B(\gamma) L^2/\pi^2 D_T$, where $B(\gamma) = (\pi/q_0 L)^2$, obtained from Fig. 2 of Ref. [13] with q_0 the wave number of the lowest mode. $B(\gamma)$ is presented in Table I for several values of ϵ relevant to the experiments and simulations in this paper. As can be seen in this table, the ratio

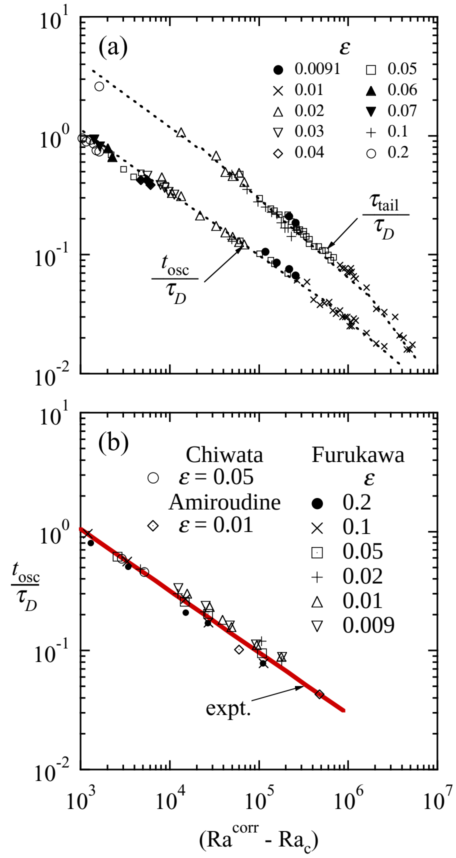

FIG. 4. The oscillation period $t_{\rm osc}$ and the time $\tau_{\rm tail}$ for the relaxation to the steady state, both scaled by the diffusion time τ_D, vs $[{\rm Ra}_{\rm corr} - {\rm Ra}_c]$. (a) Experimental data at various values of ϵ. (b) Symbols show the simulation data at different values of ϵ, and the solid line is the average of the experimental data in (a).

$\tau_0(\gamma)/\tau_0(\gamma = \infty)$ decreases with ϵ and tends to 1 for $\epsilon < 0.01$. The use of the time $\tau_D \equiv \tau_0(\gamma)\pi^2/4$ as a scale for the convection transient characteristic times is therefore preferable to t_D. It turns out that the choice of τ_D as the scaling time improves the collapsing of the experimental data points for both $t_{\rm osc}$ and t_p. In the following sections we present and compare these times obtained experimentally and from simulations.

B. Oscillatory period $t_{\rm osc}$

In Ref. [2], Fig. 4 presented $t_{\rm osc}$ scaled by t_D and plotted versus $[{\rm Ra}_{\rm corr} - {\rm Ra}_c]$. The substitution of τ_D as a scale gives a better collapsing of the data points than does t_D, particularly at the lower values of $[{\rm Ra}_{\rm corr} - {\rm Ra}_c]$ where most of the data points at the higher values of ϵ lie. Within the scatter of $\pm 15\%$ the data for $10^3 < [{\rm Ra}_{\rm corr} - {\rm Ra}_c] < 5 \times 10^5$ can be represented by a power law with an exponent of -0.52 ± 0.02. In Fig. 4(a) we present these data and in Fig. 4(b) its average, obtained by a fit to a power law with a corrective term for the larger Ra values. Also in Fig. 4(b) we show by symbols the results from the simulations by the Kyoto group (two of the present authors, A.F. and A.O., and Y. Chiwata) and by Amiroudine and Zappoli [5] at various values of ϵ. The agreement

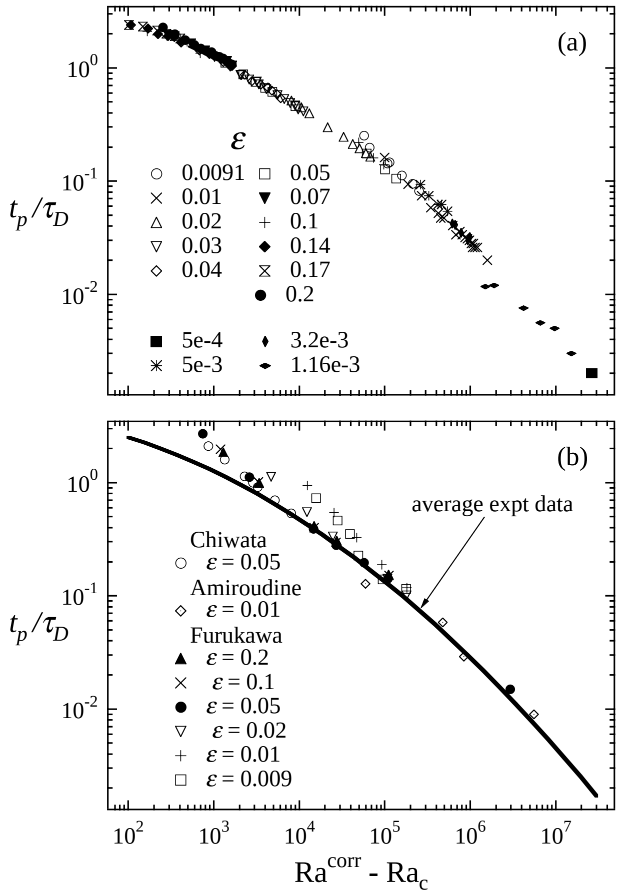

FIG. 5. The time of the first peak t_p, scaled by the diffusion time τ_D, vs $[\mathrm{Ra}_{\mathrm{corr}} - \mathrm{Ra}_c]$. (a) The experimental data, where the symbols denote the various values of ϵ at which the experiments were carried out. (b) The curve representing the average of the experimental data from (a), compared with the results from simulations, shown as symbols at the various values of ϵ.

appears to be good, though the simulations indicate some small systematic deviations from collapse on a single ‘‘universal’’ line. The basic reason for the apparent power law with an exponent of $-1/2$ remains to be understood.

C. Location of the first peak at t_p

In the experiments, the initial rise of the measured transient $\Delta T(t)$ after the start of the heat flow across the fluid layer is affected by the time constant of the thermometer circuitry, $\tau = 1.3$ s as described in Ref. [1]. This is especially so at short times, when the $\Delta T(t)$ increases rapidly, and as a result the measured value of the time at the first peak has to be corrected. This was done by comparing the shift in time between the calculated rise of $\Delta T(t)$ in the conducting regime (Eq. (3.3) of Ref. [4]) and the recorded curve for several values of ϵ and q. This shift was between 2 and 3 s and t_p was obtained after a crude correction was made by subtracting $\delta t = 2$ s from the measured time at the peak. The times t_p used in this analysis ranged from ≈ 100 to ≈ 7 s. Figure. 5(a) shows the scaled representation t_p/τ_D of the experimental times versus $[\mathrm{Ra}_{\mathrm{corr}} - \mathrm{Ra}_c]$. There is excellent collapse of the data for all the values of ϵ, even extending to the region of $\epsilon < 0.009$ where no damped oscillations are observed [2]. In Fig. 5(b) the solid curve represents the average of the experimental data, as obtained by a polynomial fit. Symbols show the results from simulations at various values of ϵ. The data from Ref. [3] at $\epsilon = 0.05$ and those in the present research are found to be internally consistent. However, contrary to the experimental results, the simulation data do not collapse on a single curve. This discrepancy is expected from the profiles $\Delta T(t)$ shown in Fig. 1 where the peak in the simulations lies at substantially longer times t_p than for the experiments. A possible source for the discrepancy between simulations and experiments will be considered below.

V. DISCUSSION

First we present general comments on transient observations in Rayleigh-Bénard convection. After the start of a heat current at constant pressure, an overshoot in $\Delta T(t)$ is routinely observed in Boussinesq fluids. The origin of the overshoot is a certain ‘‘inertia’’ of the fluid immediately after the heat flux is applied: Even though the final state is convection, initially the fluid remains stationary and $\Delta T(t)$ follows the solution of the thermal diffusion equation. As the fluid begins to move, the trace shows an overshoot with a peak at t_p followed by a transition to the steady-state value in the convective regime as described, for example, by Behringer [14]. Thus, t_p approximately describes the time that it takes for the layer to develop convective motion.

In a compressible fluid, the nonconvecting state during the initial transient is expected to persist up to a value $\Delta T_{\mathrm{instab}}$ greatly exceeding the stability threshold $\Delta T_{\mathrm{onset}}$ given by Eq. (2), derived for a linear temperature and density distribution in the vertical direction. This result for $\Delta T_{\mathrm{instab}}$ was obtained by El Khouri and Carlès [15] via a linear stability analysis and is a direct consequence of the strongly inhomogeneous vertical density and temperature distribution with pronounced boundary layers generated by the Piston effect [16]. A clear example can be seen in Fig. 1(a) of Ref. [4], which presents transient curves for $\epsilon = 0.05$. The simulated trace essentially coincides with the zero-gravity result (Eq. (3.3) in Ref. [4]) up to $\Delta T(t)$ of the order 280 μK, while the stability criterion [Eq. (2) in the present paper] gives only 7 μK. Another example is obtained by comparing the simulations in Fig. 2 of Ref. [5] at $\epsilon = 0.01$ with calculations under zero gravity, which also shows the two curves coinciding until close to t_p. Hence the simulations [4,5] are qualitatively consistent with the predictions of linear stability analysis.

We now recapitulate the main findings of the comparison between experimental data and simulations for a supercritical very compressible fluid, ^{3}He. Starting with t_{osc}, which is the characteristic time determined farthest away from the start of the heat flow, both experiments and simulations in the scaled representation are in good agreement. By contrast, as shown in Fig. 5, t_p is systematically larger in the simulations than in the experiments, the difference becoming more important as ϵ increases. An intriguing puzzle is why all the experimental t_p data at the various values of ϵ can be cast into a scaling representation, while the simulation data cannot.

This disagreement for t_p is surprising: at large ϵ where the compressibility of the fluid has become smaller, a ''simple'' Boussinesq behavior should be recovered. A possible origin of this discrepancy is the imperfection of the temperature control of the top plate during the experiment. The signal from the control thermometer [1] is recorded by a circuit with a time constant of 1.25 s and 6 dB/octave rolloff. Therefore, temperature fluctuations with frequencies above a few hertz cannot be detected. The average temperature noise is estimated to be about 1 μK/$\sqrt{\text{Hz}}$ rms. Because of very high thermal conductivity of the top plate material (oxygen-free high conductivity copper) it seems reasonable to assume that this perturbation produces no horizontal temperature gradients. It is therefore possible that small parasitic fluctuations of the top plate temperature could speed up the development of the convecting state, hence producing an overshoot with a smaller peak amplitude and t_p than the simulations do, which have no noise. We would anticipate that once the convection is almost fully developed, the influence of the fluctuations mentioned above would not be noticeable anymore. Hence they might not affect the period t_{osc} of the damped oscillations, which would explain the good agreement between experiment and simulations and also that in the steady-state condition [17].

We mention here that in the experiments [1] a rounding of the onset point on the steady-state $\Delta T(q)$ measurements was reported and also attributed to the top plate noise. The rounding was found to become more and more pronounced as the critical point was approached, i.e., the effect of the fluctuations would have to be increasing with decreasing ϵ. The effect of time-dependent boundary conditions on convection in a compressible fluid appears to be an interesting and open question. We suggest future studies, both experimental and numerical ones, that would focus on the transient response of a fluid layer in a RB cell to an externally imposed small perturbation of the top surface temperature. This could be done either via random noise with a rms amplitude of a few microkelvin or via a sinusoidal modulation at various frequencies with similar amplitude. Here we note that experiments with sinusoidal modulation of ΔT in a RB cell were carried out by Niemela and Donnelly [18], who also refer to earlier experimental and theoretical modulation work. The amplitude of their modulation, however, was much larger than in our suggested investigations, where the conditions and purpose of the perturbation are quite different from those in Ref. [18].

VI. CONCLUSIONS

A systematic comparison of the results from experimental and 2D simulation convection studies of a compressible fluid has been reported. This fluid is supercritical He^3 along the critical isochore and over a temperature range where the compressibility varies by a factor of $\approx$40. The temperature profile $\Delta T(t)$ across the fluid layer in a Rayleigh-Bénard cell after the start of a heat flow was investigated in both experiments and simulations. The damped oscillations in the transient after the start of the heat flow and with a period t_{osc} are of particular interest. They are a consequence of vertical mass flows that result from the ''piston effect'' triggered by the plumes—both of these moving up and down.

The comparison of the steady-state results, expressed in terms of the convection current versus the Rayleigh number, shows good agreement in general. However, the comparison of the transient results shows some systematic discrepancies, which appear to become more important as the compressibility decreases (i.e., as the distance from the critical point increases) and as the Rayleigh number decreases for a given ϵ. This can be clearly seen by examining the respective $\Delta T(t)$ profiles at various temperatures. The agreement is best for the oscillation period where the simulation results and the experimental data can be represented in scaled form versus the Rayleigh number. The absence of noise in the simulations, in contrast to a physical system, might be a possibility for the discrepancy. In addition, as mentioned earlier in this paper, the simulations carried out independently by two research groups [4,5] do not detect the slow relaxation of $\Delta T(t)$ to the steady-state, or the region of truncated oscillations observed in the experiments. These discrepancies remain to be understood.

Note added in proof. Recently, simulations were carried out in which a random temperature noise with a rms amplitude A was imposed on the top surface temperature of the RB cell. The most systematic studies were made at $\epsilon = 0.05$ and with $0 < A < 10$ μK. It was found that with increasing A the convection developed faster after the instability limit [15] was reached. As a result t_p decreased, bringing simulation and experiment in agreement for a value of A of the order of 1 μK rms, which is consistent with the noise estimated in the experiment (see Sec. V). We intend to describe elsewhere the results of these studies.

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