Experimental observation of dramatic differences in the dynamic response of Newtonian and Maxwellian fluids

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An experimental study of the dynamic response of a Newtonian fluid and a Maxwellian fluid under an oscillating pressure gradient is presented. Laser Doppler anemometry is used in order to determine the velocity of the fluid inside a cylindrical tube. In the case of the Newtonian fluid, the dissipative nature is observed. In the dynamic response of the Maxwellian fluid an enhancement at the frequencies predicted by theory is observed.

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I. INTRODUCTION

Since the concept of dynamic permeability was proposed several years ago [1,2], it has been extensively used in the study of practical problems such as petroleum recovery [3], soil-ground water transport [4], fluids flowing in porous media [5,6], acoustic waves in porous media [7], wave propagation in foams [8], fluid circulation in biological systems [9], etc. The use of the dynamic permeability as originally proposed [1], however, is not quite correct for some of these systems since the fluid does not behave as a Newtonian fluid but shows viscoelastic features. Theoretical analyses of viscoelastic fluids have recently predicted an interesting enhancement in the dynamic permeability of several orders of magnitude in comparison with the static permeability [10–13]. This behavior is due to the coupling between the elastic behavior of the fluid and the geometry of the container and is completely different from the pure dissipative behavior of Newtonian fluids. An increase in the flow rate of a viscoelastic fluid flowing in a tube under oscillating conditions was discovered several years ago [14–17] but up to now, the experimental measurements in terms of a frequency-dependent response had not been performed. Some theoretical studies have explored interesting consequences of the enhancement of the dynamic permeability in different systems including viscoelastic fluids [12,13,18–23]. The variety of systems where an oscillating pressure gradient can be imposed to a viscoelastic fluid is wide, for instance, from the oil recovery problem [24] to the dynamic analysis of fluid transport in animals (including the human body [25]). All previous theoretical derivations of the dynamic permeability for viscoelastic fluids have been made in terms of the averaged velocity. In this work however, we perform a detailed analysis by measuring the dynamic response at the center of a fluid column that is moved in an oscillatory way and its local velocity [26] is measured using a laser Doppler anemometer (LDA). This allows us to test a local, simple model for both Newtonian and Maxwellian fluids and our results corroborate that the simple linear model can capture the differences between the dissipative (Newtonian fluid) and resonant (Maxwellian fluid) behaviors.

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II. LINEAR THEORY

In the following, we present the pertinent equations used to obtain a local expression for the Maxwellian fluid response based on the results presented elsewhere [12]. We begin with the *linearized* momentum equation

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \nabla \cdot \overleftrightarrow{\tau} \tag{1}$$

and the continuity equation

$$\nabla \cdot \mathbf{v} = 0, \tag{2}$$

for an incompressible fluid. In the above equations, ρ denotes the mass density of the fluid, $\mathbf{v}$ the fluid velocity, p the pressure, and $\overleftrightarrow{\tau}$ represents the viscous stress tensor. To ensure the validity of Eq. (1) it is necessary to consider fluids with low Reynolds numbers, in our experimental case: Re $< 7 \times 10^{-4}$. The constitutive equation of the fluid that we use is the linearized Maxwell model:

$$t_m \frac{\partial \overleftrightarrow{\tau}}{\partial t} = -\eta \nabla \mathbf{v} - \overleftrightarrow{\tau}, \tag{3}$$

where η denotes the dynamic viscosity and t_m the relaxation time. It is necessary to stress that the linearized Maxwell model constitutes a simplification required to obtain simple analytic results, which implies neglecting nonlinear terms that may be important under certain circumstances.

Manipulating the three previous equations, applying the Fourier transform, and using cylindrical coordinates, we obtain

$$V(r,\omega) = -\frac{(1 - i\omega t_m)}{\eta \beta^2} \left(1 - \frac{J_0(\beta r)}{J_0(\beta a)} \right) \frac{dP}{dz}, \tag{4}$$

where $\beta = \sqrt{\rho[(t_m\omega)^2 + i\omega t_m]/(\eta t_m)}$, a is the cylinder radius, P and V are the pressure and the velocity in the frequency domain, respectively, ω is the angular frequency, and J_0 is the cylindrical Bessel function of zeroth order. A more detailed derivation of these expressions can be found in Refs. [12,20].

Previous theoretical analyses of the dynamic permeability of viscoelastic fluids deal with the above equations using an average velocity for the fluid. In what follows, we shall analyze the flow from a different point of view, one which is based on a "local response" defined as the response of the fluid to a pressure gradient in a particular position inside the tube. This is required by the LDA that essentially measures quasilocal particle velocities. Thus, in order to analyze the local dynamic response, and to facilitate the measurements, we evaluate Eq. (4) at the center of the cylinder ($r=0$ in cylindrical coordinates) where the amplitude of the velocity is maximum:

$$V(\omega)=-\frac{(1-i\omega t_m)}{\eta\beta^2}\left(1-\frac{1}{J_0(\beta a)}\right)\frac{dP}{dz}. \tag{5}$$

The local dynamic response is thus defined as

$$\xi=-\eta\frac{V}{dP/dz}, \tag{6}$$

where we have followed Darcy's generalized equation [10]. Substituting the velocity given by Eq. (5), we obtain

$$\xi(\omega)=\frac{(1-i\omega t_m)}{\beta^2}\left(1-\frac{1}{J_0(\beta a)}\right). \tag{7}$$

From this expression one can recover the Newtonian behavior with the substitution $t_m=0$. In order to compare the last expression with experimental results, we need an expression for the pressure gradient. In our experimental case, we have a harmonic oscillating column of fluid being moved by a piston coupled to a motor with adjustable frequency. Thus, the pressure gradient can be easily described by

$$\frac{dp(t)}{dz}=\rho z_0\omega^2\sin(\omega t), \tag{8}$$

which represents an oscillatory movement in the z direction with a displacement amplitude equal to z_0. We shall also measure the root mean square velocity v_{rms}, since the range of frequencies that can be resolved in this case is much wider than the range given by the Fourier transform of the velocity data. The experimental dynamic response of a fluid [$\xi^{\exp}(\omega)$] is thus given by

$$\xi^{\exp}=\eta\frac{v_{rms}}{dp_{rms}/dz}=\frac{\sqrt{2}\eta v_{rms}}{\rho z_0\omega^2}. \tag{9}$$

Now, since the experimental velocity is harmonic, we can directly compare the value of the local dynamic response obtained using the previous expression for each experiment with the theoretical prediction from Eq. (7).

III. EXPERIMENTAL SYSTEM

The experimental device shown in Fig. 1 consists of a vertical cylinder filled with a fluid. The oscillating movement required to have a harmonic oscillating pressure gradient, cf. Eq. (8), was produced with a piston that closes the base of the cylinder and is driven by a motor of variable frequency; as previously mentioned, the local velocity of the fluid is measured with a laser Doppler anemometer.

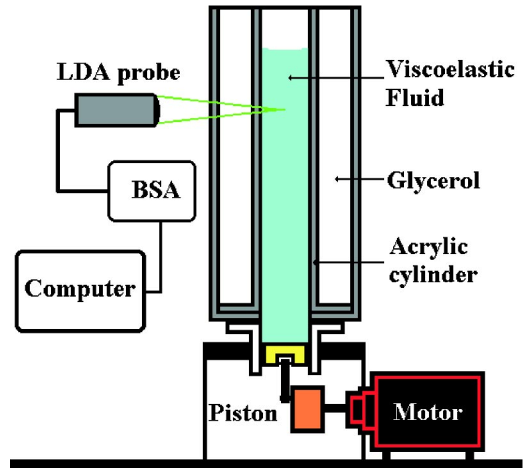

FIG. 1. Schematic view of the setup used for the study of the dynamic response with a LDA measuring system.

The cylindrical tube, made of transparent acrylic, has an inner diameter of 5 cm and a length of 50 cm. In its lower end, this tube is joined by a clamp to a stainless steel piston skirt of the same inner diameter. Within the stainless steel skirt, there is a Teflon piston that is moved by a *Siemens* motor of 1 hp, regulated by a *Varispeed 606PC3* system. This allows for a frequency of oscillation that can be varied from 1.5 to 200 Hz. The acrylic cylinder over the skirt is contained in a square recipient with parallel walls of transparent acrylic and filled with glycerol having a refractive index similar to the one of the acrylic; this is in order to avoid the cylindrical aberration due to the fact that the cylinder acts as a lens. In this and other cases, when there are cylindrical configurations, these aberrations could cause a null intersection between the LDA beams or could result in the backscattering signal not being received by the photomultiplier. The LDA used for the local velocity measurements is a widely known system [27]. Our particular LDA consists of a probe (*Dantec FiberFlow 60x20*), a photomultiplier, a Burst Spectrum Analyzer (*Dantec BSA 57N11*), and an argon laser (*Spectra Physics 177*) emitting in four different wavelengths; only the 488-nm line (200 mW) is used, and the data are processed in a PC with the *Dantec FlowManager* software. The measurement volume is of $0.64\times0.075\times0.075$ mm^3 located at the center of the acrylic cylinder. The LDA probe was set into a positioning system (precision: 1 μm) placed on a different table from the rest of the setup to prevent oscillations that could alter the experimental data. The rms velocity is calculated from the collected data.

Commercial glycerol was used as the Newtonian fluid to be tested; it has a viscosity of 1 Pa s and a density of 1250 kg/m^3. The well-known cetylpyridinium chloride and sodium salicylate solution (CPyCl/NaSal, 60/100) [28,29] was used as the Maxwellian counterpart; its properties are $\rho=1050$ kg/m^3, $\eta=60$ Pa s, and $t_m=1.9$ s [30]. All the measurements were made within the (25 ± 0.5)°C interval in order to keep constant the properties of CPyCl/NaSal solution [30]. The properties of this solution and the diameter of the tube give a Deborah's number ($\alpha=\rho a^2/\eta t_m$) of $\alpha=0.0058$. This value is much smaller than the critical value $\alpha<11.64$ for the appearance of resonances predicted by the theoretical model [12,20]. The experimental frequency range is kept within the low Reynolds number regime by setting the piston movement amplitude to 0.8 ± 0.05 mm (which assures that $\mathrm{Re}<7\times10^{-4}$). Under these circumstances we checked that the system can be described by a linearized balance momentum equation. *Dantec* 20-μm polyamid spheres were used as seeding particles; they remain suspended for a long time and cause a minimal standard deviation in the velocity data since the size of the particles is uniform. We can assure that the particles follow the flow because, using Stokes's law [31], the obtained limit frequency

$$f<0.1\frac{\eta}{\rho_P R^2}, \tag{10}$$

for which the particles follow an oscillation with a deviation up to 1%, in amplitude, is much greater than the experimental frequencies (between 1.5 Hz and 15 Hz); $f=9.7\times10^7$ Hz for glycerol and $f=5.8\times10^9$ Hz for the CPyCl/NaSal solution. The particle density ρ_P is 10.3 kg/m^3 and the radius of a particle, R, is 1×10^{-5} m [32].

To avoid transient perturbations, measurements were taken ≈ 5 min after every change in the frequency.

IV. RESULTS

For each frequency 2000 velocity data were acquired with the LDA at the cylinder axis and 40 cm above the piston, where edge effects are negligible. Experiments were carried out in the oscillating frequency range $[1.5,15]\pm0.1$ Hz. In this frequency range the viscoelastic shear modulus is nearly real valued (lossy elastic solid). The calculated rms velocities from the LDA measurements for CPyCl/NaSal and for glycerol are shown in Fig. 2, where one observes that the measured Maxwellian velocity is dramatically different from the Newtonian velocity in the studied range. Clearly the rms velocity increases with the frequency in both cases, but the velocity of the viscoelastic fluid shows well-defined peaks. An important point to be stressed is the fact that the velocity in the glycerol case is a linear function with respect to the frequency, which means a good quality of the mechanical piston movement. Moreover, this confirms that the movement of the piston is transmitted directly to the fluid.

Originally inferred by the pioneers in the analysis of the dynamic permeability [1,2], the dissipative nature of the glycerol response is clearly seen in Fig. 3, where the glycerol dynamic response and the frequency have been scaled, the first one by the value of the dynamic response at $\omega=0$ and the second by the viscous time $\tau=a^2\rho/\eta$. Also shown in Fig. 3 are the theoretical predictions derived from Eq. (7) (magnitude of ξ with $t_m=0$).

Figure 4 contains the dynamic response of the CPyCl/NaSal solution. Experimental values are plotted together with theoretical predictions [Eq. (7)]. A dramatic change in the behavior of the dimensionless Maxwellian response with respect to the dissipative Newtonian case is observed, namely resonant frequencies appear. The agreement with Eq. (7) is manifest: the positions of the peaks in Eq. (7) and in the experimental results occur at approximately the same values of the frequency ω, even though the amplitudes differ. We need to recall that the theory used for the prediction is a linear approximation and neglects the convective term and the possible nonlinearities in the viscoelastic behavior of the fluid. Although we have selected the viscoelastic fluid in order to maintain the physical conditions where resonance behavior appears, we were unable to assure the linear characteristic of the viscoelastic fluid. According to Ref. [30] the CPCl/NaSal solution behaves as a Maxwellian fluid if the shear rate is $\gamma<0.6$ s^{-1}. Transforming Eq. (4) to the time space and taking the radial derivative we have calculated the

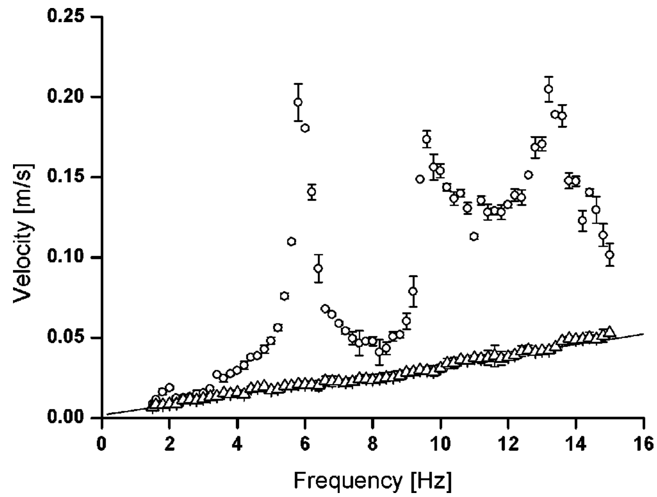

FIG. 2. Local rms velocity at the center of the cylinder. The circles correspond to the viscoelastic fluid and the triangles to the Newtonian fluid.

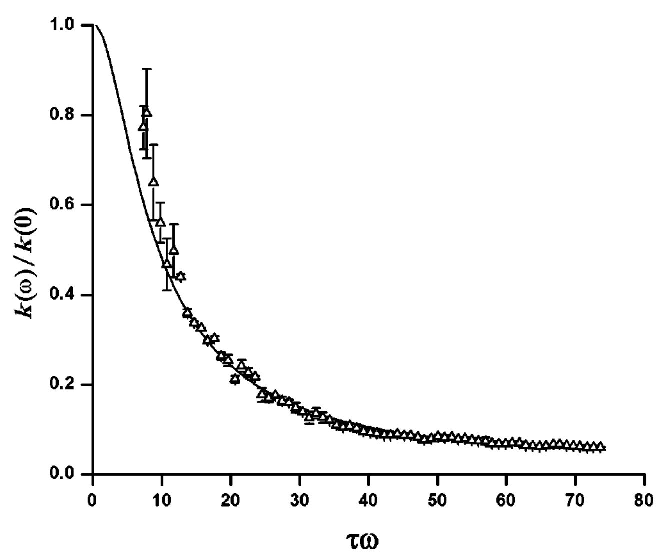

FIG. 3. Dynamic response for glycerol. Experimental values are shown by triangles [Eq. (9)] and the line represents the theoretical prediction [magnitude of ξ from Eq. (7) with $t_m=0$].

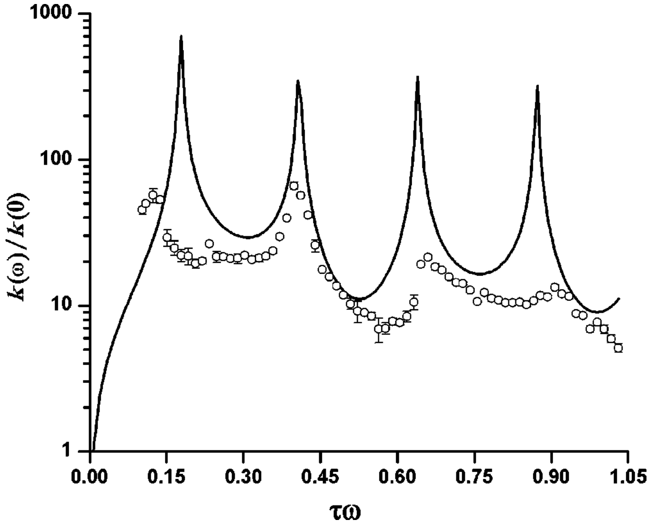

FIG. 4. Dynamic response for the CPyCl/NaSal solution. Experimental values are shown by circles [Eq. (9)] and the line represents the theoretical prediction [magnitude of ξ from Eq. (7)].

shear rate that depends on the radial coordinate and time. The maximum local shear rate increases with frequency, e.g., $\gamma_{2Hz} < 1.5\ \mathrm{s}^{-1}$, $\gamma_{6.5\ \mathrm{Hz}} < 21\ \mathrm{s}^{-1}$, $\gamma_{10\ \mathrm{Hz}} < 70\ \mathrm{s}^{-1}$, where the subscripts indicate the frequency. Then, one of the possible causes of the disagreement in the response amplitude is because the fluid was locally under higher shear rates than its Maxwellian limit. This is in agreement with the fact that if we increase the frequency, then we find larger differences between theory and experiment. Another reason for theoretical and experimental differences could be compressible effects in the viscoelastic flow, which also increase with frequency.

These results constitute the experimental evidence of the resonant behavior of a viscoelastic fluid and show the validity of the theory [12,20].

V. CONCLUSIONS

The local dynamic response of a fluid (either Newtonian or Maxwellian) has been developed following previous work [1,2,11,12]. This theoretical expression is compared with experimental LDA measurements in a harmonically oscillating pressure system. The expected dissipative behavior of Newtonian fluids is confirmed. On the other hand and although the theoretical approximation presented is a drastic simplification of the real viscoelastic behavior, it is still capable of reproducing the resonant behavior of the dynamic response of a Maxwellian fluid. The resonances appearing in the viscoelastic response, as expected, are significantly higher than the monotonic decay in the Newtonian response. In both cases, the qualitative agreement between theoretical predictions and the experimental results presented here is evident; the quantitative differences are probably due to nonlinear effects of the viscoelastic fluid used. Our experimental results show that the dynamic response of viscoelastic fluids clearly exceeds the static response for some specific frequencies, in fair agreement with the theory for such systems. Finally, we want to stress that the results obtained in this work have a wide spectrum of applications, from oil recovery problems [24] to the dynamical analysis of biological fluids [25,33,34].

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