Corrections to scaling in two-dimensional dynamic XY and fully frustrated XY models

B. Zheng,[1,2] F. Ren,[1,2] and H. Ren[1]
[1]*Physics Department, Zhejiang University, Hangzhou 310027, People's Republic of China*
[2]*FB Physik, Universität-Halle, 06099 Halle, Germany*

(Received 17 June 2003; published 21 October 2003)

With large-scale Monte Carlo simulations, we investigate the two-dimensional dynamic XY and fully frustrated XY models. Dynamic relaxation starting from a disordered or an ordered state is carefully analyzed. It is confirmed that there is a logarithmic correction to scaling for a disordered start, but a power-law correction for an ordered start. Rather accurate values of the static exponent η and the dynamic exponent z are estimated.

DOI: 10.1103/PhysRevE.68.046120 PACS number(s): 64.60.Ht, 02.70.Rr, 75.10.Hk

I. INTRODUCTION

In the last decade, many activities have been devoted to nonequilibrium relaxation of critical dynamics. Traditionally, it was believed that universal dynamic scaling behavior only exists in the long-time regime of dynamic evolution. In 1989, however, with renormalization group methods Janssen, Schaub, and Schmittmann derived a dynamic scaling form for the $O(N)$ vector model, which is valid up to the *macroscopic* short-time regime [1]. The dynamic process they considered is that the system initially at a very high temperature state with a small or zero magnetization is suddenly quenched to the critical temperature, and then released to dynamic evolution of model *A*. It is important that a new independent critical exponent must be introduced to describe the scaling behavior of the initial magnetization. This explains the anomalous behavior of the remanent magnetization in spin-glass dynamics [2].

On the other hand, the power-law decay of the magnetization in critical dynamics starting from a completely ordered state was found in rather early times, even though it was originally expected only in the long-time regime of dynamic evolution [3,4], and therefore was not referred to be the "short-time" behavior. The dynamic exponent z can be estimated from such a nonequilibrium relaxation.

Inspired and stimulated by these works, in the past years nonequilibrium short-time critical dynamics has been systematically investigated with Monte Carlo methods [5–11]. Simulations have been extended from simple spin models [9,10,12] to statistical systems with quenched disorder or frustration [13–17], XY models and Josephson junction arrays [18–25], quantum spin systems and lattice gauge theories [26–28], dynamic systems without detailed balance [29–31], melting transitions [32–34] and fluid systems [35] as well as first-order phase transitions [36–41]. More complete list of the relevant references before 1998 can be found in Ref. [9]. All these results confirm the existence of a rather general dynamic scaling form in critical dynamic systems at early times, and approximate scaling behavior in weak first-order phase transitions. The physical origin of the dynamic scaling behavior is the divergent or very large correlating time around the phase transition temperatures.

Actually, scaling behavior in nonequilibrium critical systems is not such a unique phenomenon in nature. For example, phase ordering dynamics and nonequilibrium critical dynamics share some similar features [42]. Spin-glass dynamics [2,11,14,43–45] structural glass dynamics, different kinds of growth dynamics, and aging phenomena in complex dynamic systems all may show certain scaling or quasi-scaling behavior. Concepts and methods, experiments and theories in these fields benefit from each other.

What we emphasize is that the short-time dynamic scaling form not only is conceptually interesting, but also—more interestingly and importantly, provides new techniques for the measurements of both dynamic and static critical exponents as well as the critical temperature [3,8,11,46], for a review see Ref. [9]. Since the measurements are carried out in the short-time regime, the dynamic approach does not suffer from critical slowing down. Compared with those methods developed in equilibrium, e.g., the nonlocal cluster algorithms, the dynamic approach does study the original local dynamics and can be applied to disordered or frustrated systems. Furthermore, it is very difficult to numerically solve dynamic equations with a continuous time to the long-time regime, but the short-time dynamic approach works well [10].

Recently, the idea of extracting information of the equilibrium state from nonequilibrium states has been extended to first-order phase transitions, and shows its efficiency [36–41]. Such a methodology should also be very interesting in experiments [47,48].

In understanding the universal behavior of short-time critical dynamics, it is very essential to distinguish the macroscopic and microscopic time scales. The dynamic scaling emerges only in the *macroscopic* short-time regime, after a time scale t_{mic} which is large enough in microscopic sense. t_{mic} is *not* universal. In Monte Carlo simulations, t_{mic} is rather small for the simple Ising and Potts models, e.g., from several to 100 Monte Carlo time steps [9]. However, this will not be the case for statistical systems with non-nearest-neighbor interactions, and especially with disorder, frustration, or many metastable states. For accurate measurements of the critical temperatures and critical exponents, corrections to scaling must be taken into account.

The XY model and the fully frustrated XY model have been intensively studied in the past years. The XY model is the simplest model exhibiting continuous symmetry and a Kosterlitz-Thouless phase transition in two dimensions, and may describe the critical behavior of thin films of superfluid helium. The fully frustrated XY model and its variants attract the attention of physicists because of their relevance to superconducting Josephson junction arrays in a transverse magnetic field. The dynamic approach has been found to be rather efficient and successful in dealing with the XY and fully frustrated XY models [18–25].

Bray, Briant, and Jervis have theoretically shown that there is a logarithmic correction for the two-dimensional XY model in the dynamic process starting from a disordered state [19] (see also Ref. [49]). It is believed that the logarithmic correction is induced by the vortex pair annihilation. However, the presented numerical data in Ref. [19] cannot distinguish the two *Ansätze*, a possible bigger z or a logarithmic correction. On the other hand, there has been some controversy over the value of the dynamic exponent z (see, e.g., Ref. [20] and references therein). In the case with a logarithmic correction to scaling, standard measurements of the critical exponents without taking into account the correction to scaling could be correct only asymptotically in the limit $t \to \infty$. Therefore, it is very essential to clarify the logarithmic correction.

In a recent paper [21], careful Monte Carlo simulations have been performed for the two-dimensional dynamic XY model at a temperature $T=0.89$, possible corrections to scaling in dynamic processes starting from both ordered and disordered states are examined, and relevant critical exponents are determined relatively accurately.

In this paper, simulations are extended to several temperatures below the transition temperature T_{KT}, and more systematic analysis of the data will be presented, including that of the nonequilibrium spatial correlation function. Furthermore, to reveal the effect of frustration, simulations for the two-dimensional dynamic fully frustrated XY model have been carried out. Dynamics of a statistical system with a Kosterlitz-Thouless phase transition and with frustration should be rather complicated. Our results show that dynamic scaling behavior does exist, even though corrections to scaling are much stronger than in the case without frustration. To fully understand the scaling behavior, however, the ground states of the system should be known. Fortunately, this is the case for the fully frustrated XY model we consider in this paper. Our approach is a first trial in this direction. We aim not only to reveal the dynamic scaling behavior, but also to provide relatively accurate measurements of the critical exponents, because simulations of the systems with a Kosterlitz-Thouless phase transition and with frustration in equilibrium is rather difficult.

The models and the scaling analysis of the dynamic behavior are described in Sec. II. Numerical simulations are presented in Sec. III. The final section contains the conclusions.

II. SCALING BEHAVIOR AND CORRECTIONS TO SCALING

A. Models

The two-dimensional XY model and fully frustrated XY (FFXY) model can be defined by the Hamiltonian

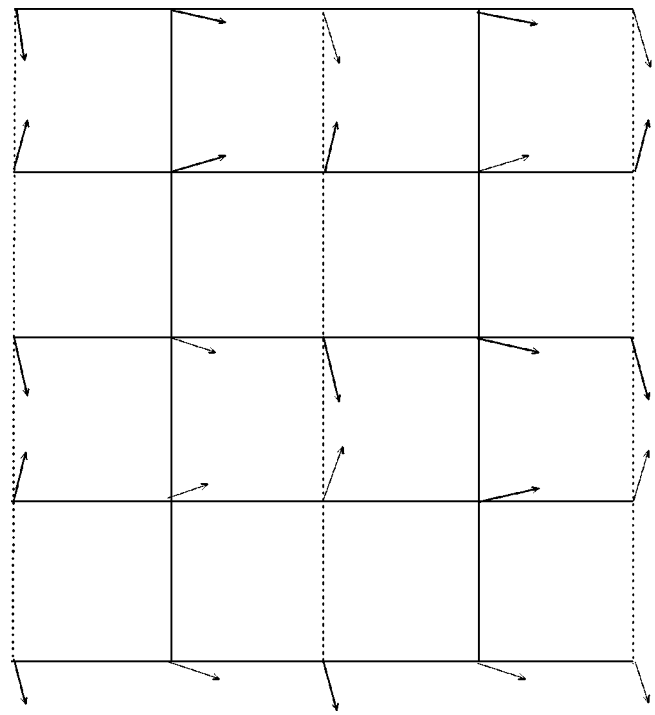

FIG. 1. A ground state of the 2D FFXY model. Dotted lines denote negative links, while solid lines correspond to positive links. The lattice is divided into four sublattices by the orientations of spins.

$$\frac{1}{kT}H=-K\sum_{\langle ij\rangle} f_{ij}\vec{S}_i\cdot\vec{S}_j, \tag{1}$$

where $\vec{S}_i=(S_{i,x},S_{i,y})$ is a planar unit vector at site i of a square lattice, the sum is over the nearest neighbors, and T is the temperature. For the XY model, $f_{ij}=1$ on all links. A simple realization of the FFXY model is by taking $f_{ij}=-1$ on half of the vertical links (negative links) and $+1$ on the others (positive links) [50], as is shown in Fig. 1. It is well known that the two-dimensional (2D) XY and FFXY model undergo a Kosterlitz-Thouless (KT) phase transition. In literature, the transition temperature T_{KT} is reported to be between 0.89 and 0.90 for the 2D XY model [18,22,51,52], while between 0.440 and 0.446 for the 2D FFXY model [53,54]. For the FFXY model, there is also a second-order phase transition in connection with chiral degrees of freedom. But in this paper, only the KT transition is concerned.

Since $\vec{S}_i$ is a planar unit vector, the Hamiltonian does not contain intrinsic dynamics. In this paper, we consider the Monte Carlo dynamics, which is believed to be in the same universality class of the Langevin dynamics. Following Refs. [19,21], we adopt the "heat-bath" algorithm in which a trial move is accepted with probability $1/[1+\exp(\Delta E/T)]$, where ΔE is the energy change associated with the move. This algorithm is somewhat faster than the standard Metropolis algorithm in a state far from equilibrium. The dynamic process we simulate is that the system initially in a completely ordered or disordered state is suddenly quenched to the KT transition temperature T_{KT} or below, and then released to dynamic evolution of model A.

Denoting a spin at the time t as $\vec{S}_i(t)$, as usual, we define the magnetization, its second moment, the autocorrelation, and the spatial correlation of the XY model at the time t as

$$\vec{M}(t) \equiv \left\langle \sum_i \vec{S}_i(t) \right\rangle \bigg/ L^2, \tag{2}$$

$$M^{(2)}(t) \equiv \left\langle \left[\sum_i \vec{S}_i(t) \right]^2 \right\rangle \bigg/ L^4, \tag{3}$$

$$A(t) \equiv \left\langle \sum_i \vec{S}_i(0) \cdot \vec{S}_i(t) \right\rangle \bigg/ L^2, \tag{4}$$

and

$$C(x,t) \equiv \left\langle \sum_i \vec{S}_i(t) \cdot \vec{S}_{i+x}(t) \right\rangle \bigg/ L^2, \tag{5}$$

respectively. Here L is the lattice size.

Due to the frustration of the couplings, spins in the ground state of the FFXY model do not orient in the same direction as in the XY model, rather the lattice is divided into four sublattices and spins on these four sublattices have different orientations. This is also shown in Fig. 1. Another ground state is obtained by translating the configuration in Fig. 1 by one lattice spacing in the vertical direction.

For the FFXY model, the magnetization is defined as the projection of the spins on the configuration of the ground state, and the second moment as well as the spatial correction function are calculated separately for each sublattice. But the autocorrelation function remains the same as in Eq. (4). Here it is very important that the definitions of the magnetization and its moments, and therefore the macroscopic initial states all rely on the ground state. If the ground state is not known, the ‘‘order parameter’’ must be defined differently. Then the dynamic scaling behavior may not be so simple as analyzed below.

B. Quench with ordered start

For the dynamic process quenched from a completely ordered state (an ordered start), e.g., $\vec{M}(0)=(1,0)$, we assume a universal dynamic scaling form in the *macroscopic short-time* regime, for example, for the kth moment of the magnetization

$$M^{(k)}(t,L)=\lambda^{-k\eta/2}M^{(k)}(\lambda^{-z}t,\lambda^{-1}L), \quad k=1,2. \tag{6}$$

Here $M(t)\equiv M^{(1)}(t)$ is the x component of the magnetization vector, η is the usual static exponent, z is the dynamic exponent, and λ is an arbitrary scale factor. Taking $\lambda=t^{1/z}$ and neglecting the finite size effect, one immediately obtains the power-law behavior

$$M(t)\sim t^{-\eta/2z}. \tag{7}$$

To determine z *independently*, we introduce a time-dependent Binder cumulant

$$U(t,L)=M^{(2)}/M^2-1. \tag{8}$$

When the nonequilibrium spatial correlation length at the time t is much smaller than the lattice size L, $U\sim 1/L^d$. Simple finite size scaling analysis leads to

$$U(t,L)\sim t^{d/z}. \tag{9}$$

Here $d=2$ is the spatial dimension.

In general, there may exist corrections to scaling in the early times, for example, the power-law corrections to scaling [21]

$$M(t)\sim t^{-\eta/2z}(1+c/t^b), \tag{10}$$

$$U(t)\sim t^{d/z}(1+c/t^b). \tag{11}$$

In the cases of the simple Ising and Potts models, corrections to scaling are rather weak [9]. For the models with many metastable states such as systems with disorder, frustration or KT transitions, however, corrections to scaling could be strong. For accurate estimate of critical exponents, one needs to take into account corrections to scaling.

C. Quench with disordered start

For the dynamic process quenched from a completely disordered state (a disordered start) with a zero or *small* initial magnetization $\vec{M}(0)=(m_0,0)$, a generalized dynamic scaling form can be written down, e.g., for the kth moment of the magnetization

$$M^{(k)}(t,m_0,L)=\lambda^{-k\eta/2}M^{(k)}(\lambda^{-z}t,\lambda^{x_0}m_0,\lambda^{-1}L), \quad k=1,2. \tag{12}$$

Here x_0 is an independent exponent describing the scaling behavior of m_0.

For a quench with a disordered start, corrections to scaling are very strong for the 2D XY model. In Ref. [19], it is shown that there should be logarithmic corrections to scaling. It is believed that the logarithmic corrections are related to the vortex pair annihilation, and do not disappear within early times [19,49].

We first consider the case of $m_0=0$ and with a sufficiently large lattice. Assuming a logarithmic correction for the nonequilibrium spatial correlation length, from scaling analysis and finite size scaling analysis, the second moment should behave [19] like

$$M^{(2)}(t)\sim\{t/[1+c\ln(t)]\}^{(2-\eta)/z}, \tag{13}$$

and the autocorrelation

$$A(t)\sim\{t/[1+c\ln(t)]\}^{\theta-d/z}. \tag{14}$$

Similarly, the scaling behavior of the spatial correlation function with a logarithmic correction to scaling is

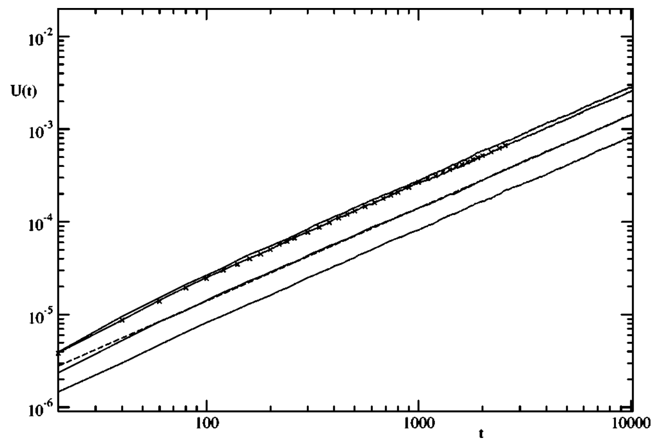

FIG. 2. The time-dependent Binder cumulant of the 2D dynamic XY model with an ordered start. Solid lines are for temperatures $T=0.90$, 0.89, 0.80, and 0.70 (from above) with a lattice size $L=256$. The dashed line shows a power-law fit for $T=0.80$. The crossed line is obtained with $L=128$ for $T=0.89$.

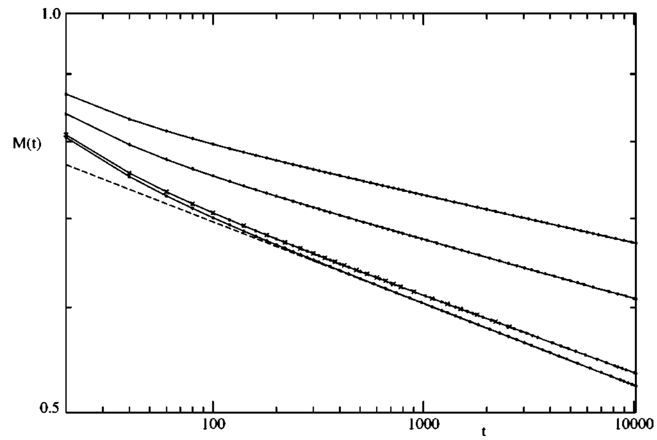

FIG. 3. The magnetization of the 2D dynamic XY model with an ordered start. Solid lines are for temperatures $T=0.90$, 0.89, 0.80, and 0.70 (from below) with a lattice size $L=256$. The dashed line shows a power-law fit for $T=0.90$. Dots fitted to the solid lines are with power-law corrections to scaling. The crossed line is obtained with $L=128$ for $T=0.89$.

$$C(x,t)=\{t/[1+c\ \ln(t)]\}^{-\eta/z}F(x/\{t/[1+c\ \ln(t)]\}^{1/z}). \tag{15}$$

For a nonzero but sufficiently small m_0, one can deduce from Eq. (12)

$$M(m_0,t)\sim t^{\theta}, \tag{16}$$

θ is related to x_0 by $\theta=(x_0-\eta/2)/z$ [1,9]. If the lattice size L is big enough, the above power-law behavior holds in a time scale $t_0\sim m_0^{-z/x_0}$. Typically, the exponent θ is positive. Therefore, this anomalous behavior is also called a critical initial increase of the magnetization.

Usually, the correction to scaling for $M(m_0,t)$ is weak because the nonzero m_0 could suppress the effect of the vortex pairs. Even if there is a correction, it does not affect so much our estimate of the dynamic exponents z and η, for the value of θ usually is rather small.

III. NUMERICAL SIMULATIONS

In order to detect any corrections to scaling and obtain accurate values of the critical exponents, we have performed the simulations up to $t=10\,240$ Monte Carlo time steps with a lattice size $L=256$. An exceptional case is for the disordered start with small m_0, where it is only up to $t=1000$. To investigate the finite size effect, some simulations are also performed for $L=128$ and 512 maximally to $t=40\,960$. Samples of the initial configurations for averaging are from 12 000 to 24 000, depending on the models, temperatures, and initial states. To estimate the errors, samples are divided into some subsamples. In addition, errors induced by fluctuations along the time direction are also taken into account.

A. Quench with ordered start for *XY* model

In Figs. 2 and 3, the Binder cumulant and magnetization of the 2D dynamic XY model with an ordered start are displayed with solid lines on a log-log scale. To uncover possible corrections to scaling, we measure the slope of the curves of $U(t)$ and $M(t)$ in a time interval $[t_1,10\,240]$, with t_1 varying from 50 to 800. The results are listed in Table I.

For the Binder cumulant $U(t)$, the slope for different t_1 fluctuates within 0.5%, comparable to statistical errors.

TABLE I. The slope of the curves of $U(t)$, $M(t)$, $A(t)$, and $M^{(2)}(t)$ in Figs. 2–5 measured in a time interval $[t_1,10\,240]$ for the 2D dynamic XY model. The transition temperature T_{KT} is believed to be between 0.90 and 0.89.

	t_1	$T=0.90$	0.89	0.80	0.70
	50	1.005	0.999	1.000	0.998
	100	1.004	0.997	0.999	0.997
$U(t)$	200	1.002	0.995	0.999	0.996
	400	1.001	0.995	1.001	0.995
	800	1.000	0.997	1.004	0.995
	50	0.0623	0.0592	0.0452	0.0365
	100	0.0620	0.0589	0.0450	0.0363
$M(t)$	200	0.0618	0.0587	0.0448	0.0361
	400	0.0617	0.0585	0.0446	0.0360
	800	0.0616	0.0584	0.0444	0.0359
	50	0.640	0.631	0.583	0.563
	100	0.643	0.634	0.586	0.566
$A(t)$	200	0.645	0.636	0.589	0.570
	400	0.648	0.639	0.592	0.574
	800	0.651	0.643	0.596	0.579
	50	0.771	0.765	0.774	0.778
	100	0.773	0.767	0.776	0.781
$M^{(2)}(t)$	200	0.776	0.770	0.777	0.784
	400	0.779	0.772	0.780	0.787
	800	0.783	0.778	0.784	0.792

TABLE II. The extracted exponents for the 2D dynamic XY model after taking into account the power-law corrections for $M(t)$ with an ordered start, and logarithmic corrections for $M^{(2)}(t)$ and $A(t)$ with a disordered start. $\eta/2z$ in the third row for $M(t)$ is obtained with a fixed correction exponent $b=1$. The value z_1 of the dynamic exponent z is estimated from d/z, η is calculated from $\eta/2z$ by taking z_1 as input, z_2 is from $(d-\eta)/z$ with η as input, and z_3 is calculated from $d/z-\theta$ and θ.

		$T=0.90$	0.89	0.80	0.70
$U(t)$	d/z	1.000(10)	0.995(5)	0.999(4)	0.995(5)
	z_1	2.00(2)	2.01(1)	2.00(1)	2.01(1)
$M(t)$	$\eta/2z$	0.0614(4)	0.0581(2)	0.0441(3)	0.0358(2)
	b	1.13	1.03	0.95	1.07
	$\eta/2z$	0.0611	0.0580	0.0442	0.0357
	Fixed b	1	1	1	1
	η	0.246(3)	0.234(2)	0.176(2)	0.144(1)
Ref. [51]	η	0.239	0.229	0.179	0.146
$M^{(2)}(t)$	$(d-\eta)/z$	0.860(12)	0.877(9)	0.897(10)	0.920(8)
	z_2	2.04(3)	2.01(2)	2.03(2)	2.02(2)
$A(t)$	$d/z-\theta$	0.756(5)	0.738(4)	0.711(5)	0.695(6)
$M(m_0,t)$	θ	0.241(2)	0.249(2)	0.263(4)	0.280(4)
	z_3	2.01(2)	2.02(2)	2.05(2)	2.05(2)

Therefore, corrections to scaling are negligible here. Even if we fit the curves with the *Ansatz* in Eq. (11), it gives the same results as without corrections to scaling. In Fig. 2, the dashed line shows a power-law fit to the curve of $T=0.80$. The fit is almost perfect starting from $t=50$.

For the magnetization $M(t)$, however, the slope for different t_1 shows a definite decreasing trend. This trend could induce an error of 2% or 3% in the measurements of the critical exponents. In Fig. 3, the dashed line is a power-law fit to the curve of $T=0.90$. Obviously, the curve of $T=0.90$ deviates visibly from the power-law behavior in the first some hundred time steps. To describe the corrections to scaling, we fit the curves to Eq. (10). In Fig. 3, dots represent the curves with the corrections to scaling, and fit nicely to the simulation data (solid lines) starting already from $t=20$. The resulting values of $\eta/2z$ and b are listed in the first two rows of the sector $M(t)$ in Table II.

Looking at the values of b for different temperatures, one finds that they are around 1. One may wonder whether the correction exponent b here is "universal" or not. We have performed the fitting with a fixed $b=1$ for all temperatures. The corresponding values of $\eta/2z$ are given in the third row of the sector $M(t)$ in Table II. Within errors, they are consistent with those values without fixing b. Therefore, the correction exponent b may be indeed universal for different temperatures.

To investigate the finite size effect, we have simulated the dynamic process for the temperature $T=0.89$ with a lattice size $L=128$. The Binder cumulant and magnetization have been plotted with crossed lines in Figs. 2 and 3. For comparison, the Binder cumulant has been divided by a factor of 4. Up to the time $t=2560$, the curves for $L=128$ and 256 overlap almost completely. Since the time scale for a finite system is $\sim L^z$ and z is about 2, we conclude that the finite size effect for $L=256$ up to $t=10\,240$ should be negligibly small in our simulations.

In Table II, the dynamic exponent z and static exponent η calculated from d/z and $\eta/2z$ are listed, and values of η estimated from simulations in equilibrium are taken from Ref. [51] for comparison. The dynamic exponent z is very close to the theoretical value $z=2$. Our η is somewhat bigger than that in Ref. [51] at temperatures around T_{KT}, but smaller at lower temperatures. If we linearly interpolate the value of η to $T=0.8933$, it is $0.238(3)$, about 2% smaller than the value $0.243(4)$ in a recent paper [52]. But these two values are still consistent if the errors are taken into account.

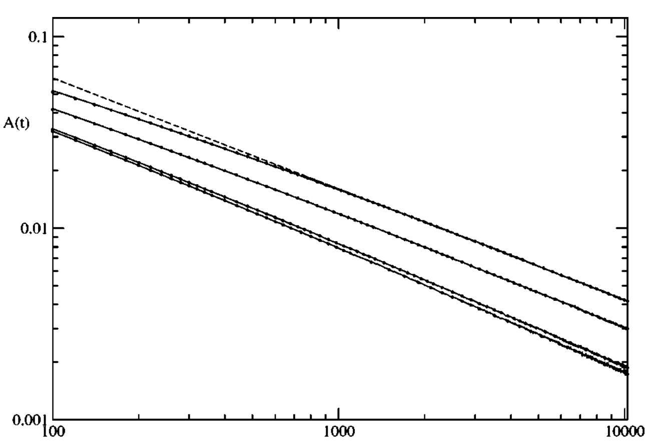

FIG. 4. The autocorrelation of the 2D dynamic XY model with a disordered start. Solid lines are for temperatures $T=0.90$, 0.89, 0.80, and 0.70 (from below). The dashed line shows a power-law fit to $T=0.70$. Dots fitted to the solid lines are with logarithmic corrections to scaling. But the slope of the dashed line is 0.579, far from 0.695 with a logarithmic correction to scaling.

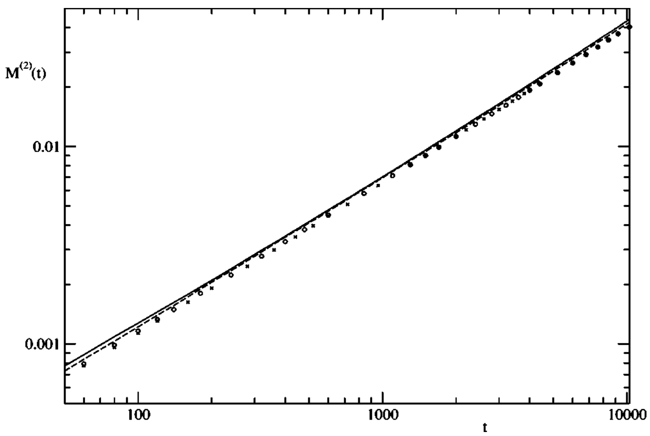

FIG. 5. The second moment of the 2D dynamic XY model with a disordered start. Crossed, circled, dashed, and solid lines are for temperatures $T=0.90$, 0.89, 0.80, and 0.70, respectively.

B. Quench with disordered start for XY model

In Figs. 4 and 5, the autocorrelation and the second moment of the 2D dynamic XY model with a disordered start are displayed on a log-log scale. Looking at the curves by eyes, they are not too far from a power-law behavior. In Fig. 4, for example, the dashed line shows a power-law fit to the curve of $T=0.70$. It seems that the fit is rather good starting from $t\sim 800$, but the situation is actually not so simple.

To reveal the corrections to scaling, we have also measured the slope of the curves of $A(t)$ and $M^{(2)}(t)$ in a time interval $[t_1, 10\,240]$, with t_1 varying from 50 to 800. The results are listed in Table I. For both $A(t)$ and $M^{(2)}(t)$, the slope shows an increasing trend. The difference among slopes with different t_1 is about 2% or 3%, comparable with that for $M(t)$ in the preceding section. If one fits the curves with power-law corrections to scaling, however, the correction exponent b is rather small. According to the argument in Ref. [19], the corrections are logarithmic, i.e., the limiting case of $b\rightarrow 0$. In Fig. 4, dots represent the curves with the logarithmic corrections to scaling, and fit to the numerical data (solid lines) from rather early times. In Table II, the resulting values of $(2-\eta)/z$ and $d/z-\theta$ are given.

It is very important to observed that the slope of a power-law fit in a time interval of $[800, 10\,240]$ as shown with the dashed line in Fig. 4 is still different by 10–15% from that with the logarithmic corrections to scaling. The logarithmic correction is so strong such that the effective exponent obtained with a power-law fit would be correct only in the limit of $t\rightarrow\infty$. In the measurements of the critical exponents, therefore, it is extremely important to take into account the corrections to scaling.

To further confirm and clarify the logarithmic corrections to scaling, we plot the data collapse of the nonequilibrium spatial correlation function $C(x,t)$ in Fig. 6. Taking $z=2.01$, $\eta=0.234$, and $c=0.704$ obtained from $M^{(2)}(t)$ as input, data of different time t rescaled suitably according to Eq. (15) collapse nicely to the curve of $t=160$, except for some departure for $t<100$.

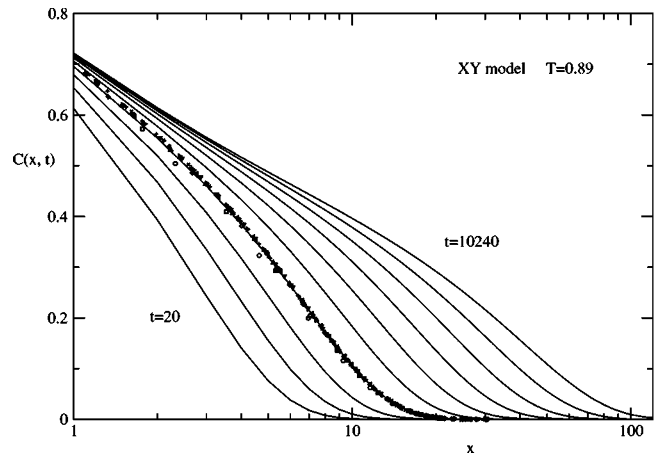

FIG. 6. Data collapse of the correlation function $C(x,t)$ of the 2D dynamic XY model with a disordered start. Solid lines are for $t=20$, 40, 80, 160, 320, 640, 1280, 2560, 5120, and 10 240 (from left). Circles, squares, diamonds, triangles up, triangles left, triangles down, triangles right, pluses, and crosses fitted to the curve of $t=160$ are curves of $t=20$, 40, 80, 320, 640, 1280, 2560, 5120 and 10 240, but rescaled according to Eq. (15) with $z=2.01$, $\eta=0.234$, and $c=0.704$.

In principle, the dynamic exponent z and the static exponent η as well as the constant c may be also extracted from the data collapse of $C(x,t)$. But the accuracy is not as high as in the measurements from $A(t)$ and $M^{(2)}(t)$.

To complete our investigation, especially to verify the scaling behavior of $A(t)$ with an exponent $d/z-\theta$, we finally perform simulations with a disordered start but a small nonzero initial magnetization m_0. Since we need a small initial magnetization m_0 and therefore suffer from large fluctuation in longer times, the simulations are performed only up to $t=1000$. In Fig. 7, $M(m_0,t)$ is displayed with solid lines on log-log scale. From these data, we can not detect a logarithmic correction. In a time interval $[100,1000]$, direct measurements of the slope yield the same exponents as with a power-law correction.

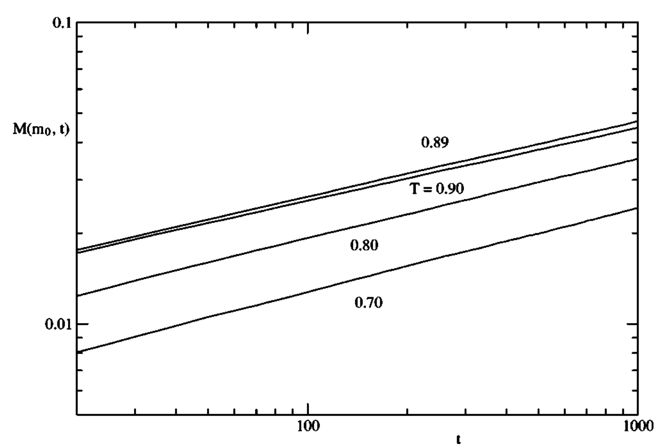

FIG. 7. The initial increase of the magnetization of the 2D dynamic XY model with a disordered start. The initial values of m_0 for temperatures $T=0.90$, 0.89, 0.80, and 0.70 are 0.008, 0.008, 0.005, and 0.003, respectively.

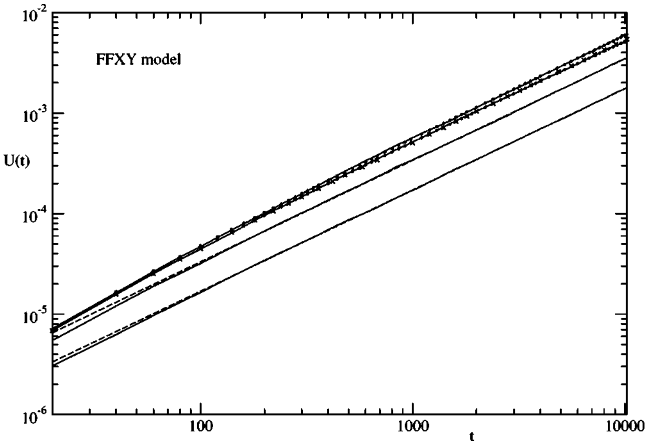

FIG. 8. The time-dependent Binder cumulant of the 2D dynamic FFXY model with an ordered start. Solid lines are for temperatures $T=0.446$, 0.440, 0.40, and 0.30 (from above). Dashed lines show power-law fits to $T=0.30$ and 0.40 (from below). Dots are with power-law corrections to scaling for $T=0.440$ and 0.446 (from below). The crossed line is obtained with $L=128$ for $T=0.440$.

In Table II, the dynamic exponent z extracted from $A(t)$, $M^{(2)}(t)$, and $M(m_0,t)$ is given. The values are bigger than 2 by 2% or 3%. This probably indicates that the logarithmic correction is still not perfect in the time intervals we simulate.

In general, for the quench with an ordered start, corrections to scaling are stronger at temperatures around T_{KT}, while for the quench with a disordered start, corrections to scaling are stronger at lower temperatures. These phenomena are understandable since the vortices and vortex pairs play an essential role around the KT transition temperature.

C. Quench with ordered start for FFXY model

In Figs. 8 and 9, the Binder cumulant and magnetization of the 2D dynamic FFXY model with an ordered start are displayed on a log-log scale. To uncover possible corrections

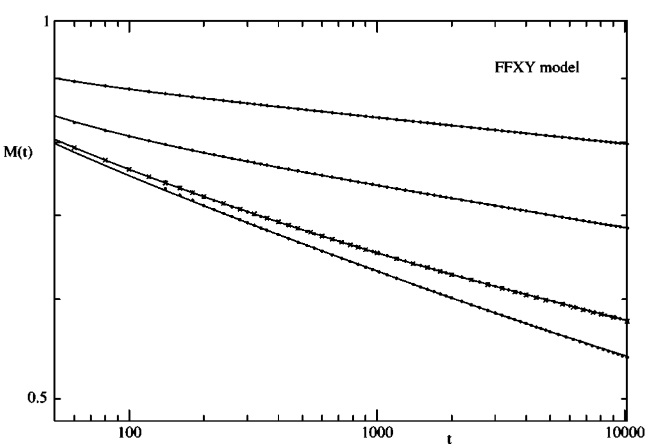

FIG. 9. The magnetization of the 2D dynamic FFXY model with an ordered start. Solid lines are for temperatures $T=0.446$, 0.440, 0.40, and 0.30 (from below). Dots fitted to the solid lines are with power-law corrections to scaling. The crossed line is obtained with $L=128$ for $T=0.440$.

TABLE III. The slope of the curves of $U(t)$ and $M(t)$ in Figs. 8 and 9 measured in a time interval $[t_1, 10\,240]$ for the 2D dynamic FFXY model. The transition temperature T_{KT} is believed to be between 0.446 and 0.440.

	t_1	$T=0.446$	0.440	0.40	0.30
	50	1.034	1.014	1.011	1.009
	100	1.031	1.011	1.010	1.007
$U(t)$	200	1.028	1.007	1.009	1.007
	400	1.025	1.003	1.010	1.007
	800	1.025	0.999	1.012	1.008
	50	0.0695	0.0563	0.0349	0.0212
	100	0.0689	0.0556	0.0345	0.0211
$M(t)$	200	0.0683	0.0547	0.0342	0.0210
	400	0.0676	0.0537	0.0340	0.0209
	800	0.0668	0.0527	0.0338	0.0209

to scaling, we again measure the slope of the curves of $U(t)$ and $M(t)$ in a time interval $[t_1, 10\,240]$, with t_1 varying from 50 to 800. The results are listed in Table III.

For the Binder cumulant, as shown by the dashed lines in Fig. 8, the corrections to scaling are small for $T=0.40$ and 0.30. But for $T=0.446$ and $T=0.440$, there exist some. In addition, the resulting exponent d/z and the correction exponent b for $T=0.440$ fluctuate a little, depending on the time interval $[t_1, 10\,240]$ in which fitting is carried out. But the universal value $b=1$ is still rather reasonable. The fit with a power-law correction to scaling is shown with dots in Fig. 8. The final values of the exponent d/z are listed in TableIV. Here the values of d/z for $T=0.446$ and 0.440 are estimated with a fixed correction exponent $b=1$, and the errors include those with an unfixed b.

For the magnetization, the corrections to scaling depend also on the temperatures. The higher the temperature is, the stronger the correction to scaling will be. The fit to a power-law correction to scaling for $T=0.30$ and 0.40 yields a correction exponent b very close to 1. But the resulting values of $\eta/2z$ and b for $T=0.446$ and 0.440 fluctuate for different time intervals $[t_1, 10\,240]$ in which fitting is carried out. Therefore, we have additionally performed some simulations for $T=0.440$ and 0.446 with $L=512$ up to the time $t=40960$. As is discussed in Sec. III A and this section, the

TABLE IV. The extracted exponents for the 2D dynamic FFXY model after taking into account the power-law corrections for $M(t)$ and $U(t)$ with an ordered start. The correction exponent is fixed to be $b=1$. The dynamic exponent z is estimated from d/z; η is calculated from $\eta/2z$ by taking z as input.

		$T=0.446$	0.440	0.40	0.30
$U(t)$	d/z	1.019(6)	0.994(10)	1.010(3)	1.007(3)
	z	1.96(1)	2.01(2)	1.98(1)	1.99(1)
$M(t)$	$\eta/2z$	0.0581(18)	0.0506(4)	0.0334(3)	0.0207(2)
	η	0.228(7)	0.203(3)	0.132(2)	0.0824(9)
Ref. [55]	η		0.196	0.122	0.064

results with $L=512$ tend to confirm that the correction exponent b takes a universal value $b=1$, even though it seems not very clear for $T=0.446$. Therefore, the values given in Table IV are with a fixed $b=1$, but the errors include those with an unfixed b, and with different lattices.

We have also simulated the dynamic process for the temperature $T=0.440$ with a lattice size $L=128$. The Binder cumulant and magnetization have been plotted with crossed lines in Figs. 8 and 9. Analyzing the data with different L carefully, we conclude that the finite size effect for $L=256$ up to $t=10\,240$ is negligible small.

In Table IV, the dynamic exponent z and static exponent η calculated from d/z and $\eta/2z$ are listed, and values of η estimated from simulations in equilibrium are taken from Ref. [55] for comparison. The dynamic exponent z is also very close to the value $z=2$. For $T=0.446$, the value $z=1.96(1)$ should indicate that the transition temperature T_{KT} may be slightly below 0.446. Our value η is bigger than that in Ref. [55].

IV. CONCLUSIONS

In conclusions, with large-scale Monte Carlo simulations we have investigated corrections to scaling in the nonequilibrium dynamic processes starting from both ordered and disordered states for the two-dimensional XY and FFXY models. The results confirm that there is a logarithmic correction to scaling in case of starting from a disordered state, but a power-law correction in case of starting from an ordered state. Rather accurate values of the static exponent η and the dynamic exponent z have been obtained. The correction exponent b in the case with an ordered start is about 1, and the estimated dynamic exponent z is very close to 2. The static exponent η carries an error of about 1% (somewhat bigger for the FFXY model at the temperature $T=0.446$). The values of z estimated from the dynamic process with a disordered start are slightly bigger than 2, but it should only indicate that the logarithmic corrections to scaling have not been perfect in the time interval we simulate.

Since the dynamic process starting from a disordered state for the FFXY model is rather complicated, we have not been able to understand it, and further investigation is needed.

ACKNOWLEDGMENTS

This work was supported in part by the DFG (Germany) under Grant No. TR 300/3-3, NNSF (China) under Grant No. 10275054, and the SRFDP (China).

[1] H.K. Janssen, B. Schaub, and B. Schmittmann, Z. Phys. B: Condens. Matter **73**, 539 (1989).
[2] D.A. Huse, Phys. Rev. B **40**, 304 (1989).
[3] D. Stauffer, Physica A **186**, 197 (1992).
[4] N. Ito, Physica A **196**, 591 (1993).
[5] K. Humayun and A.J. Bray, J. Phys. A **24**, 1915 (1991).
[6] Z.B. Li, U. Ritschel, and B. Zheng, J. Phys. A **27**, L837 (1994).
[7] L. Schülke and B. Zheng, Phys. Lett. A **204**, 295 (1995).
[8] P. Grassberger, Physica A **214**, 547 (1995).
[9] B. Zheng, Int. J. Mod. Phys. B **12**, 1419 (1998), review article.
[10] B. Zheng, M. Schulz, and S. Trimper, Phys. Rev. Lett. **82**, 1891 (1999).
[11] R.E. Blundell, K. Humayun, and A.J. Bray, J. Phys. A **25**, L733 (1992).
[12] R. da Silva, N.A. Alves, and J.R.D. de Felicio, Phys. Rev. E **66**, 026130 (2002).
[13] H.J. Luo, L. Schülke, and B. Zheng, Phys. Rev. Lett. **81**, 180 (1998).
[14] H.J. Luo, L. Schülke, and B. Zheng, Mod. Phys. Lett. B **13**, 417 (1999).
[15] H.P. Ying and K. Harada, Phys. Rev. E **62**, 174 (2000).
[16] N. Ito and Y. Ozeki, Physica A **321**, 262 (2003).
[17] H.P. Ying, H. Ren, H.J. Luo, and L. Schülke, Phys. Lett. A **298**, 60 (2002).
[18] B. Zheng, M. Schulz, and S. Trimper, Phys. Rev. E **59**, R1351 (1999).
[19] A.J. Bray, A.J. Briant, and D.K. Jervis, Phys. Rev. Lett. **84**, 1503 (2000).
[20] L.M. Jensen, B.J. Kim, and P. Minnhagen, Phys. Rev. B **61**, 15412 (2000).
[21] H.P. Ying, B. Zheng, Y. Yu, and S. Trimper, Phys. Rev. E **63**, R35101 (2001).
[22] Y. Ozeki, K. Ogawa, and N. Ito, Phys. Rev. E **67**, 026702 (2003).
[23] K. Medvedyeva, P. Holme, P. Minnhagen, and B.J. Kim, Phys. Rev. E **66**, 026130 (2002).
[24] M.B. Luo, Q.H. Chen, S.S. Feng, and Z.K. Jiao, J. Phys.: Condens. Matter **14**, 6483 (2002).
[25] Q.H. Chen, M.B. Luo, and Z.K. Jiao, Phys. Rev. B **64**, 212403 (2001).
[26] H.P. Ying, H.J. Luo, and L. Schülke, and B. Zheng, Mod. Phys. Lett. B **12**, 1237 (1998).
[27] K. Okano, L. Schülke, and B. Zheng, Phys. Rev. D **57**, 1411 (1998).
[28] M. Santos, Phys. Rev. E **61**, 7204 (2000).
[29] J.F.F. Mendes and M.A. Santos, Phys. Rev. E **57**, 108 (1998).
[30] T. Tomé and M.J. de Oliveira, Phys. Rev. E **58**, 4242 (1998).
[31] A. Brunstein and T. Tomé, Phys. Rev. E **60**, 3666 (1999).
[32] A. Jaster, Phys. Lett. A **258**, 59 (1999).
[33] A. Jaster, Physica A **277**, 106 (2000).
[34] H. Watanabe, S. Yukawa, Y. Ozeki, and N. Ito, Phys. Rev. E **66**, 041110 (2002).
[35] W.B. Zhang *et al.*, Phys. Lett. A **272**, 408 (2000).
[36] L. Schülke and B. Zheng, Phys. Rev. E **62**, 7482 (2000).
[37] E.V. Albano, Phys. Lett. A **288**, 73 (2001).
[38] R.A. Monetti and E.V. Albano, Europhys. Lett. **56**, 400 (2001).
[39] G.P. Zheng, J. Phys. A **35**, 10549 (2002).
[40] Y. Ozeki, K. Kasono, N. Ito, and S. Miyashita, Physica A **321**, 271 (2003).
[41] G.P. Saracco and E.V. Albano, J. Chem. Phys. **118**, 4157 (2003).
[42] A.J. Bray, Adv. Phys. **43**, 357 (1994), and references therein.
[43] P. Granberg, P. Svedlindh, P. Nordblad, L. Lundgren, and H.S.

Chen, Phys. Rev. B **35**, 2075 (1987).

[44] K. Fischer and J. Hertz, *Spin Glasses* (Cambridge University Press, Cambridge, 1991).

[45] J. Kisker, L. Santen, M. Schreckenberg, and H. Rieger, Europhys. Lett. **20**, 159 (1992).

[46] Z.B. Li, L. Schülke, and B. Zheng, Phys. Rev. Lett. **74**, 3396 (1995).

[47] K. Zahn and G. Maret, Phys. Rev. Lett. **85**, 3656 (2000).

[48] K. Zahn, J.M. Méndez-Alcaraz, and G. Maret, Phys. Rev. Lett. **79**, 175 (1997).

[49] A.J. Bray, Phys. Rev. E **62**, 103 (2000).

[50] A. Scheinine, Phys. Rev. B **39**, 9368 (1989).

[51] R. Gupta and C.F. Baillie, Phys. Rev. B **45**, 2883 (1992).

[52] Y. Tomita and Y. Okabe, Phys. Rev. B **65**, 184405 (2002).

[53] S. Lee and K. Lee, Phys. Rev. B **49**, 15184 (1994).

[54] P. Olsson, Phys. Rev. Lett. **75**, 2758 (1995).

[55] G. Ramirez-Santiago and J.V. José, Phys. Rev. B **49**, 9567 (1994).