Delay times and reflection in chaotic cavities with absorption

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Absorption yields an additional exponential decay in open quantum systems which can be described by shifting the (scattering) energy E along the imaginary axis, $E+i\hbar/2\tau_a$. Using the random-matrix approach, we calculate analytically the distribution of proper delay times (eigenvalues of the time-delay matrix) in chaotic systems with broken time-reversal symmetry that is valid for an arbitrary number of generally nonequivalent channels and an arbitrary absorption rate τ_a^{-1}. The relation between the average delay time and the "norm-leakage" decay function is found. Fluctuations above the average at large values of delay times are strongly suppressed by absorption. The relation of the time-delay matrix to the reflection matrix $S^\dagger S$ is established at arbitrary absorption that gives us the distribution of reflection eigenvalues. The particular case of single-channel scattering is explicitly considered in detail.

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There is a growing interest in statistical properties of the Wigner-Smith matrix $Q(E)=-i\hbar S^\dagger \partial S/\partial E$ [1,2], with $S(E)$ being the scattering matrix at the collision energy E, in the cases of chaotic scattering and transport in disordered media [3]. In the resonance scattering, the matrix element $Q_{cc'}$ describes the overlap of the internal parts of the scattering wave functions in the incident channels c and c' [2,4]. This directly relates the Wigner-Smith matrix to the effective non-Hermitian Hamiltonian $\mathcal{H}=H-(i/2)VV^\dagger$ of the unstable intermediate system as follows (henceforth $\hbar=1$) [4]:

$$Q(E)=V^\dagger \frac{1}{(E-\mathcal{H})^\dagger}\frac{1}{E-\mathcal{H}}V. \tag{1}$$

The Hermitian part H stands here for the closed counterpart of the system while the amplitudes V_n^c describe the coupling between N interior and M channel states. The random-matrix theory approach is usually adopted to simulate the complicated intrinsic motion [5–7].

The known analytical results [8–14] are restricted to the idealization neglecting absorption. The latter is, however, always present to some extent under laboratory conditions, being one of the sources of a coherence loss in quantum transport. This has dramatic consequences for the statistical observables [15,16]. Necessity of proper accounting of finite decoherence [17] was recently emphasized [18] in order to remove a certain discrepancy between theory [19,20] and experiment [16] on conductance distributions in quantum dots. Reflection in a weakly absorbing medium turned out to be directly related [15,21,22] to the time-delay matrix without absorption. Recent experiments [23] in microwave cavities demonstrated that the absorption (due to the skin effect in the walls) may be strong, leading to an exponential decay [15,23].

In this paper we show that representation (1) in terms of the effective Hamiltonian allows us to extend the consideration to the case of an arbitrary absorption. The nature of the exponential decay caused by absorption can be easily understood from the following model consideration which actually goes back to the concept of the spreading width in nuclear physics [24]; see Ref. [4] for the recent developments. In addition to coupling to continuum (scattering) states the originally closed system is considered to be also coupled to the background compound environment. The latter has a very dense spectrum with the mean level spacing $\Delta_{\rm bg}$ being much smaller than the corresponding one Δ of the closed system, $\Delta_{\rm bg}\ll\Delta$. When the coupling strength v^2 is large enough to mix background states, $v^2>\Delta_{\rm bg}^2$, the original levels acquire the damping or spreading width $\Gamma_\downarrow\equiv 2\pi v^2/\Delta_{\rm bg}$ [4,24]. Corrections to the resulting exponential decay show up at the time $t_*\sim\Delta_{\rm bg}^{-1}$ and, therefore, can be safely ignored on mesoscopic scale of the Heisenberg time $t_H\equiv 2\pi/\Delta\ll t_*$ we are interested in.

In the absorption limit of continuous spectrum of the background, when an irreversible decay into walls takes place, this description becomes equivalent to that achieved in the framework of the Büttiker's model of dephasing in mesoscopic conductors; see Fig. 1 [25]. One considers [17,19] M_ϕ fictitious scattering channels in addition to M real ones. The vanishing transmission $T_\phi\to 0$ of the fictitious channels is assumed to be compensated by their large number $M_\phi\to\infty$, the dimensionless absorption rate $\gamma=M_\phi T_\phi$ being kept fixed [20]. Then the anti-Hermitian part of the effective Hamiltonian $\mathcal{H}$, which describes coupling to (all) open channels, splits readily as $\Sigma^M_{c,\rm real}V_n^c V_m^{c\,*}+\delta_{nm}\Gamma_a$ [20] into the escape contribution (first term) and damping one with $\Gamma_a\equiv\tau_a^{-1}\equiv\gamma\Delta/2\pi$. An exponential decay, associated with the last term, lasts up to the characteristic time $t_*=t_H/\sqrt{\gamma T_\phi}$ [26] being large as compared to t_H.

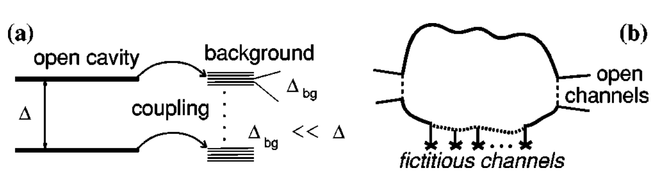

FIG. 1. An open cavity with absorption in walls modeled by coupling to (a) the compound background or (b) an infinite number of fictitious channels with vanishing transmission probabilities.

The consideration presented suggests that *nonzero* absorption is equivalent to the purely imaginary shift $E+(i/2)\Gamma_a \equiv E_\gamma$ of the energy in the Green's function $(E-\mathcal{H})^{-1}$ of the open system *without* absorption as long as resonance scattering far from the channel thresholds is concerned [27]; see also Refs. [22,28]. This is in agreement with the available data on correlations of S matrix elements in cavities with absorption [23].

In what follows we consider the time-delay matrix with absorption $Q_\gamma \equiv Q(E_\gamma)$, with Q from Eq. (1), treating $\gamma = \Gamma_a t_H$ as a phenomenological parameter. The important relation for the reflection matrix

$$R \equiv S_\gamma^\dagger S_\gamma = 1 - \Gamma_a Q_\gamma \tag{2}$$

follows directly from the definition of the scattering matrix $S_\gamma \equiv S(E_\gamma) = 1 - iV^\dagger (E_\gamma - \mathcal{H})^{-1} V$, which is subunitary ($R < 1$) at nonzero absorption. This relation gives Q_γ the meaning of the matrix of unitarity deficit and generalizes limiting expressions of Refs. [21,22] valid at weak absorption to the case of *arbitrary* Γ_a. Q_γ is an $M\times M$ Hermitian, positive-definite matrix and, therefore, has real positive eigenvalues q_c, the so-called *proper* delay times. They were recently studied in much detail for the case of zero absorption [13,14]. Even a weak absorption modifies their statistical properties significantly, as will be shown below.

We begin with the calculation of the average total delay time $q_{\rm tot} \equiv \overline{q_1 + \cdots + q_M} = \overline{{\rm tr} Q_\gamma}$, where the bar denotes the ensemble average. Making use of the invariance of the trace under cyclic permutations and the following relation $VV^\dagger = i[(E_\gamma - \mathcal{H})^\dagger - (E_\gamma - \mathcal{H})] - \Gamma_a$, one gets

$$q_{\rm tot} = {\rm Im\,Tr}\,\overline{\frac{-2}{E_\gamma - \mathcal{H}}} - \Gamma_a {\rm Tr}\overline{\left(\frac{1}{E_\gamma - \mathcal{H}}\,\frac{1}{(E_\gamma - \mathcal{H})^\dagger}\right)}, \tag{3}$$

where Tr acts in the N-dimensional intrinsic space of resonances. The first term is known [8,9] to be equal to the Heisenberg time t_H. To calculate the second one, it is instructive to go to the time domain and to exploit the well-known relation between the Green's function and the time evolution operator $\exp(-i\mathcal{H}t)$. This enables us to represent Eq. (3) in the following form:

$$\tau_{\rm tot} \equiv \frac{q_{\rm tot}}{t_H} = 1 - \Gamma_a \int_0^\infty dt\, e^{-\Gamma_a t} P(t), \tag{4}$$

where $P(t) \equiv (1/N) {\rm Tr}\overline{(e^{i\mathcal{H}^\dagger t} e^{-i\mathcal{H}t})}$ is the "norm-leakage" decay function introduced in Ref. [26]. The average delay time within the cavity becomes smaller due to additional dissolution in the walls. The average weighted-mean reflection coefficient $\langle r\rangle \equiv M^{-1}{\rm tr}\overline{R}$ is correspondingly given by $\langle r\rangle = 1 - \gamma\tau_{\rm tot}/M$.

$P(t)$ can be calculated by means of Efetov's supersymmetry technique [5,29], which becomes now a standard analytical tool. Here we only state the corresponding result for the case of preserved time-reversal symmetry (TRS):

$$P(t) = \int_{-1}^{1} d\lambda \int_1^\infty d\lambda_1 \int_1^\infty d\lambda_2 \mu(\lambda_i)\, \delta\!\left(\frac{t}{t_H} - \frac{\lambda_1\lambda_2 - \lambda}{2}\right) \times f(\lambda_i) \prod_{c=1}^{M} \left[\frac{(g_c+\lambda)^2}{(g_c + \lambda_1\lambda_2)^2 - (\lambda_1^2 - 1)(\lambda_2^2 - 1)}\right]^{1/2}, \tag{5}$$

where $\mu(\lambda_i) = (1-\lambda^2)/(\lambda_1^2 + \lambda_2^2 + \lambda^2 - 2\lambda\lambda_1\lambda_2 - 1)^2$ and $f(\lambda_i) = (2\lambda_1^2\lambda_2^2 - \lambda_1^2 - \lambda_2^2 - \lambda^2 + 1)/4$. The quantities $g_c = 2/T_c - 1 \geqslant 1$ are related to the transmission coefficients $T_c = 1 - |\overline{S_{cc}}|^2$ [5], which determine the openness strength of the system (without absorption), referring $T=1$ (0) to the completely open (closed) one. For reader's convenience, we note that the result for the case of broken TRS [26] follows from Eq. (5) by removing there the λ_2 integration and setting $\lambda_2 = 1$ everywhere in the integrand save the integration measure $\mu(\lambda_i) = (\lambda_1 - \lambda)^{-2}$ in this case [30]. It is also worth pointing out the relation between $P(t)$, Eq. (5), and the autocorrelation function of the photodissociation cross section [31]. The exact (in the RMT limit $N\to\infty$) Eq. (4) is valid for any symmetry and will also be derived below using a different way.

The norm leakage is identical to unity when the system is closed (hence the norm). Its time dependence is solely due to the openness of the system and has been thoroughly studied in Ref. [26] that allows us to understand the qualitative dependence of $q_{\rm tot}$ on absorption. The typical behavior $P(t) \sim \prod_{c=1}^{M} [1 + (2/\beta) T_c t/t_H]^{-\beta/2}$, with $\beta = 1$ (2) standing for preserved (broken) TRS, is the simple exponential $\exp(-t\Sigma_c T_c/t_H)$ at small enough times. In the so-called "diagonal approximation" [26], which neglects the nonorthogonality of the resonance wave functions and becomes asymptotically exact at large t, $P(t)$ turns out to be related by the Laplace transform $P_{\rm diag}(t) = \int_0^\infty d\Gamma\, e^{-\Gamma t} \rho(\Gamma) \equiv \langle e^{-\Gamma t}\rangle_\Gamma$ to the distribution $\rho(\Gamma)$ of resonance widths. One gets readily from Eq. (4) that $\tau_{\rm tot} = \langle \Gamma/(\Gamma + \Gamma_a)\rangle_\Gamma$ within this very approximation. The simple interpolation formula $\tau_{\rm tot} \approx (1 + \gamma/\Sigma_c T_c)^{-1}$ with corrections of the order of $\min[1/\gamma, 1/\Sigma_c T_c]$ becomes exact as the absorption rate γ and/or the total (dimensionless) escape width $\Sigma_c T_c$ grows.

We proceed further with an analysis of the distribution of the proper delay times $\mathcal{P}(q) = M^{-1}\Sigma_c \overline{\delta(q - q_c)}$. For the sake of simplicity, we restrict ourselves to the case of broken TRS (the unitary symmetry class). The factorized representation (1) of Q_γ enables us to use the same method developed in Ref. [14] to treat the zero absorption case. Thus, we skip all standard technical details, indicating only essential ones. As usual, the jump of the resolvent $G(z) = M^{-1}{\rm tr}\overline{(z - Q_\gamma)^{-1}}$ on the discontinuity line along $q = {\rm Re}\, z > 0$ determines the seeking distribution as follows: $\mathcal{P}(q) = \pi^{-1} {\rm Im}\, G(q - i0)$. Due to the factorized structure of Q_γ, $G(z)$ can then be represented in the form suitable for subsequent supersymmetry calculation [32]. We find the following expression for the determining part $K(\zeta = z/t_H) \equiv M\zeta^2 [t_H G(z) - 1/\zeta]$:

$$K(\zeta)=1+\frac{1}{2}\int_1^\infty d\lambda_1\int_{-1}^1\frac{d\lambda}{\lambda_1-\lambda}\prod_{c=1}^M\frac{g_c+\lambda}{g_c+\lambda_1}\times\left(\frac{\partial}{\partial\nu_1}-\frac{\partial}{\partial\nu}\right)b_\gamma(\lambda_1)f_\gamma(\lambda)|_{\nu_1=\nu=1}. \tag{6}$$

Here $b_\gamma(\lambda_1)=e^{(1-\gamma\zeta/2)\nu_1\lambda_1/\zeta}I_0((\nu_1/\zeta)\sqrt{(1-\gamma\zeta)(\lambda_1^2-1)})$ and $f_\gamma(\lambda)=e^{-(1-\gamma\zeta/2)\nu\lambda/\zeta}J_0((\nu/\zeta)\sqrt{(1-\gamma\zeta)(1-\lambda^2)})$, with $I_0(x)[J_0(x)]$ being the modified [usual] Bessel function. The resolvent $G(\zeta)$ given by Eq. (6) is an analytical function of the complex variable ζ for the negative values of $\mathrm{Re}\,\zeta$ and, therefore, can be expanded there in Taylor's series. One finds directly from the definition of G that $t_H G(z)=1/\zeta+\overline{\mathrm{tr}\,Q_\gamma}/(M\zeta^2 t_H)+\cdots$ for large ζ, relating thus q_w to the coefficient of the second term of this expansion. On the other hand, this coefficient is given just by $K(-\infty)$ which can easily be calculated from Eq. (6) to reproduce exactly Eq. (4).

An analytical continuation in Eq. (6) to the region of positive $\tau\equiv\mathrm{Re}\,\zeta$ requires more care as compared to the case [14] of zero absorption, where it was achieved by a proper deformation of an original integration contour. First we make the following decomposition in partial fractions:

$$\frac{1}{\lambda_1-\lambda}\prod_{c=1}^M\frac{g_c+\lambda}{g_c+\lambda_1}=\frac{1}{\lambda_1-\lambda}-\sum_{a=1}^M\frac{1}{g_a+\lambda_1}\prod_{b(\neq a)}\frac{g_b+\lambda}{g_b-g_a}.$$

The contribution from the term $(\lambda_1-\lambda)^{-1}$ leads to an exact cancellation of the first term in Eq. (6). [This is not surprising since the product term in Eq. (6), the channel factor, which determines solely the strength of system openness, reduces at $\lambda_1=\lambda$ to unity, resulting $G(z)=1/z$ in this case identically.] The integration over λ_1 gets completely decoupled from that over λ in the contribution from the rest sum. Making use of the table integrals [33], one finds that

$$\int_1^\infty\frac{d\lambda_1 e^{s\lambda_1}}{g+\lambda_1}I_0(\alpha\sqrt{\lambda_1^2-1})=\int_0^\infty dp\,\frac{e^{-gp}e^{\sqrt{(p-s)^2-\alpha^2}}}{\sqrt{(p-s)^2-\alpha^2}}, \tag{7}$$

with notations $s\equiv\tau^{-1}-\gamma/2$ and $\alpha\equiv\tau^{-1}\sqrt{1-\tau\gamma}$. Just this term (7) has a nonzero imaginary part, thus the distribution, at positive $\tau-i0$. A close inspection of the right-hand side of Eq. (7) shows that the imaginary part is determined by the integration region $s-\alpha<p<s+\alpha$, resulting at the end in $\pi I_0(\alpha\sqrt{g^2-1})\Theta(\tau^{-1}-\gamma)$, with the step function $\Theta(x)$.

We arrive finally at the following general expression for the probability distribution of the proper delay times:

$$\mathcal{P}\left(\tau=\frac{q}{t_H}\right)=\frac{1}{M}\sum_{c=1}^M\left(\frac{\partial}{\partial\nu}-\frac{\partial}{\partial\nu_1}\right)B_cF_c|_{\nu_1=\nu=1}, \tag{8}$$

for $0<\tau\leq\gamma^{-1}$, and $\mathcal{P}(\tau)\equiv0$ otherwise. Here

$$B_c=e^{-\nu_1 s g_c}I_0(\nu_1\alpha\sqrt{g_c^2-1})\prod_{a(\neq c)}\frac{1}{g_a-g_c}, \tag{9a}$$

$$F_c=\int_{-1}^{+1}\frac{d\lambda}{2}e^{-\nu s\lambda}J_0(\nu\alpha\sqrt{1-\lambda^2})\prod_{a(\neq c)}(g_a+\lambda). \tag{9b}$$

The obtained result is valid for arbitrary absorption strength and arbitrary transmission coefficients of M generally nonequivalent channels. The limit of zero absorption [14] is correctly reproduced. The case of statistically equivalent channels can easily be worked out by performing the limiting transition $g_c\to g=2/T-1$ for all c. At last, the distribution function $P_R(r)=M^{-1}\overline{\Sigma_c\delta(r-r_c)}$ of *reflection eigenvalues* $r_c=1-\gamma q_c/t_H$ follows readily from Eq. (8) as

$$P_R(r)=\gamma^{-1}\mathcal{P}(\gamma^{-1}(1-r)),\quad 0\leq r<1. \tag{10}$$

We see that the absorption rate γ enters the distribution in a highly nontrivial way. This is expected to be true for any distribution function and is contrasted with a correlation function of, say, S matrix elements $\overline{S_{ab}^*(E_\gamma)S_{a'b'}(E_\gamma+\varepsilon)}$. The corresponding form factor [23] (the Fourier transform of the correlation function) differs from that [5] of the zero absorption case simply by the presence of an additional exponential term $e^{-\gamma t/t_H}$. The most striking effect of finite absorption on the time-delay distribution is likely to consist in suppression of the universal long-time tails $\tau^{-M(\beta/2)-2}$ [10–14] at $\tau>\gamma^{-1}$ [34]. To understand this fact qualitatively, we note that the delay time $q(E)\approx\Gamma_n/[(E-E_n)^2+\frac{1}{4}(\Gamma_n+\Gamma_a)^2]$ in the vicinity of a given resonance with the energy E_n. The maximal value of this single-resonance contribution is attained at $E=E_n$, being $q_{\max}=4\Gamma_n/(\Gamma_n+\Gamma_a)^2\leq1/\Gamma_a$ for any value of the (positive) escape width Γ_n.

We analyze now the important case of single-channel scattering, $M=1$, in more detail. The explicit expression to be obtained from Eq. (8) reads as follows:

$$\mathcal{P}(\tau)=\frac{e^{-gs}}{\tau^2}\Big\{I_0(\alpha\sqrt{g^2-1})\left(\cosh\frac{\gamma}{2}-\frac{2}{\gamma}\sinh\frac{\gamma}{2}\right)+\frac{2}{\gamma}\sinh\frac{\gamma}{2}[gsI_0(\alpha\sqrt{g^2-1})-\alpha\sqrt{g^2-1}I_1(\alpha\sqrt{g^2-1})]\Big\}. \tag{11}$$

We have explicitly checked the normalization of this distribution to unity and verified relation (4) for the first moment. This function should be compared to the more simple expression $\mathcal{P}_{\gamma=0}(\tau)=\tau^{-1}(\partial/\partial\tau)e^{-g/\tau}I_0(\tau^{-1}\sqrt{g^2-1})$ [10] valid at zero absorption. Figure 2 shows the behavior of $\mathcal{P}(\tau)$ in two limiting cases of the weakly and perfectly open system. One sees in the first case that the maximum of the distribution function at the small time $\tau\sim(2g)^{-1}\approx T/4\ll1$ gets more pronounced and narrow as the absorption rate γ grows. At larger values of τ the distribution is exponentially suppressed with $\mathcal{P}(\gamma^{-1})\approx(\gamma^2/T)e^{-\gamma/T}$. The latter is contrasted with the behavior in the case of perfect coupling, $g=T=1$, when

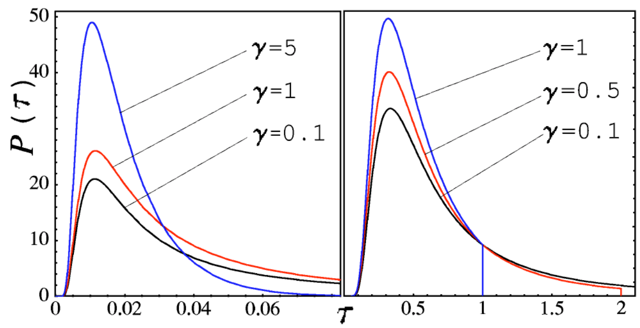

FIG. 2. (Color online) Distribution (11) of the time delay in single-channel scattering for different values of the absorption rate γ at weak ($T=0.1$, left) and perfect ($T=1$, right) coupling.

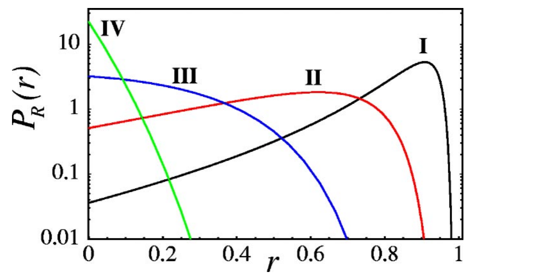

FIG. 3. (Color online) The reflection coefficient distribution in the single-channel cavity at four experimental realizations [36] of the absorption rate and transmission coefficient (see the text for details). The values $(2\gamma, T)$ correspond to I (0.56, 0.12), II (2.42, 0.75), III (8.4, 0.98), and IV (48, 0.99).

$\mathcal{P}_{T=1}(\tau)=\tau^{-2}e^{-1/\tau}[1+(1-\tau)(e^{\gamma}-1)/\tau\gamma]$ and $\mathcal{P}(\gamma^{-1})$ could be rather large. We relate these distinctions to peculiarities in fluctuations of the resonance escape widths in the two cases considered. The width distribution $\rho(\Gamma)$, which is known exactly [10] for any T and M, has the simple exponential form $e^{-\Gamma t_H/T}$ when coupling is small, $T\ll 1$, and the power law behavior $\sim\Gamma^{-2}$ at $\Gamma\gtrsim t_H^{-1}$ when $T=1$. The ratio of the widths $\Gamma\sim\Gamma_a$ determining $\mathcal{P}(\tau\sim\gamma^{-1})$ is, therefore, exponentially small in the first case and only a power in the second. This conclusion holds for any finite M.

The sharp border at $\tau=\gamma^{-1}$ of the obtained distribution is the direct consequence of Eq. (2) with the absorption rate fixed to a constant. Although, as shown above, the value $\mathcal{P}(\gamma^{-1})$ of the jump may be exponentially small when coupling is weak, a generic exponential suppression should be intuitively expected at large values of delay times $\tau\gg\gamma^{-1}$. Indeed, for the time δt a wave packet oscillating in the cavity with the frequency $\Delta/2\pi$, on an average, experiences $(\Delta/2\pi)\delta t$ collisions with the walls, yielding the probability $T_\phi(\Delta/2\pi)\delta t$ to be absorbed into one of M_ϕ fictitious channels. The total reflection is then estimated as $R\simeq[1-T_\phi(\Delta/2\pi)\delta t]^{M_\phi}$, giving $e^{-\gamma\delta t/t_H}$ in the limit of fixed $\gamma=M_\phi T_\phi$ as $M_\phi\to\infty$ and $T_\phi\to 0$. It is instructive, therefore, to define alternatively through the following relation $R\equiv e^{-\Gamma_a Q_R}$ the matrix Q_R, which we call the matrix of *reflection time delays*. The positive-definite matrix Q_R is related to Q_γ as $Q_R=-\Gamma_a^{-1}\ln(1-\Gamma_a Q_\gamma)$ that leads to the following connection:

$$\mathcal{P}_R(\tau_r)=e^{-\gamma\tau_r}\mathcal{P}(\gamma^{-1}(1-e^{-\gamma\tau_r})),\quad \tau_r>0, \tag{12}$$

between the corresponding distributions $\mathcal{P}_R(\tau_r)$ and $\mathcal{P}(\tau)$ of proper delay times (eigenvalues of Q_R and Q_γ, respectively). Both Q_R and Q_γ reduce to the same Wigner-Smith matrix (1) in the limit of vanishing absorption. The difference between them becomes noticeable at finite γ. Still both distributions coincide up to the time appreciably less than γ^{-1}. They start to differ at larger times, when $\mathcal{P}(\tau)$ has the cutoff whereas $\mathcal{P}_R(\tau\gg\gamma^{-1})\propto e^{-\gamma\tau}$ is exponentially suppressed.

Finally, we discuss the distribution $P_R(r)$ of the reflection coefficient $r=|S_\gamma|^2=1-\gamma\tau$ in the single-channel cavity. This distribution at arbitrary values of γ and T is explicitly given by Eqs. (10) and (11), reproducing exactly the recent result [35] obtained by a different method. In the particular case of perfect coupling it simplifies further to the expression $P_{T=1}(r)=(1-r)^{-3}e^{-\gamma/(1-r)}[\gamma(e^{\gamma}-1)+(1+\gamma-e^{\gamma})(1-r)]$ found earlier [22]. For the case of preserved TRS ($\beta=1$), the reflection coefficient distribution in a microwave cavity has recently been measured [36]. Our distribution $P_R(r)$ at the values of absorption and transmission realized in this experiment under compulsory (although not surprising in the RMT) rescaling γ to $\gamma\beta/2$ with $\beta=1$ is shown in Fig. 3. (This corresponds to replacing our parameter γ in Eqs. (10) and (11) with $T_w/2$ of Ref. [36].) That should roughly take into account the difference between the symmetry class of our analytical result ($\beta=2$) and that of the experiment. Such a replacement is expected to become more efficient as absorption grows. The trend is clearly seen from the distribution sharply peaked near $r\sim 1$ at weak absorption ($\gamma\ll 1$) to the Rayleigh distribution $P_R(r)\simeq(\gamma\beta/2)e^{-r\gamma\beta/2}$ [28], see also Ref. [22], reproduced correctly at strong absorption ($\gamma\gg 1$) and perfect coupling ($T=1$). Figure 3 is in good qualitative agreement with the experimental data reported in Ref. [36] (see Figs. 4 and 6 there), which becomes even quantitative as absorption gets stronger. The rigorous analytical treatment for the case of preserved TRS is still lacking, being under current investigation.

In summary, we have calculated the general distribution of proper delay times and reflection coefficients in an open chaotic system (e.g., billiard) with broken TRS at arbitrary absorption. Finite absorption leads to strong suppression of fluctuations at large values of delay times, making the distribution narrower around the mean. The latter as well as the mean reflection coefficient are found to be related to the norm-leakage decay function. The particular case of single-channel scattering is paid appreciable attention, when discussion of available experimental data is also given.

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[1] E. Wigner, Phys. Rev. **98**, 145 (1955).
[2] F.T. Smith, Phys. Rev. **118**, 349 (1960).
[3] For a general review of the concepts of the time delay and its applications, see C.A.A. de Carvalho and H.M. Nussenzveig, Phys. Rep. **364**, 83 (2002) as well as Ref. [10].
[4] V.V. Sokolov and V. Zelevinsky, Phys. Rev. C **56**, 311 (1997).
[5] J.J.M. Verbaarschot, H.A. Weidenmüller, and M.R. Zirnbauer, Phys. Rep. **129**, 367 (1985).
[6] C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).
[7] Y. Alhassid, Rev. Mod. Phys. **72**, 895 (2000).
[8] V.L. Lyuboshitz, Phys. Lett. B **72**, 41 (1977); Yad. Fiz. **27**, 948 (1978) [Sov. J. Nucl. Phys. **27**, 502 (1978)].
[9] N. Lehmann, D.V. Savin, V.V. Sokolov, and H.-J. Sommers, Physica D **86**, 572 (1995).
[10] Y.V. Fyodorov and H.-J. Sommers, J. Math. Phys. **38**, 1918 (1997); Phys. Rev. Lett. **76**, 4709 (1996).
[11] V.A. Gopar, P.A. Mello, and M. Büttiker, Phys. Rev. Lett. **77**, 3005 (1996).
[12] Y.V. Fyodorov, D.V. Savin, and H.-J. Sommers, Phys. Rev. E **55**, R4857 (1997); D.V. Savin, Y.V. Fyodorov, and H.-J. Sommers, *ibid.* **63**, 035202(R) (2001).
[13] P.W. Brouwer, K.M. Frahm, and C.W.J. Beenakker, Phys. Rev. Lett. **78**, 4737 (1997); Waves Random Media **9**, 91 (1999).
[14] H.-J. Sommers, D.V. Savin, and V.V. Sokolov, Phys. Rev. Lett. **87**, 094101 (2001).
[15] E. Doron, U. Smilansky, and A. Frenkel, Phys. Rev. Lett. **65**, 3072 (1990).
[16] A.G. Huibers, S.R. Patel, C.M. Marcus, P.W. Brouwer, C.I. Duruöz, and J.S. Harris, Jr., Phys. Rev. Lett. **81**, 1917 (1998).
[17] M. Büttiker, Phys. Rev. B **33**, 3020 (1986).
[18] E.R.P. Alves and C.H. Lewenkopf, Phys. Rev. Lett. **88**, 256805 (2002).
[19] H.U. Baranger and P.A. Mello, Phys. Rev. B **51**, 4703 (1995).
[20] P.W. Brouwer and C.W.J. Beenakker, Phys. Rev. B **55**, 4695 (1997); **66**, 209901(E) (2002).
[21] S.A. Ramakrishna and N. Kumar, Phys. Rev. B **61**, 3163 (2000).
[22] C.W.J. Beenakker and P.W. Brouwer, Physica E **9**, 463 (2001).
[23] R. Schäfer, T. Gorin, T.H. Seligman, and H.-J. Stöckmann, J. Phys. A **36**, 3289 (2003).
[24] A. Bohr and B.R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. 1.
[25] It is worth noting that the case of absorption, when dissolution in walls occurs, should be contrasted with that of dephasing, when the particle number in the cavity is conserved; see Ref. [20].
[26] D.V. Savin and V.V. Sokolov, Phys. Rev. E **56**, R4911 (1997).
[27] Only in this case the smooth energy dependence of the coupling amplitudes can be neglected, so that any scattering quantity depends explicitly on E by means of $(E-\mathcal{H})^{-1}$ only.
[28] E. Kogan, P.A. Mello, and H. Liqun, Phys. Rev. E **61**, R17 (2000).
[29] K.B. Efetov, *Supersymmetry in Disorder and Chaos* (Cambridge University Press, Cambridge, UK, 1996).
[30] This is the usual procedure in the crossover regime of gradually broken TRS; see Refs. [12,29]. Note also a misprint in Ref. [26]: $f(\lambda_i)=(\lambda_1^2-\lambda^2)/4$ is correct.
[31] Y.V. Fyodorov and Y. Alhassid, Phys. Rev. A **58**, R3375 (1998).
[32] The generating function (4) of Ref. [14] can be again written in terms of the $2N\times 2N$ determinants $\mathrm{Det}(A_\pm)$, where now $A_\pm = \frac{1}{2}VV^\dagger-(z_\pm^{-1}-\frac{1}{2}\Gamma_a)-z_\pm^{-1}\sqrt{1-z_\pm\Gamma_a}\sigma_1-i(E-H)\sigma_3$.
[33] $\int_1^\infty dt e^{-st}I_0(\pm\alpha\sqrt{t^2-1}=e^{\sqrt{s^2-\alpha^2}}/\sqrt{s^2-\alpha^2}$ for $s>\alpha\geqslant 0$. For the calculation of the imaginary part we use $\int_{-1}^1 dt e^{-\alpha g t}\cos(\alpha\sqrt{1-t^2})/\sqrt{1-t^2}=\pi I_0(a\sqrt{g^2-1})$.
[34] Similar suppression was previously observed in Ref. [21] for the time-delay distribution in single-channel reflection from one-dimensional amplifying random media.
[35] Y.V. Fyodorov, JETP Lett. **78**, 250 (2003).
[36] R.A. Méndez-Sánchez *et al.*, e-print cond-mat/0305090.