Car accidents in cellular automata models for one-lane traffic flow

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Conditions for the occurrence of car accidents are introduced in the Nagel-Schreckenberg model. These conditions are based on the thought that a real accident depends on several parameters: an unexpected action of the car ahead (sudden stop or abrupt deceleration), the gap between the two cars, the velocity of the successor car and its delayed reaction time. We discuss then the effect of this delayed reaction time on the probability of traffic accidents. We find that these conditions for the occurrence of car accidents are necessary for modeling realistic accidents.

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I. INTRODUCTION

In recent years, vehicular traffic problems have attracted much attention, and a number of cellular automata (CA) describing traffic flow have been proposed in order to consider the dynamical aspects of the traffic system [1,2]. Presently, there are two basic CA models that describe single-lane traffic flow: the Nagel-Schreckenberg (NS) model [3] and the Fukui-Ishibashi (FI) model [4]. Besides the CA models which are discrete in space and time, several other approaches to traffic flow have been discussed recently. Among these are space-continuous models in discrete time such as the model of Krauss *et al.* [5], as well as models continuous both in space and time, e.g., the macroscopic (fluid-dynamical) models [6]. These traffic flow models have successfully reproduced many qualitative features observed in real traffic systems such as traffic jams [7,8], traffic with hindrance [9], highway junctions [10], etc.

Our modern life is very much affected by car traffic, which fulfills many human daily needs. This traffic, however, represents a major everyday risk of accident injury or death. Recently, CA models have been extended to study the occurrence of car accidents [11–16]. Boccara *et al.* [11] have been the first authors to propose conditions for car accidents to occur in the deterministic NS model. The first condition is that the number of empty cells in front of the car (gap) is smaller than the speed limit, the second condition is that the car ahead is moving, and the last condition is that the moving car ahead is suddenly stopped at the next time step. Using these conditions, the exact results of the probability of a car accident are obtained in special cases [12,13]. General numerical results for the probability of car accidents are reported in the nondeterministic NS model [14]. In the FI model, the probability for an accident to occur is found to be proportional to the product of the fraction of stopped cars and the traffic flow [15]. Although, the conditions of Boccara *et al.* have made a great progress in the study of car accidents, we think that more investigations with techniques of statistical physics are highly desirable. The main aim of this paper is, on one hand, to introduce conditions for the occurrence of car accidents, based on the delayed reaction time of the successor car, and to discuss its effects on the probability of traffic accidents on the other hand.

The paper is organized as follows. In Sec. II we define the NS model. In Sec. III we discuss the original conditions for the occurrence of car accidents and present our conditions. In Sec. IV we present the results of computer simulations of the probability of the occurrence of car accidents and discuss the advantages of our conditions. Finally, we conclude with a summary in Sec. V.

II. BASIC MODEL

The basic CA model for traffic flow is the NS model. This model is a probabilistic CA of traffic flow in a one-lane roadway. It consists of N cars moving in a one-dimensional lattice of L cells with periodic boundary conditions (the number of vehicles is conserved). Each cell is either empty or occupied by just one vehicle with velocity $v=1,2,\ldots,v_{\max}$. We denote by $x(k,t)$ and $v(k,t)$ the position and the velocity of the kth car at time t respectively. The number of empty cells in front of the kth car is denoted by $d(k,t)=x(k+1,t)-x(k,t)-1$ and is called hereafter as the gap. Space and time are discrete. At each discrete time step $t\to t+1$ the system update is performed in parallel for all cars according to the following four subrules: rule 1 (acceleration)—$v(k,t+\frac{1}{3})\leftarrow\min[v(k,t)+1,v_{\max}]$; rule 2 (slowing down) (due to other cars)—$v(k,t+\frac{2}{3})\leftarrow\min[v(k,t+\frac{1}{3}),d(k,t)]$. rule 3 (randomization)—$v(k,t+1)\leftarrow\max[v(k,t+\frac{2}{3})-1,0]$ with probability p; and rule 4 (motion)—the car is moved forward according to its new velocity, $x(k,t+1)\leftarrow x(k,t)+v(k,t+1)$.

Rule 1 reflects the tendency of drivers to drive as fast as possible if allowed to do so, without exceeding the maximum speed limit. Rule 2 is intended to avoid collisions between cars. The randomization in Rule 3 takes into account the different behavioral patterns of the individual drivers, especially nondeterministic acceleration and overreaction while slowing down; this randomization is important for the spontaneous formation of traffic jams [2,8]. If the stochastic randomization p is equal to zero, the model is called the *deterministic* NS model. For a realistic description of highway traffic, the typical length of a cell is about 7.5 m, which is interpreted as the length of a vehicle plus the distance between vehicles in a jam. Each time step corresponds to ≈ 1 s of real time for $v_{\max}=5$. The vehicles have speeds that are multiples of 1 cell/s which corresponds to 27 km/h; for example, $v_{\max}=5$ corresponds to 135 km/h.

III. CONDITIONS FOR CAR ACCIDENTS TO OCCUR

In the basic model, car accidents will not occur because of the second rule which is designed to avoid accidents. However, in real traffic, car accidents often occur if the conditions for safe driving are not satisfied. Recent studies point out that dangerous situations (DSs) exist within the framework of the NS model [11–16]. These DSs concern the states of two neighborhood cars at different time steps; therefore they are correlative, both spatialy and temporal. In the following sections we shall investigate in detail, the issue of DSs within the framework of the NS model.

A. Conditions of Boccara *et al.* for the occurrence of car accidents

Boccara *et al.* [11] have extended the deterministic case of the NS model to study the occurrence of a car accident. They assume that some drivers may be careless, i.e., their driving is not careful enough. The characteristic of the careless driver is that when the car ahead is moving, he expects it to move again at the next time step, and therefore tends to drive as fast as possible and increases safety velocity by one unit.

Let p' be a probability that the driver of the kth car is careless at time t. This probability is assumed to be independent of the car and the time. It is clear that if $p'=0$, all the drivers in the road are careful. But, if $p'=1$, all the drivers are careless. Hereafter, p' is called careless driver probability. The DS between two neighborhood cars k and $k+1$ will exist at time $t+1$, if the following events occur: E_i, the gap between the cars k and $k+1$ is inferior or equal to the speed limit; E_{ii}, the $(k+1)$th car is moving at time t; and E_{iii}, the $(k+1)$th car will suddenly stop at the next time step. These three conditions could be reduced to their simplest expressions as

$$\begin{aligned} &\text{(i) } d(k,t) \leqslant v_{\max}, \\ &\text{(ii) } v(k+1,t) > 0, \\ &\text{(iii) } v(k+1,t+1) = 0. \end{aligned} \tag{1}$$

If the driver of the kth car is careless (with probability p'), then $v'(k,t+1)=v(k,t+1)+1$, v' denotes the velocity of the careless driver. Let us note that the careless driver can increase his speed two times, unlike the careful one. It is clear that under the DS, the careless driver will arrive at the position of the moving car ahead. This leads to the occurrence of car accident at time $t+1$.

It is important to note that, in the numerical simulation results given in all the previous papers [11–16], the car accident defined as a collision does not really happen. They looked for DSs on the road and took them as the indicator of the occurrence of car accidents. To show this clearly, we shall give a simple example. Suppose that in the initial configuration (time t), the velocity and the gap of the kth car are $v(k,t)=d(k,t)-1$ and $d(k,t)\leqslant v_{\max}$, respectively. The velocity of the predecessor car is $v(k+1,t)>0$. For such configuration, it is clear that $v(k,t+1)=d(k,t)$ after the update rules of the deterministic NS model. Then, we compute the velocity $v(k+1,t+1)$ of the predecessor car at time $t+1$. Hence, if this velocity is equal to zero, the cars k and $k+1$ are in a DS. Besides, if the driver of the kth car is careless, then $v'(k,t+1)=d(k,t)+1$. It is clear that within such situation, the collision between cars k and $k+1$ happens at time $t+1$. An indicator indicates then that the car accident occurs. We note that in the configuration of time $t+1$, $v(k,t+1)=d(k,t)$, and $v(k+1,t+1)=0$, the two cars do not really hit each other and the velocity $v'(k,t+1)$ is not carried out. In the numerical simulations, it is the NS model that is used, the accident indicator does not change the rules of the NS model.

The main result of Boccara *et al* is given as follows. The car accident will not occur until the density reaches a critical value. With increasing the density, the probability of accident increases, reaches a maximum, and then decreases. Later, Huang *et al.* [14] studied the probability of the occurrence of a car accident within the framework of the nondeterministic NS model. However, because the update rule of slowing down (Rule 2) is applied before the randomization step (Rule 3) even if the gap is smaller than the speed limit, with the increase of safety velocity by a careless driver, the successor car fails to reach the position of the stopped cars. To correctly determine accidents caused by careless drivers in the nondeterministic NS model, Yang *et al.* [16] changed the three conditions of Eq. (1) for the occurrence of DSs as follows:

$$\begin{aligned} &\text{(i) } v(k,t+1) = d(k,t), \\ &\text{(ii) } v(k+1,t) > 0, \\ &\text{(iii) } v(k+1,t+1) = 0. \end{aligned} \tag{2}$$

The first condition is that over the iterations of the NS rules 1–3, the speed of the car is exactly equal to the gap, which means that the careless driver can reach the car ahead. The two other conditions of Boccara *et al.* remain unchanged.

B. Conditions for the occurrence of car accidents

The Boccara *et al.* conditions seem to reproduce some realistic features of accidents in car traffic. That is, the probability of a car accident increases with the density, reaches a maximum, and then decreases with further density. However, this maximum is always located in the high-density region. In real traffic, accidents which are often caused by driving at high speeds usually occur in the low-density region. Moreover, the accidents are not caused only by stopped cars but also when cars abruptly decelerate. Besides, the careless driver of Boccara *et al.* has a greater acceleration capability than the other careful drivers. We think that this is an aggressive driving, which exists little in realistic traffic.

The above discussions lead us to present different condition for the occurrence of a car accident based on the thought that a real accident depends on several parameters: an unexpected action of the car ahead (suddenly stop or abrupt deceleration), the gap between the two cars, the velocity of the successor car, and its delayed reaction time.

1. Car accident caused by stopped cars

When a car reduces its velocity, the brake light of the car is switched on. The successor car does not react if it is far from the car ahead. But, in the contrary case, it decelerates in order to avoid collision. However, this reaction of the driver of the successor car is carried out only after a certain reaction time τ. This reaction time is defined as the time passed between the instant where the brake light of the predecessor car is switched on and the one where the successor car begins his braking maneuver. It is clear that as the reaction time τ increases, the more assurance of safe driving decreases.

In models which are continuous both in space and time, a safety condition which assures safe driving can be derived. Assume that at time t, a kth car with velocity $v(k,t)$ is following another $(k+1)$th car with velocity $v(k+1,t)$ within a distance $d(k,t)$. Here, $d(k,t)$ is the free space between the cars. Suppose that the predecessor car is moving and will stop at the next time step. Then, the safety is satisfied if

$$D_k+\tau v(k,t)\leqslant D_{k+1}+d(k,t) \tag{3}$$

holds, with D being the braking distance needed to stop. The term $\tau v(k,t)$ is the distance required to cover by the kth car during the time τ.

If we denote by $-\gamma$ the deceleration $(\gamma>0)$, Eq. (3) becomes

$$\frac{v(k,t)^2}{2\gamma_k}+\tau v(k,t)\leqslant\frac{v(k+1,t)^2}{2\gamma_{k+1}}+d(k,t). \tag{4}$$

In CA models which are discrete both in space and time, due to the unbounded deceleration capabilities of the cars, $v^2/2\gamma\to 0$ and the reaction time $\tau\to 0$. Thus, the safety equation (4) becomes

$$0\leqslant d(k,t), \tag{5}$$

indicating that the safety always holds. Consequently, no accident can occur in the basic CA models.

To extend the CA models for the occurrence of car accidents, we assume that some drivers may be careless, i.e., their driving is not careful enough. A characteristic of this careless driver is that when the car ahead is moving, he expects it to move again at the next time step, and therefore his braking maneuver is done only after a delayed reaction time τ. Thus, from Eq. (4), we derive the condition of nonsatisfaction of safety,

$$\tau v(k,t)>d(k,t). \tag{6}$$

The DS between two neighborhood cars k and $k+1$ will exist at time $t+1$, if the following events occur: E_i, the distance required to cover by the kth car during the time τ is superior to its gap; E_{ii}, the $(k+1)$th car is moving at time t; and E_{iii}, the $(k+1)$th car will suddenly stop at the next time step. These three conditions could be reduced to their simplest expressions as

$$\begin{aligned}&\text{(i)}\ \tau v(k,t)>d(k,t),\\ &\text{(ii)}\ v(k+1,t)>0,\\ &\text{(iii)}\ v(k+1,t+1)=0.\end{aligned} \tag{7}$$

As it is stated in Sec. III A, p' denotes the careless driver probability. So, if the driver of the kth car is careless, then $\tau\neq 0$. However, if the driver is careful enough, then $\tau=0$. We point out that the careless driver has the same acceleration capability as the careful driver; they only differ by their delayed reaction times τ. Unlike this, the careless driver which was proposed by Boccara *et al.* has the same reaction time $(\tau=0)$ as the careful driver, but they differ by their acceleration capabilities.

It is clear that under the DS of Eq. (7), the careless driver will crash his car through the predecessor car. This leads to the occurrence of a car accident at time $t+1$. However, for careful driving $(\tau=0)$, the DS never occurs since the gap $d(k,t)$ is always superior or equal to zero.

2. Car accident caused by great deceleration

Now let us go beyond the type of accident cited before. In a realistic traffic system, accidents frequently happen when drivers drive their cars at high speeds. Moreover, it is incontestable that a great and sudden deceleration of a car can cause an accident with its successor. So it is important to change the above conditions of the car accident [Eq. (7)] in order to incorporate the high speed effect.

Suppose that at time t the car ahead with speed $v(k+1,t)$ abruptly decelerates. At time $t+1$ its velocity will be reduced to $v(k+1,t+1)$. If the covered distance during the delayed reaction time τ of the successor car is enough to reach the next time position of the car ahead, then a DS occurs on the road. Hence, the conditions for the occurrence of a DS with respect to abrupt deceleration of the car ahead are as follows.

$$\begin{aligned}&\text{(i)}\ \tau v(k,t)>d(k,t)+v(k+1,t+1),\\ &\text{(ii)}\ v(k+1,t)-v(k+1,t+1)\geqslant v_d.\end{aligned} \tag{8}$$

If the above two conditions are satisfied, then a car accident will occur at time $t+1$ with probability p'.

The parameter v_d is the deceleration limit beyond which a risk of the occurrence of DS exists. The first condition requires that, during the time τ, the driver k can reach his predecessor, who will greatly decelerate at the next time step. The second condition requires that the driver $k+1$ greatly and suddenly decelerates at time $t+1$.

The conditions for the occurrence of a DS caused by a stopped car and a great deceleration simultaneously of the car ahead are given as follows:

$$\begin{aligned}&\text{(i)}\ \tau v(k,t)>d(k,t),\\ &\text{(ii)}\ v(k+1,t)\geqslant v_d,\\ &\text{(iii)}\ v(k+1,t+1)=0.\end{aligned} \tag{9}$$

If the above three conditions are satisfied, then a car accident will occur at time $t+1$ with probability p'.

IV. NUMERICAL RESULTS

We simulate one-lane traffic using the NS model with a one-dimensional lattice of length $L=3000$ sites with closed boundary conditions. The density ρ is defined as $\rho=N/L$, where N is the number of cars. The model parameters are given by the maximal velocity of the cars $v_{\max}$, the probability of the randomization (or noise) p, the careless driver probability p', and the deceleration limit v_d. As time scale, the delayed reaction time τ of the careless driver τ is chosen equal to 1 s ($\tau=\Delta t=1$ s).

We start with random positions and velocities of cars at the initial configurations. Next, we update the individual vehicle velocities and positions in accordance with the NS-update rules (Sec. II). For each initial configuration, results are obtained by averaging over 6×10^3 time steps after the first 2×10^3 time steps, so that the system reaches a stationary state. The procedure is then repeated for a number (80) of different realizations. The average over all the different realizations gives a mean value of a physical quantity such as flow and probability of car accidents.

As it was explained in Sec. III A, in the numerical simulation results, the car accident defined as a collision does not really happen; it is the NS model that it is used. More specifically, the car accident algorithm is divided into two independent major parts: (1) check on car accident and (2) forward movement, given as follows.

(1) The verification of car accidents is an implementation of the following. Let $[x(k,t),v(k,t)]_{k=1,N}$ denote the configuration of the system at time *t*. For each car *k*, we compute its gap $d(k,t)$ and the velocities $v(k+1,t)$ and $v(k+1,t+1)$ of the car $k+1$ at time t and $t+1$, respectively. First, we verify if the driver is careless. Then we check on the DS; for example, we verify if $\tau v(k,t)>d(k,t)$ and $v(k+1,t)>0$ and $v(k+1,t+1)=0$ [Eq. (7)]. Finally, if all these previous conditions are verified simultaneously, an indicator indicates that an accident between cars happens.

(2) The configuration of the system at the next time step $[x(k,t+1),v(k,t+1)]_{k=1,N}$ is computed from that of time t according to the four subrules of the NS model (Sec. II). That is, $[x(k,t+1),v(k,t+1)]_{k=1,N}\underset{\leftarrow}{\mathrm{NS}}[x(k,t),v(k,t)]_{k=1,N}$.

To simplify the reading of this paper, we denote the previous conditions for car accidents to occur caused by stopped cars of Eq. (1) (Refs. [11,14]) and Eq. (2) (Ref. [16]) as "$\mathrm{SCC_I}$" and "$\mathrm{SCC_{II}}$," respectively. The conditions presented here for car accidents corresponding to stopped cars of Eq. (7) are denoted by "NSCC" while those caused by the great deceleration with $v_d=i$ of Eq. (8) are denoted by "$\mathrm{GDC_i}$". The conditions of car accidents of Eq. (9) caused, simultaneously, by a stopped car and a great deceleration with $v_d=i$ are denoted by "$\mathrm{NSCGDC_i}$."

A. Probability of a car accident caused by stopped cars

In this section, we study numerically the probability *per car* and *per time step* for an accident to occur, P_{ac}, caused by stopped cars. As P_{ac} is proportional to the careless driver probability p', we shall study the quantity P_{ac}/p' and leave the probability p' unspecified.

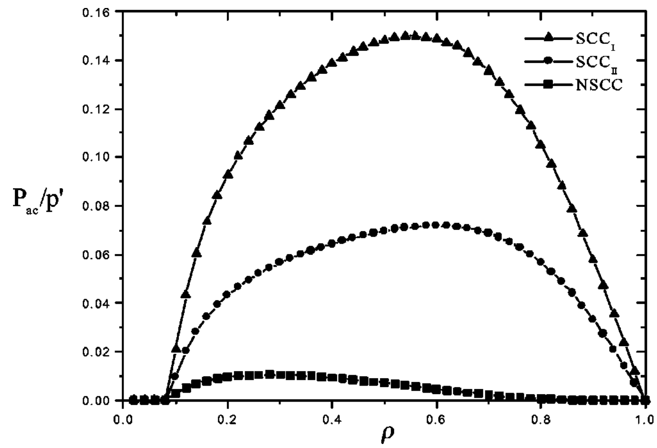

FIG. 1. The probability P_{ac} (scaled by p') of the occurrence of car accident caused by stopped cars as a function of density ρ for the three different conditions. The parameters of the NS model are chosen as $v_{\max}=5$ and $p=0.4$.

First, we shall show in Fig. 1 the differences in the probability P_{ac} calculated under the three different conditions of car accidents [Eqs. (1), (2), and (7)]. The parameters of the simulation on the NS model are $v_{\max}=5$ and the stochastic braking parameter is chosen $p=0.4$. At low densities, the car accidents will not occur until the density reaches a "critical density" ρ_c. This is because in the free-flow region ($\rho\leqslant\rho_c$) all cars move with a velocity equal to $v_{\max}$. The mean gap between cars is superior to the maximal speed $v_{\max}$ and thus no stopped cars exist. Consequently, at low densities, the probability P_{ac} is essentially determined by the probability of $\mathrm{E_{iii}}$ which concerns the stopped cars. We point out that we have put the words critical density in quotation marks because there is no consensus concerning the existence of the phase transition in the case of the nondeterministic NS model. However, in the deterministic case of the NS model, $\rho_c=1/(v_{\max}+1)$ is a critical density which corresponds to transition from a free-flow regime to a congested regime where start and stop waves dominate the dynamics of the system. This transition is usually viewed as a second-order phase transition [17–19]. Yet, in the deterministic case of the NS model, ρ_c coincides with the density of maximal flow while ρ_c is smaller to the latter in the nondeterministic case. A typical flow-density diagram is shown in Fig. 2 for the nondeterministic NS model with $v_{\max}=5$ and $p=0.4$.

From Fig. 1, we observe that the value of the "critical density" ρ_c remains unchanged with respect to the three different conditions. This is because $\mathrm{E_{iii}}$, which essentially determines P_{ac} at low densities, exists in the three different conditions of DSs [Eqs. (1), (2), and (7)].

Before continuing our discussion on the probability of an accident, let us recall that in the congested phase of the nondeterministic NS model, we can distinguish two different

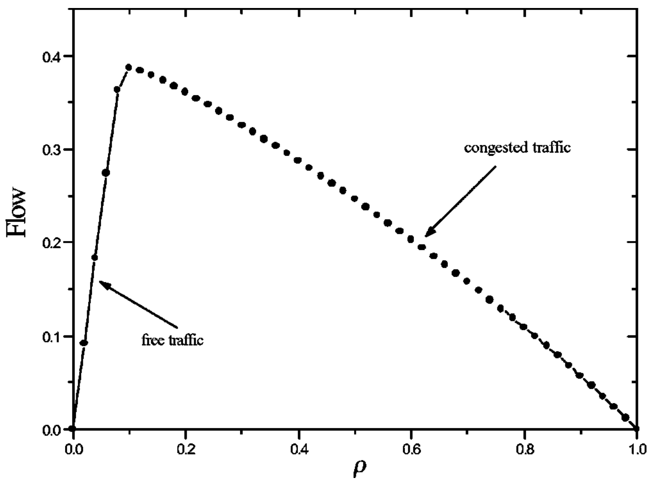

FIG. 2. Fundamental diagram of the NS model for $v_{\max}=5$ and $p=0.4$.

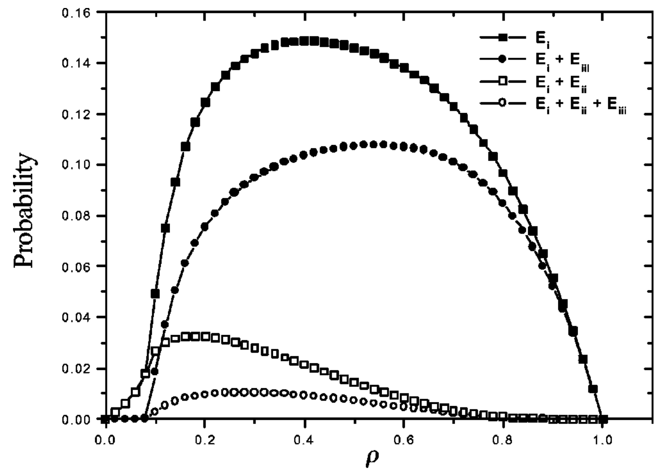

FIG. 3. The probability of the occurrence of the events which mainly contribute to NSCC in the high-density region. For example, $P(\mathrm{E_i}+\mathrm{E_{ii}})$ is the probability that $\mathrm{E_i}$ and $\mathrm{E_{ii}}$ [Eq. (7)] occur simultaneously. The parameters of the NS model are chosen as $v_{\max}=5$ and $p=0.4$.

regimes [20]. For relatively low density (above ρ_c), the jamming regime occurs where a coexistence of free flow and jamming exists. In this regime, the configuration of the system is usually formed by spontaneous jams which dissolve after a while and also moving cars with maximum velocity $v_{\max}$. For high density, a superjamming exists where the hole system is congested; the jamming waves become connected and form an infinite wave.

Above the "critical density," P_{ac} increases with the density, reaches a maximum, and then decreases with further density. The density $\rho_{\max}$ corresponding to the maximum of P_{ac} ($P_{ac}^{\max}$) is considered as the most probable density, at which accidents occur most frequently. Comparing the three different conditions, we find that the NSCC leads to weak values of P_{ac} than those of $\mathrm{SCC_{II}}$ which in turn are inferior than those of $\mathrm{SCC_I}$. Also, the maximum of P_{ac} in NSCC is reached at relatively low density ($\rho_{\max}\simeq 0.28$), which could be situated in the jamming regime. This is compared to that of $\mathrm{SCC_I}$ ($\rho_{\max}\simeq 0.56$) and $\mathrm{SCC_{II}}$ ($\rho_{\max}\simeq 0.60$) which could be situated in the superjamming regime. From our daily experiences of the traffic on highways, we know that accidents usually occur when the cars are partially in jams and free flow, i.e., jamming regime. Our description of car accident approaches the reality better than the one considered by Boccara *et al.*

In the very high-density region, the difference between NSCC and the previous conditions ($\mathrm{SCC_I}$, $\mathrm{SCC_{II}}$) is noticeable. That is, P_{ac} decreases more rapidly in NSCC than in the previous conditions. Moreover, in contrast to the cases $\mathrm{SCC_I}$ and $\mathrm{SCC_{II}}$, where P_{ac} vanishes at saturated density ($\rho=1$), the NSCC leads to another density limit ($\rho_h \simeq 0.90$) above which no accident can occur. To further discuss this behavior, we plot in Fig. 3 the probability of the occurrence of the events which mainly contribute to NSCC in the high-density region. Hence, we find that at high density, P_{ac} is mainly determined by $P(\mathrm{E_i}+\mathrm{E_{ii}})$. Here, $P(\mathrm{E_i}+\mathrm{E_{ii}})$ denotes the probability that $\mathrm{E_i}$ and $\mathrm{E_{ii}}$ occur simultaneously. Hence, in the superjamming regime, as the density increases, the likehood for the simultaneous movement of two successive cars decreases. This can explain the vanishing of P_{ac} at densities above ρ_h. In Fig. 4, we have plotted the probability of a traffic accident as a function of the density, for different values of size L ($L=3000$, 6000, and 9000). It shows clearly that the density limit ρ_h is not affected by finite-size effects and is solely due to fact that $\mathrm{E_i}$ and $\mathrm{E_{ii}}$ cannot occur simultaneously at densities above ρ_h.

Now, we shall study the effect of the parameters of the system on the probability of a car accident, P_{ac}, caused by stopped cars. Here and hereafter we shall use only the NSCC and for comparison with $\mathrm{SCC_I}$ and $\mathrm{SCC_{II}}$, the reader should see Ref. [14] and Ref. [16], respectively.

To show how P_{ac} depends on the stochastic braking parameter p, we plot in Fig. 5 the values of P_{ac} against the car density ρ with maximal speed $v_{\max}=5$ and for various values of p. Hence, the value of P_{ac} is enhanced in the low-density region. Moreover, we observe that the "critical density" ρ_c decreases when increasing p. Another qualitative result is

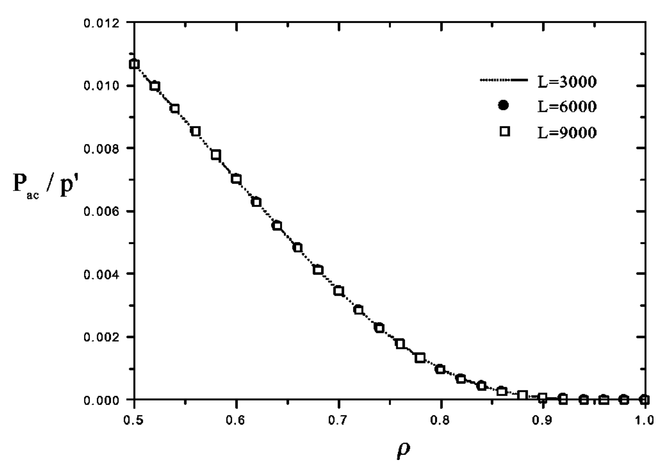

FIG. 4. The probability P_{ac} (scaled by p') with the NSCC as a function of density ρ and for various length L of the circuit. The parameters of the NS model are chosen as $v_{\max}=5$ and $p=0.4$.

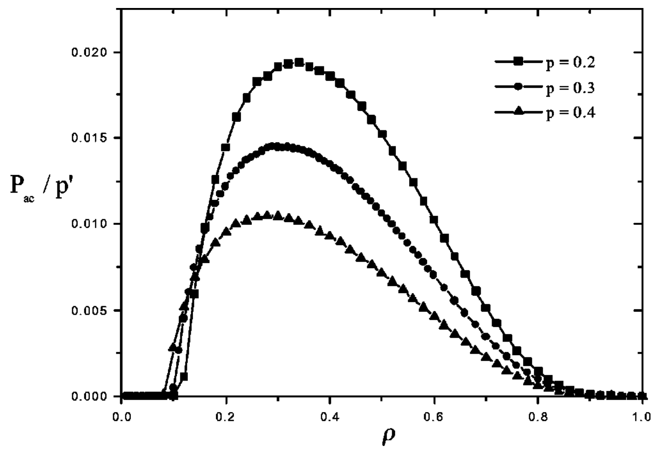

FIG. 5. The probability P_{ac} (scaled by p') with the NSCC as a function of density ρ for $v_{\max}=5$ and for different values of p.

that beyond ρ_c, the probability P_{ac} increases when decreasing p. These results could be explained by the fact that when the stochastic driving behavior is important, the conduct of cars becomes slow. In a certain way, the driving is more careful. Finally, we found that the density limit ρ_h remains unchanged when we change the values of the stochastic braking p.

As the speed limit $v_{\max}$ increases, the probability of the occurrence of car accidents increases and the "critical density" ρ_c is shifted towards the low-density region. As it was found in Ref. [14] ($\mathrm{SCC_I}$), a kind of scaling relation is expected in the high-density region. Yet, with $\mathrm{SCC_I}$ (or $\mathrm{SCC_{II}}$) the density $\rho_{\max}$ and $P_{ac}^{\max}$ are found independent of the speed limit, especially when $v_{\max}\geqslant 3$. Unlike this, with NSCC, $\rho_{\max}$ decreases while $P_{ac}^{\max}$ increases with $v_{\max}$. Certainly, this result agrees with realistic traffic since the accidents often occur when the drivers drive their cars with high speeds, even if the density is very low. The density limit ρ_h, above which no accidents can occur, is found independent of the speed limit. The results are shown in Fig. 6.

B. Probability of a car accident caused by great deceleration

In this section, we shall study the more general and realistic types for conditions for the occurrence of car accidents.

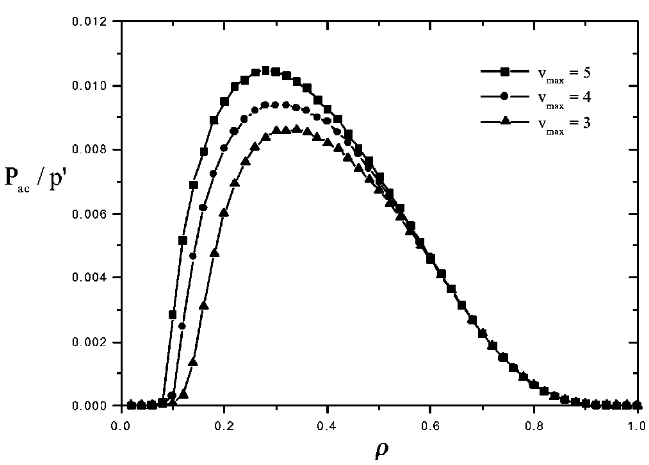

FIG. 6. The probability P_{ac} (scaled by p') with the NSCC as a function of density ρ for $p=0.4$ and for different values of $v_{\max}$.

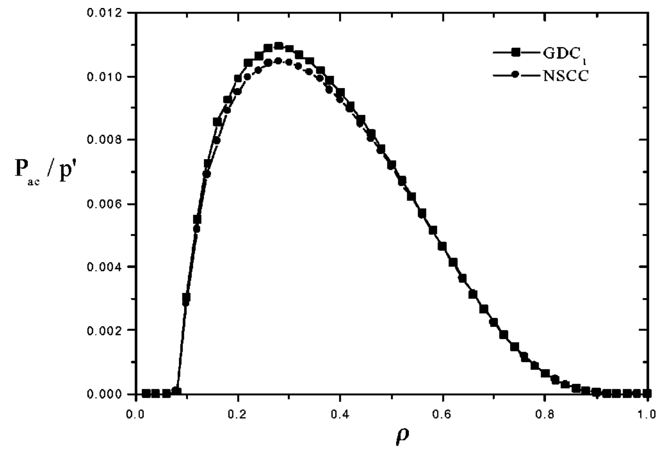

FIG. 7. The probability P_{ac} (scaled by p') as a function of density ρ with the NSCC (circles) and the geat decelerations with $v_d=1$ $\mathrm{GDC_1}$ (squares) for $v_{\max}=5$ and $p=0.4$.

As it was mentioned before, these accidents are caused by a great and sudden deceleration of the cars ahead [Eqs. (8) and (9)]. In order to clarify more these conditions of accident, we consider the case $\mathrm{GDC_1}$ where the deceleration limit v_d is equal to one. In this case, DSs can occur when the car ahead slowed down by at least a unity. It is clear that all the conditions of a car accident cited before are included in $\mathrm{GDC_1}$ [see Eqs. (7)–(9)]. For example, the inclusion relations between the following conditions hold:

$$\mathrm{GDC_4}\subset\mathrm{GDC_3}\subset\mathrm{GDC_2}\subset\mathrm{GDC_1}\mathrm{NSCGDC_4}$$
$$\subset\mathrm{NSCGDC_3}\subset\mathrm{NSCGDC_2}\subset\mathrm{NSCC}\subset\mathrm{GDC_1}. \qquad (10)$$

We also note that the condition of car accident NSCC is the same as $\mathrm{NSCGDC_1}$.

In Fig. 7, we plot the probability of car accident P_{ac} against the car density ρ corresponding to $\mathrm{GDC_1}$ and NSCC. The maximal speed is $v_{\max}=5$ and the stochastic braking parameter is chosen as $p=0.4$. We observe that the two curves are confounded for all car densities except in the interval [0.2;0.4] where a slight difference could be noted. This shows that the majority of the accidents in $\mathrm{GDC_1}$ are provoked by stopped cars.

The probability of the occurrence of car accidents provoked, simultaneously, by a stopped car and a great deceleration ($\mathrm{NSCGDC_i}$) is shown in Fig. 8 for various values of the deceleration limit v_d. The "critical density" (ρ_c, below which no accident occurs) is found independent of v_d. Above this "critical density," the values of P_{ac} decrease when increasing the deceleration limit. Thus, if we believe that a simple braking of the car ahead (deceleration by one unit) cannot really cause an accident, then the values of P_{ac} for NSCC are overestimated. In the congested region, where the traffic is a stop-and-go wave, the density limit (ρ_h, above of which no accident occurs) is shifted towards the low-density region. For example, we found for $\mathrm{NSCGDC_1}$ the value $\rho_h\simeq 0.90$ while for $\mathrm{NSCGDC_3}$ $\rho_h\simeq 0.60$.

Now we shall study the behavior of the probability of a car accident caused by an abrupt deceleration ($\mathrm{GDC_i}$, i

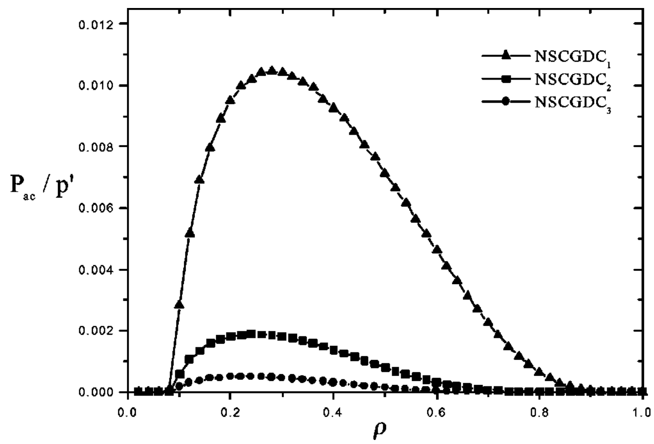

FIG. 8. The probability P_{ac} (scaled by p') as a function of density ρ with, simultaneously, a stopped car and a great deceleration NSCGDC$_i$ for various v_d. The values of the NS parameters are $v_{max}=5$ and $p=0.4$.

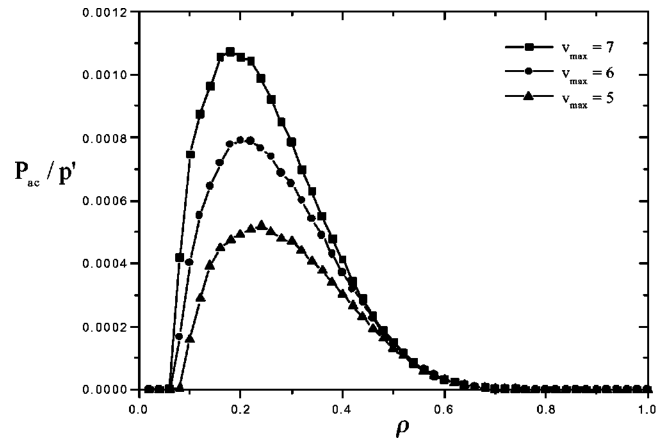

FIG. 10. The probability P_{ac} (scaled by p') as a function of density ρ with a great deceleration condition with $v_d=3$ GDC$_3$ for various v_{max}. The braking probability is $p=0.4$.

>1), i.e., the situation where a car ahead decelerated by one unit is not considered dangerous for the drivers. To show the influence of the different behavioral patterns of the individual drivers on car accidents, we plot in Fig. 9 the results of P_{ac} against the density ρ for various values of p. The parameters of the simulation are $v_{max}=5$ and $v_d=3$. Here also, we observe that the ''critical density'' ρ_c decreases when increasing p. Moreover, the probability of car accidents increases when decreasing p. In brief, the great deceleration conditions lead to the same behavior of P_{ac} with respect to the variation of the stochastic braking than those corresponding to the stopped cars (NSCC). However, the values of P_{ac} in GDC$_i$ are, about, ten times weaker than those of NSCC.

In Fig. 10, we show the variations of the probability P_{ac} against the car density with $v_d=3$ for various values of v_{max}. As in the case of stopped cars (NSCC), when the maximal speed is increased, ρ_{max} (the most probable density, at which accidents occur most frequently) is shifted towards the free-flow region and the maximum probability P_{ac}^{max} is increased. The effect of the deceleration limit v_d on the probability of a car accident is shown in Fig. 11 for $v_{max}=5$ and $p=0.4$. Hence, we find that ρ_c does not depend on the parameter v_d. Moreover, the values of probability P_{ac} decreased when increasing the deceleration limit v_d. However, the density limit ρ_h moves towards the low-density region when increasing v_d.

V. CONCLUSION

In summary, we have studied car accidents in the one-lane traffic model described by the well-known Nagel-Schreckenberg model. We have introduced two different conditions of car accidents based on the delayed reaction time of the successor car. The first one corresponds to car accidents caused by stopped cars while in the second condition, the accidents are caused by great and sudden deceleration of cars. So, we have investigated the effect of variations of different parameters on the probability of a car accident to occur. With our conditions of a car accidents, there exists a ''critical density'' ρ_c situated in the low-density region, below which no accident can occur. With increasing the car

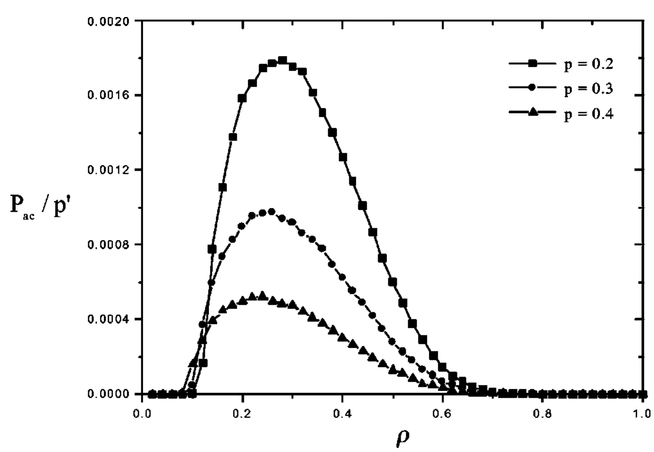

FIG. 9. The probability P_{ac} (scaled by p') as a function of density ρ with a great deceleration condition with $v_d=3$ GDC$_3$ for various p. The maximum speed is $v_{max}=5$.

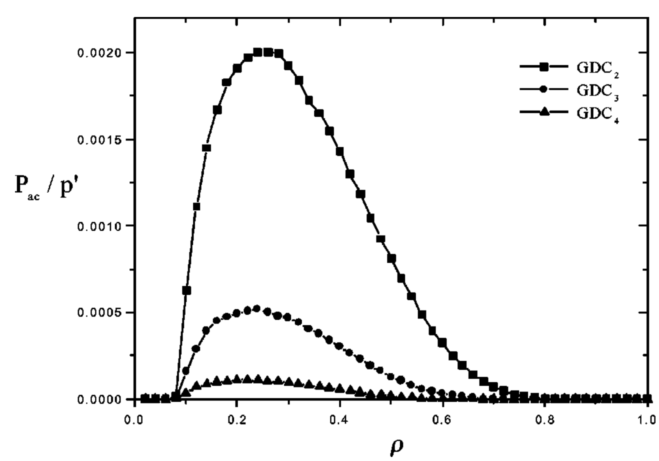

FIG. 11. The probability P_{ac} (scaled by p') as a function of density ρ with a great deceleration condition GDC$_i$ for various v_d. The values of the NS parameters are $v_{max}=5$ and $p=0.4$.

density ρ, the probability of accident, P_{ac}, increases, reaches a maximum, and then decreases. However, in the very high-density region, we found that the accidents cannot occur if the density is superior to a density limit ρ_h. For DSs caused by stopped cars given by our conditions, the density $\rho_{\max}$, at which accidents can occur most frequently, is located in the jamming regime. Unlike this, with the previous conditions, this density is located in the superjamming regime. Finally, the deceleration limit v_d which plays an important role in the probability of the occurrence of a car accident could be considered as a generalization of the conditions of DSs caused by stopped cars.

From our results given in this paper, some suggestions about how to avoid car accidents can be derived. These suggestions are given as follows.

Suggestion 1. If the drivers increase their maximal speeds, the ‘‘critical density’’ is shifted towards the low density. This increases the risk of accident even if the density of cars is low. Hence, this suggests that the driver should not considerably increase his maximal speed.

Suggestion 2. The density $\rho_{\max}$, at which accidents can occur most frequently, is located in the jamming regime. Therefore, the formation of jams increases the risk of accidents between cars. Thus, we advise the driver to slow down when he/she approaches a jam.

Suggestion 3. A slow car, which moves with low maximal speed, can produce phase separated stationary states at low densities. These states consist of a large jam behind the slowest vehicle and a large gap in front of the slowest car [21]. Thus, drivers are advised not to considerably decrease their maximal speeds for avoiding formation of jams.

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[1] S. Wolfram, *Theory and Application of Cellular Automata* (World Scientific, Singapore, 1986).
[2] D. Chowdhury, L. Santen, and A. Schadschneider, Phys. Rep. **329**, 199 (2000).
[3] K. Nagel and M. Schreckenberg, J. Phys. I **2**, 2221 (1992).
[4] M. Fukui and Y. Ishibashi, J. Phys. Soc. Jpn. **65**, 1868 (1996).
[5] S. Krauss, P. Wagner, and C. Gawron, Phys. Rev. E **54**, 3707 (1996); **55**, 5597 (1997).
[6] B. Kerner, S.L. Klenov, and P. Konhäuser, Phys. Rev. E **56**, 4200 (1997).
[7] L. Neubert, H.Y. Lee, and M. Schreckenberg, J. Phys. A **32**, 6517 (1999).
[8] T. Nagatani, Rep. Prog. Phys. **65**, 1331 (2002).
[9] A. Pottmeier, R. Barlovic, W. Knospe, A. Schadschneider, and M. Schreckenberg, Physica A **308**, 471 (2002).
[10] S.C. Benjamin, N.F. Johnson, and P.M. Hui, J. Phys. A **29**, 3119 (1996).
[11] N. Boccara, H. Fuks, and Q. Zeng, J. Phys. A **30**, 3329 (1997).
[12] D.W. Huang, J. Phys. A **31**, 6167 (1998).
[13] D.W. Huang and W.C. Tseng, Phys. Rev. E **64**, 057106 (2001).
[14] D.W. Huang and Y.P. Wu, Phys. Rev. E **63**, 022301 (2001).
[15] Xian-qing Yang and Yu-qiang Ma, J. Phys. A **35**, 10539 (2002).
[16] Xian-qing Yang and Yu-qiang Ma, Mod. Phys. Lett. B **16**, 333 (2002).
[17] N. Boccara and H. Fuks, J. Phys. A **33**, 3407 (2000).
[18] B. Eisenblaetter, L. Santen, A. Schadschneider, and M. Schreckenberg, Phys. Rev. E **57**, 1309 (1998).
[19] S. Lübeck, M. Schreckenberg, and K.D. Usadel, Phys. Rev. E **57**, 1171 (1998).
[20] S. Cheybani, J. Kertész, and M. Schreckenberg, J. Phys. A **31**, 9787 (1998).
[21] W. Knospe, L. Santen, A. Schadschneider, and M. Schreckenberg, in *Traffic and Granular Flow ’97*, edited by M. Schreckenberg and D.E. Wolf (Springer, Berlin, 1998), p. 349.