Resonance effects in the electron distribution function formation in spatially periodic fields in inert gases

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The calculations of the electron distribution function (EDF) in striationlike, sinusoidally modulated electric fields were performed to determine the dependence on spatial period length. The calculations were done for a discharge in neon at $pR=2$ Torr cm, $i/R=5$ mA/cm, and electric field $E/p=1.9$ V cm^{-1} Torr^{-1}. The presence of the resonances in the EDF and macroscopic parameters has been demonstrated. These resonances correspond to *S* and *P* striations observed in experiments. An interpretation of the results is proposed based on an analytical approximation of the numerical solution. Decomposition of EDF into two factors—amplitude and body—is carried out. The amplitude of the EDF is shown to be resonantly dependent on the value of the spatial period. One maximum in the EDF is formed at the value of the spatial period corresponding to the *S* striation, and two maxima at the value which corresponds to the *P* striation. The experimental measurements of the EDF in *S* and *P* striations with high spatial resolution showed agreement between the theoretical and the experimental results. Resonance effects in the EDF formation are considered based on the linear theory in the weakly modulated electric fields.

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I. INTRODUCTION

The description of the electron component behavior in the spatially periodic electric fields is much needed in the problems of plasma stratification, i.e., the existence of plasma in the form of standing or moving ionization waves.

Electron distribution function (EDF) resonance formation in spatially periodic electric fields occurs if the electron energy balance is governed by the inelastic processes, and energy losses in elastic collisions over the length of the spatial period are small. This situation occurs in the inert gas discharges at small currents and low pressures, when large values of reduced electric field E/p are present.

With increasing pressure, when values of the reduced electric field become small, the picture of the movement of electrons differs essentially. The electron energy relaxation length $\lambda_\varepsilon=\lambda\sqrt{M/m}$ (λ is the electron mean free path, M and m are the masses of an atom and an electron, respectively) becomes much smaller than the minimum length required for an electron to obtain energy exceeding the excitation threshold U_{ex}, $L_0=U_{ex}/eE_0$. In this case, elastic collisions play a remarkable role in the electrons' energy balance, and resonance effects do not influence the formation of the EDF. A hydrodynamic description of the electrons' behavior is then applicable.

In numerous papers, which were reviewed in Refs. [1,2], a hydrodynamic approach was used for the description of mechanisms of striations' occurrence and propagation.

Starting from Ref. [3], a kinetic approach is applied for the description of stratification phenomena. A series of papers have been devoted to investigation of the resonance behavior of the EDF. Tsendin [4] obtained an analytical solution of the Boltzmann kinetic equation in the homogeneous and inhomogeneous fields for the case of inelastic energy balance. It was shown that the relaxation process of an arbitrary initial EDF in the homogeneous electric field has a form of damped oscillations with the energy period $U_L^{\rm res}$ and spatial period $L_{\rm res}$. It is possible to consider the electrons as being accelerated in the electric field with approximate conservation of their total energy $\varepsilon=U+e\varphi(x)$ [kinetic energy U plus potential energy $e\varphi(x)$] until they acquire a kinetic energy equal to the excitation threshold. Then they undergo inelastic collisions with simultaneous loss of a quantum of energy equal to the excitation threshold, and continue their motion with a smaller value of the total energy. This stepwise mechanism determines the characteristic periodicity scale in energy space, $U_L^{\rm res}=U_{\rm ex}+\Delta U$ ($U_{\rm ex}$ is the excitation energy and ΔU is the small energy losses in elastic collisions) and the spatial period $L_{\rm res}=U_L^{\rm res}/E_0$ (E_0 is the period-averaged electric field). The spatial scale $L_{\rm res}$ fixes the resonance length of the periodic field $E(x)$ (the resonance field). Namely, the resonance field is formed self-consistently in the stratified positive column and defines the fundamental mode of the wave.

Different kinds of striations are discovered experimentally in inert gases under low pressures and small currents. In Ref. [1], a detailed nomenclature of the observed waves according

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to their dispersion properties and to the potential fall over a wavelength is given. In particular, Ref. [5] shows that in different inert gases there exists a wave with a potential drop of the order of magnitude of the first excitation threshold. This wave was called *S* striation. Along with the *S* striation, a wave was observed with the potential drop two times smaller than that in *S* striation. This wave was called *P* striation.

The fall in potential over the length of *S* striation is equal to U_L^{res}. The potential fall over the length of *P* striation is equal to $U_L^{\mathrm{res}}/2$, as long as the wavelength of *P* striation is two times smaller than that of *S* striation. The electrons in the *P* striations should travel two spatial periods in order to acquire an energy equal to the excitation threshold.

Relaxation of an initial EDF in the spatially periodic electric field with small modulation degree leads to an establishment of the periodic EDF with an amplitude depending resonantly on the spatial period of the field [4]. In electric fields with a large degree of modulation, a bunching effect [4] was observed, which is the constriction of the EDF towards the resonance trajectories on the plane (ε,x). The amplitude of the EDF has, in this case, the form of a narrow Gaussian.

Relaxation processes of arbitrary initial EDFs injected into the homogeneous and sinusoidally modulated electric fields are illustrated in Ref. [6] on the basis of a numerical solution of the kinetic equation, accounting for elastic collisions and excitation of several levels. The electric field was strongly modulated and the spatial period was taken equal to the resonance length (corresponding to an *S* striation) and to the half of that (corresponding to a *P* striation). Similar results have been obtained in Ref. [7] on the basis of solving analytically the kinetic equation. The EDFs obtained in these fields demonstrate the bunching effect that is caused both by small energy losses in elastic collisions and by the presence of the several excited levels. In Ref. [8], the EDFs formed in the resonance electric fields and also in the fields with small deviations of the spatial period towards smaller and larger values are discussed.

The objective of the present work is to analyze the EDFs in spatially periodic electric fields to determine the dependence on the period of the field. Scanning over a wide range of the field periods will permit us to elucidate the essence of the resonance formation of the EDF and its influence on the macroscopic quantities. The origin of the resonances is pointed out by the decomposition of the EDF into two factors with help of numerically solving the Boltzmann kinetic equation in strongly modulated fields and by applying linear analysis for weakly modulated fields. The measurements of distribution functions with high spatial resolution in *S* and *P* striations confirm the resonant character of the EDF formation.

II. THE EDF AND MACROSCOPIC QUANTITIES IN THE SPATIALLY PERIODIC ELECTRIC FIELDS

In Ref. [8], the EDF behavior was analyzed in the vicinity of the resonance length L_{res}. The calculations were performed for the discharge in neon at $E_0/p = 2\ \mathrm{V\,cm^{-1}\,Torr^{-1}}$ and field modulation degree $\alpha = 0.9$. The resonance length was found to be equal to $L_{\mathrm{res}}p = 9.67\ \mathrm{cm\,Torr}$. The distribution functions were shown for the values of $L = 0.9L_{\mathrm{res}}$, $L = 1.0L_{\mathrm{res}}$, and $L = 1.1L_{\mathrm{res}}$, and the strong variation of the EDF appearance in the given range of L was noted.

It is possible to obtain sharper resonances if one increases strongly the frequency of inelastic collisions. This leads to a much steeper reduction of the EDF in the inelastic region at energies exceeding the excitation threshold. The deformation of the EDF at small energies caused by the backscattering processes is also reduced. Consequently, the numerical solution of the kinetic equation will strive to the "black wall" approximation, which is the solution with the zero boundary condition at the excitation threshold.

In the present paper, calculations are performed for a sinusoidally modulated electric field with modulation degree $\alpha = 0.9$ and a period-averaged value of $E_0/p = 1.9\ \mathrm{V\,cm^{-1}\,Torr^{-1}}$, which corresponds to a discharge in neon at a pressure $p = 1$ Torr, current $i = 10$ mA, and $R = 2$ cm.

The Boltzmann kinetic equation for the isotropic part of the EDF, $f_0(\varepsilon,x)$, in terms of the variables total energy ε and coordinate x can be written as

$$\frac{\partial}{\partial x}D_\varepsilon(v)\frac{\partial f_0(\varepsilon,x)}{\partial x}+\frac{\partial}{\partial \varepsilon}V_\varepsilon(v)f_0(\varepsilon,x) = v\,\nu^*(v)f_0(\varepsilon,x)-v'\,\nu^*(v')f_0(\varepsilon+U_{ex},x), \tag{1}$$

where $D_\varepsilon = v^3/3\nu(v)$ is the diffusion coefficient, $V_\varepsilon = 2m^2/M\,\nu(v)v^3$ is the drift coefficient in energy space due to elastic collisions, m and M are the masses of an electron and an atom, $\nu^*(v)$ is the total frequency of inelastic processes, and $\nu(v)$ is the frequency of elastic collisions. Velocities v and v' are related by the energy conservation law, $mv'^2/2 = mv^2/2 + U_{\mathrm{ex}}$.

We shall consider the spatial relaxation of an arbitrary initial EDF, $f_0^{\mathrm{init}}(\varepsilon)|_{x=0} = f_0^{\mathrm{init}}(U)$, in the homogeneous electric field E_0 and in the spatially modulated electric fields of the form

$$E(x) = E_0\left[1+\alpha\cos\left(\frac{2\pi x}{L}\right)\right], \tag{2}$$

with the potential $e\varphi(x) = \int_0^x E(x)dx$, where L is the spatial period. The kinetic equation (1) has to be supplemented by the appropriate boundary conditions:

$$f_0(\varepsilon,x)|_{U\to\infty} = 0, \quad \left.\frac{\partial f_0(\varepsilon,x)}{\partial x}\right|_{U=0} = 0. \tag{3}$$

The Crank-Nickolson algorithm [9] was used for the numerical analysis of Eq. (1) with the boundary conditions (3). The distribution function was normalized using the constant value of the electron current density

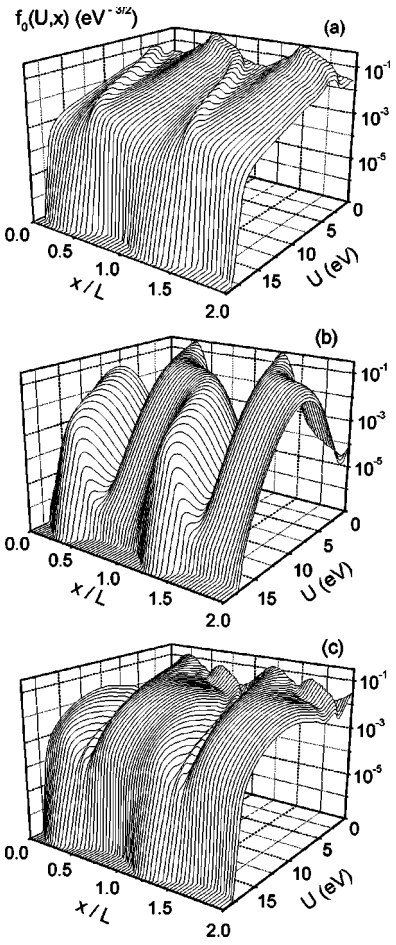

FIG. 1. Electron distribution function in the sinusoidally modulated electric field [Eq. (2)] in the vicinity of the resonance. (a) $L=0.9L_{\mathrm{res}}$, (b) $L=L_{\mathrm{res}}$, (c) $L=1.1L_{\mathrm{res}}$.

$$j=\frac{1}{3}\sqrt{\frac{2}{m}}\int_0^\infty U f_1(U)dU,$$

where $f_1(U)$ is the directed part of the distribution function. The frequencies of inelastic collisions were taken two orders of magnitude higher than those in Ref. [8] in order to demonstrate more sharp resonance behavior of the EDF formation.

At first Eq. (1) was solved for the case of the homogeneous electric field E_0 in order to obtain the value of the resonance spatial period L_{res}. The resonance length determined as the period of damped oscillations in the EDF was found to be $L_{\mathrm{res}}=9.8$ cm. In the second step, Eq. (1) was solved in the inhomogeneous electric field (2) for the different values of the spatial period L belonging to the range 4 -12 cm. It was found that at $L=L_{\mathrm{res}}$ and $L=L_{\mathrm{res}}/2$ the pronounced structures in the EDF appear which correspond to the two resonances at these values of L.

The three-dimensional (3D) plots for the EDF in the vicinity of the resonance are shown in Fig. 1 by analogy to Ref. [8]. It is seen that a small detuning in the period length changes significantly the form of the EDF.

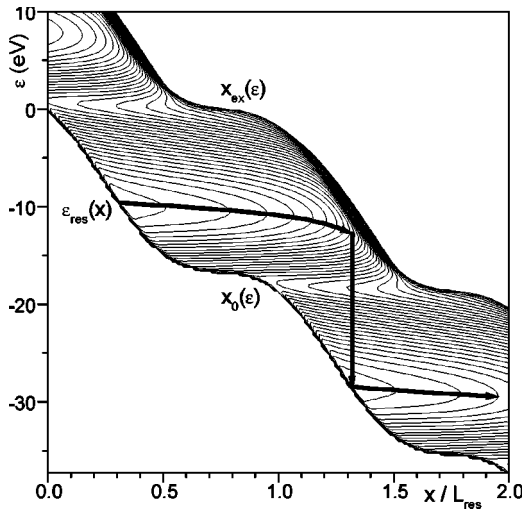

FIG. 2. Contour plot of the resonance EDF on the plane (ε,x). $\varepsilon_{\mathrm{res}}(x)$ is the resonance trajectory. At the curves $x_0(\varepsilon)$ and $x_{\mathrm{ex}}(\varepsilon)$, the kinetic energy of the electrons equals zero and U_{ex}, respectively.

The resonance at $L=L_{\mathrm{res}}$, the EDF is compressed into a peak which moves on the plane (U,x) as, it is seen in Fig. 1(b). In the coordinates (ε,x), this maximum moves along the trajectory $\varepsilon_{\mathrm{res}}(x)$ shown in Fig. 2, where the contour plot of the distribution function [Fig. 1(b)] is given. At smaller [Fig. 1(a)] or larger [Fig. 1(c)] values of L, the modulation in the EDF is significantly decreased.

In order to illustrate the resonance behavior of the macroscopic quantities, the modulation degrees of the electron density n and the mean electron energy $\overline{U}$ were calculated:

$$m_n(L)=\frac{n_{\max}-n_{\min}}{n_{\max}+n_{\min}}\%,$$

$$m_{\overline{U}}(L)=\frac{\overline{U}_{\max}-\overline{U}_{\min}}{\overline{U}_{\max}+\overline{U}_{\min}}\%,$$

where $n(x)=\int_0^\infty \sqrt{U}f_0(U,x)dU$ and $\overline{U}(x)=n^{-1}\int_0^\infty U^{3/2}f_0(U,x)dU$. Indices max and min correspond to the maximal and the minimal values of these parameters at the period.

In Fig. 3, the results of calculation for the modulation degrees of electron density and mean energy are represented as the functions of the spatial period L. This figure reveals the presence of two resonances at $L=L_{\mathrm{res}}$ and $L=L_{\mathrm{res}}/2$ which corresponds to S and P striations observed in the experiments. It is also seen that the resonances on the curve for the mean energy (curve B) are more pronounced than for the density (curve A). Transition to the ''black wall'' approximation results in sharper resonance on the mean energy, which can be seen from the comparison of the curve B with the data of Ref. [8].

It is seen from Fig. 1 that the EDF dependence on the spatial period L is highly complicated. It is necessary to select a parameter which will characterize the form of the EDF and be sensitive to the alteration of the spatial period L. This

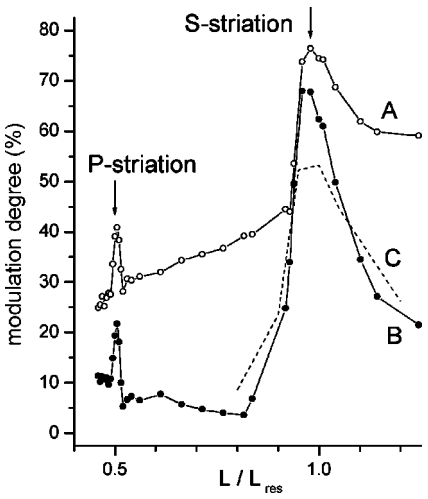

FIG. 3. Degrees of modulation of the electron density (curve *A*) and mean electron energy (curve *B*) versus spatial period length. Comparison with the data of Ref. [8] for the modulation degree of the mean energy (curve *C*).

parameter can be chosen on the basis of the following considerations. An analytical solution of Eq. (1) under the ''black wall'' approximation, i.e., with the boundary conditions $f_0(\varepsilon,x)|_{U=U_{ex}}=0$, can be obtained [4] in the form of the series expansion relative to the small parameter $\delta=6m^2\nu^2(v_1)U_{ex}/M(eE_0)^2$ $[v_1=(2U_{ex}/m)^{1/2}]$ as follows:

$$f_0(\varepsilon,x)=\sum_{i=0} f_0^{(i)}(\varepsilon,x)\,\delta^i. \tag{4}$$

The leading term $f_0^{(0)}(\varepsilon,x)$ can be written as

$$f_0^{(0)}=\Phi(\varepsilon)\int_x^{x_{ex}(\varepsilon)}\frac{dx}{D_\varepsilon}\equiv\Phi(\varepsilon)F(\varepsilon,x), \tag{5}$$

where $x_{ex}(\varepsilon)$ is the curve on the plane (ε,x), at which the kinetic energy of the electrons is equal to U_{ex}, $\Phi(\varepsilon)$ is the amplitude of the distribution function, and $F(\varepsilon,x)$ is the distribution function which is formed in the electric field (2) when the energy losses in elastic collisions are neglected. This function can be easily calculated according to its definition.

Accurate numerical solution of Eq. (1) can be approximated by expression (4). In this case, the amplitude $\Phi(\varepsilon)$ can be obtained from the relation

$$\Phi(\varepsilon)=\frac{f_0(\varepsilon,x)}{F(\varepsilon,x)}, \tag{6}$$

where $f_0(\varepsilon,x)$ is the strict numerical solution of Eq. (1) in the electric fields with different spatial periods L, and $F(\varepsilon,x)$ is given by expression (5) and depends on the spatial period L.

This approach gives a representation of the EDF as a product of two factors, one of which, $F(\varepsilon,x)$, is almost insensitive to the alteration of the spatial period L and the other, $\Phi(\varepsilon)$, reacts strongly on the variations in L. Function

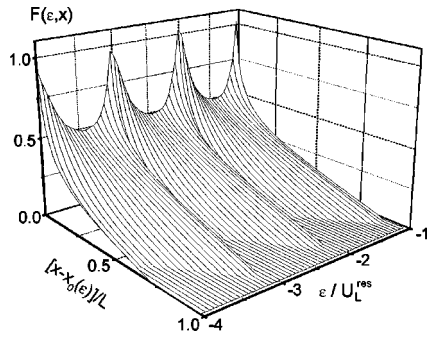

FIG. 4. The function $F(\varepsilon,x)$ [Eq. (5)] which depends weakly on the spatial period L of the field.

$F(\varepsilon,x/L_{res})$ is shown in Fig. 4 on the plane $[\varepsilon,x-x_0(\varepsilon)]$ for the case when $L=L_{res}$. For other values of L, the function $F(\varepsilon,x/L)$ has almost the same appearance. The scale of the spatial decrease is the only parameter that is varied.

It is seen from Fig. 5 that $\Phi(\varepsilon,L)$ depends resonantly on the value of the spatial period L. A sharp Gaussian maximum in $\Phi(\varepsilon)$ is formed in the vicinity of $L=L_{res}$. This maximum defines the final form of the EDF. The procedure of the EDF decomposition into two factors is a sort of approximation which leads to a small shift of the maximum value of $\Phi(\varepsilon)$ relative to $L=L_{res}$. An analysis of the 3D plots shows that the peak of $\Phi(\varepsilon)$ corresponding to the resonance EDF is placed at $L=L_{res}$ and $\varepsilon=0.6U_L^{res}$. The value $\varepsilon=0.6U_L^{res}$ gives the position of the resonance trajectory on the plane (ε,x) along which the total energy losses in elastic and inelastic collisions $(U_{ex}+\Delta U)$ are equal to the fall of potential eE_0L_{res} over the period. It is also seen in Fig. 5 that two maxima are formed in $\Phi(\varepsilon)$ at a value of $L=L_{res}/2$ and energies $\varepsilon=0.3U_L^{res}$ and $\varepsilon=0.8U_L^{res}$. These maxima define two resonance trajectories on the plane (ε,x). The electrons in an electric field with spatial period $L_{res}/2$ are bunching to these trajectories.

This procedure describes bunching in spatially periodic, strongly modulated resonance fields.

III. EXPERIMENTAL MEASUREMENTS OF THE EDF IN *S* AND *P* STRIATIONS

The EDF formation mechanism described above is confirmed by the experimental data giving the EDF measure-

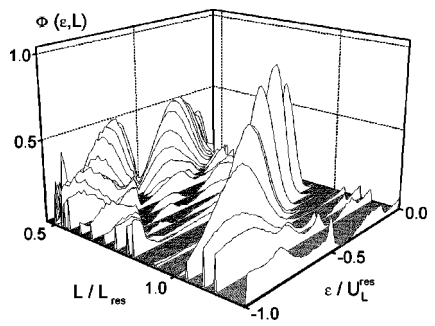

FIG. 5. The amplitude $\Phi(\varepsilon)$ [Eq. (6)] versus spatial period of the electric field.

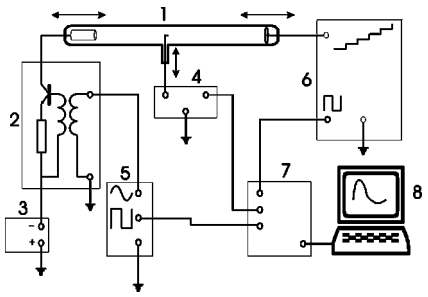

FIG. 6. Experimental setup for the measurements of the EDF in striations. 1 is the discharge tube with movable anode and cathode, 2 is the current stabilizer and modulator, 3 is the high voltage supply, 4 is the current-voltage convertor, 5 is the function generator, 6 is the stepwise voltage source, 7 is the analog-to-digital converter, and 8 is the personal computer.

ments in *S* and *P* striations, carried out with high spatial resolution.

The measurements were conducted in a discharge tube with inner diameter 40 mm and distance between electrodes 55 cm. The electrodes were supplied with screw displacement units that allowed the discharge to be moved relative to the stationary probe in steps of 0.7 mm at one full screw rotation. The resulting spatial resolution in the axial direction was limited by the probe length of 1.5 mm. The experimental setup is shown in Fig. 6. The EDF was measured by the well-known technique of double differentiation of current-voltage characteristics of the probe. The time resolution was 24 μs.

The measurements were conducted as follows. The stepwise potential in steps of 0.1 V was supplied to the probe at one fixed probe position. For each step of the potential, the dependence of the probe current on time was measured. The total measurement procedure for a given spatial position took 120 S. The time evolution of the EDF at a given spatial position was thus obtained.

Assuming that the plasma potential is defined by the zero of the second derivative, it is possible also to determine the spatiotemporal distribution of the plasma potential in the ionization wave.

The measurements were performed in a neon discharge at a pressure of $p=1.5$ Torr for *S* striation and $p=1.0$ Torr for *P* striation. The discharge current was equal to 10 mA in both cases. The fall of the potential in *S* striation was equal to $F_{L_S}=19$ V on the period $L_S=10.15$ cm, and in *P* striation it was equal to $F_{L_P}=9.5$ V on the period $L_P=5.1$ cm. The period-averaged value of the electric field was $E_0=1.9$ V/cm in both cases.

The measurements and calculations of the EDFs in the experimentally measured electric field $E(x)$ are shown in Figs. 7(a,b) for *S* striation and in Figs. 8(a,b) for *P* striation.

It is seen from the figures that these measured and calculated EDFs correlate well. The form of EDF in striations is defined by the formation of one resonance peak on $\Phi(\varepsilon)$ at $\varepsilon=0.6U_L^{\rm res}$ in *S* striation and two peaks at $\varepsilon=0.3U_L^{\rm res}$ and $\varepsilon=0.8U_L^{\rm res}$ in *P* striation. The experimental EDFs are smoother than the calculated ones, which can be attributed to the instrumental widening effects encountered in the measurement.

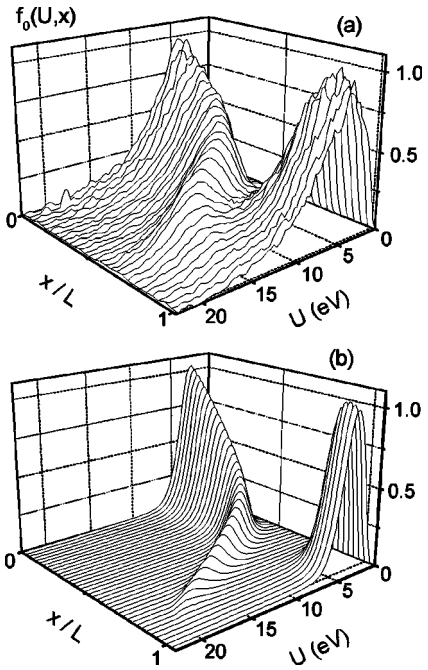

FIG. 7. Comparison of the measured (a) and calculated (b) EDFs in *S* striation. Neon, $pR=3.0$ Torr cm, $i/R=5$ mA/cm.

IV. LINEAR THEORY OF THE EDF FORMATION IN THE SPATIALLY PERIODIC WEAKLY MODULATED ELECTRIC FIELDS

The previous sections were devoted to the analysis of the kinetic equation in electric fields with an arbitrary degree of

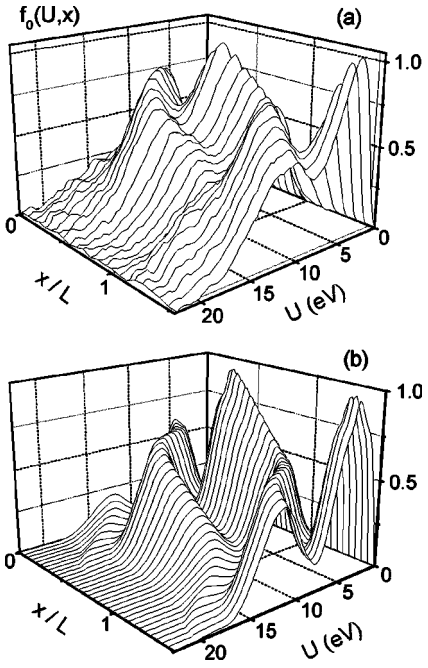

FIG. 8. Comparison of the measured (a) and calculated (b) EDFs in *P* striation. Neon, $pR=2.0$ Torr cm, $i/R=5$ mA/cm.

modulation based on the approximation of the numerical solution by an analytical expression. It was thus possible to observe the dependence of the EDF resonance behavior on the spatial period of the electric field.

In weakly modulated electric fields, a strict analytical approach to the kinetic equation is possible. This approach illustrates the resonance nature of the distribution function formation, proposed by Tsendin [4].

The distribution function $f_0^{(0)}(\tilde{\varepsilon},x)$ can be represented in form (5) where the amplitude $\Phi(\tilde{\varepsilon})$ satisfies the equation (see the Appendix)

$$\Phi(\tilde{\varepsilon}-1-\delta A)-\Phi(\tilde{\varepsilon})-\delta^2 B\frac{\partial^2\Phi}{\partial\tilde{\varepsilon}^2}=\delta\chi\frac{\partial}{\partial\tilde{\varepsilon}}\Phi(\tilde{\varepsilon})\exp(-ik\tilde{\varepsilon}), \tag{7}$$

where $\tilde{\varepsilon}$ is the dimensionless total energy measured in the U_{ex} units. Equation (7) describes the evolution of the initial distribution function injected in the homogeneous field (if $\alpha=0$ and $\chi=0$) or periodic field with the period $L=(2\pi/k)(U_{\mathrm{ex}}/E_0)$.

The potential energy in the spatially periodic electric fields with a small degree of modulation is $e\tilde{\varphi}(\tilde{x})=-\tilde{x}+(\alpha/2\pi)\exp(ik\tilde{x})$, where the dimensionless variables $\tilde{x}=x/L_{\mathrm{res}}$ and $\tilde{\varphi}=\varphi/U_{\mathrm{ex}}$ are used. In what follows the tilde sign will be omitted. In this field, the relative energy losses in elastic collisions can be expressed as a function of the total energy ε,

$$\Psi(\varepsilon)=A+\chi\exp(-ik\varepsilon). \tag{8}$$

The weakly modulated function $B(\varepsilon)$ can be considered approximately as a constant (see the Appendix). The relaxation of a distribution function injected in the homogeneous electric field ($\alpha=0,\chi=0$) is described by the homogeneous equation (7) with the right-hand side equal to zero.

The solution of the homogeneous equation can be represented as a Fourier series,

$$\Phi_{\mathrm{hom}}(\varepsilon)=\sum_n \Phi_{\mathrm{hom}}^n\exp[(ik_n+\gamma_n)\varepsilon], \tag{9}$$

$$k_n=\frac{2\pi n}{1+\delta A},\quad \gamma_n=(2\pi n\delta)^2 B,$$

$$\Phi_{\mathrm{hom}}^n=\int_0^1\Phi^{\mathrm{init}}(\varepsilon)\exp(-ik_n\varepsilon)d\varepsilon.$$

It is seen from Eq. (9) that all harmonics with $n\neq 0$ are damp (negative values of the total energy ε are assumed). Relaxation has the form of damped oscillations with period defined by the first mode ($n=1$), i.e., with the resonance length $L_{\mathrm{res}}=(1+\delta A)U_{\mathrm{ex}}/eE_0$, as it is can be seen from expression (9). The final result of the relaxation is the establishment of the EDF independent of the initial condition and homogeneous in space, which is the solution of kinetic equation (1) in the homogeneous electric field.

The relaxation process of an EDF injected in the weakly modulated electric field is described by the inhomogeneous equation (7) with the right-hand side being treated as the periodic external force with the period $2\pi/k$. If we write the unknown solution of the equation in the form

$$\Phi_{\mathrm{var}}(\varepsilon)=\Phi_{\mathrm{hom}}(\varepsilon)+\delta\tilde{\Phi}(\varepsilon), \tag{10}$$

we come to the following expression for $\tilde{\Phi}(\varepsilon)$:

$$\tilde{\Phi}(\varepsilon,k)=\sum_n \tilde{\Phi}^{(n)}(k)\exp\{[i(k_n-k)+\gamma_n]\varepsilon\},$$

$$\tilde{\Phi}^{(n)}(k)=\Phi_{\mathrm{hom}}^{(n)}\{\chi[i(k_n-k)+\gamma_n]\}[\delta^2 B(k_n-k)^2-1+\exp\{-[i(k_n-k)+\gamma_n](1+\delta A)\}]^{-1}. \tag{11}$$

It is seen from Eq. (11) that at values k, satisfying the expression $(k_n-k)(1+\delta A)=2\pi m$, the resonances are formed. In this case only the terms connected with the nonzero Q factor are retained in the denominator of the right-hand side of Eq. (11),

$$\exp[-\gamma_n(1+\delta A)]-1+\delta^2 B(k_n-k)^2\sim\gamma_m-\gamma_n.$$

Given that all harmonics with $n\neq 0$ are damp, the final result is

$$\tilde{\Phi}^{(0)}(k)=\Phi_{\mathrm{hom}}^{(0)}\frac{-ik\chi}{\exp[ik(1+\delta A)]-1+\delta^2 Bk^2}. \tag{12}$$

Expression (12) describes the amplitude of the EDF established in a spatially periodic electric field with an arbitrary value k. The EDF in this case can be written as

$$f_0(\varepsilon,x,k)=[\Phi_{\mathrm{hom}}^{(0)}+\delta\tilde{\Phi}^{(0)}(k)\exp(ik\varepsilon)]F_0(\varepsilon,x,k). \tag{13}$$

When k takes on the values $k=2\pi n/(1+\delta A)$ corresponding to the energy periods $\varepsilon=U_L^{\mathrm{res}}/n$ and spatial periods $L=L_{\mathrm{res}}/n$, the exponential term in Eq. (11) becomes equal to unity which describes the resonances on the function $\tilde{\Phi}^{(0)}(k)$.

Figure 9 shows the dependence of the fundamental harmonics $\tilde{\Phi}^{(0)}$ on the spatial period L. The figure demonstrates the resonant nature of the harmonic at $L=L_{\mathrm{res}}/n$. The dependence of the real part of $\tilde{\Phi}(\varepsilon)$, which describes the form of the EDF in the spatially periodic electric fields (13), is shown in Fig. 10. It is seen from Fig. 10 that at $L=L_{\mathrm{res}}$ and $\varepsilon=0.5U_L^{\mathrm{res}}$ one sinusoidal maximum is formed, and at $L=L_{\mathrm{res}}/2$ and $\varepsilon=0.25U_L^{\mathrm{res}}$, $\varepsilon=0.75U_L^{\mathrm{res}}$ two maxima are formed.

The presented theory qualitatively describes the peculiarities of the EDF formation which can be seen from the comparison of Figs. 10 and 5. It is also seen from expressions (11) and (13) that the alternating addition to the amplitude of the EDF has a very small value of the order of $\delta\chi$ and therefore results in a very weak modulation of the solution in the homogeneous field.

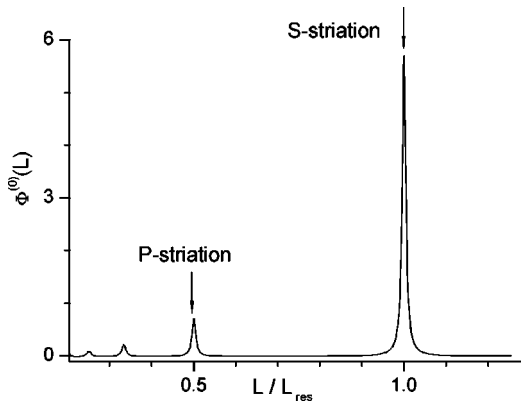

FIG. 9. Real part of the alternating undamped part of the amplitude $\tilde{\Phi}^{(0)}(k)$ [Eq. (12)] versus spatial period length L.

In Ref. [4], the case of the moderately modulated field was also considered. It was shown that in this case the amplitude of the EDF has the form of a Gaussian with a maximum similar to that shown in Fig. 5.

V. CONCLUSION

In this paper, the calculations of the EDF in spatially periodic, sinusoidally modulated electric fields with a large modulation degree have been performed. The dependence on the spatial period length L was investigated. The calculations were done for the conditions which correspond to the neon discharge at pressure $pR=2$ Torr cm, current $i/R=5$ mA/cm, and the period-averaged electric field strength $E=1.9$ V/cm. The inelastic collisions dominate in the energy balance of the electrons under the chosen conditions. This fact results in a resonant behavior of the EDF. The form of the EDF in the vicinity of the resonance is shown. The calculations of the EDF and the macroscopic parameters of the plasma have shown the presence of two resonances which correspond to S and P striations observed in experiments. An interpretation of the results based on the analytical approximation of the numerical solution is proposed. The amplitude of the EDF, $\Phi(\varepsilon)$, dependent on the total energy ε and spatial period L has been introduced. It is shown that at $L=L_{\mathrm{res}}$ one Gaussian maximum and at $L=L_{\mathrm{res}}/2$ two Gaussian maxima are formed on the amplitude $\Phi(\varepsilon)$. The experimental measurements of the EDF in S and P striations reveal good agreement between the experimental and theoretical data. The calculations of the resonant behavior of the EDF based on the linear theory proposed in Ref. [4] are shown.

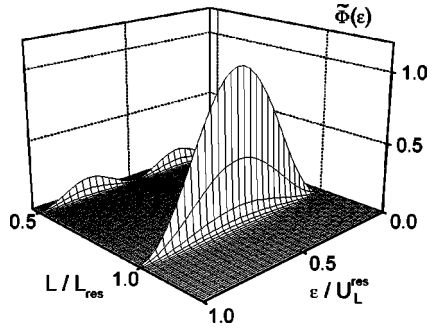

FIG. 10. Amplitude $\tilde{\Phi}(\varepsilon)$ [Eq. (10)] versus total energy and spatial period of the electric field L.

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APPENDIX

According to Ref. [4], the kinetic equation with appropriate boundary conditions can be written in dimensionless variables as

$$\frac{\partial}{\partial x}\tilde{D}_\varepsilon\frac{\partial f_0(\varepsilon,x)}{\partial x}+\delta\frac{\partial}{\partial\varepsilon}\tilde{V}_\varepsilon(U)f_0(\varepsilon,x)=0, \tag{A1}$$

$$f_0(\varepsilon,x)\big|_{U=U_{\mathrm{ex}}}=0,$$

$$\tilde{D}_\varepsilon\frac{\partial f_0(\varepsilon,x)}{\partial x}\bigg|_{x=x_0(\varepsilon)}=\tilde{D}_\varepsilon\frac{\partial f_0(\varepsilon,x)}{\partial x}\bigg|_{x=x_{\mathrm{ex}}(\varepsilon)}, \tag{A2}$$

where the energy and potential are measured in units U_{ex}, length is measured in units U_{ex}/eE_0, $\tilde{D}_\varepsilon(U)=D_\varepsilon(U)/D_\varepsilon(U_{\mathrm{ex}})$, $\tilde{V}_\varepsilon(U)=V_\varepsilon(U)/V_\varepsilon(U_{\mathrm{ex}})$. The solution of Eq. (A1) can be written in the series expansion up to the terms squared in parameter δ,

$$f_0(\varepsilon,x)=f_0^{(0)}(\varepsilon,x)+\delta f_0^{(1)}(\varepsilon,x)+\delta^2 f_0^{(2)}(\varepsilon,x).$$

After substitution of the expansion in Eq. (A1) and making use of the first boundary condition, we obtain

$$f_0^{(0)}(\varepsilon,x)=\Phi(\varepsilon)F(\varepsilon,x),$$

$$F(\varepsilon,x)=\int_x^{x_{\mathrm{ex}}(\varepsilon)}\frac{dx}{\tilde{D}_\varepsilon(U)},$$

$$f_0^{(1)}(\varepsilon,x)=-\int_x^{x_{\mathrm{ex}}(\varepsilon)}\frac{dx'}{\tilde{D}_\varepsilon(U')}\frac{\partial}{\partial\varepsilon}\int_{x_0(\varepsilon)}^{x'}\tilde{V}_\varepsilon(U)f_0^{(0)}(\varepsilon,x)dx,$$

$$f_0^{(2)}(\varepsilon,x)=-\int_x^{x_{\mathrm{ex}}(\varepsilon)}\frac{dx'}{\tilde{D}_\varepsilon(U')}\frac{\partial}{\partial\varepsilon}\int_{x_0(\varepsilon)}^{x'}\tilde{V}_\varepsilon(U)f_0^{(1)}(\varepsilon,x)dx.$$

In order to satisfy the second boundary condition (A2), it is necessary to calculate the derivative of $f_0(\varepsilon,x)$ at upper and lower limits,

$$\tilde{D}_\varepsilon \frac{\partial f_0(\varepsilon,x)}{\partial x}\bigg|_{x=x_{ex}(\varepsilon)} = \Phi(\varepsilon) + \delta\frac{\partial}{\partial\varepsilon}\Phi(\varepsilon)\int_{x_0(\varepsilon)}^{x_{ex}(\varepsilon)}\tilde{V}_\varepsilon F(\varepsilon,x)dx + \delta^2\frac{\partial}{\partial\varepsilon}\int_{x_0(\varepsilon)}^{x_{ex}(\varepsilon)}\tilde{V}_\varepsilon dx\int_x^{x_{ex}(\varepsilon)}\frac{dx'}{\tilde{D}_\varepsilon}\frac{\partial}{\partial\varepsilon}\Phi(\varepsilon) \times\int_{x_0(\varepsilon)}^{x'}\tilde{V}_\varepsilon F(\varepsilon,x'')dx''.$$

Taking into account that $x_0(\varepsilon-1)=x_{ex}(\varepsilon)$, we obtain

$$\tilde{D}_\varepsilon \frac{\partial f_0(\varepsilon,x)}{\partial x}\bigg|_{x=x_0(\varepsilon)} = \Phi(\varepsilon-1),$$

and according to the second boundary condition (A2) we obtain an equation for the amplitude $\Phi(\varepsilon)$,

$$\Phi(\varepsilon-1)=\Phi(\varepsilon)+\delta\frac{\partial}{\partial\varepsilon}\Phi(\varepsilon)\Psi(\varepsilon)+\delta^2\frac{\partial^2}{\partial\varepsilon^2}\Phi(\varepsilon)C(\varepsilon), \tag{A3}$$

where

$$\Psi(\varepsilon)=\int_{x_0(\varepsilon)}^{x_{ex}(\varepsilon)}\tilde{V}_\varepsilon(U)F(\varepsilon,x)dx,$$

$$C(\varepsilon)=\int_{x_0(\varepsilon)}^{x_{ex}(\varepsilon)}\tilde{V}_\varepsilon\,dx\int_x^{x_{ex}(\varepsilon)}\frac{dx'}{\tilde{D}_\varepsilon(U')} \times\int_{x_0(\varepsilon)}^{x'}\tilde{V}_\varepsilon(U'')F(\varepsilon,x'')dx''. \tag{A4}$$

In the homogeneous electric field, the functions $\Psi(\varepsilon)$ and $C(\varepsilon)$ are constants. In the weakly modulated electric fields $\varphi=-x+\tilde{\varphi}\exp(ikx)$, the function Ψ can be represented as $\Psi(\varepsilon)=A+\chi\exp(-ik\varepsilon)$ and the weakly modulated function $C(\varepsilon)$ can be considered as constant, since after triple integration (A4), required for its calculation, inhomogeneities vanish.

To solve Eq. (A3), it is expedient to shift the argument ε by the value δA. Taking into account the terms squared in δ, Eq. (A3) can be written as

$$\Phi(\varepsilon-1-\delta A)-\Phi(\varepsilon)-\delta^2 B\frac{\partial^2\Phi(\varepsilon)}{\partial\varepsilon^2} =\delta\chi\frac{\partial}{\partial\varepsilon}\Phi(\varepsilon)\exp(-ik\varepsilon), \tag{A5}$$

where $B=C-A^2/2$. This equation can be used for solving the electron kinetics problems in inhomogeneous electric fields of arbitrary configuration.

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