Direct electron acceleration by stochastic laser fields in the presence of self-generated magnetic fields

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A simple direct acceleration model is proposed, taking into account the stochastic phase disturbance of the coherent driving laser fields. A relativistic single particle simulation shows that plasma electrons are efficiently accelerated far above the ponderomotive energy. The energy and momentum distributions of the accelerated electrons are derived to examine the effects of the self-generated magnetic field on the characteristics of the electron beams. In addition to the beam collimation effect, the magnetic field is found to further enhance the electron acceleration, resulting in the generation of ultrahigh energy electrons.

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I. INTRODUCTION

Recent remarkable progress in ultrahigh power and ultrashort pulse laser technology has stimulated innovative activities in scientific and engineering fields such as astrophysics, fast ignition of inertial confinement fusion targets, and high-energy particle accelerators. Collective acceleration of electrons by electron plasma waves excited by intense laser pulses has been intensively studied both theoretically and experimentally [1] since the first proposal of the fundamental concept [2].

In contrast to the collective acceleration schemes, alternative scenarios of the direct particle acceleration by laser fields have been proposed and studied, that are valid in vacuum without the plasma waves in principles [3]. Generally there is no net energy transfer to a single electron after the interaction with a planar laser light pulse in vacuum without the help of additional lasers [4,5] or disturbances in the coherent electron motion. Electron acceleration by intense laser pulses in the presence of stochastic perturbations has been studied by introducing some artificial anisotropic friction terms into the equations of motion [6].

In the present paper, we propose and study another stochastic acceleration mechanism based on phase-jump disturbances of the driving laser fields [7]. The presence of such disturbances is highly probable in the intense laser-plasma interactions where various plasma instabilities may readily produce the complex structure of the distribution of the electron plasma density. In addition, we consider the effect of the self-generated magnetic field on the characteristics of the electron beams accelerated in the plasma channels. A fully relativistic single particle code is used to study the fundamental characteristics of the electron beams, especially the momentum and energy distributions. Although such a numerical analysis is not fully self-consistent, it gives us the essential insight into the physical processes involved in the laser plasma interactions by introducing the appropriate modeling even without elaborate particle-in-cell (PIC) simulations.

In Sec. II the theoretical model is shown to describe the direct electron acceleration by the stochastic phase-jump disturbance of the laser field. The general formulation of the acceleration process and the energy distribution function are given. In Sec. III the numerical model and the results are shown. Ultrahigh energy electron generation is studied as a result of the combined effect of the phase-jump acceleration and the magnetic focusing. Section IV discusses the factors affecting the distribution functions of the electron energy, including statistics of the phase-jump events. In the Appendix the axial current and the azimuthal magnetic field induced by the drifting electrons are discussed.

II. ELECTRON ACCELERATION IN THE PRESENCE OF STOCHASTIC PHASE JUMPS OF ELECTROMAGNETIC LASER FIELDS

A. Relativistic electron dynamics in plane electromagnetic fields

First we review the dynamics of relativistic electrons in a linearly polarized plane light wave propagating in the z direction with the electric field linearly polarized in the x direction. The temporal evolution of the normalized electron momenta (p_x, p_z) and energy $(\gamma - 1)$ are simply defined by the normalized vector potential $a = (eA/m_e c^2)$ as functions of $\tau (= t - z(t)/c)$:

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$$p_x(\tau)-a(\tau)=p_x(0)-a(0), \tag{1}$$

$$\gamma(\tau)-p_z(\tau)=\gamma(0)-p_z(0), \tag{2}$$

where $\gamma^2=1+p_x^2+p_z^2$. The constants of motion are defined by the initial conditions at $\tau=0$.

For the electron initially at rest before the interaction, we find

$$p_x(\tau)=a(\tau), \tag{3}$$

$$p_z(\tau)=\tfrac{1}{2}a^2(\tau), \tag{4}$$

$$\gamma(\tau)=1+p_z(\tau), \tag{5}$$

$$p_z(\tau)=\tfrac{1}{2}p_x^2(\tau), \tag{6}$$

and therefore the kinetic energy of the electron is given by $\gamma(\tau)-1=\frac{1}{2}a^2(\tau)$. There is no net energy transfer to these electrons after leaving the laser pulse because the vector potential returns to the initial value $a(0)=0$.

Now we introduce a disturbance in the laser fields to examine its effects as a potential mechanism for efficient electron acceleration. We model the disturbance by stochastic phase jumps in the coherently oscillating electromagnetic fields. For an electron initially at rest, the relation between the momenta $p_z(\tau)=\frac{1}{2}p_x^2(\tau)$ is retained throughout even in the presence of the phase jumps. Scattered electrons with energy $\gamma-1$ have therefore the deflection angle of θ:

$$\tan\theta=\frac{p_x}{p_z}=\sqrt{\frac{2}{\gamma-1}}. \tag{7}$$

This formula is a general relation for the case of the ponderomotive scattering [8,9]. The small angular spread of these electrons may be observed only at a strongly relativistic laser intensity ($\gamma\gg1$).

The evolution of the electron motion is completely described by the vector potential $a(\tau)$, which is derived from the integration of the laser electric field $\varepsilon_L(\tau)$ disturbed by the phase jumps:

$$a(\tau)=-\int_0^\tau \varepsilon_L(\tau)d\tau, \tag{8}$$

$$d\tau=dt-\frac{dz}{c}=(1-\beta_z)dt, \quad \beta_z=\frac{v_z}{c}.$$

The vector potential is a continuous function even if the electric fields are discontinuous due to the introduction of the phase jumps. We write the phase of the laser field as $\phi(\tau)=\omega_L\tau+\phi_0(\tau)$ with ω_L and $\phi_0(\tau)$ being the angular laser frequency and the disturbed phase term, respectively. Now consider the electron dynamics in the presence of the stochastic phase jumps. With the amplitude of the normalized vector potential, a_L, the normalized laser electric field is given by $\varepsilon_L(\tau)=a_L\sin[\phi(\tau)]$.

B. Electron acceleration by stochastic phase jumps in laser fields

As mentioned above, there is no net energy transfer from the laser fields to the electrons in the absence of some disturbances to the harmonic motion of the quivering electrons.

As well as particle collisions, abrupt changes in the phase of the laser fields disturb the coherent motion of the electrons and yield the finite residual value of the vector potential given by Eq. (8), resulting in net electron acceleration or heating. Different plasma instabilities may readily produce the complex structure of the distribution of the electron plasma density in the intense laser-plasma interactions. As a result, it is probably the case that not only the phase but also the amplitude of the laser experiences fluctuations in experiments. The resultant disturbance in the driving laser fields leads to the electron acceleration. Making no mention of the potential driving sources of these phase disturbances, we describe their effects on the coherent laser fields by introduction of a set of phase jumps and theoretically study the resultant electron acceleration process. We theoretically study the electron acceleration driven by stochastic phase jumps induced in the laser fields. In the present theoretical model, the number and the magnitude of the phase jumps m is essentially arbitrary. However, in practice, the value of m depends on the physical interaction processes in the plasma channels and defines the energy gain or the energy distribution of the accelerated electron beams. In the present study, we simply estimate the maximum number of phase jumps.

We consider the scale length of the potential plasma instabilities induced in the laser-plasma interactions. In highly relativistic or nonlinear laser-plasma interactions, copious instabilities may be induced, which generate an inhomogeneous distribution in the plasma density of a short spatial scale length comparable to electron plasma waves. Besides the lasers, induced electron beams may further produce additional beam-plasma instabilities. Therefore, we assume that the shortest characteristic length of the disturbance (λ_{min}) in the plasma channel may be represented by the magnetic skin depth or the wavelength of the electron plasma wave (c/ω_{pe}), where ω_{pe} is the electron plasma frequency. During the interaction with the coherent fields of a Gaussian laser pulse of duration τ_L, the electrons travel the distance z_d at the drift velocity v_d approximately given by

$$z_d=v_d\tau_L, \tag{9}$$

$$\left(\frac{v_d}{c}\right)=\left(\frac{a_L^2}{4+a_L^2}\right). \tag{10}$$

Considering that the maximum number of the phase jumps m_{max} is equivalent to the collision number over the distance z_d for the free path (λ_{min}), we obtain the following simple scaling:

$$m_{max}=\frac{z_d}{\lambda_{min}}=2\pi\left(\frac{a_L^2}{4+a_L^2}\right)\left(\frac{\omega_{pe}}{\omega_L}\right)\left(\frac{c\tau_L}{\lambda_L}\right). \tag{11}$$

The maximum number of the phase jumps m_{max} may further increase above the predicted value in the case of the interaction with a weakly relativistic laser pulse ($a_L \lesssim 1$), since the effective drift velocity v_d may increase substantially due to the preceding phase-jump events. As the number of the phase jumps is a dominant factor specifying the effective temperature and the maximum energy of the electron distribution for a given laser intensity, a proper choice of the number of the phase jumps should be examined by comparing the model prediction with the experimental results.

First we analytically study the characteristics of the electron beams accelerated in the presence of the stochastic phase jumps and examine the profile of the energy distribution function and the effective temperature. For the slowly varying amplitude of the leading and trailing edge of the laser pulse, we introduce m phase jumps from ϕ_i^- to ϕ_i^+ by $\Delta\phi_i$ ($\Delta\phi_i = \phi_i^+ - \phi_i^-$ for $i=1,2,3,\ldots,m$) in the laser field of the constant amplitude a_L. We obtain the vector potential for the electrons interacting with the laser having the specified phase jumps,

$$a(\tau) = -\int_0^\tau \varepsilon_L(\tau)d\tau = -\int_0^\tau a_L \sin[\phi(\tau)]d\tau = a_L \sum_{i=1}^{m} [\cos(\phi_i^-) - \cos(\phi_i^+)]. \tag{12}$$

Without loss of generality, we assume that all the m phase jumps occur randomly in τ space and have arbitrary step heights. The evolution of the vector potential can equivalently be analyzed based on the $2m$-step random walk with the step distance of a_L:

$$a(\tau) = a_L \sum_{j=1}^{2m} \cos(\theta_j) = a_L \sum_{j=1}^{2m} \mathrm{Re}[\exp(i\theta_j)], \tag{13}$$

where θ_j's are $2m$ random phases in $[0,2\pi]$. Here, we consider the complex plane of the polar coordinates $[r,\theta]$. For an ensemble of a sufficient number of test particles, this process is described by the time-dependent solution of the radial diffusion equation

$$f(r) = \frac{1}{4\pi Dt} \exp\left(-\frac{r^2}{4Dt}\right), \tag{14}$$

where the diffusion coefficient $D = (\frac{1}{4})\lambda^2\nu$ is defined by the mean-free-path λ and the collision frequency ν. The number of the collisions is νt. Considering the correspondence of $\nu t = 2m$ and $\lambda = a_L$, we obtain the radial distribution of the resultant vector potential of amplitude r_a in the complex plane $[x, iy]$:

$$f(r_a) = \left(\frac{1}{2\pi m a_L^2}\right) \exp\left(-\frac{r_a^2}{2m a_L^2}\right) = \left(\frac{1}{\pi r_D^2}\right) \exp\left(-\frac{r_a^2}{r_D^2}\right)$$

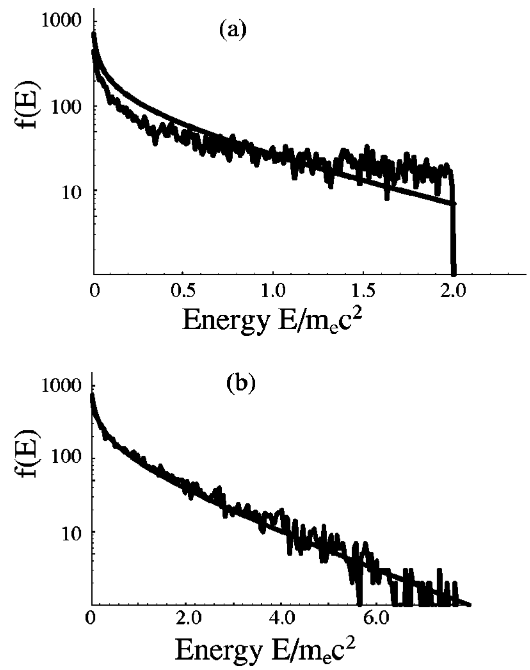

FIG. 1. Energy distributions of 10 000 accelerated electrons in the stochastic laser fields for the number of the phase jumps $m=1$ (a) and $m=2$ (b) with $a_L=1$, predicted by the random-walk model. The smooth reference function is given by $f(E) = f_0/\sqrt{E/E_0}\exp(-E/E_0)$ with $E_0 = m a_L^2$.

where $r_D^2 = 2m a_L^2$.

Noting that the kinetic energy of the electron is $E = \frac{1}{2}(r_a \cos\theta)^2 = \frac{1}{2}x^2$ with $x = r_a\cos\theta$ and $y = r_a \sin\theta$, the energy distribution is given by

$$\left[\int_{-\infty}^{+\infty} f(x,y)dy\right]dx = \frac{1}{\sqrt{\pi} r_D} \exp\left(-\frac{x^2}{r_D^2}\right)dx.$$

From $E = (\frac{1}{2})x^2$ and $dx = dE/\sqrt{2E}$, we obtain the energy distribution

$$f(E)dE = \left(\frac{1}{\sqrt{2\pi E} r_D}\right) \exp\left(-\frac{E}{\frac{1}{2}r_D^2}\right) dE = \frac{f_0}{\sqrt{\frac{E}{E_0}}} \exp\left(-\frac{E}{E_0}\right) dE, \tag{15}$$

where $f_0 = 1/(2\sqrt{\pi}E_0)$.

This is the distribution function of the energy observed along a given direction for the isotropic three-dimensional Maxwellian system of the equivalent temperature of $kT = E_0 = \frac{1}{2}r_D^2 = m a_L^2$. From the result of simple numerical Monte Carlo simulations shown in Fig. 1, it is seen that this model description reproduces the numerical results well for the phase-jump number $m \geqslant 2$. The maximum available energy is given by

$$E_{max} = \gamma_{max} - 1 = 2m^2 a_L^2, \tag{16}$$

as expected in the case of a straight walk. For the number of the phase jumps $m=1$, the corresponding maximum energy appears at $E_{max}=2.0$ as a descending step. This smooths out for larger m.

III. NUMERICAL SIMULATION MODEL OF SINGLE ELECTRON MOTION DRIVEN BY A LASER PULSE IN THE PRESENCE OF STOCHASTIC PHASE JUMPS

The Monte Carlo simulations have been carried out to examine the characteristics of the accelerated electron beams. We consider the dynamics of a single relativistic electron in the electromagnetic fields of a laser pulse propagating in the z direction. With the electric field linearly polarized in the x-z plane, the equations of motion are written as follows [10]:

$$m_e c^2 \frac{d\gamma}{dt} = -e\mathbf{u}\cdot\mathbf{E}, \tag{17}$$

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E}+\mathbf{u}\times\mathbf{B}), \tag{18}$$

$$\mathbf{u} = \frac{d\mathbf{r}}{dt}. \tag{19}$$

The electric and magnetic fields are given by

$$\mathbf{E} = \mathbf{e}_x\left(\frac{m_e\omega_L c}{e}\right)a, \tag{20}$$

$$\mathbf{B} = \mathbf{e}_y\left(\frac{m_e\omega_L c}{e}\right)a + \mathbf{a}_B, \tag{21}$$

$$a(t,\mathbf{r}) = a_L f(t,\mathbf{r})\sin[\omega_L t - k_L z + \varphi(z)],$$

$$\mathbf{a}_B = \left[\mathbf{e}_x\left(\frac{y}{r}\right) + \mathbf{e}_y\left(\frac{x}{r}\right)\right]\left(\frac{B(r,z)}{m_e c\,\omega_L}\right) f^2(t,\mathbf{r}),$$

$$f(t,\mathbf{r}) = \frac{\exp\left[-\left(\frac{t-(z/c)}{\tau_L}\right)^2\right]\exp\left[-\left(\frac{r}{w_0}\right)^2\right]}{\sqrt{1+\left(\frac{z}{Z_R}\right)^2}},$$

$$Z_R = \frac{\pi w_0^2}{\lambda_L},$$

where τ_L is the laser pulse width, ω_L the laser angular frequency, k_L the laser wave number, and λ_L the laser wavelength. Although the electromagnetic fields of the laser are described by a plane wave, a focused geometry is taken into account to define the laser beam profile in the plasma channel. w_0 is the laser beam radius at the focus center and Z_R is the Rayleigh length. The spatial distribution function of the normalized self-generated magnetic field $\mathbf{a}_B$ is defined by the laser pulse profile $f(t,\mathbf{r})$ and the function $B(r,z)$ derived in the Appendix. The phase jumps $\Delta\phi_i$ are introduced into the phase term $\varphi(z)$ at the points statistically specified along the plasma channel.

For efficient computation without loss of the main physical features, we studied the 2D electron dynamics in the x-z plane containing the laser wave vector and the electric field vector.

Referring to the conditions of one of the typical electron-acceleration experiments carried out in the laser gas-jet interaction [11], we have adopted the following parameters for the first demonstrative numerical simulation. The amplitude of the normalized vector potential a_L is 0.7, corresponding to the intensity of a Ti:sapphire laser ($\lambda_L = 790$ nm) of about 10^{18} W/cm^2. The Gaussian laser beam radius w_0 and pulse width τ_L are given for 7 μm and 100 fs full width at half maximum (FWHM), respectively. The Rayleigh length Z_R is 200 μm. For the specified number of phase jumps m, the z coordinates of the phase-jump events are randomly distributed in the region $[0,z_d]$ along the z axis, where z_d is the characteristic drift distance given by Eq. (9).

For simplicity the step heights of the phase jumps are given, which are arbitrary in $[0,2\pi]$, which is the same as the random-walk process discussed in Sec. II B. The electrons are initially at rest on the axis of the laser beam. The radius of the Alfven current channel (r_c) is about 0.7 μm for the electron density $n_e = 0.17 n_c$, where n_c is the critical electron density of 1.6×10^{21} cm^{-3}. The maximum magnetic field surrounding the current channel ($I_A = 2.1$ kA) is $B_c = 6.0$ MG.

As mentioned in the preceding section, the number of the phase jumps is arbitrary in principle but its maximum value m_{max} is practically estimated from the simple physical scaling [Eq. (11)]. Noting that we have approximately $m_{max} = 10$ for parameters of the experimental conditions adopted here, we show the representative numerical results obtained for tentative values of $m=2$ and 4 in the following.

First the dynamics of 4000 electrons are studied for the case of the normalized vector potential $a_L = 0.7$ and a fixed number of the phase jumps $m=2$. The momentum and energy distribution of the ejected electrons are shown in Fig. 2.

Comparing the electron distribution in momentum space with the parabolic relation given in Eq. (6), it is seen that the angular spread is suppressed by the collimating effect of the surrounding magnetic field. The energy distribution shows a Maxwellian profile well described by Eq. (15) based on the simple random-walk model. Also, the normalized cutoff energy is consistent with the theoretical maximum energy of $E_{max} = 3.9$ given by Eq. (16). Therefore, the energy gain is not affected significantly by the magnetic field in this case.

In order to clearly see the acceleration processes due to the phase disturbance, the phase jump number is increased to $m=4$. The results are shown in Fig. 3. The high-energy tail is observed to extend up to $E_{max} = 30.9$, far beyond the Maxwell distribution of the bulk electrons. These energetic electrons are observed only when both the phase jumps and the surrounding magnetic field are introduced. The high-energy tail is enhanced by increasing the strength of the magnetic field as well.

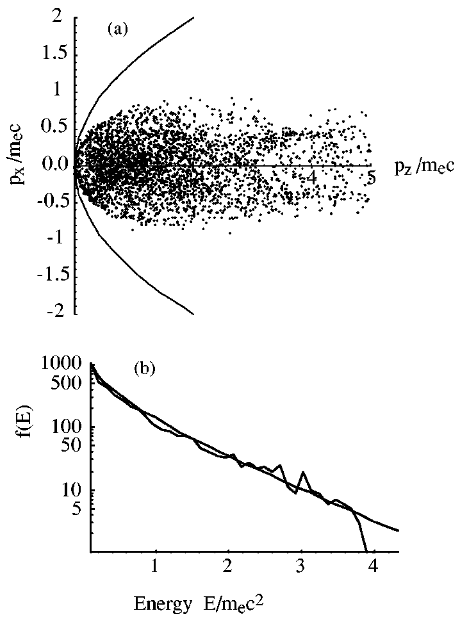

FIG. 2. The momentum (a) and energy (b) distributions of 4000 electrons accelerated in the stochastic laser field with a phase-jump number of $m=2$. The theoretically predicted maximum energy is $E_{max}=2m^2a_L^2=3.9$. The smooth reference functions shown in the upper and the lower figure are the parabolic relation given by Eq. (6) and the Maxwellian distribution given by Eq. (15), respectively.

The trajectories of these energetic electrons indicate the repetitive interaction of the electrons with the laser pulse. On the phase jump the electrons are first accelerated to a moderately high energy level and then leave the laser pulse transversely. However, as they still travel near the laser pulse, some of them are deflected back to the laser pulse by the surrounding self-induced azimuthal magnetic field and enter again into the laser pulse. If the electrons ride the laser field in the proper phase, they gain substantial energy, traveling with the driving laser pulse. This process is similar to the *B*-loop acceleration proposed by A. Pukov and J. Meyer-ter-Vehn [12]. Such an electron acceleration becomes more probable and is enhanced as the number of phase jumps as well as the strength of the magnetic field increases.

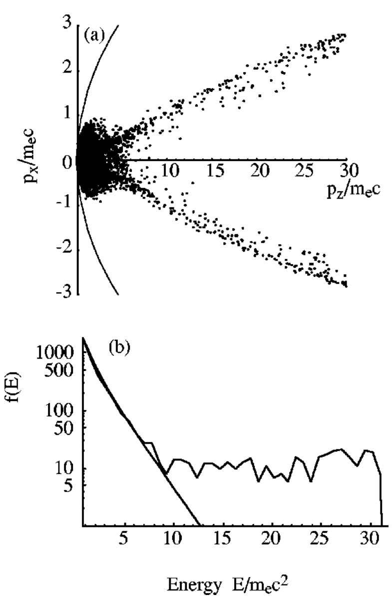

FIG. 3. The momentum (a) and energy (b) distributions of 4000 electrons accelerated in the stochastic laser field with a phase-jump number of $m=4$. The smooth reference functions are defined as in Fig. 2.

IV. DISCUSSION AND CONCLUSION

Based on a simple model it is shown that electrons are effectively accelerated to relativistic energies far above the ponderomotive energy in the presence of a stochastic phase-jump disturbance in the driving laser fields. For a small number of phase jumps $(m\leqslant 2)$, the energy distribution of the accelerated electrons is shown to be nearly Maxwellian with an effective temperature given by $kT=ma_L^2$. When the self-generated magnetic field is introduced, the beams are collimated within a narrowed angular cone but the electron distribution still remains Maxwellian. As the number of phase jumps increases $(m\geqslant 4)$, ultrahigh energy electrons are generated, yielding an extended high-energy tail. The generation of these energetic electrons is observed only in the presence of the magnetic field and enhanced by the increase in the strength of the magnetic field.

Considering the statistical nature of an encounter with the phase jumps, we now give the distribution of the phase jump numbers based on the mean-free-path model usually adopted to describe a statistical collision process. In this model the probability that an electron traveling from the point $z=0$ along the z axis encounters a phase jump in the path region in the plasma channel $[z,z+dz]$ is given by

$$p(z)dz=\frac{1}{\lambda_{pj}}e^{-(z/\lambda_{pj})}dz. \tag{22}$$

The effective mean-free-path for the phase jump is $\lambda_{pj}=\int_0^{+\infty}zp(z)\,dz$ and the averaged number of the phase jumps over the path length z_p is $m_0=(z_p/\lambda_{pj})$.

A numerical calculation is carried out for the conditions that for the individual test electrons the phase-jump numbers m are given statistically by the mean-free-path model with an averaged phase-jump number of $m_0=2$. In addition, the initial positions of the electrons are randomly distributed within the plasma channel of a critical Alfven radius r_c. The other conditions are kept the same as adopted to obtain Fig. 2. The number distribution of the phase jumps, and the energy and angular distributions of the accelerated electrons are shown in Fig. 4.

For the sake of reference, the fitted power law of the form of $f(E)=f_0(E/E_0)^k$ for $k=-3$ is compared with the numerically obtained energy distribution. The characteristic energy is $E_0=m_0a_L^2$. The angular distribution is given for the electron energy above the threshold of 10 keV and the angular spread is about 20°(FWHM).

The experimentally observed energy distributions are usually compared with a Maxwellian function to define the effective temperature(s) [13] or otherwise with a power-law function [11]. When the statistical nature of the distribution

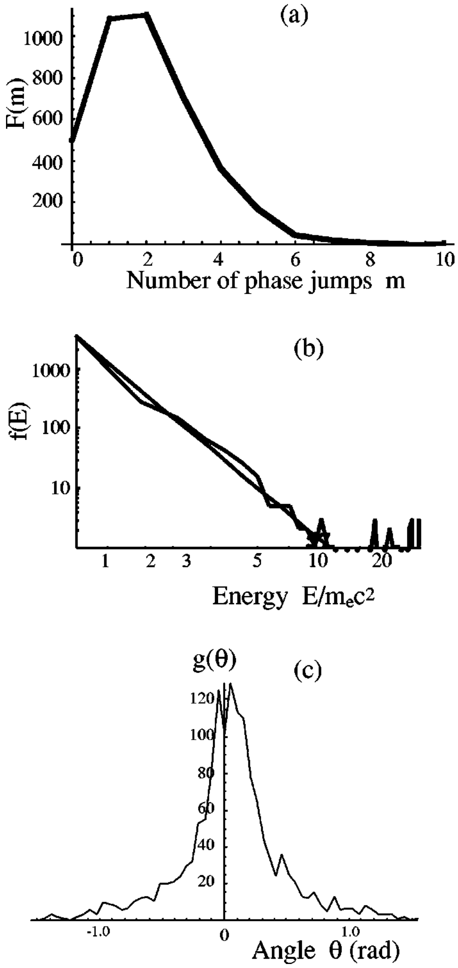

FIG. 4. The distribution of the phase-jump numbers given by the mean-free-path model for the averaged phase-jump number of $m_0 = 2$ (a) and the energy distribution (b) of 4000 electrons accelerated in the stochastic laser field with the self-generated magnetic field along the plasma channel, respectively. The angular distribution (c) for the electrons with energy above 10 keV. In the middle, the reference energy distribution function is given by $f(E) = f_0(E/E_0)^{-3}$ with $E_0 = m_0 a_L^2$.

of the phase-jump numbers is introduced into the present model, a part of the electrons may encounter phase jumps more often than the average. The additional acceleration leads to the high-energy tail on the original Maxwellian energy distribution. The random distribution of the initial electron position enhances this tendency as well, leading to the modified energy distribution apparently fitted by a power-law scaling. The averaged number of the phase jumps m_0 determines the effective electron temperature or the maximum electron energy for a given laser field intensity. As m_0 increases, the distribution of the phase jumps concentrates at the narrower Gaussian peak with a $1/e$ full width, $2\sqrt{2m_0}$, at $m = m_0$ in the mean-free-path model. Therefore, the shift from the Maxwellian to the power-law scaling becomes to be less enhanced.

Although different factors may influence the observed energy distribution functions in practice, the present simple model of direct laser acceleration offers one of the potential mechanisms prevalent in intense laser-plasma interactions. It is desirable to self-consistently study the acceleration processes by a full 3D particle-code simulation in order to get more detailed insight.

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APPENDIX: AZIMUTHAL MAGNETIC FIELD GENERATION BY DRIFTING ELECTRONS IN A LASER PULSE

We consider the current induced by the forward $v \times B_L$-drift motion of the background electrons driven by the relativistic laser pulse, where v and B_L are the electron quiver velocity and the laser magnetic field, respectively. In the plane-polarized laser fields of the normalized vector potential a_L, the electron motion is described by a figure-eight oscillation around the center drifting along the laser beam axis with a constant velocity $v_d = [a_L^2/(4+a_L^2)]c$. The current density induced by these drifting electrons is given by

$$j = e n_e v_d = j_c \left(\frac{n_e}{n_c}\right)\left(\frac{v_d}{c}\right) = j_c \left(\frac{n_e}{n_c}\right)\left(\frac{a_L^2}{4+a_L^2}\right),$$

where n_e and n_c are the electron density and the critical electron density, respectively, and $j_c = e n_c c$.

Generally the current level of the electron beams propagating in plasmas is limited by the self-generated magnetic field below the value of the Alfven current I_A given by [14]

$$I_A = 17\beta_0\gamma_0 \quad [\text{kA}],$$

where β_0 is the forward electron velocity normalized by c and the relativistic mass factor γ_0 is given by $(1-\beta_0^2)^{-1/2}$. Using approximate expressions

$$\beta_0 = \left\langle \frac{v_z}{c} \right\rangle = \left(\frac{v_d}{c}\right) = a_L^2/(4+a_L^2), \quad \gamma_0 = \langle \gamma \rangle = 1 + \frac{a_L^2}{4},$$

the Alfven current is simply given by

$$I_A\ [\text{kA}] = \left(\frac{17}{4}\right) a_L^2.$$

From the relation $I_A = \pi r_c^2 j$, we obtain the critical electron-beam radius

$$r_c\ [\mu\text{m}] = 0.34\lambda_L \sqrt{\left(1+\frac{a_L^2}{4}\right) \Big/ \left(\frac{n_e}{n_c}\right)},$$

where λ_L is the wavelength of the laser in μm. We have assumed that the laser beam radius r_L is sufficiently large compared to r_c. When the laser pulse (the electron beam) is long enough compared to the beam radius, the magnetic field at the current beam boundary ($r = r_c$) is approximately given by

$$B_c\,[\mathrm{MG}] = \frac{25}{\lambda_L} a_L^2 \sqrt{\left(\frac{n_e}{n_c}\right) \Big/ \left(1 + \frac{a_L^2}{4}\right)}.$$

This maximum strength of the self-magntic field is proportional to the laser intensity, if the laser is not strongly relativistic ($a_L^2 \ll 4$).

Now we calculate the spatial profiles of the magnetic field induced by the laser-driven electron beam pulses of finite length. First we consider an infinitely thin current. From the Biot-Savart law, the azimuthal magnetic field induced at point $P(r,z)$ by a uniform current filament of length s located along the z axis $[-s/2, +s/2]$ is given analytically by

$$B(r,z) = \left(\frac{B_0}{2}\right)\left(\frac{r_0}{r}\right) \times \left[\frac{z + \frac{s}{2}}{\sqrt{r^2 + \left(z + \frac{s}{2}\right)^2}} - \frac{z - \frac{s}{2}}{\sqrt{r^2 + \left(z - \frac{s}{2}\right)^2}} \right],$$

where $B_0 = \mu_0 I / 2\pi r_0$ is the magnetic field strength at a reference radius r_0.

In the limit of an infinitely long current filament we simply find

$$B(r,z) \rightarrow B_0\left(\frac{r_0}{r}\right) = \frac{\mu_0 I}{2\pi r} \quad (s \rightarrow \infty).$$

For $s \ll |r|$ and $s \ll |z|$,

$$B(r,z) = \left(\frac{B_0}{2}\right)\left(\frac{r_0}{r}\right)\frac{s r^2}{(r^2 + z^2)^{3/2}}$$

and further for $z^2 \ll r^2$,

$$B(r) = \left(\frac{B_0}{2}\right)\left(\frac{r_0}{r}\right)\left(\frac{s}{r}\right).$$

Next we derive the self-induced magnetic field, taking into account of the finite radius and length of the beam current. We consider the traveling current element whose radius and length are equal to the critical radius $r_0 = r_c$ and the laser pulse length $2s = c\tau_L$, respectively. Then it follows that $B_0 = B_c$. Further we assume the uniform current distribution within the beam radius r_0 and, therefore, we approximately write the profile of the magnetic field as follows.

(1) Within the current channel, $r \leqslant r_0$,

$$B(r,z) = \left(\frac{r_0}{r}\right) B(r_0, z) = \left(\frac{B_0}{2}\right)\left(\frac{r}{r_0}\right) \left[\frac{z + \frac{s}{2}}{\sqrt{r_0^2 + \left(z + \frac{s}{2}\right)^2}} - \frac{z - \frac{s}{2}}{\sqrt{r_0^2 + \left(z - \frac{s}{2}\right)^2}} \right].$$

(2) Outside the current channel, $r \geqslant r_0$,

$$B(r,z) = \left(\frac{B_0}{2}\right)\left(\frac{r_0}{r}\right) \times \left[\frac{z + \frac{s}{2}}{\sqrt{r^2 + \left(z + \frac{s}{2}\right)^2}} - \frac{z - \frac{s}{2}}{\sqrt{r^2 + \left(z - \frac{s}{2}\right)^2}} \right].$$

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