Phase diffusion and random walk interpretation of electromagnetic scattering

Hasan Bahcivan,* David L. Hysell,† and Michael C. Kelley
School of Electrical and Computer Engineering, Cornell University, Ithaca, New York 14850, USA
(Received 4 February 2003; revised manuscript received 3 April 2003; published 1 August 2003)

The relaxation behavior of phase observables for different particle diffusion models is found to establish a ground for radioscience interpretations of coherent backscatter spectra. The characteristic function for a random walk process at twice the incident radiation wave number is associated with the complex amplitude of the scattered field from a medium containing refractive index fluctuations. The phase relaxation function can be connected to the evolution of the characteristic function and may describe the average regression of the scattered field from a spontaneous fluctuation undergoing turbulent mixing. This connection holds when we assume that the stochastic description of particle movements based on a diffusion model is valid. The phase relaxation function, when identified as the generalized susceptibility function of the fluctuation dissipation theorem, is related to the spectral density of the scattered field from steady-state fluctuations.

DOI: 10.1103/PhysRevE.68.021101 PACS number(s): 05.40.Fb, 47.27.Qb, 41.20.Jb

I. INTRODUCTION

Coherent scattering of radiowaves from the ionosphere, middle, and lower atmosphere has been attributed to refractive index fluctuations associated with turbulence. According to the theory developed by Villars and Weisskopf [1], pressure fluctuations in the middle and lower atmosphere produce corresponding density fluctuations, which in turn produce fluctuations in refractive index and hence, a scattered field. Electron density fluctuations are the source of scattered fields from the ionosphere [2].

In this study, we generalize a theory relating the spectra of the scattered fields to the stochastic motions of individual particles in a statistical flow. We analyze the spread of the particles due to diffusion and obtain characteristic functions corresponding to evolving density distributions as a consequence of this diffusional spread. Diffusional spread of a quantity can go on indefinitely if the space in which it takes place is unrestricted. However, the relevant space for electromagnetic scattering is what we term as *phase space* and it is finite. The term ''phase space'' is used here as the reduced space representing the behavior of the periodic phase variable. We want to emphasize here that it is something very different from the one typically used in mechanics and in statistical physics. When observed at a fixed wave number in k space, the motion of particles in a scattering medium may be described by a process termed *phase diffusion*, which can be visualized as a mixing event at a fixed scale. In a closed system or in the absence of energy input from lower wave numbers in a turbulent medium, all the energy will eventually be dissipated due to molecular viscosity or diffusion.

A detailed investigation on the relaxation of characteristic function due to phase diffusion on the basis of several stochastic diffusion models has only recently been made available by Talkner [3]. We realize that this diffusional relaxation behavior of characteristic functions can be connected to the attenuation of scattered field from subsiding turbulent fluctuations. What we exactly mean by subsiding turbulence will be discussed in the following sections. This connection will be our main point in the present paper to obtain a theoretically sound formulation of the time evolution, i.e., spectral density, of the scattered field from steady-state turbulent mediums. We should emphasize here that we are trying to understand the spectral characteristics of backscatter from statistical flows. While the term ''turbulence'' implies different things in different contexts, we will find it convenient to use on several occasions to provide better insight into concepts here.

This paper is organized as follows. In Sec. II, we develop simple diffusion and scattering concepts to describe the spectra of the scattered fields. We start by a definition of characteristic function and explain the basis of how it can be related to the scattered field amplitude. Next, we describe the phase relaxation function and how it can be identified as the generalized susceptibility of the fluctuation-dissipation theorem. Then, a simple relation is obtained to connect the spectra of the scattered field from the steady-state fluctuations to the phase relaxation function. In Sec. III, the Levy random walk process is used to provide an example in which we obtain the phase relaxation function and the scattered field spectra.

II. DIFFUSION AND SCATTERED FIELD EVOLUTION

Diffusion can be viewed as an accumulation of increments, as in a random walk. In the following analysis, we assume that the configuration states or distributions of particles in space are equivalent. That is, the individual stochastic motions of particles undergoing diffusion are assumed to be independent of ensemble space-velocity configuration. The diffusion process will be stationary in that each particle will perform the characteristic random walk scheme at all times and wherever it goes. With this conception of motion, the governing algebraic equations will be in effect at all times.

A. Equivalency of the characteristic function to the complex amplitude of the scattered field

The characteristic function corresponding to the electron density distribution $\rho(x,t)$ at time t is given by

*Electronic address: hb53@ece.cornell.edu
†Also at Earth and Atmospheric Sciences, Cornell University.

$$\Phi(k,t)=\int_{-\infty}^{\infty}dxe^{ikx}\rho(x,t), \tag{1}$$

where x is the *extended* space variable along the radar line of sight. If the observation point is far from the whole scattering system, the characteristic function $\Phi(k,t)$ yields the complex amplitude of the scattered field for an incident electromagnetic wave with wave number $k/2$ at time t. The halving of the wave number comes from the Bragg condition. This simple relation is the most important for the present paper.

The above definition of the characteristic function involves $\rho(x,t)$, which is a probability density function, not an instantaneous electron density or a refractive index fluctuation. Even if $\rho(x,t)$ is uniform, yet there will be backscatter from fluctuations that are not represented by $\rho(x,t)$. For this reason, it is not appropriate to say that the characteristic function directly yields the scattered field amplitude. However, given certain assumptions, we can think of the random motion of the scatterers in terms of the time evolution of a configuration of pseudomaterial (as a configuration corresponding to a spontaneous fluctuation) which starts out concentrated at a point and then diffuses outward. Only in this regard, $\rho(x,t)$ can be understood as the instantaneous electron density distribution, which is an average outcome of all possible traces of diffusion from a configuration state. It will be argued in the following sections that the spectra of the scattered field, given certain assumptions, can be related to the phase relaxation of the characteristic function at half the radiation wave number.

1. Phase-space representation of density distribution

The calculation of the scattered field from an electron involves only the phase. Hence, relocation of this electron in steps of half a wavelength along the radar line of sight should make no difference. In view of the foregoing considerations, let us relocate all electrons in steps of half a wavelength to points nearest to the origin on the x axis. This operation is nearly equivalent to taking the half wavelength *modulus* of particle positions along the radar line of sight. Then, $\rho(x,t)=0$ for $x<-\lambda/4$ and $x>\lambda/4$ and particles are distributed in the phase range $-\pi\leqslant\theta<\pi$. Hereinafter, $\rho(x,t)$ will be termed *phase-space density distribution*, while we will sometimes call it *phase-space diagram*. Considering that the dynamics of particles does not change with the above relocation of particles, extended-space and phase-space density distributions are equivalent in that the corresponding characteristic functions or the scattered fields will be identical. That is, *for scattering calculations, every instantaneous electron density distribution in the extended space can be represented by a compact electron blob on a line a half wavelength long, as long as the phase of each electron is preserved*. This point will be useful in the last section discussing Levy processes, where we model initial electron density distributions as small blobs expanding via diffusion.

2. Subsiding turbulence

We now consider a refractive index medium initially in equilibrium at some time in the distant past, which is brought into steady-state inertial range turbulence by application of an appropriate set of thermodynamics forces. In this steady-state, a constant turbulent energy flux ϵ per unit mass streams across scales down to dissipation ranges. For example, considering Kolmogorov-type turbulence, the energy distribution in the inertial range within which the energy flux is constant is given by

$$E(k)=\alpha\epsilon^{2/3}k^{-5/3}, \tag{2}$$

where the dimensionless coefficient α is about 1.5 as revealed by measurements. Once the volume containing this turbulent system is closed so that no continuous paths exist for the transport of mass or energy, the system will no longer be able to support steady-state turbulent motions due to irreversible processes such as diffusion and viscous dissipation. Supposing that no reverse turbulent energy flux occurs in k space and that no energy exists in wave numbers smaller than k once the system is closed, the system will be called to be in a state of *subsiding turbulence* with energy flux cutoff at k. In the following sections, subsiding turbulence will be used to described the state of a medium displaying a linear response to a spontaneous fluctuation as in the fluctuation-dissipation theorem [4,5].

We need to emphasize here that a turbulent medium or any statistical flow may not have any refractive index fluctuations, even in the presence of velocity fluctuations. However, turbulent velocity fluctuations tend to destroy or mix any variation in the refractive index. Therefore, the evolution of the scattered field $\Phi(k,t)$ at a fixed wave number k is governed by the characteristics of velocity fluctuations. Our aim here is to represent the effect of the velocity fluctuations using well-formulated random walk models. Again, the reader should keep in mind that our discussion is general in that the ideas here may be relevant to not only inertial range turbulent fields but also to any statistical flow.

3. Phase diffusion and phase relaxation

Let us think of the dynamics of a flow in steady-state inertial range turbulence in extended space. From the density distribution in the extended space, we obtain the phase-space density distribution for a wave number k in the inertial range. At the cessation of energy input from larger scales the turbulence subsides and the diffusive flux due to both turbulent and thermal motions drives the system towards a uniform phase-space density distribution. The phase of each particle undergoes diffusion only for a short time scale and since, by our definition, a phase can take values only in the range $-\pi\leqslant\theta<\pi$ or in the interval of half a wavelength around the origin, an ever increasing spread of the variance of x is impossible. Therefore, phase diffusion, as an irreversible thermodynamic process, gives rise to *phase relaxation* of the scattered field, a phase observable, in subsiding turbulence. Once the system comes to equilibrium, the phase-space density distribution will be uniform and there will be no coherent scattering due to absence of fluctuations. Meanwhile, some examples of periodic or quasiperiodic motions are displayed by atmospheric and ionospheric waves where individual particles undergo a coherent and periodic motion. In this case, the phase may not tend to be distributed according to the equipartition in the phase-space $[-\pi, \pi)$.

4. Phase relaxation function

We have discussed that the characteristic function at half the electromagnetic wave number yields the measured scattered field. In the absence of energy input from larger scales, the characteristic function for an initially compact electron population with a given reference distribution is in a state of subsiding turbulence. The scattering field relaxes as the scatterers undergo phase diffusion. The corresponding relaxation curve for the value of the characteristic function at a *fixed* wave number k will be termed *phase relaxation function*, $m(k,\tau)$ for $\tau>0$. An initial nonuniform phase-space distribution for a compact electron blob approaches the uniform equilibrium distribution due to phase diffusion and absorption of energy in the dissipation scales. Phase relaxation curves for different random walk models have recently been studied [3] and will be reviewed on the basis of turbulent scattering in the last section.

B. Relation between spectral density of fluctuations and phase relaxation function

We consider a medium that displays steady-state turbulent fluctuations. The system is thought to be displaying a linear response to a set of thermodynamic forces.

1. Correlation of fluctuations in time

The complex scattered field quantity of a steady-state turbulent medium, $\Phi(k,t)$, will undergo variations in time, fluctuating about its zero mean value. There is some correlation between the values of $\Phi(k,t)$ at different instants. We can characterize the time correlation by the mean value of the product

$$c(k,t,t')=\langle \Phi(k,t)\Phi^*(k,t')\rangle. \tag{3}$$

The statistical averaging above is equivalent to time averaging. Thus, the correlation function above depends only on the time difference $\tau=t-t'$. Therefore, it can also be written as

$$c(k,\tau)=\langle \Phi(k,0)\Phi^*(k,\tau)\rangle. \tag{4}$$

Because of obvious symmetry in Eq. (3), $c(k,\tau)$ is an even function,

$$c(k,\tau)=c(k,-\tau). \tag{5}$$

Partial equilibrium is defined as the state in which the relaxation time for the establishment of partial equilibrium for a given value of $\Phi(k,t)$ is assumed to be much less than that required to reach the equilibrium value of $\Phi(k,t)$ itself. By choosing a specific value for $\Phi(k,t)$, a definite state of partial equilibrium can be characterized. The fluctuations of this type show quasistationary behavior and have a correlation function [6]

$$c(k,\tau)=\langle |\Phi(k)|^2\rangle e^{-\lambda|\tau|}. \tag{6}$$

We do not require that the scattered field from a turbulent field be quasistationary. It is not possible to derive a general formula for the correlation function or spectral density of arbitrary fluctuations analogous to the above formulation for the quasistationary fluctuations. However, it is possible to relate the properties of fluctuations to quantities describing the behavior of the system perturbed by external forces.

In the presence of an externally perturbing generalized force of time $f(t)$, the fluctuation $\Phi(k,t)$ can be related to this force as [4]

$$\Phi(k,t)=\int_0^\infty \alpha(t)f(t-\tau)d\tau. \tag{7}$$

The above relation can also be expressed in terms of the Fourier components of the force and the fluctuation

$$\Phi(k,\omega)=\alpha(\omega)f_\omega(\omega), \tag{8}$$

where $\alpha(\omega)$ is the generalized susceptibility [4,6] and is taken to describe the diffusional characteristics of the system

$$\alpha(\omega)=\int_0^\infty \alpha(t)e^{i\omega t}dt. \tag{9}$$

If this function is specified, the behavior of the system under a given perturbation is completely determined.

2. Fluctuation-dissipation theorem

The classical limit of the fluctuation-dissipation theorem relates the spectrum density of the fluctuations to the imaginary part of the generalized susceptibility function as [4]

$$c(k,\omega)=\int_{-\infty}^\infty c(k,\tau)e^{i\omega\tau}d\tau=\frac{2T}{\omega}\alpha''(\omega). \tag{10}$$

T in the above equation is related to the mean square of the fluctuation by

$$\langle |\Phi(k)|^2\rangle=\frac{2T}{\pi}\int_0^\infty \frac{\alpha''(\omega)}{\omega}d\omega. \tag{11}$$

The above formulas describe the fluctuations under monochromatic perturbation f at frequency ω. These formulas can also be viewed as the equation for fluctuations of the scattered field $\Phi(k,t)$ in an equilibrium closed system under the action of random force f. It should be emphasized that the formulation of the above thermodynamic fluctuation theory is valid for fluctuations of arbitrary size [4,6]. The absence of restrictions on the permissible values of the scattered field amplitude allows us to apply the fluctuation-dissipation theorem to weak incoherent or Thompson scattering as well as strong coherent scattering.

Consider first a one-dimensional system in equilibrium having a uniform phase-space density distribution $\rho(k,x)$. The accompanying scattering field vanishes due to the homogeneity of the scattering medium. Now, there exists a one-to-one correspondence between the decay of a displacement from the equilibrium distribution and the decay of the corresponding scattering field. The equilibrium fluctuation dissipation theorem can be regarded as a consequence of this one-to-one correspondence. In the absence of reverse turbulent energy flux, a system in subsiding turbulence and the corresponding scattering field can be considered to be a response of the system to a spontaneous fluctuation. In this regard, the generalized susceptibility function $\alpha(\omega)$ will be treated as the average regression of a spontaneous fluctuation.

In order to provide further insight into the diffusional regression of the scattered field, we consider N discrete scatterers and obtain the complex amplitude of the scattered field by

$$\Phi(k,t)=\sum_{j}^{N} e^{ikx_j(t)}, \tag{12}$$

where x_j is the distance of jth scatterer. After some delay τ, the position of each particle is advanced by δx_j such that the scattered field after such delay can be formulated as

$$\Phi(k,t+\tau)=\sum_{j}^{N} e^{ikx_j(t)}e^{ik\delta x_j(\tau)}. \tag{13}$$

We assume that the statistical properties of increments δx_j can be described by a random walk process. In the absence of any external thermodynamic forces such as energy flux from lower wave numbers, the diffusion described by a particular random walk scheme will clearly drive the scattered field toward zero. We proceed to obtain the correlation function by performing statistical averaging in time,

$$\begin{aligned}
c(k,\tau)&=\langle\Phi(k,t)\Phi^*(k,t+\tau)\rangle\\
&=\left\langle \sum_{j}^{N} e^{ikx_j(t)}\sum_{m}^{N} e^{-ikx_m(t)}e^{-ik\delta x_m(\tau)}\right\rangle\\
&=\left\langle \sum_{j}^{N} e^{ikx_j(t)}\sum_{m}^{N} e^{-ikx_m(t)}\sum_{n}^{N} \frac{e^{-ik\delta x_n(\tau)}}{N}\right\rangle\\
&=\left\langle \sum_{j}^{N} e^{ikx_j(t)}\sum_{m}^{N} e^{-ikx_m(t)}\right\rangle\left\langle \sum_{n}^{N} \frac{e^{-ik\delta x_n(\tau)}}{N}\right\rangle\\
&=\langle|\Phi(k)|^2\rangle\left\langle \sum_{n}^{N} \frac{e^{-ik\delta x_n(\tau)}}{N}\right\rangle.
\end{aligned} \tag{14}$$

The above derivation assumes uncorrelated positions and velocities. The second multiplier term on the right-hand side in the last equation above is what we previously described as the phase relaxation function,

$$m(k,\tau)=\left\langle \sum_{n}^{N} \frac{e^{-ik\delta x_n(\tau)}}{N}\right\rangle, \tag{15}$$

where the averaging is over all possible traces of $\delta x(\tau)$. We now take the Fourier transform of the phase relaxation function and express it in terms of real and imaginary parts,

$$m(k,\omega)=m'(k,\omega)+im''(k,\omega)=\int_0^\infty m(k,\tau)e^{i\omega\tau}d\tau. \tag{16}$$

Representing a turbulent medium as a dissipative system, we define its generalized susceptibility function by the Fourier transform of the phase relaxation function,

$$\alpha(k,\omega)=m(k,\omega). \tag{17}$$

We assume that the average shape of a spontaneous fluctuation pulse is identical with the observed shape of an irreversible decay of the scattered field from subsiding turbulence toward its zero equilibrium value. In this regard, phase relaxation function $m(k,\tau)$ can be understood as the generalized susceptance of the system.

For stationary scattered fields and invoking the principle of microscopic reversibility [7,8], we obtain the spectral density of the scattered field for an incident radiation at the wave number $k/2$ as [4]

$$c(k,\omega)=(2T/\omega)m''(k,\omega). \tag{18}$$

The proof of the above relation employs certain theories of random variables, which can be found in Ref. [4].

C. On the role of initial phase-space diagrams

It is evident from the description of the phase-space density distribution that if there is only a linear cluster of electrons and it spreads along its line, the phase-space diagram will want to expand along the x axis. But it will fail to do so since any electron exiting at one end, e.g., $x=-\lambda/4$, will enter at the opposite end, e.g., $x=+\lambda/4$. Since the total number of electrons will be preserved, the area under it must be a constant. The final equilibrium state can be expressed by a straight line. Suppose that an equilibrium phase-space diagram is perturbed such that an initial displacement $\rho'(k,x)$ from equilibrium occurs, starting at time zero:

$$\rho(k,x,\tau)=h+\rho'(k,x,\tau), \tag{19}$$

where h is the height of the equilibrium diagram. The scattered field is given by

$$\Phi(k,t)=\int_{-\infty}^{\infty}dx e^{ix}\rho(k,x,t)=\int_{-\infty}^{\infty}dx e^{ix}\rho'(k,x,t). \tag{20}$$

Now suppose that diffusion takes place. Can we find the changes to the scattered field normalized by its initial value,

$$\frac{\Phi(k,\tau)}{\Phi(k,0)}, \tag{21}$$

for $\tau > 0$, if we know the precise stochastic behavior of the scatterers? Not from this information alone, since diffusion is also controlled by the gradients of a phase-space diagram. Therefore, we need a description of $\rho'(k,x,0)$ to find a stochastic solution for the relaxation of the scattered field from its perturbed value. In this instance, we have no information about what $\rho'(k,x,0)$, on average, looks like for a turbulent medium. Therefore, in the example in the following section, we will assume special forms for the initial displacement of the phase-space diagram and will continue our discussion on the basis of Levy random walk processes.

III. A STOCHASTIC DIFFUSION MODEL AND CORRESPONDING SPECTRAL DENSITY

Phase diffusion and relaxation for several random walk models has recently been studied by Talkner [3], including processes with independent increments, self-similar processes, and continuous time random walks. In this section, we will discuss only Levy processes which might establish a good example for the use of phase diffusion in the interpretation of steady-state turbulent or nonturbulent scattering. Again, the fundamental idea of this paper is the relation between the spectral density of the scattered field and the phase relaxation function. We are in no way committing ourselves to turbulent or nonturbulent flow, since the discussion here is general and may be relevant to any statistical flow. Limitations of time and competence preclude a detailed discussion of alternative random walk schemes. Although the discussion below on Levy processes closely follows that of Ref. [3], we find it extremely useful to include it in order to provide a self-consistent explanation of the ideas here.

Levy processes are a group of self-similar processes with a space and time evolution of a reference distribution in a scaling fashion. We do not know what kind of reference phase-space density distribution would be appropriate, if it matters at all, for our analysis (see section on initial phase-space distributions). Once an initial distribution is chosen, the evolution of characteristic function depends on how this initial distribution spreads in time. Since Levy processes are self-similar, the probability density function of this process varies in time as

$$\rho(x,\lambda t) = \lambda^{-\alpha/2}\rho(\lambda^{-\alpha/2}x,t), \tag{22}$$

where λ is redefined to be the scaling factor. Then, starting with a reference density function $\rho_0(x,t_0)$ at time t_0, the density function at a later time t can be obtained by setting $\lambda = t/t_0$,

$$\rho(x,t) = \rho_0\left[\left(\frac{t_0}{t}\right)^{\alpha/2}x\right]\left(\frac{t_0}{t}\right)^{\alpha/2}. \tag{23}$$

Since the characteristic function, $\Phi(k,t)$, is the Fourier transform of $\rho(x,t)$, rescaling the x variant of the density function corresponds to rescaling the k domain in $\Phi(k,t)$. Let $\Phi_0(k,t_0)$ be the characteristic function of reference distribution ρ_0 at time t_0. Then, the phase relaxation function of the scattered field from a medium in a state of subsiding turbulence can be obtained from the reference characteristic function as

$$m(k,\tau) = \Phi_0\left[k\left(\frac{\tau}{t_0}\right)^{\alpha/2}\right]. \tag{24}$$

For example, we take the case of symmetric Levy distributions with reference characteristic function

$$\Phi_0(k) = \exp[-\sigma_0^{\gamma}k^{\gamma}], \tag{25}$$

where $\sigma_0 > 0$ is a reference scale and the exponent γ is restricted to $0 < \gamma \leq 2$. Then, the phase relaxation function is obtained from Eq. (24) as

$$m(k,\tau) = \exp\left[-\sigma_0^{\gamma}k^{\gamma}\left(\frac{\tau}{t_0}\right)^{\gamma\alpha}\right]. \tag{26}$$

A requirement for a stationary process is that $\gamma\alpha \leq 2$ for $0 < \gamma < 1$ and $\alpha \leq 1$ for $1 \leq \gamma \leq 2$ [3]. In the above equation, the choice for t_0 is not clear but by no means determines the exponent of the time delay and can be incorporated into the constant σ_0.

The generalized susceptance can now be obtained from the phase relaxation function by a Fourier transform operation, as in Eq. (10). However, an analytical form of $m(k,\omega)$ is not easy to obtain except in a case where $\gamma\alpha = 1$. Given that the stochastic motion of particles in a statistical flow can be described as a Levy process with $\gamma\alpha = 1$, the generalized susceptance can be obtained using Eq. (10) as

$$m(k,\omega) = \frac{1}{k - i\omega}, \tag{27}$$

where the parameter κ is defined as

$$\kappa = \frac{\sigma_0^{\gamma}k^{\gamma}}{t_0}. \tag{28}$$

Finally, we can obtain the spectral density of the scattered field fluctuations in light of the fluctuation-dissipation theorem using Eq. (19)

$$c(k,\omega) = \frac{2T}{\kappa^2 + \omega^2}. \tag{29}$$

The above form of spectral density is Lorentzian and reminiscent of normal diffusion processes. While what matters most is the shape of the spectrum which is controlled by κ, T on the right-hand side can be calculated by measuring the mean square of fluctuations and solving Eq. (12) for T.

The phase relaxation function may take a variety of forms from algebraic decay to exponential and to faster than exponential depending on the type of random walk process using [3]. Accordingly, we anticipate a wide range of spectrum densities. A Gaussian form of spectral density that is commonly used to interpret turbulent Doppler spectrum from atmospheric processes [9] would establish only a particular case. We think that this wide class of random walk processes and understanding what kind of statistical flow they represent deserve a separate study.

IV. CONCLUSION

In this paper, we have investigated how the stochastic motion of one scatterer can be related to the scattered field spectra from a medium containing many such scatterers. The scatterers in the medium can be collapsed into a small compact volume, i.e., phase space, when the phase of each scatterer is preserved. In this regard, we can release a collection of scatterers in several reference configurations and calculate the evolution of the scattered field for different random walk schemes. The average regression of the scattered field from a subsiding turbulent flow is described by the phase relaxation function. For arbitrary perturbations to the equilibrium phase-space diagram, the phase relaxation function is related to the spectral density of the scattered field from steady-state turbulent fluctuations under consideration of the fluctuation-dissipation theorem. For the Levy process example we studied, and for other random walk models recently studied, the phase relaxation function displays a wide range of behavior from algebraic decay to faster than exponential. While we have obtained an analytical form of spectral density only for a particular case of the Levy process, we expect the spectral density to display a wide range of behavior. The spectral density is not necessarily Gaussian, although it is often assumed to be this way by the research community.

In this paper we have formulated the connection between the spectral density of the scattered field from steady-state statistical flows and the phase relaxation function. We believe this connection establishes a sound starting point for interpreting coherent backscatter spectra from refractive index fluctuations in a scattering medium. The phase relaxation approach on the basis of several diffusion models may allow us to obtain a *dynamical* picture of particle processes which would facilitate the study of other such dynamical processes, including shear instabilities and mixing in turbulent flows. The ideas here can ultimately serve as the foundation for more comprehensive ones connecting the time signatures of scattered fields to the physics of atmospheric processes.

Research at Cornell University was supported in part by the Office of Naval Research under grant N00014-00-1-0658. H.B. thanks Jorge Chau for discussions on the interpretation of coherent backscatter.

[1] F. Villars and V. F. Weisskopf, Phys. Rev. **94**, 2 (1953); **94**, 232 (1953).
[2] W. Gordon, Proc. IRE **46**, 1823 (1958).
[3] P. Talkner, Phys. Rev. E **64**, 061101 (2001); **64**, 1 (2001).
[4] H. B. Callen and R. F. Greene, Phys. Rev. **86**, 702 (1952).
[5] W. Bernard, Phys. Rev. **128**, 1 (1962); **128**, 421 (1962).
[6] L. D. Landau and E. M. Lifshitz, *Statistical Physics*, 3rd ed. (Nauka, Moscow, 1976), Pt. 1, p. 377.
[7] L. Onsager, Phys. Rev. **37**, 405 (1931).
[8] L. Onsager, Phys. Rev. **38**, 2265 (1931).
[9] D. L. Hysell and J. L. Chau (unpublished).