Group delay, stored energy, and the tunneling of evanescent electromagnetic waves

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A general relation between group delay and stored electric and magnetic energies is presented for two-port networks. It generalizes the results of Dicke to situations where electric and magnetic stored energies differ. The general result is applied to tunneling evanescent waves in cutoff waveguides. It is shown explicitly that the group delay is equal to the dwell time plus a self-interference delay which is proportional to the net reactive stored energy. The Hartman effect, the saturation of group delay with length in cutoff waveguides, is explained on the basis of saturation of stored energy with guide length. It is pointed out that the anomalously short delays observed in tunneling experiments are not propagation delays and should not be associated with superluminal velocities. A strictly luminal energy velocity is derived and a method is suggested for the measurement of dwell time and energy velocity.

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I. INTRODUCTION

It is surprising that a problem as classical as the tunneling of a pulse of electromagnetic energy through an obstacle in a waveguide has become the subject of ongoing debate in modern physics [1–3]. Experiments by Enders and Nimtz appeared to show electromagnetic pulses traveling with group velocities in excess of the vacuum speed of light as they tunneled through a constriction in a waveguide [4]. This led to notions [5] and disputations [6–8] regarding the possibility of superluminal signaling and information transfer. Other experiments with photonic band-gap structures also showed apparent superluminal barrier traversal [9,10]. It was found that the traversal time of a pulse across a thick barrier becomes independent of the barrier width [4,10], a phenomenon called the Hartman effect [11]. These observations as well as the apparent agreement with theoretical predictions have led to a widespread belief that tunneling electromagnetic waves travel with group velocities that exceed c and can even become infinite [8,12]. Various arguments, mostly based on pulse reshaping, are then advanced to explain why such anomalous group velocities do not violate causality. It is widely believed that the entire transmitted pulse comes from just the small early tail of the much larger incident pulse [6,12,13]. However, many have also expressed discomfort with the current explanations. Landauer states that ‘‘at a fundamental theoretical level the easy explanation suggested is unsatisfying.’’ [3] With regard to the whole pulse being reconstructed by the early tail, F.E. Low speaks for many when he says ‘‘how and why it happens is not understood, at least by me.’’ [14]

A fundamental difficulty with these superluminal group velocities is that the wave vector of a tunneling wave is an imaginary quantity. The group velocity, defined as the derivative of angular frequency with respect to wave number, is likewise imaginary and devoid of physical meaning. Because the wave vector is purely imaginary for an evanescent wave, this also means that the mechanism advanced by Japha and Kurizki [13] as a universal explanation of ‘‘superluminal’’ propagation cannot possibly apply to tunneling evanescent waves. The work cited uses a summation of propagating waves with real wave vectors and appeals to destructive interference between these propagating waves. While this may provide a mechanism in regions of allowed propagation, it has no relevance for regions of true evanescence (e.g., for a waveguide below cutoff) where there are no propagating modes. Of course the evanescent field does enter the waveguide by means of the high-frequency *propagating* components at the pulse front at the time of turn on. These components are strictly limited by the vacuum speed of light as discussed by Sommerfeld and Brillouin [15]. Once the transient front traverses the guide there is no longer propagation, only evanescence: exponential decay of storage fields that merely stand and wave, akin to breathers [16]. Operationally, superluminal velocities have been inferred by measuring the delay time between the appearance of a pulse peak at the input and a pulse peak at the output of a barrier. We have recently shown, however, that the peak of a tunneling pulse does not even enter the barrier and hence the input and output pulse peaks are not related by causal propagation [16]. The question then is what is the appropriate time scale with which to characterize the tunneling of electromagnetic waves and what is the meaning of the observed delay? The influential review article by Hauge and Støvneng [17] lists seven tunneling time definitions of which two, the group delay (defined by the frequency derivative of a transmission phase shift) and the dwell time (defined by an integrated probability or stored energy) are considered ‘‘well established.’’ They do remain contentious, however, and even the relation between these two time scales is controversial, with some denying any connection between them [18].

In recent papers we have suggested an explanation of these ‘‘faster-than-light’’ phenomena as the modulation of the stored energy in a cavity by the slow envelope of the input pulse [16,19]. We have used the concept of energy storage and release to resolve the paradox of the Hartman effect [19]. In that work, we proved the equality of the group delay and the dwell time for a situation where the barrier was surrounded by free space so that the approach to the barrier involved nondispersive propagation. For the problem of a

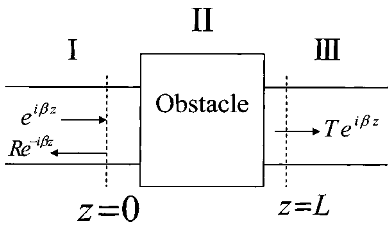

FIG. 1. Schematic of the process of scattering by an obstacle in a waveguide.

constricted waveguide, which is more analogous to quantum tunneling, the approach to the barrier occurs in a dispersive waveguide and that can affect the overall delay. In this paper, we derive an explicit relation between group delay and the stored electric and magnetic energies in a waveguide with a discontinuity. This is an important result that generalizes relations obtained by Dicke [20] and Carlin [21] to situations in which the stored electric and magnetic energies differ and where the waves beyond the scattering region are dispersive. It demonstrates the importance of the net reactive stored energy in situations that involve evanescent fields. We show that this expression for the group delay in terms of stored energy yields the same result as that based on the frequency derivative of the transmission phase shift. It is shown that the group delay equals the dwell time plus a self-interference delay, thus unifying these two tunneling time definitions. The Hartman effect for the undersized waveguide is explained on the basis of the saturation of stored energy with increasing length. We calculate an energy velocity, show that it is strictly subluminal for tunneling pulses, and present a recipe for its measurement and for the determination of the dwell time.

II. THE MODEL

When a pulse of electromagnetic energy encounters a discontinuity in a waveguide, it is partially reflected and partially transmitted. In the process some of the energy may end up in higher-order evanescent modes which, being nonpropagating, are confined to the vicinity of the obstacle. Far away from the obstacle, its effects can be completely described by its reflection coefficient, its impedance, or its scattering matrix [22,23]. A key simplification is that the scatterer does not alter the polarization state of the incident wave so that one may use scalar analysis. Consider a waveguide junction (a two-port network) to which is connected two uniform waveguides filled with lossless, isotropic, dispersionless, nonmagnetic material of refractive index n_1 (Fig. 1). For simplicity, we assume the waveguides support only the dominant TE_{10} mode. This is the situation of greatest practical interest [4]. The electric and magnetic fields for this mode can be written

$$\mathbf{E}(x,z,\omega)=\hat{\mathbf{y}}\sin(\gamma x)\,\psi(z), \tag{1a}$$

$$\mathbf{H_t}(x,z,\omega)=\hat{\mathbf{x}}\sin(\gamma x)\,\frac{i}{\omega\mu_0}\,\frac{d\psi}{dz}, \tag{1b}$$

$$\mathbf{H_z}(x,z,\omega)=-\hat{\mathbf{z}}\cos(\gamma x)\,\frac{i\gamma}{\omega\mu_0}\,\frac{d\psi}{dz}, \tag{1c}$$

where $\gamma=\pi/a$ is the eigenvalue of the transverse mode, a is the width of the guide, z is the propagation direction, and a harmonic time dependence of $\exp(-i\omega t)$ has been assumed. The wave function ψ satisfies the Helmholtz equation

$$\frac{d^2\psi}{dz^2}+\beta^2\psi=0, \tag{2}$$

where the propagation constant β is given by

$$\beta^2=n_1^2k_0^2-\gamma^2, \tag{3}$$

with $k_0=\omega/c$. There is a cutoff angular frequency $\omega_1=\gamma c/n_1$ for which $\beta=0$ and below which the waveguide does not support propagating modes. We will assume operation above this cutoff frequency for the waveguides I and III connected to the junction II. For now we will leave the nature of region II unspecified, beyond requiring that it be lossless. Our goal is to calculate the group delay a narrowband pulse suffers in transmission through the junction and to relate it to the energy stored.

III. RELATION BETWEEN GROUP DELAY AND STORED ENERGY

The group delay is given by the derivative of the phase of the transmitted pulse with respect to angular frequency. To obtain a relation between group delay and stored energy we use a variational theorem (also known as the energy theorem) which follows from taking frequency derivatives of the complex Maxwell's equations [23],

$$\oint_S\left[\frac{\partial\mathbf{E}}{\partial\omega}\times\mathbf{H}^*+\mathbf{E}^*\times\frac{\partial\mathbf{H}}{\partial\omega}\right]\cdot\mathbf{ds}=4i\langle U\rangle. \tag{4}$$

It relates the frequency derivatives of the electric and magnetic fields on a closed surface to the total time-average stored energy $\langle U\rangle=\langle U_m\rangle+\langle U_e\rangle$ within the volume bounded by the surface. The electric and magnetic stored energies are given by

$$\langle U_e\rangle=\frac{1}{4}\int_V\mathbf{E}\cdot\mathbf{E}^*\frac{\partial\omega\varepsilon}{\partial\omega}dv,\ \ \langle U_m\rangle=\frac{1}{4}\int_V\mathbf{H}\cdot\mathbf{H}^*\frac{\partial\omega\mu}{\partial\omega}dv, \tag{5}$$

expressions which hold for general dispersive media. The surface integration is carried out over the metal walls of the waveguide and planes $z=0$ and $z=L$ located in waveguides I and III. Only the transverse components of the fields contribute to the surface integral. On the other hand, the volume integrals that yield the total energy include both transverse and longitudinal components of the electromagnetic field.

With use of the mode fields of Eq. (1), the integrand in Eq. (4) becomes

$$\hat{\mathbf{z}}\,\frac{i}{\omega\mu_0}\left[\frac{\partial\psi}{\partial\omega}\,\frac{\partial\psi^*}{\partial z}+\frac{\psi^*}{\omega}\,\frac{\partial\psi}{\partial z}-\psi^*\,\frac{\partial^2\psi}{\partial\omega\partial z}\right]. \tag{6}$$

At the entrance surface the field consists of an incident and reflected wave

$$\psi_I = E_0(e^{i\beta z} + Re^{-i\beta z}), \tag{7}$$

while at the exit the field has only a transmitted component

$$\psi_{III} = TE_0 e^{i\beta z}. \tag{8}$$

In terms of their magnitudes and phases, we have $R = |R|e^{i\phi_r}$ and $T = |T|e^{i\phi_t}$. For lossless media the magnitudes of the reflection and transmission coefficients satisfy

$$|R|^2 + |T|^2 = 1, \tag{9}$$

while for symmetric barriers, which we will now assume, $\phi_r - (\phi_t + \beta L) = \pm \pi/2$. Evaluating the surface integrals at $z=0$ and $z=L$, we obtain

$$\frac{A|E_0|^2}{2\omega\mu_0}\left\{-2i\,\mathrm{Im}(R)\left(\frac{\beta}{\omega} - \frac{d\beta}{d\omega}\right) + 2i\beta\frac{d\phi_0}{d\omega} + 2\beta\left[|R|^2\frac{d\ln|R|}{d\omega} + |T|^2\frac{d\ln|T|}{d\omega}\right]\right\} = 4i\langle U\rangle, \tag{10}$$

where $\phi_0 = \beta L + \phi_t$ is the total phase of the transmitted wave and $A = ab$ is the cross-sectional area of the waveguide. The group delay is the frequency derivative of the phase of the transmitted wave, hence we find

$$\tau_g \equiv \frac{d\phi_0}{d\omega} = \frac{\langle U\rangle}{P_{in}} + \frac{\mathrm{Im}(R)}{\beta}\left(\frac{\beta}{\omega} - \frac{d\beta}{d\omega}\right), \tag{11}$$

where $P_{in} = \varepsilon_0|E_0|^2 Ac^2\beta/4\omega$ is the time averaged incident power and Im(R) is the imaginary part of the reflection coefficient. Because of the symmetry of the barrier, the reflection group delay $\tau_r = d\phi_r/d\omega$ is equal to the transmission group delay τ_g.

The first term in Eq. (11) is the standard result of Dicke relating group delay to the total time average stored energy in a termination [20,21]. The second term is a self-interference term arising from the overlap between incident and reflected waves in the region before the obstacle [17]. It depends not only on the reflectivity of the obstacle but also on the dispersion in the connecting waveguides. It vanishes if the waveguides are dispersionless since in that case the interference pattern (envelope) travels with the same velocity as the phase fronts and there is no extra delay. It also vanishes when the reflection coefficient is zero (at transmission resonances) or is purely real. Clearly the standard result is incomplete as it fails to account for this coherent term. Because this term is proportional to the inverse of the propagation constant, its impact will be substantial near the cutoff of the dominant mode where the propagation constant goes to zero.

The imaginary part of the reflection coefficient can be related to a difference between stored magnetic and electric energies through the complex Poynting theorem for lossless media [23]:

$$\mathrm{Im}\oint_S (\mathbf{E}\times\mathbf{H}^*)\cdot \mathbf{ds} = 4\omega(\langle U_m\rangle - \langle U_e\rangle). \tag{12}$$

Upon inserting the expressions for the fields at the entrance and exit surfaces, we find

$$\mathrm{Im}(R) = -\omega\frac{(\langle U_m\rangle - \langle U_e\rangle)}{P_{in}}. \tag{13}$$

This expression is reminiscent of the definition of the Q of a cavity resonator:

$$Q = \frac{\omega\times(\text{time-average energy stored in system})}{(\text{energy loss per second in system})}. \tag{14}$$

In Eq. (13), however, the numerator is proportional to the net reactive energy stored in the junction while the denominator is the energy per second incident on the junction. It is in essence an external Q. Indeed the barrier region forms an evanescent mode resonator with a finite decay time. With use of Eq. (13) the group delay can now be written

$$\tau_g \equiv \frac{d\phi_0}{d\omega} = \frac{\langle U\rangle}{P_{in}} + \frac{\langle U_m\rangle - \langle U_e\rangle}{P_{in}}\left(\frac{v_{p1}^0}{v_{g1}^0} - 1\right). \tag{15}$$

Here $v_{p1}^0 = \omega/\beta$ is the phase velocity in the region outside the junction while $v_{g1}^0 = d\omega/d\beta$ is likewise the group velocity outside the junction, assuming an infinite waveguide. From the dispersion relation (3) we find that these velocities are related through

$$v_{p1}^0 v_{g1}^0 = c^2/n_1^2. \tag{16}$$

For the infinite waveguide the phase and group velocities satisfy the inequality $v_{g1}^0 < c/n_1 < v_{p1}^0$.

Equation (15) relates the group delay to the total time average stored energy and the net reactive energy in cylindrical waveguides with discontinuities, such as the undersized waveguides used in tunneling experiments. It reduces to the well known expression [20,21] under resonance conditions when stored electric and magnetic energies are equal, which is the case for propagating modes, or when the junction is surrounded by a medium without dispersion, or far above cutoff in a waveguide. The junction is inductive if the magnetic energy exceeds the electric energy and is capacitive otherwise. Increasing the stored magnetic energy compared to the stored electric energy will increase the group delay. Conversely, increasing the electric energy will decrease the group delay. For the cutoff dominant mode, the magnetic energy exceeds the electric energy. (Elsewhere we consider capacitive obstacles for which the interference delay is negative.) Above cutoff, the barrier is no longer a barrier but an "accelerator" since the phase velocity is higher in this region than outside. Under these conditions the stored electric energy can exceed the stored magnetic energy and thus negative delays become possible. This should not be surprising since electromagnetic waves travel faster in lower index material.

IV. RELATION BETWEEN GROUP DELAY (PHASE TIME) AND DWELL TIME

The tunneling literature defines a dwell time as [18,24]

$$\tau_d = \langle U \rangle / P_{in}. \tag{17}$$

This is a measure of the average time spent by a wave packet in a given region of space. This dwell time is not quite the same thing as the lifetime Q first introduced by Smith. There was a term that Smith eliminated through an averaging procedure because it was oscillatory with distance. That term turns out to be the self-interference term in Eqs. (11) and (15). Because tunneling without distortion requires that a pulse be much longer than the barrier width [16], an incident pulse will always interfere with the reflected portion of itself in front of the barrier [25]. This self-interference gives rise to a pulsating reactive contribution that must be taken into account when defining an overall delay time. In sum, the group delay is seen to consist of two contributions. The first term is the usual dwell time. The second term is due to reactive stored energy, instantaneous changes in the net stored energy resulting from self-interference. We can thus write the group delay as

$$\tau_g = \tau_d + \tau_i, \tag{18}$$

where τ_d is the usual dwell time and τ_i is the self-interference delay. This simple result is contrary to the often stated view that there is no relation between the dwell time and the phase time [18]. When the reflectivity is high the incident pulse spends much of its time "dwelling" in front of the barrier as it interferes with itself during the tunneling process. If we include this excess dwell time and interpret dwell time as a difference between time spent in the presence of the barrier minus time spent in its absence, we recover the lifetime Q as originally intended by Smith. This generalized dwell time is identical to the group delay. Although the effect of self-interference in the tunneling process has been appreciated, it had been thought that its contribution to the overall group delay could not be disentangled [25,26]. Here we have succeeded in disentangling the self-interference delay from the dwell time and shown that the group delay has an unambiguous meaning in terms of energy storage.

In Ref. [19], we showed that the group delay was identical to the dwell time for a photonic band-gap structure (PBG). There we assumed that the medium surrounding the PBG was dispersionless and had the same refractive index as the average index of the PBG. This led to the vanishing of the self-interference term. In related work on PBG's, it has been recognized that there is a term associated with a difference between stored electric and magnetic energies [27]. However, those authors associate this term plus the dwell term with the inverse of an energy velocity rather than with the group delay and hence do not make the link between group delay and stored energy. A self-interference term similar to the one we have described here also appears in quantum tunneling [28]. That term happens to be the Lagrangian that leads to the Schrödinger equation. It is interesting to note that the self-interference term here is also proportional to the free-field Lagrangian for the electromagnetic field.

We now comment on some of the other tunneling time definitions often mentioned in the literature [17]. The in-plane Larmor time τ_y, originally defined for tunneling quantum particles, measures the spin precession in a plane perpendicular to an auxiliary magnetic field appended to the barrier [18]. A related approach based on Faraday rotation was suggested for electromagnetic waves [29]. The Larmor time, however, has been shown to agree with the dwell time [18] and hence provides no new information. Büttiker [18] introduced another Larmor time $\tau_B = \sqrt{\tau_y^2 + \tau_z^2}$, which takes into account the rotation of the precessing spin towards the direction z of the applied magnetic field. That time agrees with another time scale τ_{BL} introduced by Büttiker and Landauer which requires an extra modulation of the barrier [30]. These two times, however, have been shown to be more the back-reaction of a measurement than an intrinsic tunneling time scale and so we do not comment further on them [31].

V. APPLICATION TO TUNNELING EVANESCENT WAVES

The general relations between group delay, dwell time, and stored energy derived in the previous sections will now be tested by applying them to the specific problem of the tunneling of evanescent waves through a waveguide below cutoff. We will obtain the group delay by two methods: (i) by calculating the frequency derivative of the phase shift and (ii) by calculating the time average stored energy and the net reactive energy.

The barrier region (region II in Fig. 1) is taken as a waveguide of the same transverse dimensions as the connecting waveguides I and III. It contains a nondispersive material of refractive index $n_2 < n_1$. The frequency of the incident wave is chosen so that it is below the cutoff frequency of this central waveguide. In that case, it acts as an attenuator with an attenuation constant κ given by

$$\kappa^2 = \gamma^2 - n_2^2 k_0^2. \tag{19}$$

The cutoff angular frequency of the barrier is $\omega_2 = \gamma c / n_2$. This problem has previously been analyzed by Martin and Landauer [32]. However, we find it useful to repeat the analysis here since the results of [32] contain a few misprints. The field inside the barrier is a solution of the Helmholtz equation (3) with β^2 replaced by $-\kappa^2$. It can be written as a sum of forward and backward evanescent waves:

$$\psi_{II} = C e^{-\kappa z} + D e^{\kappa z}. \tag{20}$$

The constants C and D as well as the transmission and reflection coefficients are determined by requiring the continuity of ψ and $d\psi/dz$ at the boundaries $z=0$ and $z=L$. They are given by

$$C = (1 - i\beta/\kappa) e^{\kappa L} / 2g, \tag{21a}$$

$$D = (1 + i\beta/\kappa) e^{-\kappa L} / 2g, \tag{21b}$$

$$T = e^{-i\beta L} / g, \tag{22}$$

$$R = -i[(\kappa/\beta + \beta/\kappa)\sinh \kappa L]/2g, \tag{23}$$

where $g = \cosh \kappa L + i\Delta \sinh \kappa L$ and $\Delta = (\kappa/\beta - \beta/\kappa)/2$. The transmissivity $|T|^2$ of the barrier as a function of frequency is shown in Fig. 2 for several values of the barrier strength γL. The stop band is the frequency region $\omega_1 < \omega < \omega_2$, where $\omega_2 = (n_1/n_2)\omega_1$.

The phase of the transmitted wave is

$$\phi_0 = \arg(T) + \beta L = -\tan^{-1}(\Delta \tanh \kappa L). \tag{24}$$

From this we find the group delay

$$\tau_g = \frac{d\phi_0}{d\omega} = \frac{L}{v_{g1}^0}\frac{\cos^2\phi_0}{2}\left\{\frac{\omega_1^2}{\omega^2}\left(\frac{\beta}{\kappa}+\frac{\kappa}{\beta}\right)^2\frac{\tanh \kappa L}{\kappa L} - \frac{n_2^2}{n_1^2}\left(\frac{\beta^2}{\kappa^2}-1\right)\text{sech}^2\,\kappa L\right\}, \tag{25}$$

where

$$\cos^2\phi_0 = \frac{1}{1+\Delta^2\tanh^2\kappa L} \tag{26}$$

and

$$v_{g1}^0 = (c/n_1)\sqrt{1-(\omega_1/\omega)^2}. \tag{27}$$

Taking the energy approach, we find

$$\langle U\rangle = \left(\frac{\varepsilon_0|E_0|^2A}{4}\right)\frac{\cos^2\phi_0}{2}\left\{\frac{\gamma^2}{k_0^2}\left(\frac{\kappa^2+\beta^2}{\kappa^2}\right)\frac{\tanh\kappa L}{\kappa} - Ln_2^2\left(\frac{\beta^2}{\kappa^2}-1\right)\text{sech}^2\,\kappa L\right\} \tag{28}$$

for the time average stored energy and

$$\langle U_m\rangle - \langle U_e\rangle = \left(\frac{\varepsilon_0|E_0|^2A}{4}\right)\frac{\cos^2\phi_0}{2}\left(\frac{\kappa^2+\beta^2}{k_0^2}\right)\frac{\tanh\kappa L}{\kappa} \tag{29}$$

for the net reactive stored energy. Recognizing that the time averaged incident power is

$$P_{in} = \frac{\varepsilon_0|E_0|^2Ac\beta}{4k_0}, \tag{30}$$

we obtain from Eqs. (15), (28), and (29) the dwell time

$$\tau_d = \frac{L}{v_{g1}^0}\left(\frac{\cos^2\phi_0}{2}\right)\left\{\frac{\omega_1^2}{\omega^2}\left(1+\frac{\beta^2}{\kappa^2}\right)\frac{\tanh\kappa L}{\kappa L} - \frac{n_2^2}{n_1^2}\left(\frac{\beta^2}{\kappa^2}-1\right)\text{sech}^2\,\kappa L\right\} \tag{31}$$

and the self-interference time

$$\tau_i = \frac{L}{v_{g1}^0}\left(\frac{\cos^2\phi_0}{2}\right)\frac{\omega_1^2}{\omega^2}\left(1+\frac{\kappa^2}{\beta^2}\right)\frac{\tanh\kappa L}{\kappa L}. \tag{32}$$

Their sum yields the group delay which is seen to be identical to Eq. (25). It is indeed gratifying to find that the group delays calculated by two such different approaches are in complete agreement. This highlights the intimate connection between group delay and stored energy.

Figure 3 shows the group delay (solid line), the dwell time, and the interference delay. It is seen that outside the stop band the dwell time is identical to the group delay. The two differ in the stop band where the reflectivity is high. The interference delay diverges as the incident wave approaches cutoff whereas the dwell time goes to zero. This is because the incident wave spends all of its time being reflected by the barrier and nothing penetrates. The times are normalized by $\tau_0 = L/c$, the transit time of a light front across a distance L in vacuum. Normalized delays less than L/c have been called superluminal. However, these are not propagation delays and should not be associated with velocities.

VI. HARTMAN EFFECT AND ITS ORIGIN

For an opaque barrier the group delay saturates with increasing barrier length [11]. This phenomenon, termed the Hartman effect, has been taken to imply superluminal, and indeed infinite velocities of propagation for tunneling wave packets [4]. We disagree with this interpretation. The very

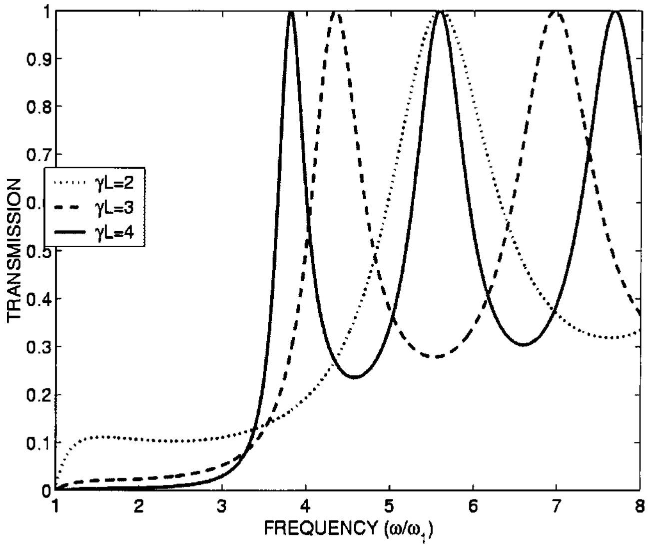

FIG. 2. Transmissivity of a section of waveguide below cutoff. Here $n_1=3, n_2=1$.

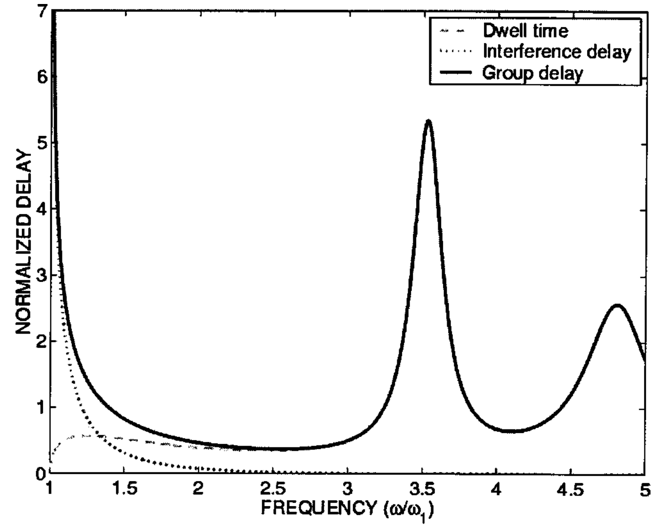

FIG. 3. (Color online) Dwell time, self-interference delay, and group delay for a barrier with $\gamma L=5, n_1=3, n_2=1$.

fact that the delay saturates with increasing barrier length means that it cannot be a propagation delay and should not be associated with a velocity of propagation. In fact, we have shown through numerical solutions of the Klein-Gordon equation that the peak of an incident narrowband pulse does not actually propagate from input to exit [16]. Input and output peaks are not necessarily related by causal propagation, as pointed out by Büttiker and Landauer [30]. We can explain the Hartman effect on the basis of saturation of stored energy with increasing length of the barrier [19]. Since the group delay is proportional to the energy stored, it saturates as the energy saturates. While we originally derived this result for a photonic band-gap structure, it should hold for any barrier in which there is an exponential decay of stored energy density with distance. Here we derive the limiting values of the stored electric and magnetic energies in the cutoff waveguide and show that they explain the saturation of group delay.

From Eqs. (28) and (29) we find

$$\lim_{L\to\infty}\langle U\rangle=\frac{U_0\beta^2\gamma^2}{\kappa(\kappa^2+\beta^2)k_0^2}, \tag{33a}$$

$$\lim_{L\to\infty}(\langle U_m\rangle-\langle U_e\rangle)=\frac{U_0\beta^2\kappa^2}{\kappa(\kappa^2+\beta^2)k_0^2}, \tag{33b}$$

where $U_0=\varepsilon_0 A|E_0|^2/4$. Using these saturated stored energies in Eq. (16), we obtain in the limit $L\to\infty$,

$$\tau_d=\frac{2}{\kappa v_{g1}^0}\left(\frac{\omega_1}{\omega}\right)^2\frac{\kappa^2}{\kappa^2+\beta^2},\qquad \tau_i=\frac{2}{\kappa v_{g1}^0}\left(\frac{\omega_1}{\omega}\right)^2\frac{\beta^2}{\kappa^2+\beta^2},$$

$$\tau_g=\tau_d+\tau_i=\frac{2}{\kappa v_{g1}^0}\left(\frac{\omega_1}{\omega}\right)^2. \tag{34}$$

Thus the Hartman effect in a cutoff waveguide is the result of the saturation of stored energy with increasing length. Because of the exponential decay of the electromagnetic field with distance, beyond a certain decay length $1/\kappa$, any additional barrier length adds little to the total integrated stored energy. It is this stored energy that determines the delay of the output peak. An output peak is the result of the barrier releasing energy it has stored from the instant the pulse was first turned on. An anomalously short delay in the appearance of a peak is more an indication of a cavity lifetime than of a propagation delay. The experimental results of Nimtz and others [4] as well as the theoretical prediction of Hartman can thus be understood without having to appeal to superluminal velocities. These anomalously short delays are always preceded by a lengthy cavity filling time during which the exponential standing wave mode is set up [33]. This filling time should be accounted for in any discussion of causal relationships between input and output pulses. Note that above the cutoff frequency of the barrier the hyperbolic functions in the tunneling time become trigonometric functions and hence there is no saturation with length in that case. In this allowed region of propagation there are still delays that can be less than L/c. These occur in the low transmission parts of the barrier when the guide wavelength is much greater than the barrier thickness. It is for those regions that the interference explanation of anomalous delays may be applicable [13].

VII. ENERGY VELOCITY

If neither the group delay nor the dwell time can be associated with a traversal velocity through the barrier, is there some physically meaningful velocity that leads to sensible results? After all, some energy does get transmitted through the barrier. The output pulse does contain energy otherwise it could not be detected. A single purely evanescent mode cannot transmit any time averaged power. That is because the electric and magnetic fields are in quadrature (90° out of phase). There is, however, a reactive power flow that reverses direction twice per cycle and has no time average value. So long as the quadrature relationship is maintained there will be no time average power flow. This is the case in an infinitely long evanescent region. In a finite region, however, there will be some reflection of the forward attenuating evanescent mode at the exit. Because the cutoff waveguide has a purely imaginary characteristic impedance, the reflection coefficient will have a phase shift associated with it. The reflected backward evanescent mode has a phase shift so that the total electric field is no longer in quadrature with the magnetic field. Thus the interference between the forward and backward evanescent modes gives rise to a nonzero time average power flow. This power flow is not due to a propagating wave but can be seen as the beating between two evanescent cavity modes. In addition to the purely reactive pulsations of energy, there is a time averaged contribution due to the fact that net energy escapes through the boundary during each cycle. This energy is radiated away and does not return to the source. That energy must be replenished by a real power flow into the volume. Another way of saying the same thing is that the wave impedance which is purely imaginary for a single decaying evanescent mode (hence purely reactive) acquires a real part when a reflected, phase-shifted antievanescent contribution is added. This real part of the impedance governs the transfer of time-averaged power from one end of the structure to the other.

The local energy velocity relates to the flow of real power. It is defined as [15,34]

$$v_E=\frac{P_t}{\langle W\rangle}, \tag{35}$$

where $\langle W\rangle$ $(\mathrm{J\,m^{-1}})$ is the time averaged energy density integrated over the cross section and $P_t=|T|^2P_{in}$ (W) is the constant average power flow through that area. A global or average energy velocity through the barrier can be defined as

$$V_E=\frac{\dfrac{1}{L}\displaystyle\int_0^L P_t dz}{\dfrac{1}{L}\displaystyle\int_0^L \langle W\rangle dz}. \tag{36}$$

The local and global energy velocities must be bounded from above by the speed of light in vacuum c. Here we demonstrate by using the solutions for the fields within the evanescent region that the energy velocities are indeed subluminal. The energy per unit length is given by

$$W=\frac{1}{4}\int_S[\varepsilon(\mathbf{E}\cdot\mathbf{E}^*)+\mu(\mathbf{H}\cdot\mathbf{H}^*)]ds. \tag{37}$$

Using the field solutions in the barrier we find

$$W=\frac{\varepsilon_0|E_0|^2A}{8k_0^2|g|^2}\{(n_1^2+n_2^2)k_0^2\cosh^2\kappa(z-L)$$
$$+[(\beta/\kappa)^2(k_0^2n_2^2+\gamma^2)+\kappa^2]\sinh^2\kappa(z-L)\}.$$

The maximum of v_E occurs where the denominator W is a minimum, since the numerator is constant in z. Clearly the minimum of W is at $z=L$. Hence

$$(v_E)_{\max}=\frac{|T|^2P_{in}}{A\varepsilon_0|E_0|^2(n_1^2+n_2^2)/8|g|^2}$$
$$=c\left(\frac{2n_1}{n_1^2+n_2^2}\right)\sqrt{1-(\omega_1/\omega)^2}\leqslant c. \tag{38}$$

Thus the local energy velocity is always less than c, increasing from zero at the cutoff frequency of the external region to $c[2n_1/(n_1^2+n_2^2)]$ when far above cutoff. Of course, we cannot go very far above cutoff if the wave is to be evanescent in the barrier region.

Having shown that the local energy velocity is strictly subluminal, we now consider the global or average energy velocity defined above. Again, since the numerator is constant, V_E is maximized when the denominator is minimum. Hence

$$(V_E)_{\max}=\frac{LP_t}{\min\displaystyle\int_0^L W dz}\leqslant\frac{LP_t}{L\min(W)}, \tag{39}$$

where the inequality follows from the positive definite nature of W. Recognizing the last term as $(v_E)_{\max}$, we can then state the following inequalities:

$$(V_E)_{\max}\leqslant(v_E)_{\max}\leqslant c. \tag{40}$$

Thus both the local and global energy velocities are nicely bounded from above by the vacuum speed of light. In recent work Diener has proposed an unconventional definition of an energy transport velocity which seeks to separate stored, nonpropagating energy from propagating energy in the waveguide [35]. We believe that this approach is fundamentally flawed since the energy is stored in the entire electromagnetic field. One cannot separate longitudinal field energy from transverse field energy since the field components are coupled into one electromagnetic wave.

Figure 4 shows the energy velocity in the barrier region normalized by the vacuum speed of light. The dotted curve is the group velocity (and energy velocity) in an infinite length of the external waveguide. It is asymptotic to c/n_1. The dashed curve is the group velocity (and energy velocity) in an infinite length of the central waveguide for frequencies above the cutoff frequency. It is asymptotic to c/n_2. The energy velocity in the finite barrier is much less than the speed of light for frequencies $\omega_1<\omega<\omega_2$ within the stop band. Outside the stop band the global energy velocity rises to an asymptotic value between v_{g1}^0 and v_{g2}^0.

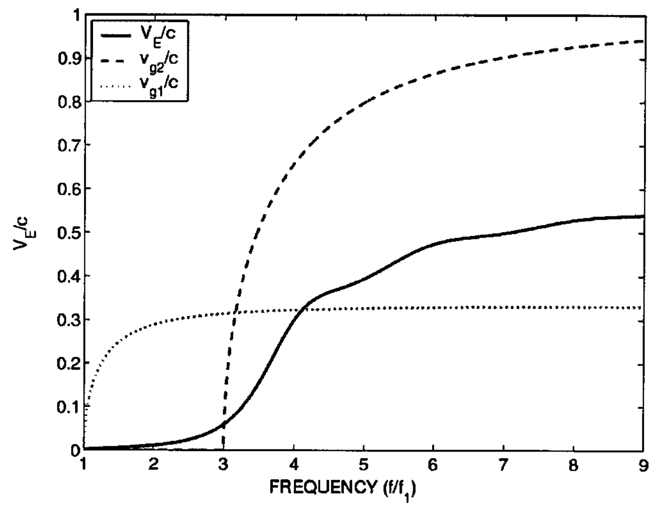

FIG. 4. (Color online) Energy velocity for a tunneling evanescent mode (solid curve). The stopband is the frequency range $1<f/f_1<3$. Here $\gamma L=5$, $n_1=3$, $n_2=1$.

VIII. A RECIPE FOR MEASURING DWELL TIME AND ENERGY VELOCITY

The general relation linking the group delay to the dwell time can be used to measure the energy velocity. The group delay is a well defined, measurable quantity for narrowband pulses that satisfy the conditions for the validity of the group delay notion. It is determined by measuring the time difference between an input pulse peak and an output pulse peak. In fiber-optic communications, group delay is also routinely measured by monitoring the phase shift between an output modulation and an input modulation. The self-interference delay is also a measurable quantity determined by measuring the complex reflection coefficient of the barrier. (One way is by making cw standing wave measurements with a slotted line which yields the magnitude and phase of the reflection coefficient.) From these two measurements we obtain the dwell time as $\tau_d=\tau_g-\tau_i$. From Eq. (36), the global energy velocity is

$$V_E=\frac{|T|^2L}{\tau_d}. \tag{41}$$

Hence a series of cw measurements yields all the information we need to characterize the dynamic response of the barrier. We may also define an energy transit time as $\tau_E=L/V_E=\tau_d/|T|^2$. This is the time it takes for all the energy stored in the barrier to leave through the exit at $z=L$, assuming it is transported with the average energy velocity. This time is much longer than the group delay, which is a $1/e$ lifetime of stored energy escaping through both ends.

IX. GROUP DELAY VERSUS GROUP VELOCITY

We assert that no one has yet measured a superluminal tunneling *velocity*. What have been measured are group *delays*. The distinction is not just semantic. A delay time by itself makes no assumption about the mechanism responsible for the delay. It could be due to absorption and reemission, storage, multiple reflection, transmutation, even perhaps propagation. A velocity, on the other hand, assumes that the phenomenon in question, in this case a pulse peak, actually propagates. In doing so, it passes continuously through every point along a trajectory. For tunneling pulses, we have shown through direct numerical solutions of the wave equation that the pulse peak does not propagate from input to output, therefore the notion of group velocity is not relevant. It is often said that evanescent waves propagate with superluminal velocities. That by itself is a contradiction in terms. Evanescent waves, by definition, do not propagate. They are storage fields that oscillate in place. If they are traveling they cannot be evanescent.

Distortionless tunneling is a quasistatic process where the pulse length greatly exceeds the barrier length [16]. This is true in all experiments where tunneling without distortion has been observed. It is true in numerical simulations where "infinite" tunneling velocities have been claimed. In Ref. [36], for example, a pulse of width 37.32 ns was said to have tunneled with infinite speed through a barrier of length 32.96 mm. Given that the spatial extent of the tunneling pulse was 11.2 m, which is *three hundred and forty times the barrier width*, it is safe to say that this was a steady state process and that the peak was always at the exit as well as at the input. In fact, with respect to the barrier, this pulse is not a localizable object for which one can speak of a transit time. This quasistatic nature of the interaction is also true in other "superluminal" contexts such as in gain media where a 720-m-long pulse was used to probe a 6-cm sample [37]. This in itself raises questions about the possibility of localizing a peak so broad in a sample so small. To what accuracy can such a measurement be made? In the adiabatic tunneling process, there is a long build up time as the front of the pulse fills the barrier with energy [33]. This filling process takes a couple of transit times. As the main part of the pulse arrives it cannot propagate through the barrier because it is below the cutoff frequency. It can only modulate the stored energy in the barrier. Some of the stored energy leaks out and is converted to propagating energy. The output simply follows the input with a small delay owing to energy storage. Because of the quasistatic nature of the tunneling process, what is actually measured is a phase shift of an amplitude modulation.

X. CONCLUSION

In summary, we have derived an explicit relation between group delay and stored magnetic and electric energies in waveguide junctions such as the ones used in tunneling experiments. For evanescent modes, the electric and magnetic energies differ. This leads to an additional term in the relation between group delay and stored energy for waveguide terminations that was derived by Dicke some time ago. This additional term is also related to a self-interference delay experienced by a tunneling pulse. We are thus able to express the group delay as a sum of a dwell time inside the barrier and a self-interference delay, something that had previously been thought impossible. Both of these delays saturate with distance because the stored energy densities, being exponentially decaying functions of distance, result in saturated integrated energies. Because they saturate with distance they cannot possibly be propagation delays, unless we assume the waves are smart enough to adjust their velocities so they can cover increasing distances in the same amount of time. The anomalous group delays seen in barrier tunneling are not propagation delays but a measure of cavity lifetime. On the other hand, one can define a strictly luminal energy velocity which can actually be determined experimentally by measurements of group delay and self-interference delay.

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