Screening of dust grains in a weakly ionized gas: Effects of charging by plasma currents

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Screening and charging of a grain in a weakly ionized plasma background are studied within the drift-diffusion approximation. The computations evidence that the account of grain charging results in a distinct qualitative change in the screened field as compared to the thermodynamically equilibrium case of a grain with a constant charge. The stationary grain charge as well as the field within the sheath around the grain are shown to be almost independent of the type of boundary conditions (for relatively low ionization rates and small grain sizes), whereas the asymptotical behavior of the effective field is rather sensitive to them.

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Screening and charging of objects immersed in a plasma background are important problems in plasma physics, and have received the attention of researchers for decades. Recently, the interest in these problems has been renewed due to the experimental studies of dusty plasmas (DP), i.e., plasmas containing a large number of highly charged dust grains. The experiments revealed a number of collective effects in DP, in particular, the formation of an ordered state in the colloidal component associated with the strong Coulomb coupling in the grain subsystem [1,2]. The processes of grain charging and screening play therewith an important role, since these determine effective grain-grain interactions responsible for the above mentioned collective phenomena. In contrast to the systems with a fixed grain charge (e.g., charged colloids), the charge of a dust grain immersed in a plasma background is maintained by ion and electron currents to the grain surface. This makes the conventional thermodynamically equilibrium approaches, such as Poisson-Boltzmann or linear Debye-Hückel theory, inapplicable. The problem of grain screening in DP with allowance for charging by plasma currents attracted considerable interest of researchers in recent years. In most of the literature, the collisionless or weakly collisional plasma background is considered [3–8], which is typical for laboratory experiments on DP and astrophysical observations. The opposite limit of the strongly collisional background (e.g., a weakly ionized high pressure gas) is less examined [8–10]. Besides its fundamental significance, this case also has industrial aspects [11].

The aim of this work is to study the screening of a spherical grain charged by plasma currents in a weakly ionized high pressure gas. As will be shown below, the properties of grain screening in this case substantially depend on the type of boundary conditions (BC). In contrast to the works [9,10], where a complicated semirealistic multigrain system with relevant specific BC is considered, we are going to examine the simplest case of a *single* grain with the emphasis on the basic features of this problem.

Thus, we consider a single spherical grain of radius a imbedded in a weakly ionized high pressure gas. In this case, it is natural to use the drift-diffusion (DD) approach, because collisions of plasma particles with neutrals play here a dominant role. Assuming two types of plasma particles (ions and electrons) only, we write the general time-dependent equations for the unknown ion (electron) densities $n_{i,e}$ and self-consistent potential ϕ in the form

$$\frac{\partial n_{i,e}}{\partial t} = -\mathrm{div}\mathbf{j}_{i,e} + I_0 - \alpha n_i n_e , \tag{1}$$

$$\Delta \phi = -4\pi e(n_i - n_e). \tag{2}$$

Here, e is the absolute value of the electron charge, α is the coefficient of recombination, I_0 is the intensity of plasma ionization (we examine the case of uniformly distributed plasma sources). The expression for the current densities $\mathbf{j}_{i,e}$ has the form

$$\mathbf{j}_{i,e} = -\mu_{i,e} n_{i,e} \boldsymbol{\nabla}\phi - D_{i,e} \boldsymbol{\nabla} n_{i,e} ,$$

where $\mu_{i,e}$ and $D_{i,e}$ are the ionic (electronic) mobility and diffusivity, respectively. These latter are assumed to be related due to Einstein's equation $\mu_{i,e} = z_{i,e} e_{i,e} D_{i,e}/k_B T$ [here $z_{i,e} = \pm 1$ is the ion (electron) charge number]. In a weakly ionized gas with dominating plasma-neutrals collisions, it is reasonable to assume that the ion and electron temperatures are equal. Thus, we consider below only the case where $T_i = T_e = T$. The grain charge emerges as a result of plasma currents due to the difference between electron and ion diffusivities. With regard to spherical symmetry, the relevant equation for the grain charge number $Z = Q_{grain}/e$ reads as

$$\frac{dZ}{dt} = -4\pi a^2 (j_{(r)i} - j_{(r)e}), \tag{3}$$

where the subscript (r) denotes the radial component of the current.

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In order to formulate the BC, we admit that the system is confined in a spherical volume of sufficiently large radius $R \simeq (50\text{–}500) r_D$ (where r_D is the Debye screening length) with the grain placed at the center. The BC are specified at the surface of this sphere and at the surface of the grain. In our simulations, we consider the two basic cases and two types of BC, respectively. In the first case, case (I), the sources of plasma ionization, which compensate the loss of plasma particles due to the absorption on the grain surface, are assumed to be far from the grain (outside the spherical volume). The action of these sources is modeled by maintaining constant electron and ion densities on the surface of the sphere, $n_i = n_e = n_0$. According to this, we write the BC for the densities $n_{i,e}$ as

$$n_{i,e} = n_0 \quad \text{at} \quad r = R,$$

and assume the rates of plasma ionization and recombination over the volume I_0 and α to be equal to zero. In the second case, case (II), we examine the problem with uniformly distributed plasma sources ($I_0 \neq 0$) with allowance for the plasma recombination over the volume ($\alpha \neq 0$). A possible physical realization could be, for instance, the ionization of a neutral gas by UV radiation. Note that in this case, the quantities I_0 and α are related to the unperturbed bulk plasma density n_0 by the equation $I_0 = \alpha n_0^2$ valid in the absence of the grain. The relevant BC read as

$$\frac{\partial n_e}{\partial r} = \frac{\partial n_i}{\partial r} = 0 \quad \text{at} \quad r = R.$$

The BC for the potential at the grain surface have the form

$$\frac{\partial \phi}{\partial r} = -\frac{Z(t)e}{a^2} \quad \text{at} \quad r = a.$$

For the densities $n_{i,e}$, we use the BC of Ref. [9]

$$n_{i,e} = 0 \quad \text{at} \quad r = a$$

appropriate for the case of strongly collisional background.

We solved the above system of equations by means of lines and Gear's method. The numerical codes were tested to reproduce the results of the nonlinear Poisson-Boltzmann theory (and Derjaguin-Landau-Verwey-Overbeek theory in the case of weak plasma-grain coupling, see Refs. [12–14]) in the thermodynamically equilibrium case of the grain with a fixed charge. In addition, we performed a limited number of Brownian dynamics (BD) simulations based on particle-in-cell method [15] with spherically symmetric concentric cells and the BC corresponding to case (I). In these simulations, the plasma background is modeled by finite numbers of particles of two sorts representing the ion and electron components. The dynamics of the system is governed by the reduced Langevin equations of overdamped motion

$$h \frac{d\mathbf{x}_k}{dt} = -\nabla_k U + \mathbf{F}_k(t).$$

Here $\mathbf{x}_k$ is the radius vector of the kth particle and U is the potential energy of the configuration. The friction coefficient h and the random force $\mathbf{F}_k(t)$ are determined by the properties of the heatbath (in our case, the high pressure neutral gas plays the role of heatbath). Random force acting on kth particle is specified by the Gaussian distribution

$$P(\mathbf{B}_k(\Delta t)) = \frac{1}{(4\pi h^2 D \Delta t)^{3/2}} \exp\left[-\frac{|\mathbf{B}_k(\Delta t)|^2}{4h^2 D \Delta t} \right],$$

which determines the probability for the momentum

$$\mathbf{B}_k(\Delta t) = \int_t^{t+\Delta t} \mathbf{F}_k(t)\, dt$$

to be transferred to the kth plasma particle during the time span Δt. The random forces, which act on different plasma particles, are uncorrelated. It is clear that the quantities h and D related to the ion and electron components are different. In the above expressions, we omitted the subscripts for simplicity. Note that the friction coefficient h can be expressed via diffusivity and temperature, $hD = k_B T$, which enables one to establish the correspondence with the continuous DD approach. A detailed presentation of the issues concerning BD and its relation to the continuous probabilistic approaches, such as Fokker-Planck and Smolukhovsky equations, can be found in Refs. [16,17]. Here we would like to point out that the overdamped BD represents the direct microscopic analog to the DD approach, since the latter can be derived from the Smolukhovsky equations for one-particle distributions (i.e., within the additional mean field approximation). The aim of BD simulations was to test the results of DD approximation.

Equations (1)–(3) were solved numerically, with regard to the spherical symmetry, for the range of parameters characteristic of the DP experiments in high pressure weakly ionized noble gases such as Ne or Ar. The typical values are: plasma background coupling $\Gamma \simeq 10^{-3}$, plasma density $n_0 \simeq 10^{10}$ cm^{-3}, the density of the neutrals $n \simeq 10^{18}$ cm^{-3}, radius of the grain $a \simeq 10^{-3}$ cm, electron-ion recombination coefficient $\alpha \simeq 10^{-7}$ cm^3/sec, the ratio of the Debye length to the grain radius $r_D/a \simeq 0.1\text{–}50$. The ratio of diffusivities in all computations was fixed, $A = D_e/D_i = 10^3$ (with exception for the BD simulations). The goal of simulations was the final time-independent density and charge distributions, which establish themselves after a sufficiently long period of relaxation.

In the problem of grain screening in the spherically symmetric case, it is convenient to introduce the charge distribution function $Q(r)$ defined as the total charge residing within a sphere of radius r. Note that in the case of the unscreened Coulomb field, this function is constant due to the Ostrogradsky-Gauss theorem.

The results of computations are presented in the figures. In Fig. 1, we give the relative charge distributions $Q(r)/Q_{grain}$ for case (I) (the plasma sources being far from the grain, no ionization and no recombination over the volume). Remarkably, we observe the Coulomb-type asymptotical behavior of the screened field with the effective charge determined by the asymptotical value of the charge distribu-

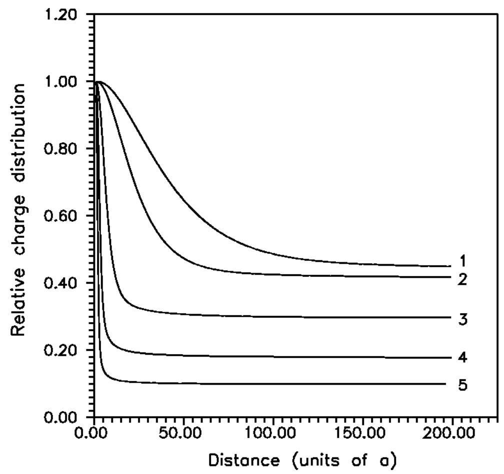

FIG. 1. Charge distributions for the case in which the plasma sources are placed at infinity, BC [case (I)]. The Debye length is $r_D/a=20(1)$, $10(2)$, $2(3)$, $0.5(4)$, and 0.16(5).

tion. It should be pointed out that a series of computations performed with subsequently increasing the radius of the spherical volume, $R=100r_D, 200r_D, \ldots, 500r_D$, resulted in the same charge distributions and magnitudes of the effective charge, i.e., these latter do not depend on the position of the right boundary in the limit $R\rightarrow\infty$. Besides, we performed a number of computations for the same sets of plasma parameters, starting from different initial distributions. As one would expect, the final solutions turned out to be independent of the initial distributions. Note that the Coulomb-type asymptotical behavior of the screened field may be viewed as a consequence of the Ohm's law for the problem under consideration. The computations evidence that the relative charge distributions $Q(r)/Q_{grain}$ depend on the ratio r_D/a. This is due to the fact that the stationary solutions to the problem allow a scaling transformation. As can be seen from Eqs. (1)–(3), a simultaneous transformation of bulk plasma density n_0 and the coupling Γ_{bg}, which keeps the Debye length invariant, yields just a scaled solution to the problem. The relevant dependence for the relative effective charge is displayed in Fig. 2. In contrast to case (I), the screening in

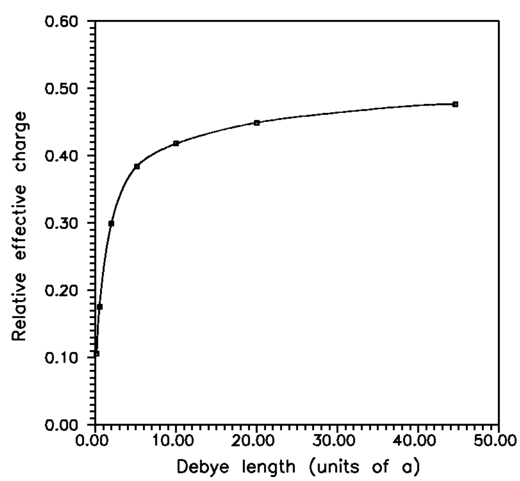

FIG. 2. Effective charge vs Debye length for case (I).

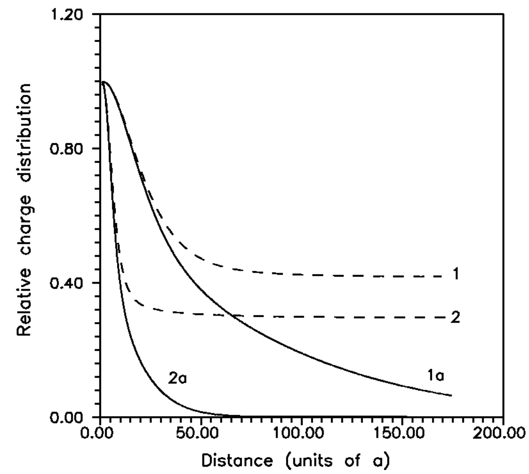

FIG. 3. Comparison of charge distributions for different types of BC for the same stationary bulk plasma parameters. The Debye length is $r_D/a=10$ for curves 1 and 1a, and $r_D/a=2$ for (2) and (2a). Dashed and solid lines relate to cases (I) and (II), respectively.

the case of ionization over the volume has a finite screening length $\simeq(10–50)r_D$, Fig. 3. The computations performed for the same plasma parameters (in particular, for the same steady bulk density n_0 at long distances) for cases (I) and (II) indicate that there exists a sheath ranging up to $10r_D$ independent of the type of BC (provided that the ionization rate is relatively low). At longer distances, a distinct difference in the asymptotical behavior is observed.

Figure 4 illustrates the behavior of the electric field as dependent on the rate of ionization. The bulk plasma density is held constant therewith due to the simultaneous appropriate change of the recombination coefficients. The approximate straightness of the lines outside the sheath (on log

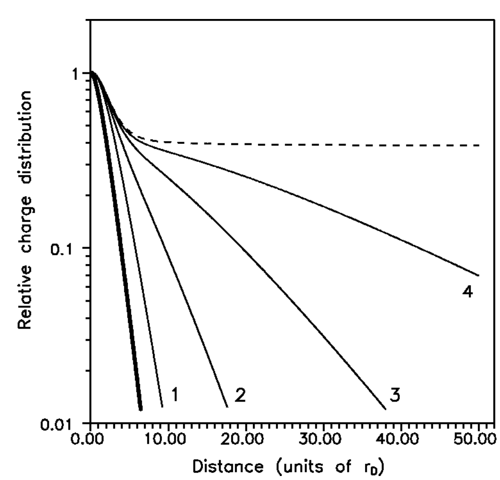

FIG. 4. Relative charge distributions for BC [case (II)] as dependent on the ionization rates at fixed bulk plasma density. The dimensionless intensity of plasma sources over the volume $i_0=I_0a^5/D_i$ is (1) 1.25×10^{-2}, (2) 2.5×10^{-3}, (3) 5×10^{-4}, (4) 10^{-4}. The bold line relates to the linear Debye-Hückel theory and the dashed line is the DD approach for BC [case (I)]. The bulk plasma parameters in all cases are the same; the grain radius a/r_D is 0.158.

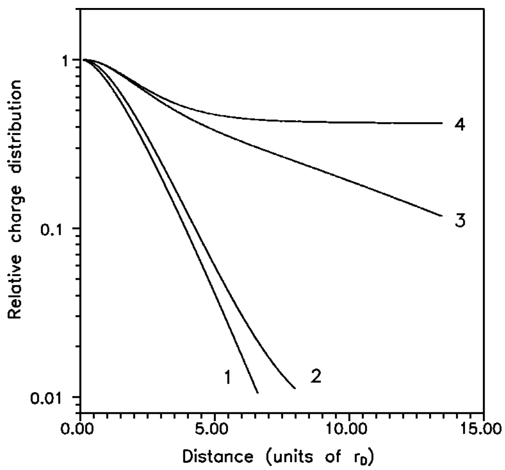

FIG. 5. Comparison of the relative charge distributions for the same bulk plasma parameters: (1) Debye-Hückel theory; (2) collisionless OML theory (calculations are performed in a way presented in Ref. [5] with no bound ionic states taken into account); (3) drift-diffusion approach, BC [case (II)]; and (4) drift-diffusion approach, BC [case (I)]. The grain radius a/r_D is 0.1.

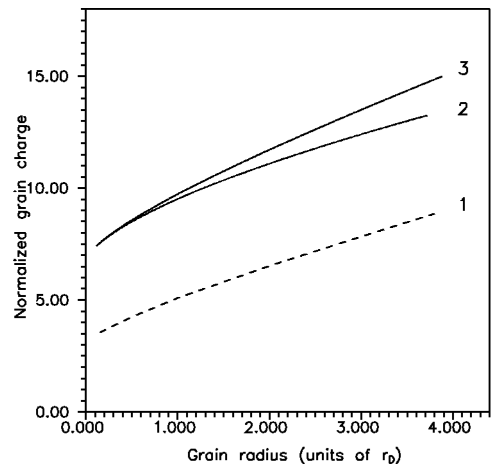

FIG. 6. Normalized grain charge vs grain radius a/r_D. (1) The collisionless OML theory (based on the approach presented in Ref. [5] with no bound ionic states taken into account), (2) DD approach BC [case (I)], and (3) DD approach BC [case (II)] with the intensity of plasma sources $i_0 = I_0 a^5/D_i = 10^{-3}$.

scale) suggests the exponential type of the screening at distances. Different rates of ionization (and recombination) correspond to the different slopes and the screening lengths, respectively. The higher the intensity of the ionization, the shorter is the length of screening. At higher rates, the relative indifference of the sheath is likely to break down, and the properties of screening approach the predictions of the Debye-Hückel (DH) theory (the bold line in Fig. 4). A comparison of DD approximation with the electrostatic DH and collisionless orbit motion limited (OML) theories indicates that, typically, the charging plasma currents in the presence of collisions result in increase in the length of screening, Fig. 5. Note that the above results correlate qualitatively with those of Ref. [10] dealing with a more complicated case of nonisothermic nitrogen plasma.

The behavior of the normalized grain charge $Ze^2/k_B Ta$ as dependent on the grain radius a/r_D, along with the comparison to the OML theory, is presented in Fig. 6. As is seen from it, the presence of collisions gives rise to the enhancement of charging, and both types of BC yield close magnitudes of the stationary grain charge, in particular, for small grain sizes ($a \leqslant r_D$).

Typical density distributions around the grain are given in Fig. 7. It is interesting to note that though the densities for the BC (I) and (II) are distinctly different, they produce the same electric field within the sheath (for the same set of parameters).

In order to test the results of computations, we performed a number of BD simulations with the relevant parameters and the BC corresponding to case (I) of DD approach. Since these simulations are rather difficult, they were performed for a moderate average number of particles within a spherical volume, $N_p \simeq 1300$–1500, and the following parameters: $A = 10.0$, $a/r_D = 0.373$, $R/a = 15$. A comparison of the continuous DD approach and the microscopic BD simulations is given in Figs. 8 and 9. The qualitative behavior in both cases is quite similar. Some discrepancy (DD approach yields $\approx 10\%$ higher absolute value of the stationary grain charge) is, apparently, the result of microscopic effects in the plasma background in BD simulations.

To conclude, we studied the screening and charging of a grain in a weakly ionized plasma background within the drift-diffusion approximation. The computations evidence that the account of grain charging results in a distinct qualitative change in the asymptotical behavior of the screened field as compared to the thermodynamically equilibrium Debye-Hückel theory for a grain with constant charge. In the case where the plasma sources are placed at infinity, we observe at long distances the Coulomb field with a certain effective charge. The effect of screening manifests itself in the decrease of this effective charge as compared to the stationary grain charge. The smaller the ratio of the Debye length to

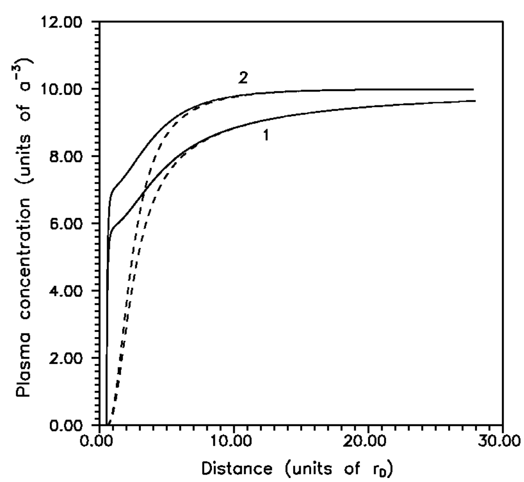

FIG. 7. Comparison of plasma densities for different types of BC for the same stationary bulk plasma parameters. The lines (1) and (2) relate to cases (I) and (II), respectively. The grain radius is $a/r_D = 0.5$. Dashed and solid lines correspond to the electrons and ions, respectively.

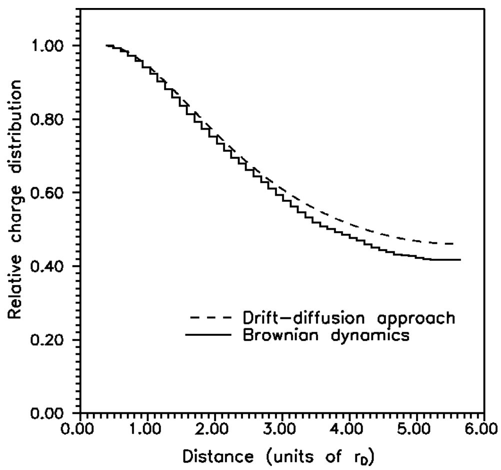

FIG. 8. Comparison of charge distributions obtained in DD and BD simulations for the same parameters, $a/r_D=0.373$, $A=10.0$.

the grain size, the smaller the effective charge is observed. In the case where the plasma sources are distributed uniformly over the volume, there exists a finite screening length depending on the rate of ionization. Typically, this screening length in the presence of plasma currents and strong collisions considerably exceeds the Debye radius. The stationary grain charge as well as the field within the sheath around the grain ($\simeq 10r_D$) does not depend on the type of BC and the ionization rate provided that this latter is relatively low and the grain size is not too large ($a \leqslant r_D$). At higher ionization rates, the properties of screening approach the predictions of Debye-Hückel theory. The tests based on the microscopic

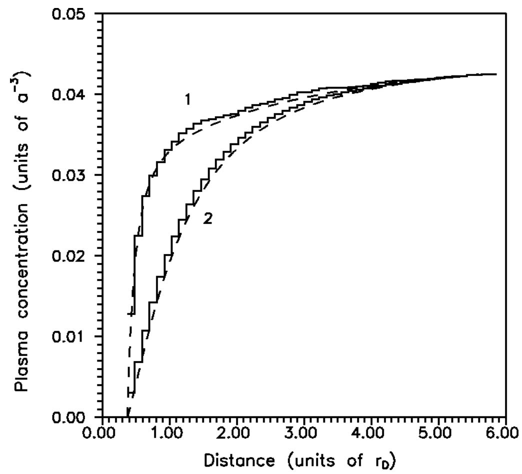

FIG. 9. Comparison of ion (1) and electron (2) densities obtained in the DD approximation (dashed lines) and in BD simulations (solid lines) for the same parameters; $a/r_D=0.373$, $A=10.0$.

Brownian dynamics simulations correlate well with the continuous drift-diffusion approach. The microscopicity of the plasma background results in some decrease in the steady charge acquired by the grain.

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