Guided modes in negative-refractive-index waveguides

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We study linear guided waves propagating in a slab waveguide made up of a negative-refractive-index material, the so-called *left-handed waveguide*. We reveal that the guided waves in left-handed waveguides possess a number of peculiar properties such as the absence of the fundamental modes, mode double degeneracy, and sign-varying energy flux. In particular, we predict the guided waves with *a dipole-vortex structure* of their Poynting vector.

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Recent experimental demonstration of novel composite materials with a negative index of refraction [1] opens up a unique possibility to design novel types of devices where electromagnetic waves propagate in a nonconventional way. The history of these materials begins with the paper by Veselago [2], who studied the wave propagation in a hypothetical material with simultaneously negative dielectric permittivity ϵ and magnetic permeability μ. Such media are usually termed as *left-handed materials*, since the electric and magnetic fields form a left set of vectors with the wave vector. Already back in 1968, Veselago predicted a number of remarkable properties of waves in left-handed materials, including negative refraction. However, the structures with both negative ϵ and μ have not been known until recently, although materials with negative dielectric permittivity are known (e.g., a metal below the plasma frequency).

The study of microstructured metallic materials for magnetic resonance imaging [3] has shown that such structures can be described in terms of effective magnetic permeability that becomes negative in the vicinity of a resonance frequency. It was expected that mixing the composite materials with negative magnetic permeability [3] with those possessing negative dielectric permittivity [4] would allow us to create a different type of *metamaterials* with a negative index of refraction. Indeed, both numerical simulations [5] and experimental results [1,6] confirmed that such left-handed (or negative-index refraction) materials can readily be fabricated.

One of the first applications of the negative-refraction materials was suggested by Pendry [7], who demonstrated that a slab of a lossless negative-refraction material can provide a perfect image of a point source. Although the perfect image is a result of an ideal theoretical model used in Ref. [7], the resolution limit was shown to be independent of the wavelength of electromagnetic waves (but can be determined by other factors such as loss, spatial dispersion, etc.), and direct numerical simulations [8] indicate that the resolution limit can be better than that of a conventional lens; however, this problem is still under intense investigation [9].

The improved resolution of a slab of the negative-refraction material can be explained by the excitation of surface waves at both interfaces of the slab. Therefore, it is important to study the properties of surface waves at the interfaces between the negative-refraction and conventional materials. So far, only the frequency dispersion of surface waves at a single interface was analyzed [10], and some modes of a slab waveguide were calculated numerically for particular medium parameters [11].

In this paper, we study the structure and basic properties of electromagnetic waves guided by a left-handed waveguide. In order to emphasize the unusual and somewhat exotic properties of such waves, we compare them with the guided waves of conventional planar dielectric waveguides. We reveal that the guided modes in left-handed waveguides differ dramatically from conventional guided waves, and they possess a number of unusual properties, including the absence of the fundamental modes, double degeneracy of the modes, the sign-varying energy flux, etc. In particular, we predict the existence of different types of guided waves with a dipole-vortex structure of the energy flux and the corresponding Poynting vector.

We consider a symmetric slab waveguide in a conventional planar geometry [see, e.g., the top left inset of Fig. 1(a)]. In a general case, a slab of the thickness $2L$ is made up of a material with dielectric permittivity ϵ_2 and magnetic permeability μ_2, which both can be negative or positive. We assume that the surrounding medium is right handed, and therefore it is characterized by both positive ϵ_1 and μ_1. It is well known that a slab waveguide made up of a conventional (right-handed) dielectric material with $\epsilon_2>0$ and $\mu_2>0$ creates a nonleaky waveguide for electromagnetic waves, provided the refractive index of a slab is higher than that of the surrounding dielectric medium, i.e., $\epsilon_2\mu_2>\epsilon_1\mu_1$. However, in the following, we demonstrate that this simple criterion cannot be applied to the waveguides made up of a left-handed material.

To be specific, below we describe the properties of the TE guided modes in which the electric field $\mathbf{E}$ is polarized along the y axis. A similar analysis can be carried out for the TM modes. From Maxwell's equations, it follows that stationary TE modes are described by the following scalar equation for the electric field $E=E_y$:

$$\left[\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial x^2}+\frac{\omega^2}{c^2}\epsilon(x)\mu(x)-\frac{1}{\mu(x)}\frac{\partial \mu}{\partial x}\frac{\partial}{\partial x}\right]E=0, \quad (1)$$

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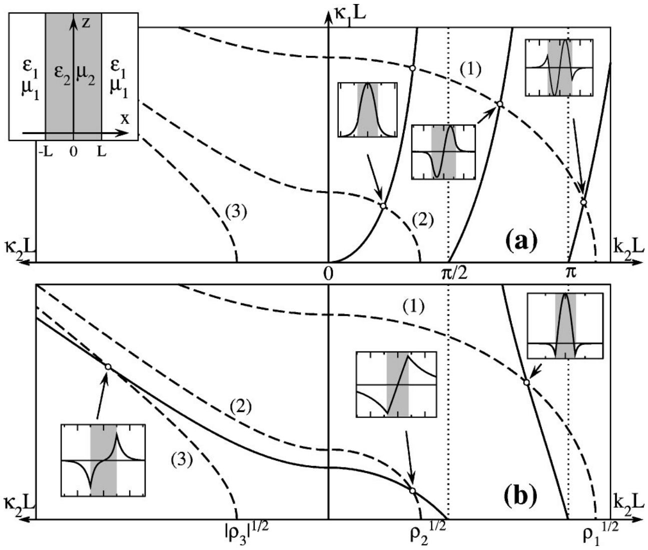

FIG. 1. A comparison between the conventional (a) and left-handed (b) guided modes of a slab waveguide. The dashed and solid curves correspond to the left- and right-hand sides of the dispersion relations in Eqs. (4) and (5), respectively. Intersections of these curves indicate the existence of guided modes. Three dashed lines in each plot correspond to waveguides with different parameters $\rho_1 > \rho_2 > \rho_3$, but the fixed ratio μ_2/μ_1. Insets show the waveguide geometry and the transverse profiles of the guided modes.

where ω is the angular frequency of the field.

The guided modes are stationary solutions of Eq. (1) of the form

$$E(x,z) = E_0(x) e^{ihz}, \tag{2}$$

where real h is the wave propagation constant and $E_0(x)$ is the spatially localized transverse profile of the mode. Substituting Eq. (2) into Eq. (1), we obtain an eigenvalue problem that possesses spatially localized solutions for

$$\kappa_1^2 = h^2 - \frac{\omega^2}{c^2}\epsilon_1\mu_1 > 0, \tag{3}$$

because only in this case the mode amplitude decays away from the waveguide, $E_0(x) \sim \exp(-\kappa_1|x|)$.

We solve the eigenvalue problem in each of the homogeneous layers, and then employ the corresponding boundary conditions following from Eq. (1). As a result, we obtain the dispersion relations that define a set of allowed eigenvalues h,

$$(\kappa_1 L) = \pm\frac{\mu_1}{\mu_2}(k_2L)\tan^{\pm 1}(k_2L), \tag{4}$$

where $(+)$ and $(-)$ correspond to the symmetric and antisymmetric guided modes, respectively, and $k_2 = [(\omega^2/c^2)\epsilon_2\mu_2 - h^2]^{1/2}$. When k_2 is real, the corresponding modes can be identified as "fast waves," since their phase velocities ω/h are larger than the phase velocity in an homogeneous medium with the same ϵ_2 and μ_2.

The parameter k_2 becomes purely imaginary for "slow waves," when the propagation constant h exceeds a critical value; such waves resemble surface waves in metal films [12]. Then, it is convenient to present Eq. (4) in an equivalent form using the notation $\kappa_2 = ik_2$,

$$(\kappa_1 L) = -\frac{\mu_1}{\mu_2}(\kappa_2 L)\tanh^{\pm 1}(\kappa_2 L). \tag{5}$$

Following a standard analysis (see, e.g., Ref. [13]), we consider the parameter plane $(k_2L, \kappa_1 L)$, and also extend it by including the imaginary values of k_2 using the auxiliary parameter plane $(\kappa_2 L, \kappa_1 L)$. In Figs. 1(a) and 1(b), we plot the dependencies described by the left-hand (dashed) and right-hand (solid) sides of Eqs. (4) and (5), using the parameter

$$(\kappa_1 L)^2 + (k_2 L)^2 = L^2(\omega^2/c^2)(\epsilon_2\mu_2 - \epsilon_1\mu_1) \equiv \rho. \tag{6}$$

In Figs. 1(a) and 1(b), three dashed lines, (1)–(3), correspond to different slab waveguides having the same ratio μ_1/μ_2. The intersections of a dashed curve with solid curves indicate the existence of solutions for guided modes. We present results for a conventional (right-handed) waveguide in Fig. 1(a), in order to compare them directly with the corresponding dependencies for a left-handed slab waveguide in Fig. 1(b).

First of all, the analysis of Eqs. (4) and (5) confirms the well-known result that a right-handed slab waveguide can only support fast guided modes, which exist when the waveguide core has a higher refractive index than its cladding, i.e., for $\epsilon_2\mu_2 > \epsilon_1\mu_1$. In this case, there always exists *the fundamental guided mode*, whose profile does not contain zeros. The conventional waveguide can also support higher-order modes, their number depends on the value $2\rho^{1/2}/\pi$. These various regimes of the mode guiding are presented in Fig. 1(a) by different dashed lines.

The properties of the left-handed slab waveguides are found to be very different. First, such waveguides can support slow modes, and they are either symmetric (node less) or antisymmetric (one zero). Such solutions represent *in-phase* or *out-of-phase* bound states of surface modes, localized at two interfaces between right and left media. In the conventional case of both positive ϵ and μ, such surface waves do not exist, however, they appear when the magnetic permeability changes its sign (for the TE polarization). Thus, the guided modes can be supported by *both low-index and high-index* left-handed slab waveguides.

Second, the conventional hierarchy of fast modes disappears. Specifically, (i) the fundamental node-less mode does not exist at all, (ii) the first-order mode exists only in a particular range of ρ, and it always disappears in wide waveguides when ρ exceeds a critical value, and (iii) two modes having the same number of nodes can coexist in the same waveguide. We illustrate some of these nontrivial features in Fig. 1(b).

Frequency dispersion of the guided waves in the left-handed waveguides should be studied by taking into account the dispersion of both ϵ_2 and μ_2, since this is an essential

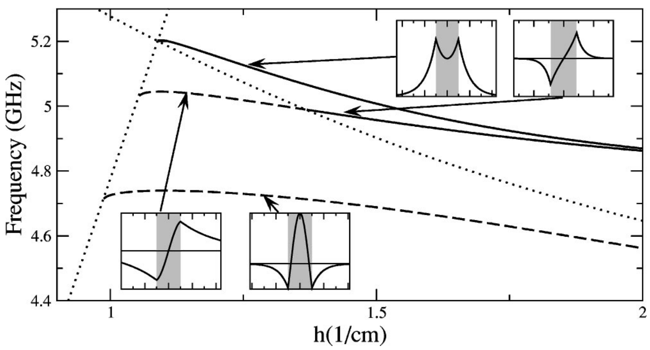

FIG. 2. Frequency dispersion curves for the three lower-order guided modes of the left-handed slab waveguide ($L=2$ cm). Insets show the characteristic mode profiles.

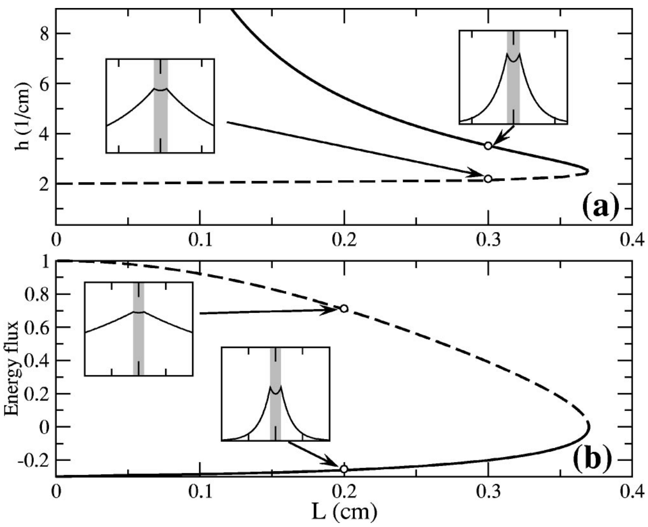

FIG. 3. Surface waves in a slab waveguide for the case $\epsilon_2\mu_2<\epsilon_1\mu_1$ and $\mu_2<\mu_1$. Shown are (a) the propagation constant h and (b) the normalized energy flux P vs the slab thickness parameter L. Solid and dashed lines correspond to strongly and weakly localized modes, respectively. Insets show the characteristic mode profiles.

property of such materials [2]. We consider the following frequency dependencies of the effective left-handed medium characteristics,

$$\epsilon_2(\omega)=1-\frac{\omega_p^2}{\omega^2},\quad \mu_2(\omega)=1-\frac{F\omega^2}{\omega^2-\omega_0^2}, \tag{7}$$

where the parameters $\omega_p/2\pi=10$ GHz, $\omega_0/2\pi=4$ GHz, and $F=0.56$. Our choice is motivated by the experimental results. The region of simultaneously negative permittivity and permeability in this case ranges from 4 GHz to 6 GHz. Dispersion curves for the first three guided modes in a slab waveguide with the thickness parameter $L=2$ cm are shown in Fig. 2, where dashed curves correspond to fast modes and solid curves to slow modes. We find that the fundamental slow mode exists only at higher frequencies, whereas the second-order fast mode appears at lower frequencies. Both modes can have either positive or negative group-velocity dispersion in different parameter regions. Properties of the first-order antisymmetric mode are different. The type of this mode seamlessly changes from fast to slow as the wave number grows. This transition occurs when the condition $k_2=0$ is satisfied, which is a boundary separating the two types of modes, as shown in Fig. 2 by a dotted line. The high-order fast modes exist at the frequencies close to the resonant frequency at $\omega=4$ GHz.

In the left-handed materials, the electromagnetic waves are backward, since the energy flux and wave vector have the opposite directions [2], whereas these vectors are parallel in the conventional (right-handed) homogeneous materials. The energy flux is characterized by the Poynting vector averaged over the period $T=2\pi/\omega$ and defined as $\mathbf{S}=(c/8\pi)\,\mathrm{Re}[\mathbf{E}\times\mathbf{H}^*]$. A monochromatic guided mode has, by definition, a stationary transverse profile, and the averaged energy flux is directed along the waveguide only. It follows from Maxwell's equations and Eq. (2) that the z component of the energy flux is found as $S_z=c^2hE_0^2/8\pi\omega\mu(x)$.

The total power flux through the waveguide core and cladding can be found as $P_2=\int_{-L}^{L}S_z\,dx$ and $P_1=2\int_L^{+\infty}S_z\,dx$, respectively. We find that the energy flux distribution for the waves guided along the left-handed slab is rather unusual. Indeed, the energy flux inside the slab (with $\mu<0$) is opposite to that in the cladding (with $\mu>0$). This occurs because the normalized wave vector component along the waveguide (h) is fixed in a guided mode according to Eq. (2). An important information about the guided modes can be extracted from the study of the normalized energy flux $P=(P_1+P_2)/(|P_1|+|P_2|)$. This parameter is bounded, $|P|<1$, $P\to 1$ when the mode is weakly localized ($|P_1|\gg|P_2|$), whereas $P<0$ for modes that are highly confined inside the left-handed slab.

We have performed a detailed analysis of the slow guided modes and identified following four distinct cases.

(i) $\epsilon_2\mu_2>\epsilon_1\mu_1$, $\mu_2>\mu_1$. Only odd mode exists below the threshold, $\rho<\mu_1^2/\mu_2^2$. The corresponding critical value of the slab thickness L below which the odd mode exists is found as

$$L_{\mathrm{cr}}=\frac{c}{\omega}\frac{\mu_1}{\mu_2\sqrt{\epsilon_2\mu_2-\epsilon_1\mu_1}}. \tag{8}$$

The energy flux P is positive for all values of L. The modes are forward propagating, i.e., the total energy flux along the waveguide is codirected with the wave vector.

(ii) $\epsilon_2\mu_2>\epsilon_1\mu_1$, $\mu_2<\mu_1$. Even mode exists for all values of ρ; odd modes can appear only when a threshold parameter value is exceeded, $\rho>\mu_1^2/\mu_2^2$. Accordingly, the critical value (8) determines the lower boundary of the existence region for odd modes. The total energy flux is negative for all L, and the modes are backward. The energy is mostly localized inside the slab.

(iii) $\epsilon_2\mu_2<\epsilon_1\mu_1$, $\mu_2>\mu_1$. Both odd and even modes exist at all values of ρ and L, and the modes are forward.

(iv) $\epsilon_2\mu_2<\epsilon_1\mu_1$, $\mu_2<\mu_1$. Only even modes exist below the threshold value of ρ that can be found from Eq. (5). Characteristic dependences of the wave number, and normalized power versus the slab width are shown in Figs. 3(a) and

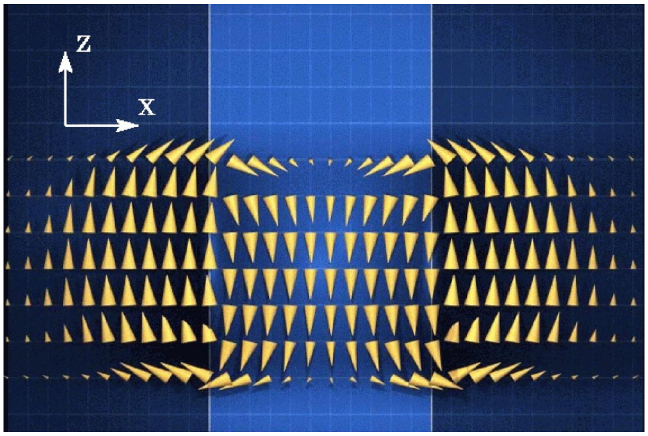

FIG. 4. (Color online) Structure of the Poynting vector field in a localized surface wave propagating along a left-handed slab.

3(b). For any slab thickness below a critical value, two modes always coexist. One of the modes is forward and weakly localized, but the other one is backward and more confined. When the slab width approaches the critical value, the branches corresponding to different modes merge and the energy flux vanishes. In this special case, the energy fluxes inside and outside the slab exactly compensate each other.

Since the energy fluxes are oppositely directed inside the guided modes, it might initially seem that such waves can only be sustained by two continuously operating sources positioned at the opposite ends of the waveguide. Therefore, it is important to understand whether wave packets of finite temporal and spatial extensions can exist in left-handed waveguides. We calculate the Poynting vector averaged over the period of the pulse carrier frequency, and present the characteristic structure of the energy flow in Fig. 4. Due to the unique double-vortex structure of the energy flow, most of the energy remains localized inside the wave packet, and it does not disintegrate. The group velocity is proportional to the total energy flux P, and it can therefore be made very small or even zero by a proper choice of the waveguide parameters as demonstrated above. On the other hand, the group-velocity dispersion, which determines the rate of pulse broadening, can also be controlled. This flexibility looks very promising for potential applications.

Finally, we note that recent numerical simulations demonstrated that the phenomenon of the negative refraction, similar to that found for the left-handed metamaterials, can be observed in *photonic crystals* [8,14]. Although in this case the wavelength is of the order of the period of the dielectric structure (and, therefore, the analysis in terms of the effective medium approximation is not justified), we expect that similar mechanisms of wave localization will remain generally valid.

In conclusion, we have described for the first time, to the best of our knowledge, guided waves in left-handed slab waveguides. We have demonstrated a number of exotic properties of such waves, including the absence of fundamental modes and the sign-varying energy flux, and we have predicted the existence of the fundamentally different classes of guided waves with a vortex-type internal structure.

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