Nonlinear drift waves in electron-positron-ion plasmas

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It is suggested that low-frequency drift waves can play an important role in the dynamics of electron-positron plasmas comprising some concentration of ions. In the electromagnetic case the drift wave couples with the shear Alfvén wave in an electron-positron-ion plasma. The drift wave frequency can be very low in such plasmas depending on the concentration and density scale lengths of the plasma components. In the nonlinear regime these waves can give rise to dipolar vortices in both electrostatic and electromagnetic limits. The velocity of the nonlinear structure turns out to be different compared to the case of an electron-ion plasma.

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The investigation of collective phenomena in electron-positron (pair) plasmas in the past has been the subject of studies dealing mainly with some astrophysical objects, such as active galactic nuclei, early stage in the evolution of Universe, as well as with pulsar magnetospheres (with extreme physical conditions which include magnetic fields of the magnitudes up to 10^8 T, and magnetic-field-aligned electric fields of about 10^{10} V/m). In the pair plasma the plasma frequency ω_p is modified [1], i.e., it is $\omega_p^2 = 2\omega_e^2$, where $\omega_e = n_0 e^2/\varepsilon_0 m_e$, as well as some other quantities such as the Debye length, the Alfvén velocity, etc. The other very important difference, in comparison to standard plasmas, is certainly the annihilation process. In some astrophysical situations the analysis of corresponding annihilation spectra allows for the estimate of the conditions in the environment where the annihilation takes place. It can be shown [2] that usually the pair plasma will last sufficiently long for the collective interaction to take place; otherwise a process of creation of pairs is required to balance the annihilation rate. This is the situation in laboratory environments as well, where in fact the annihilation is not of much importance; the annihilation time turns out to be of the order of 1 s at temperatures of about 1 eV and the electron concentration of 10^{12} cm^{-3} [3]. As for the investigation of waves in pair plasmas it started as far back as in 1978 [4]. A review of standard modes in a pair, unmagnetized and magnetized, plasma can be found in Ref. [2]. In laboratory conditions positrons are most efficiently accumulated from some radioactive source, and then cooled to room temperature in a few seconds [3].

Experimental studies and the corresponding analytical modeling can be used for the understanding of physical processes in astrophysical situations as well. In such situations the presence of ions should be taken into account. This is clear bearing in mind the recent progress in producing antihydrogen in laboratory conditions [5,6]. Various aspects of the presence of ions in pair plasmas have been discussed in Refs. [7–12].

Due to the presence of ions, low-frequency electrostatic ion acoustic waves can exist in electron-positron-ion plasmas. The nonlinear study of this mode has shown that the amplitude of density humps can reduce due to the presence of positrons in electron-ion plasmas [8]. The effects of stationary ions were incorporated into the study of nonlinear coupling of low-frequency electrostatic and electromagnetic waves in a strongly magnetized nonuniform electron-positron plasmas [9]. In Ref. [11], the nonlinear dynamics of drift-Alfvén waves in an inhomogeneous electron-positron plasma with a small admixture of heavy ions has been studied. It has been shown that electromagnetic perturbations in the presence of heavy ions in a relativistic electron-positron plasma can saturate into two-dimensional dipolar vortices. The vortices appear due to the vector nonlinearity that follows from the convective derivatives in corresponding momentum equations.

In the case of standard electron-ion plasmas, vortices may appear in the presence of a nonlinear electrostatic drift wave [13]. This is a low-frequency wave, in comparison with the ion gyrofrequency $\Omega_i = eB_0/m_i$, where m_i is the ion mass, e is electron charge, and B_0 is the external magnetic field, with the perpendicular (with respect to the magnetic field) wave number $k_\perp$ which is much larger than the parallel wave number $k_\parallel$. Theory of drift waves has begun about 40 years ago by theoretical predictions [14,15] and experimental verifications [16], and has been developed intensively in the past several decades [17–19]. The nonlinear electromagnetic drift waves were studied in an electron-ion plasma long ago [18]. The electron and ion mass difference, and correspondingly their slow and rapid response to the electric field in the presence of a density gradient, is a typical feature of the drift wave. Therefore in equal mass plasmas, such as electron-positron (pair) plasmas, these waves do not appear.

In this study we show that the presence of ions in a nonuniform electron-positron plasma can introduce important low-frequency phenomenon, the drift wave. Both the nonlinear dispersion relation and the nonlinear wave structure of the electrostatic drift wave are modified in the electron-positron-ion plasma, similar to the ion acoustic wave studied a few years ago [8]. The drift wave can also couple with Alfvén waves and hence the electromagnetic drift waves can appear in the electron-positron-ion plasmas as well.

We consider an electron-positron-ion plasma embedded in a uniform magnetic field $\vec{B}_0 = \vec{e}_z B_0$, and having density gradient along the x axis. The background densities of all three plasma species obey the quasineutrality condition in the equilibrium:

$$n_{e0}(x)=n_{i0}(x)+n_{p0}(x). \tag{1}$$

The perturbed $\vec{E}$ and $\vec{B}$ are written as

$$\vec{E}=-\vec{\nabla}\phi-\frac{1}{c}\frac{\partial A_z}{\partial t}\vec{e}_z,\quad \vec{B}=-\vec{e}_z\times\vec{\nabla}_\perp A_z,\quad \vec{A}_z=A_z\vec{e}_z.$$

Ampère's law gives

$$\vec{\nabla}\times\vec{B}=\frac{4\pi}{c}\vec{j}\quad\text{and}\quad j_z\approx-\frac{c}{4\pi}\nabla_\perp^2 A_z.$$

The equation of motion for the jth species is

$$\left(\frac{\partial}{\partial t}+\vec{v}_j\cdot\vec{\nabla}\right)\vec{v}_j=\frac{q_j}{m_j}\vec{E}+\frac{q_jB_0}{m_j}\vec{v}_j\times\vec{e}_z-\frac{1}{n_jm_j}\vec{\nabla}p_j, \tag{2}$$

where $j=i,e,p$. With the help of this equation, the perpendicular velocity components can be written as

$$\vec{v}_{\perp e}\approx\frac{c}{B_0}\vec{E}\times\vec{e}_z-\frac{cT_e}{eB_0n_e}\vec{e}_z\times\vec{\nabla}n_e+v_{ez}\frac{\vec{B}_\perp}{\vec{B}_0}, \tag{3}$$

$$\vec{v}_{\perp p}\approx\frac{c}{B_0}\vec{E}\times\vec{e}_z+\frac{cT_p}{eB_0n_e}\vec{e}_z\times\vec{\nabla}n_e+v_{pz}\frac{\vec{B}_\perp}{\vec{B}_0}, \tag{4}$$

$$\vec{v}_{\perp i}\approx\frac{c}{B_0}\vec{E}\times\vec{e}_z+\frac{cT_i}{eB_0n_e}\vec{e}_z\times\vec{\nabla}n_e+\frac{c}{B_0\Omega_i}\left[\frac{\partial}{\partial t}+\frac{c}{B_0}\vec{e}_z\times\vec{\nabla}_\perp\phi\cdot\vec{\nabla}_\perp+v_{jz}\frac{\partial}{\partial z}\right]\vec{\nabla}_\perp\phi. \tag{5}$$

The charge conservation equation becomes

$$\frac{\partial}{\partial t}(n_i+n_p-n_e)+\vec{\nabla}_\perp\cdot(n_i\vec{v}_{\perp i}+n_p\vec{v}_{\perp p}-n_e\vec{v}_{\perp e})+\frac{\partial}{\partial z}\left(\frac{j_z}{e}\right)=0. \tag{6}$$

Using the quasineutrality $n_i+n_p\sim n_e$ and Ampère's law, the charge conservation equation yields

$$\frac{cn_{i0}}{B_0\Omega_i}d_t\nabla_\perp^2\phi=-\frac{c}{4\pi e}\partial_z\nabla_\perp^2A_z, \tag{7}$$

where $d_t=\partial/\partial t+\vec{v}_E\cdot\vec{\nabla}$ and the ion continuity equation can be written as

$$d_tn_i-\frac{c}{B_0}(\vec{e}_z\times\vec{\nabla}n_i\cdot\vec{\nabla}_\perp)\phi-\frac{cn_{i0}}{B_0\Omega_i}d_t\nabla_\perp^2\phi=0. \tag{8}$$

The parallel equation of motion for electrons and positrons becomes

$$\left(\frac{\partial}{\partial t}+\vec{v}_j\cdot\vec{\nabla}\right)j_z\approx\frac{(n_{e0}+n_{p0})}{mc}\left[-cd_z\phi-\frac{cT}{e^2}d_z\left(\frac{n_p-n_e}{n_{e0}+n_{p0}}\right)-\left(\frac{\partial}{\partial t}-\frac{cT}{eB_0}\frac{\vec{e}_z\times\vec{\nabla}n_{i0}}{n_{e0}+n_{p0}}\cdot\vec{\nabla}\right)A_z\right] \tag{9}$$

or

$$\left(\frac{\partial}{\partial t}+N_0v_0^*\frac{\partial}{\partial y}\right)A_z=\lambda_{ep}^2d_t\nabla_\perp^2A_z-cd_z\left(\phi-\frac{T}{e}N_0\frac{n_i}{n_{i0}}\right). \tag{10}$$

Here $N_0=n_{i0}/(n_{e0}+n_{p0})$, $d_z=\partial/\partial z+(\vec{\nabla}_\perp A_z\times\vec{e}_z\cdot\vec{\nabla}_\perp)/B_0$, $v_0^*=cT|\kappa_i|/eB_0$, and $\kappa_i=-dn_{i0}/n_{i0}dx$, $\lambda_{ep}^2=c^2/(\omega_{pe}^2+\omega_{pp}^2)$ with $\omega_{pe}^2=4\pi n_{e0}e^2/m$, and $\omega_{pp}^2=4\pi n_{p0}e^2/m$. In deriving these equations we have taken $T_e=T_p$ as a natural condition for the equal mass particles and ignored the parallel velocity of ions and the temperature T_i, while $\vec{V}_E\cdot\vec{\nabla}_\perp\gg v_{jz}\partial_z$ has been assumed. Note that $\vec{V}_E=c\vec{E}\times\vec{e}_z/B_0$ is the electric drift.

In the linear domain, perturbations proportional to $\exp[i(k_yy+k_zz-\omega t)]$ are assumed. From Eqs. (7), (8), and (10) we obtain the linear dispersion relation of electromagnetic drift waves in the electron-positron-ion plasma as

$$\left(\omega^2-\frac{N_0\omega\omega^*}{a_{ep}}-\frac{\omega_A^2\rho_s^2k_y^2}{a_{ep}}\right)\omega=\frac{\omega_A^2}{a_{ep}}(\omega-N_0\omega^*), \tag{11}$$

where $\omega^*=v_0^*k_y$, $\omega_A^2=v_A^2k_z^2$, and $v_A^2=B_0^2/(4\pi n_{i0}m_i)$. For $N_0=1$ and $a_{ep}=1+\lambda_e^2k_\perp^2$, with $\lambda_e^2=c^2/\omega_{pe}^2$, Eq. (11) will become the linear dispersion relation for the case of pure electron-ion plasma.

Now we look for traveling solutions that are stationary in the reference frame $\xi=y+\alpha z-ut$, which moves with the velocity u. Nonlinear equations (7), (8), and (10) can be written, respectively, as

$$\hat{D}_\phi\nabla_\perp^2\bar{\phi}=-\frac{v_A^2}{cu}\alpha\hat{D}_A\nabla_\perp^2\bar{A}_z, \tag{12}$$

$$\hat{D}_\phi\left[\frac{n_i}{n_{i0}}-\left(\frac{v_0^*}{u}+\rho_s^2\nabla_\perp^2\right)\phi)\right]=0, \tag{13}$$

and

$$c\alpha\hat{D}_A\left[\left(\frac{N_0v_0^*-u}{c\alpha}\right)A_z+\left(\phi-N_0\frac{n_i}{n_{i0}}\right)\right]=0. \tag{14}$$

Here $\bar{\phi}=e\phi/T$, $\bar{A}_z=eA_z/T$, $\hat{D}_\phi=\{\partial_\xi+C(\partial_x\phi\partial_\xi-\partial_\xi\phi\partial_x)/MB_0\}$, and

$$\hat{D}_A=\left\{\partial_\xi+\frac{1}{\alpha B_0}(\partial_xA_z\partial_\xi-\partial_\xi A_z\partial_x)\right\}.$$

The $\hat{D}_\phi$ and $\hat{D}_A$ operators satisfy the relation

$$\hat{D}_\phi\left(\phi-\frac{u}{\alpha c}A_z\right)=\hat{D}_A\left(\phi-\frac{u}{\alpha c}A_z\right). \tag{15}$$

The trivial solution of Eq. (13) yields

$$\frac{n_i}{n_{i0}}=\frac{v_0^*}{u}\bar{\phi}+\rho_s^2\nabla_\perp^2\bar{\phi}.$$

Using the above relation for n_i/n_{i0} in Eq. (14), we can obtain as a trivial solution, the following relation:

$$\nabla_\perp^2 \overline{\phi} = G_0\left(\overline{A_z} - \frac{c\alpha}{u}\overline{\phi}\right), \tag{16}$$

where

$$G_0 = \frac{1}{N_0\rho_s^2}\left(\frac{N_0 v_0^* - u}{c\alpha}\right).$$

Equations (12), (15), and (16) yield

$$\hat{D}_A\left[\nabla_\perp^2 \overline{A_z} - P_0\left(\overline{A_z} - \frac{c\alpha}{u}\overline{\phi}\right)\right] = 0.$$

Here $P_0 = cuG_0/v_A^2$, and its solution can be written as

$$\nabla_\perp^2 \overline{A_z} - P_0\left(\overline{A_z} - \frac{c\alpha}{u}\overline{\phi}\right) = f(\overline{A_z} + \alpha B_0 x). \tag{17}$$

The function $f = f(\overline{A_z} + \alpha B_0 x)$ for simplicity can be assumed to be linear such that $f = C_1(\overline{A_z} + \alpha B_0 x)$, where C_1 is a constant. Equations (16) and (17) yield

$$\nabla_\perp^4 \overline{\phi} + b\nabla_\perp^2 \overline{\phi} + d\overline{\phi} - C_1\alpha B_0 x = 0, \tag{18}$$

where

$$b = \left(\frac{c\alpha}{u}G_0 - P_0 - \frac{C_1}{G_0}\right) \quad \text{and} \quad d = -\frac{c\alpha}{u}C_1.$$

To find the solution of Eq. (18) we divide the space by a circle of radius r_0, and search for solutions independently in the outside $(r > r_0)$ and the inside region $(r < r_0)$.

In the outside region, to avoid growing solutions, we choose $C_1 = 0$, and Eq. (18) becomes

$$\nabla_\perp^4 \overline{\phi} - b_1^2 \nabla_\perp^2 \overline{\phi} = 0, \tag{19}$$

where $b_1^2 = c\alpha G_0/u - P_0$. Its solution is

$$\overline{\phi_{out}}(r,\theta) = [A_1 K_1(b_1 r) + A_2/r]\cos\theta, \tag{20}$$

where A_1 and A_2 are arbitrary constants and K_1 is the modified Bessel function. It should be noted that $b_1 > 0$ for $C_1 = 0$, which is the condition for the localized solution.

For the inside solution, the x dependence should be taken into account as well. For this purpose we take $C_1 \neq 0$. The linear combination of the solutions of homogeneous and nonhomogeneous parts of the equation gives

$$\overline{\phi_{in}}(r,\theta) = \left[A_3 J_1(b_2 r) + A_4 I_1(b_3 r) + \frac{C_1\alpha B_0}{d} r\right]\cos\theta, \tag{21}$$

where

$$b_{2,3}^2 = \tfrac{1}{2}[\sqrt{b^2 - 4d} \pm b],$$

with the condition $d < 0$. Here J_1 and I_1 are the ordinary and modified Bessel functions, respectively.

We can find the corresponding vector potential from Eq. (16) as

$$\overline{A_{z,out}}(r,\theta) = \left[\frac{1}{G_0}\left(b_1^2 + \frac{G_0 c\alpha}{u}\right)A_1 K_1(b_1 r) + \frac{G_0 c\alpha}{ur}A_2\right]\cos\theta, \tag{22}$$

and

$$\overline{A_{z,in}}(r,\theta) = \left[\frac{1}{G_0}\left(-b_2^2 + \frac{G_0 c\alpha}{u}\right)A_3 J_1(b_2 r) + \left(b_3^2 + \frac{G_0 c\alpha}{ur}\right)A_4\right]\cos\theta. \tag{23}$$

The arbitrary constants A_1, A_2, A_3, A_4 are to be found from the corresponding continuity conditions for $\overline{\phi}, \partial_r\overline{\phi}, \nabla^2\overline{\phi}$, and for $\overline{A}, \partial_r\overline{A_z}, \nabla^2\overline{A_z}$ at $r = r_0$. The solutions presented by Eqs. (20)–(23) are for the case of electromagnetic drift waves.

In the electrostatic limit, we ignore the electron and positron inertia, i.e., they follow the Boltzmann distributions. Then we have

$$n_e = n_{e0}\exp\left(\frac{e\phi}{T_e}\right), \quad n_p = n_{p0}\exp\left(-\frac{e\phi}{T_p}\right). \tag{24}$$

The quasineutrality in the perturbed state yields

$$n_i = n_{e0}(x)\exp\left(\frac{e\phi}{T_e}\right)\left\{1 + \alpha_0\exp\left[-\left(1 + \frac{T_e}{T_p}\right)\frac{e\phi}{T_e}\right]\right\}, \tag{25}$$

where $\alpha_0 = -n_{p0}/n_{e0}$. The ion equation of motion, along with the Eqs. (24) and (25), yields

$$\left(\frac{\partial}{\partial t} + \frac{c}{B_0}\vec{e}_z \times \vec{\nabla}\phi \cdot \vec{\nabla}\right)\left[\frac{T}{e}\ln n_{i0} + \left(1 + 2\frac{n_{p0}}{n_{i0}}\right)\phi - \rho_s^2\nabla^2\phi\right] = 0, \tag{26}$$

which is a modified Hasegawa-Mima equation [13]. In deriving Eq. (26) we have again assumed $T_e = T_p$. From Eq. (26) we obtain the linear dispersion relation for electrostatic drift waves in the electron-positron-ion plasma as

$$\omega = -\frac{c_s^2 k_y}{\Omega_i}\frac{n_{i0}'}{n_{i0}}\frac{1}{1 + 2\dfrac{n_{p0}}{n_{i0}} + \rho_s^2 k_y^2} \equiv \frac{c_s^2 k_y}{\Omega_i n_{i0}\left(1 + 2\dfrac{n_{p0}}{n_{i0}} + \rho_s^2 k_y^2\right)}(n_{e0}' - n_{p0}'), \tag{27}$$

or

$$\omega=\frac{\omega^*}{\left(1+2\frac{n_{p0}}{n_{i0}}+\rho_s^2k_y^2\right)},$$

where $c_s^2=T/m_i$, $\rho_s=c_s/\Omega_i$, and the prime denotes derivatives in the x direction. In standard two-component quasineutral plasmas, the drift frequency is determined by the plasma density gradient; here, however, the drift frequency can vary depending on the gradients of the plasma species. In the limit $n'_{e0}\to n'_{p0}$, the frequency goes to zero. Physically, the meaning of the right hand side of Eq. (27) is clear bearing in mind that the propagation of a drift wave is closely related to the density gradient, as well as to mobility of lighter particles (electrons and positrons) along the magnetic-field lines. The perturbation of density is proportional to the perturbation of electrostatic potential, which causes the ion oscillation in the perpendicular (to the wave vector) direction. However, the light particles are of the opposite sign and with the same mass (i.e., mobility) that will influence the magnitude of the potential perturbation, and consequently the ion motion as well. In the absence of positrons Eq. (27) reduces to the well-known dispersion relation of the drift wave in the standard electron-ion plasma.

The localized solution of Eq. (26) for the outer region $(r>r_0)$ can be written in terms of the modified Bessel function $K_1(r)$ as

$$\phi_{out}(r,\theta)=Q_1K_1(\lambda_1 r)\cos\theta, \tag{28}$$

where Q_1 is the integration constant and

$$\lambda_1^2=\frac{-v_0^*+\left(1+2\frac{n_{p0}}{n_{i0}}\right)u}{u\rho_s^2}>0. \tag{29}$$

The inner solution $(r<r_0)$ turns out to be

$$\phi_{in}(r,\theta)=\left[Q_2J_1(\lambda_2 r)+\frac{\lambda_1^2+\lambda_2^2}{\lambda_2^2}ruB_{0/c}\right]\cos\theta, \tag{30}$$

where J_1 is the Bessel function, and Q_2 and λ_2^2 are new constants of integration that should be, together with Q_1, determined from physically justified continuity conditions of the solution at boundary of the circle, i.e., from the continuity of ϕ, $\vec{\nabla}\phi$, and $\nabla^2\phi$.

In conclusion, we have studied the drift waves in an electron-positron-ion plasma. It is important to note that the low-frequency drift waves do not occur in electron-positron plasmas. However, a small concentration of ions in such a system can introduce these low-frequency fluctuations provided that the density is nonuniform. In the standard (electron-ion) plasma case the electrostatic vortex propagates with the velocity that exceeds the phase speed v_0^* of the linear mode, or the propagation is in opposite direction. Here, we see from the condition of localized solutions (29) that it is satisfied even if $u=v_0^*$. Otherwise we find that $u>v_0^*/(1+2n_{p0}/n_{i0})$, i.e., the magnitude of the propagating speed can be less than the phase speed of the linear mode. The frequency of the drift mode can be very small in the electron-positron-ion plasmas if $n'_{e0}\to n'_{p0}$. In this situation some higher-order effects should be taken into account as well, such as the parallel ion motion, which will introduce the ion acoustic wave dynamics. Thus the drift-wave dynamics may be of importance for the wave propagation in laboratory pair plasmas as well as in astrophysical objects. Such plasmas support low-frequency drift-wave-type fluctuations as long as ions are present. Therefore, we believe that further study in this direction should be interesting.

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