Simple model of the aging effect in heart interbeat time series

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In this work, we calculate the fractal dimension of heart interbeat time series of some healthy young and elderly individuals. As has been found by means of other methods (detrended fluctuation and spectral analyses), we also find that interbeat series of healthy young subjects can be characterized by only one scaling exponent and a crossover behavior in it is observed with aging. By means of a zoom over the hinges of the crossover region, interesting effects of aging are presented. Our results with real interbeat time series are reasonably reproduced by using a simple model based on combinations of noisy first-order autoregressive series.

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I. INTRODUCTION

Heart rate dynamics is related to a large number of control mechanisms. Heartbeat fluctuations are a very complex manifestation of regulatory neuroautonomic feedback loops [1]. In recent years, fluctuations of this physiological signal have been studied by means of several methods derived from nonlinear dynamics and statistical physics, such as detrended fluctuation analysis (DFA) [2,3], spectral analysis [4,5,15], entropy (approximate and sample) [6,7], and correlation dimension [8]. In particular, fractal methods have been proved to assess diverse characteristics and changes in heart rate dynamics. These methods are strongly related to the fact that irregularity of the beat-to-beat time series for the case of healthy human heartbeat exhibits an absence of characteristic time scales compatible with the concept of adaptability understood as a system's repertoire of responses to environmental stimuli. Heart rate variability has been proposed as an important marker of changes at the level of neuroautonomic control [9,10]. Declination in the neuroautonomic control of heart as a process occurring with aging and some heart failure has been proposed [5,11]. An important question related with aging is to quantify the loss of $1/f$-like behavior (long-range correlations) as a synonym of healthy heart variability towards degradated regimes (as that proposed by Iyengar *et al.* [5] to model healthy very elderly subjects). These authors reported by means of DFA and spectral analysis that with aging a crossover phenomenon appears with respect to the former monofractal behavior corresponding to young healthy individuals. This crossover behavior occurs in the interbeat scaling exponents, from a higher value of α (the DFA exponent), close to Brownian noise for fluctuations on small time scales, to a lower value of α (close to white noise) for large time scales. Iyengar *et al.* modeled this type of crossover behavior by a simple stochastic model consisting in a noisy first-order autoregressive process that gives a reasonable fit with the fluctuations of interbeat interval for four healthy elderly subjects (three of them the oldest in their sample, 76, 77, and 81 yr). In the present paper, within the spirit of the findings of Iyengar *et al.*, we propose a method to study the evolution of interbeat time series with aging. Our approach is based on the fractal analysis proposed by Higuchi [12] and we find the crossover phenomena associated with aging and model them by means of combinations of first-order autoregressive processes that mimic both the young $1/f$-like behavior and the evolution of interbeat time series with aging. This paper is organized as follows. In Sec. II, we briefly introduce the Higuchi's method and apply it to heart interbeat time series of two groups of individuals: Healthy young and healthy elderly subjects. In Sec. III, we propose a numerical model to simulate the results observed in the preceding section and finally, we give some conclusions in Sec. IV.

II. FRACTAL APPROACH TO RR-TIME SERIES

As asserted by Goldberger *et al.* [9], the output of healthy living systems, under certain parameter conditions, reveals a type of complex variability associated with long-range (fractal) correlations. Although nowadays it is recognized that heart interbeat (RR-) time series display multifractal properties [10], in a first approximation one can study them by means of a monofractal approach [3,5]. Higuchi [12] proposed a technique to measure the fractal dimension which gives stable indices even for small number of data. The method consists in considering a finite set of data taken at an interval $\nu_1, \nu_2, \ldots, \nu_N$. From this series, we construct new time series ν_m^k, defined as

$$\nu(m), \nu(m+k), \nu(m+2k), \ldots, \nu\left(m+\left[\frac{N-k}{k}\right]k\right)$$

$$\text{with } m=1,2,3,\ldots,k, \tag{1}$$

where [ ] denotes Gauss' notation, that is, the bigger integer, and m and k are integers that indicate the initial time and the interval time, respectively. The length of the curve v_m^k is defined as

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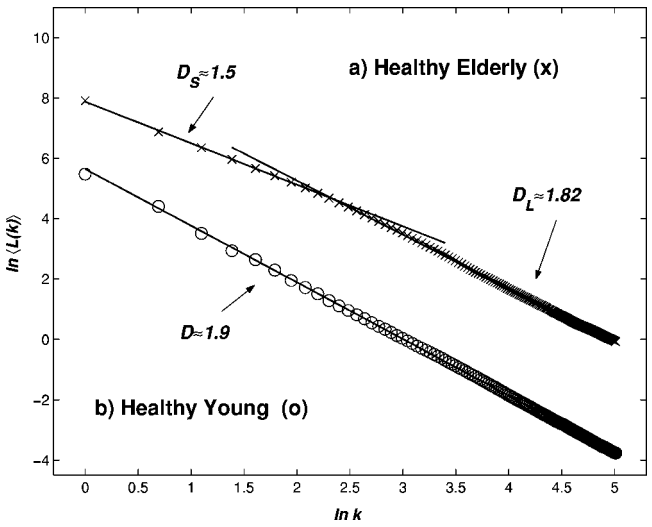

FIG. 1. Log-log plot of $\langle L(k)\rangle$ vs k for representative cases of (a) healthy elderly and (b) healthy young subjects.

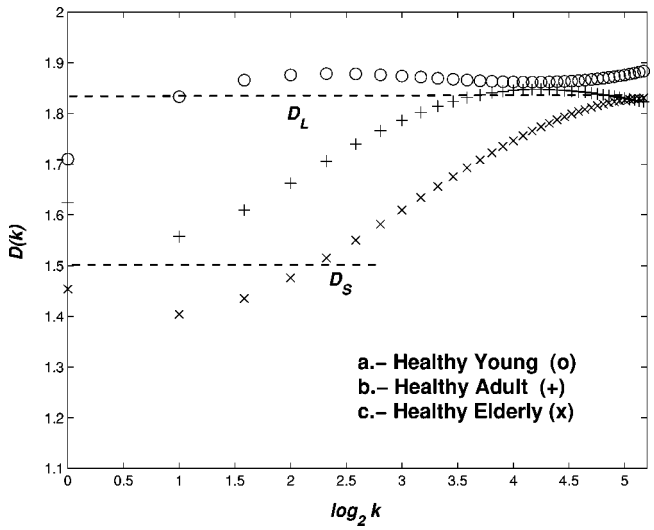

FIG. 2. Plot of $D(k)$ vs $\log_2 k$, in the region of short scales for three representative cases: healthy young, healthy adult, and healthy elderly subjects. In this region, the aging effect is quite evident.

$$L_m(k)=\frac{1}{k}\left[\left(\sum_{i=1}^{[(N-m)/k]}\right|\nu(m+ik)-\nu[m+(i-1)k]\left|\right)\frac{N-1}{\left[\frac{N-m}{k}\right]k}\right] \tag{2}$$

and the term $(N-1)/[(N-m)/k]k$ represents a normalization factor. Then, the length of the curve for the time interval k is given by $\langle L(k)\rangle$: the average value over k sets $L_m(k)$. Finally, if $\langle L(k)\rangle \propto k^{-D}$, then the curve is fractal with dimension D. For the case of self-affine curves, this fractal dimension is related to the spectral exponent β by means of $\beta=5-2D$. If D is in the interval $1<D<2$ then $1<\beta<3$ [12]. Higuchi showed that this method provides an accurate estimation of the fractal dimension and has advantages over conventional methods. One important feature of this method is that is very sensitive to changes in the self-organization (fractal) and it may reflect this fact through changes of the fractal dimension over several scales, giving an important tool to study the crossover phenomena.

We analyze beat-to-beat time series obtained from ten healthy young subjects (age 21–31 yr), eight healthy elderly subjects (age 70–81 yr), and one 58 yr old healthy individual [13]. All records were sampled at 250 Hz under repose conditions. In the present study, we analyze only short segments of ECG's (2 h), equivalent to $\approx$8000 beats. By using the Higuchi's algorithm described above, we calculate the fractal dimension of all series. In Fig. 1, we present log-log plots of $\langle L(k)\rangle$ versus k for representative cases from each group; (a) a healthy elderly subject, (b) a healthy young subject. In all of the healthy young subject cases, a single fractal dimension value is needed to fit the data, but in the cases of healthy elderly subjects, two fractal dimension values are required. The fractal dimension associated with healthy young subjects lies within the range of $D\approx 1.874\pm 0.0213$ (fractal dimension value$\pm$standard deviation), which corresponds to a spectral exponent $\beta\approx 1.26$, within the range of $1/f$-like behavior. This monofractal behavior for healthy young individuals has been reported by means of other methods such as DFA and power spectral analysis [3,5]. On the other hand, healthy elderly individuals present a clear crossover phenomenon, which has been reported by means of DFA analysis [5]. In this case we find two regions: over short scales ($k<k_C \approx 9$) fractal dimension is in the range $D_S\approx 1.519\,59 \pm 0.0841$, whereas for lags $k>k_C\approx 9$, $D_L\approx 1.826\,57 \pm 0.0931$. By using Student's t-test, we find that there is a highly significant difference between D_S and D_L for elderly subjects ($P=0.0001$), but not for the young ($P=0.137$). We present an additional numerical study of the crossover phenomenon observed in the fractal dimension. Although apparently the two-segment curve drawn in the case presented in Fig. 1(a) satisfactorily fit the $\log(k)$ versus $\log\langle L(k)\rangle$ data, these plots show a round corner around a certain k_c. Since the fractal dimension D is defined by minus the slope of the straight line fitted to the $\log\langle L(k)\rangle$ versus $\log(k)$ points, we assume that the fractal dimension can be written as [12]

$$D(k)=-\frac{d\ln\langle L(k)\rangle}{d\ln(k)}. \tag{3}$$

In Fig. 2, the behavior of $D(k)$ with respect to $\log_2(k)$ is plotted [14]. The two dotted horizontal lines superposed in the figure indicate the values of D_L and D_S, which are obtained by fitting the two-segment curve showed in Fig. 1(a). In the case of healthy elderly subjects [Fig. 2(c)], clearly, $D(k)$ gradually becomes larger as k increases, and then saturates at $D\approx 1.82$, a small increase is observed at large k. It is noteworthy that $D(k)$ does not discontinuously change from D_S to D_L as a step function, but it shows a gradual increase as k increases. It is remarkable that in the plane $D(k)$ versus $\log_2 k$, as can be seen in Fig. 2, the aging effect over the

beat-to-beat time series is quite evident. In this figure, one can observe how for a healthy young individual [age 29 yr, Fig. 2(a)] the crossover follows a very soft small-slope path (this path smoothly tends to $D \approx 1.9$), while for a healthy adult person [age 58 yr, Fig. 2(b)] the transition is more remarkable, but less dramatic than in the case of a healthy elderly person [age 81 yr, Fig. 2(c)]. We believe that in the plane $D(k)$ versus $\log_2(k)$, one can observe the aging effect over RR-time series in a very clear fashion and apparently a correlation between the vertical deviations of $D(k)$ and the age is observed.

III. THE AUTOREGRESSIVE MODEL

A simple model of $1/f$ noise is a stochastic process composed of a superposition of many modes with exponential decay associated with different time constants [16]. One time constant can be obtained from a single first-order autoregressive process,

$$X_{t+\tau+1} = aX_{t-\tau} + \varepsilon_{t-\tau}, \tag{4}$$

where ϵ is a Gaussian distributed random variable and a is a coefficient that is related to correlations of events. We are interested in the case where $0 \leqslant a \leqslant 1$. Correlations between different events can be calculated as $C(\tau) = Aa^{\tau} = Ae^{\tau \ln a}$, with A a constant. Time constants are related to the correlation function as the characteristic time in which the correlation has decayed $1/e$. Thus, the autocorrelation function for a stochastic process with a single characteristic time is $C(\tau) = Ae^{-\tau/\tau_o}$, with $\tau_0 = -1/\ln a$, clearly τ_0 goes from zero to infinite while a varies from 0 to 1. By using the Weiner-Khinchine theorem [17] it is easy to show that the power spectrum of such a process is given by

$$s(f) = \frac{4A\tau_0}{1 + (2\pi f \tau_0)^2}. \tag{5}$$

This spectrum shows two different zones; for low frequencies ($f \ll 1/2\pi\tau_0$) it is constant with a white noise behavior and for high frequencies ($f \gg 1/2\pi\tau_0$) is a Brownian motion. It has been proposed that a linear superposition of many independent characteristic times with hyperbolic distribution leads to $1/f$ noise in a certain region [17]. The sum of many power spectra given by single $\tau_0' s$ is

$$S(f) = \int_0^{\infty} d\tau_0 s(f) P(\tau_0), \tag{6}$$

where $P(\tau_0)$ is the characteristic-time distribution of the form

$$P(\tau_0) = \begin{cases} c/\tau_0 & \text{if} \quad 0 < \tau_1 \leqslant \tau_0 \leqslant \tau_2 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

with c a normalization constant and τ_1, τ_2 being the lower and upper time interval limits, respectively. The integration of Eq. (6) leads to

$$S(f) = \frac{2Ac}{\pi f}[\arctan(2\pi\tau_2 f) - \arctan(2\pi\tau_1 f)]. \tag{8}$$

This expression can be separated in three regions;

$$S(f) \approx \begin{cases} 4Ac\Delta\tau, & 0 < f \ll \dfrac{1}{2\pi\tau_2} \ll \dfrac{1}{2\pi\tau_1} \\ \dfrac{Ac}{f}, & \dfrac{1}{2\pi\tau_2} \ll f \ll \dfrac{1}{2\pi\tau_1} \\ \dfrac{Ac\Delta\tau}{\pi^2\tau_1\tau_2 f^2}, & \dfrac{1}{2\pi\tau_2} \ll \dfrac{1}{2\pi\tau_1} \ll f, \end{cases} \tag{9}$$

where $\Delta\tau = \tau_2 - \tau_1$. In the first region (very low frequencies) the process is white noise type, with a flat power spectrum; in the second region ($1/2\pi\tau_2 \ll f \ll 1/2\pi\tau_1$) the process is $1/f$ type; and in the third region (very high frequencies) it is Brownian type.

As was reported by Iyengar *et al.* [5], healthy-elderly heart rate dynamics can be resembled by a single first-order autoregressive relation with a single characteristic time. We are interested in recuperating healthy heart rate dynamics and how it evolves to senescence. We use the simple model of superposition of many modes with exponential decay such as described above. The repertoire of characteristic times is obtained from the variation of the parameter a in the interval $0 \leqslant a \leqslant 1$. In case (i), we take a linear superposition of 18 time constants of the first-order autoregressive model given by Eq. (4), chosen equally spaced in the interval $[a_1, a_2] = [0.15, 0.95]$ (note that time constants are not equally spaced). In case (ii), we reduce the interval of parameter a (equally spaced) to $[a_1, a_2] = [0.65, 0.95]$, and take only nine time constants to perform the superposition. In case (iii), we consider only six time constants from the interval $[a_1, a_2] = [0.85, 0.95]$ to perform the superposition. The Higuchi analysis of cases (i) and (iii) and their comparison with real data is presented in Fig. 3. In this figure, one can observe that a single fractal dimension can be associated with the simulated case of a healthy young and a good agreement is observed with real data. Also, in the simulated case of a healthy elderly subject, we obtain a crossover as is observed in real data. It is interesting to note that in the case of the simulation of a healthy elderly subject, the left region of the separation generated by the crossover is Brownian type and a good agreement is observed with real data. It is important to note that the crossover point is given at $k_c \approx 9$ in both cases. By performing a zoom on the crossover point in simulated cases (Fig. 4), we roughly recover the behavior reported in real cases (see Fig. 2). In the case of healthy-young-simulated behavior, a very soft path is observed. As the interval and the number of time constants are reduced, a gradual decrease and a nonstep transition are observed around the crossover point.

IV. CONCLUSIONS

By means of the Higuchi's fractal approach, we find that RR-time series of young healthy individuals have a reason-

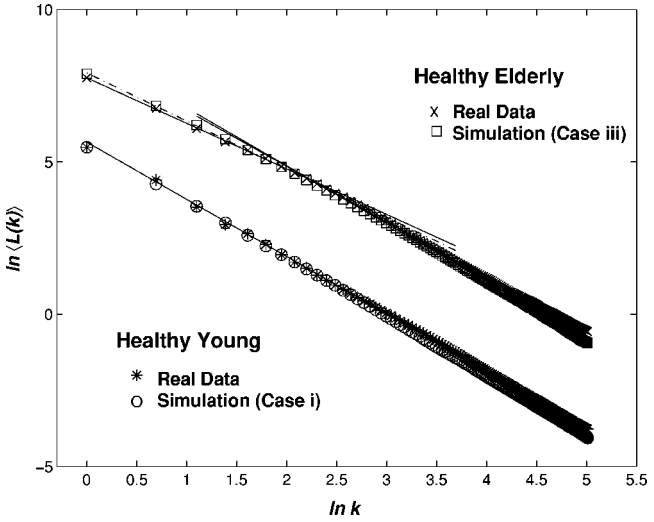

FIG. 3. Log-log plot of $\langle L(k)\rangle$ vs k. A comparison between simulation and real data for healthy elderly and healthy young persons is depicted.

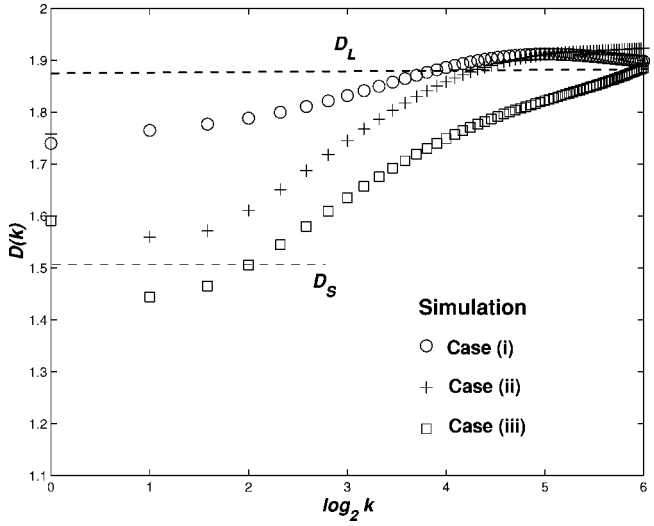

FIG. 4. Zoom on the region of short scales for simulated cases (i), (ii), and (iii).

able monofractal behavior with long-range correlations. This behavior is usually taken as a sign of cardiac health that is gradually lost with aging. This is apparently expressed through the appearance of the crossover phenomena in the RR-time series. We also observe this feature in the fractal dimension. When we apply a zoom over the hinges corresponding to the crossover points, we find that the size of the transition of D from the region of low lags to that of greater lags can work, probably, as an auxiliary biomarker of physiological aging. Some of these properties of actual RR-time series are resembled by means of a simple statistical model based on first-order autoregressive processes. This simple model suggests to consider aging as a gradual loss of heart adaptability understood as a system's repertoire of responses to environmental stimuli. This is expressed as the diminution of the number of characteristic times needed to simulate the RR-time series and also as the diminution of the a-coefficient interval.

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