PHYSICAL REVIEW E **67**, 051924 (2003)

# Estimate of the vibrational frequencies of spherical virus particles

L. H. Ford*
*Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155*
(Received 21 March 2003; published 23 May 2003)

The possible normal modes of vibration of a nearly spherical virus particle are discussed. Two simple models for the particle are treated, a liquid drop model and an elastic sphere model. Some estimates for the lowest vibrational frequency are given for each model. It is concluded that this frequency is likely to be of the order of a few GHz for particles with a radius of the order of 50 nm.

DOI: 10.1103/PhysRevE.67.051924 PACS number(s): 87.50.Kk, 43.64.+r, 87.15.La

## I. INTRODUCTION

Virus particles (virions) come in a variety of sizes and shapes. However, approximately spherical shapes with diameters in the range between 50 nm and 100 nm are especially common. Many nearly spherical viruses are revealed by x-ray crystallography to have icosahedral symmetry. A typical virus particle contains genetic material, RNA or DNA, surrounded by a protein coat (capsid). Such an object should have reasonably distinct vibrational frequencies, the study of which may be of interest. Excitation of these vibrations could have applications in either the diagnosis or treatment of viral diseases. To this author's knowledge, the sole discussion of these vibrational modes in the literature is that of Babincová *et al.* [1]. These authors discuss the conjecture that the ultrasound in the GHz range could be resonantly absorbed by HIV virus particles, leading to their destruction. Cooper *et al.* [2] have recently reported the detection of viruses by acoustic oscillations. However, the process of "rupture event scanning," which these authors report, involves the separation of a virus particle from antibodies by ultrasound. This is distinct from the excitation of the vibrational modes of the virus particle itself, and occurs at much lower frequencies.

There have also been some experimental studies of ultrasonic absorption by empty viral capsids [3,4]. These experiments reveal an enhanced absorption in the MHz range as proteins reassemble into a capsid, but do not find a resonant peak in this frequency range. Witz and Brown [5] have emphasized that these and other results show that viral capsids are flexible and change size or shape in response to vibrations or to changes in temperature or $p$H.

The purpose of the present paper is to provide some estimates of the lowest vibrational frequencies of a spherical virus particle. The simplest estimate is to take this frequency to be of the order of a characteristic speed of sound divided by the size of the virus particle. This is the estimate used in Ref. [1]. For the purpose of giving a more accurate estimate, we will examine two models, which treat the particle (1) as a liquid drop and (2) as a uniform elastic sphere. Similar models have been used by Bulatov *et al.* [6] to estimate the vibrational frequencies of nanoclusters.

*Email address: ford@cosmos.phy.tufts.edu

## II. A LIQUID DROP MODEL

Consider a sphere of radius $a$ filled with a nonviscous liquid with surface tension $\gamma$ and mass density $\rho$. The lowest vibrational mode of this sphere will be a quadrupole mode with frequency [7]

$$\nu = \frac{1}{\pi}\sqrt{\frac{2\gamma}{\rho a^3}}, \tag{1}$$

which can be written as

$$\nu = 3.4\times 10^8 \ \text{Hz}\left(\frac{50\ \text{nm}}{a}\right)^{3/2}\left(\frac{\rho_W}{\rho}\right)^{1/2}\left(\frac{\gamma}{\gamma_W}\right)^{1/2}, \tag{2}$$

where $\rho_W = 10^3$ kg/m$^3$ and $\gamma_W = 0.073$ Nm are the mass density and surface tension for water, respectively. The surface tension and mass density, along with the lowest vibrational frequency derived from Eq. (2) for $a = 50$ nm, are given in Table I for several liquids. Recall that Eq. (2) assumes a nonviscous liquid. In fact, the viscosity of many of the liquids in Table I cannot be neglected for such a small drop. When viscosity is sufficiently large, the drop will not oscillate, but rather undergo overdamped motion [7]. The main lesson from the liquid drop model is that a *nonviscous* liquid drop of $a = 50$ nm with a typical surface tension and mass density would have a lowest vibrational frequency of the order of a few times $10^8$ Hz.

TABLE I. The mass density, surface tension, and the lowest vibrational frequency predicted by Eq. (2) for drops of various liquids with a radius of $a = 50$ nm. The data for Benzene and Diethylene glycol [8] are for droplets in air at room temperature. The data for the three proteins [9] are for aqueous solutions at approximately 50 °C.

| Liquid | $\gamma/\gamma_W$ | $\rho/\rho_W$ | $\nu/(10^8$ Hz$)$ |
|---|---|---|---|
| Benzene | 0.397 | 0.88 | 2.3 |
| Diethylene glycol | 0.62 | 1.12 | 2.5 |
| Trehalose | 0.95 | 1.63 | 2.6 |
| Lysine hydrochloride | 0.90 | 1.38 | 2.7 |
| Arginine hydrochloride | 0.95 | 1.44 | 2.8 |

 

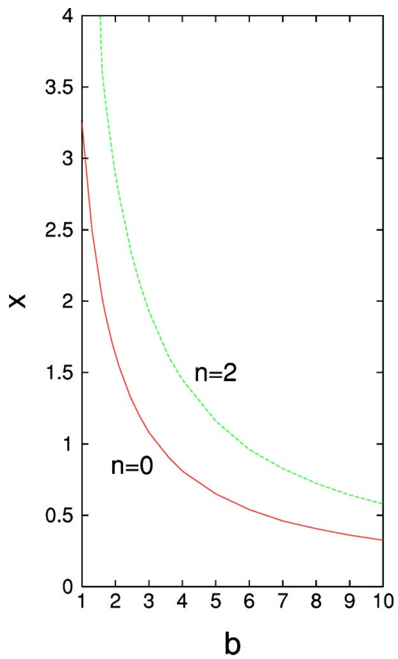


FIG. 1. The root $x$ of Eq. (3) is plotted as a function of $b=c_S/c_P$ for $n=0$ (compression mode) and $n=2$ (quadrupole deformation).

## III. AN ELASTIC SPHERE MODEL

A better model for a virus particle is to treat it as a uniform elastic sphere. The three independent parameters that characterize such a sphere can be taken to be the radius $a$, the speed of pressure waves $c_P$, and the speed of shear waves $c_S$. The oscillations of an elastic sphere are treated in detail by Pao and Mow [10]. Here we quote their results in the notation of Ref. [6]. The eigenfrequencies of the normal modes are given by the vanishing of the determinant

$$\begin{vmatrix} S_{rp}(n,x,y) & S_{rs}(n,y) \\ S_{tp}(n,x) & S_{ts}(n,y) \end{vmatrix} = 0, \tag{3}$$

where

$$S_{rp}(n,x,y) = \left(n^2 - n - \frac{1}{2}y^2\right)^2 j_n(x) + 2x j_{n+1}(x), \tag{4}$$

$$S_{rs}(n,y) = n(n+1)[(n-1)j_n(y) - x j_{n+1}(y)], \tag{5}$$

$$S_{tp}(n,x) = (n-1)j_n(x) - x j_{n+1}(x), \tag{6}$$

$$S_{ts}(n,y) = -\left(n^2 - n - \frac{1}{2}x^2\right)^2 j_n(x) - x j_{n+1}(x). \tag{7}$$

Here $x=\omega a/c_P$, $y=\omega a/c_S$, and the $j_n$ are the spherical Bessel functions. Let

$$b = \frac{c_S}{c_P} \tag{8}$$

be the ratio of the speed of the shear wave to that of the pressure wave, so that $y=bx$. Given $b$ and $n$, we can solve Eq. (3) for $x$ and hence for the frequency of the normal mode, $\nu = x\, c_P/(2\pi a)$. The smallest roots for $x$ for the $n=0$ and $n=2$ modes are plotted in Fig. 1. The corresponding frequencies of oscillation can be expressed as

$$\nu = 4.8\times 10^9 \ \text{Hz}\left(\frac{50 \ \text{nm}}{a}\right)\left(\frac{c_P}{1500 \ \text{m/s}}\right) x. \tag{9}$$

The frequencies obtained from this equation for the $n=0$ mode are given in Table II for various materials. We can see that for a wide range of materials, the lowest mode of vibration is a purely radial mode with a frequency of the order of a few times $10^9$ Hz for a sphere of radius $a=50$ nm.

TABLE II. The speeds of sound (in m/s) and the lowest vibrational frequency for the $n=0$ mode predicted by Eq. (9) for spheres of various materials with a radius of $a=50$ nm. In cases where no data for $c_S$ are available, the frequency is given as a multiple of $x$, which is likely to be less than 1 for these materials.

| Material | $c_P$ | $c_S$ | $\nu/(10^9$ Hz) |
|---|---|---|---|
| Nylon [11] | 2620 | 1070 | 11.4 |
| Polystyrene [11] | 2350 | 1120 | 11.3 |
| Polyethylene [11] | 1850 | 540 | 5.6 |
| Neoprene rubber [11] | 1600 | | 5 $x$ |
| Polynucleotides [12] | 1700–1900 | | (5.3–6) $x$ |
| Amino acids [12] | 1900–2400 | | (6–7.7) $x$ |
| Globular proteins [12] | 1700–1800 | | (5.3–5.7) $x$ |

## IV. CONCLUSIONS

In the previous sections, we have examined two models for a spherical virus particle, a liquid drop model and an elastic sphere model. It is of interest that the two models yield estimates for the lowest vibrational frequency which differ by only about 1 order of magnitude. Of the two models, the elastic sphere model is probably the better description of a virus particle. An even better model might be the one in which the particle has a liquid core (DNA or RNA) surrounded by an elastic outer shell (the capsid). Such a model would probably yield vibrational frequencies intermediate between those predicted by the two models discussed in this paper. In any case, we obtain an estimate for the lowest vibrational frequency of the same order of magnitude as that given in Ref. [1], in the range of a few GHz for particles with a size of about 100 nm. Of course, the existence of a resonance requires damping below the critical value, above which the overdamped motion occurs. Even if this condition is fulfilled, it is difficult to predict the width of the resonance. This remains a question for experimental investigation. The existence of well-defined resonances could prove valuable both for basic science and for medicine. Thus, this is a potentially fruitful area for further research.

## ACKNOWLEDGMENTS

I would like to thank V. Ford and D. Weaver for helpful comments. This work was supported in part by the National Science Foundation under Grant No. PHY-9800965.

[1] M. Babincová, P. Sourivong, and P. Babinec, Med. Hypotheses **55**, 450 (2000).
[2] M.A. Cooper, F.N. Dultsev, T. Minson, V.P. Ostanin, C. Abell, and D. Klenerman, Nat. Biotechnol. **19**, 833 (2001).
[3] R. Cerf, B. Michels, J.A. Schulz, J. Witz, P. Pfeiffer, and L. Hirth, Proc. Natl. Acad. Sci. U.S.A. **76**, 1780 (1979).
[4] B. Michels, Y. Dormoy, R. Cerf, and J.A. Schulz, J. Mol. Biol. **181**, 103 (1985).
[5] J. Witz and F. Brown, Arch. Virol. **146**, 2263 (2001).
[6] V.L. Bulatov, R.W. Grimes, and A.H. Harker, Philos. Mag. Lett. **77**, 267 (1998).
[7] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* (Oxford University Press, London, 1961) Sec. 99.
[8] *CRC Handbook of Chemistry and Physics*, edited by D.R. Lide (CRC Press, Boca Raton, FL, 2002), pp. 3-26, 3-159, 6-149, 6-150.
[9] T-Y Lin and S.N. Timasheff, Protein Sci. **5**, 372 (1996).
[10] Y-H Pao and C-C Mow, *Diffraction of Elastic Waves and Dynamic Stress Concentrations* (Crane Russak, New York, 1973), Chap. 6, Sec. 2.
[11] *CRC Handbook of Chemistry and Physics*, edited by D.R. Lide (CRC Press, Boca Raton, FL, 2002), pp. 14-41.
[12] A.P. Sarvazyan, Annu. Rev. Biophys. Biophys. Chem. **20**, 321 (1991).