Anomaly of the height-height correlation functions in self-flattening surface growth

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By Monte Carlo simulations and scaling theories, we consider the height-height correlation function $G(r,t;L)$ of the one-dimensional equilibrium self-flattening surface growths, where the deposition (evaporation) attempt only at the globally highest (lowest) site is suppressed. $G(r,t:L)$ is shown to satisfy the anomalous scaling behavior $G(r,t;L)=L^{2\alpha}g_1(r/L^{\delta},t/L^z)$ or $G(r,t;L)=t^{2\beta}g_2(r/t^{1/z'},L/t^{1/z})$. Here α, β, and z are the roughness, growth, and dynamic exponents, respectively, for the surface width, with $\alpha=1/3$ and $z=\alpha/\beta=3/2$. Anomalous exponents z' and δ are found to satisfy $z'=9/4$ and $\delta=z/z'$. We also show that anomalous behavior of $G(r,t;L)$ can be understood from a scaling theory based on the competition between local random-walk-like behavior and the global-length-scale suppression.

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The equilibrium and dynamical properties of surfaces have been studied extensively due to the theoretical importance of the scaling behaviors of statistical systems in addition to practical reasons of predicting the long time, large scale surface morphology [1]. For most surface models with a stochastic dynamics such as growths, evaporations, or fluctuations with thermal noises, it is known that the surface configurations show scaling behaviors. The dynamic scaling hypothesis is that in a finite system of lateral size L, the standard deviation or the root mean square (rms) fluctuation W of the surface height starting from a flat substrate scales as [1–3]

$$W^2(t;L)=L^{2\alpha}g_W(t/L^z), \tag{1}$$

where α and z are the roughness and its dynamic exponents respectively. The scaling function has the asymptotic form of $g_W(x)\sim x^{2\beta}$ for $x\ll 1$ and $g_W(x)\sim$const for $x\gg 1$ so that the width $W(t;L)$ increases as t^{β} initially ($t\ll L^z$) and saturates to L^{α} for $t\gg L^z$, where $\beta=\alpha/z$.

Another way to characterize the roughness of a surface is through the height-height correlation function $G(r,t)$, defined by the rms height differences between two sites separated by the distance r. For stochastic dynamic models [1], $G(r,t)$ also shows a similar dynamic scaling behavior,

$$G(r,t)=r^{2\alpha'}g_G(t/r^{z'}), \tag{2}$$

where α' and z' are the wandering exponent and the wandering dynamic exponent, respectively. Normally $\alpha'=\alpha$ and $z'=z$ are expected. The scaling function is also normally expected to have the following, asymptotic behavior:

$$g_G(x)\sim\begin{cases}x^{2\beta} & \text{for} \quad x\ll 1\\ \text{const} & \text{for} \quad x\gg 1.\end{cases} \tag{3}$$

However, $g_G(x)$ of some growth models [1,4–6] have been shown to have an anomalous behavior when $x\gg 1$, such as

$$g_G(x)\sim x^{\kappa/z} \quad \text{for} \quad x\gg 1. \tag{4}$$

This anomalous behavior means that the corresponding growth models have different roughening behaviors for local or short length scales $l(\ll L)$ from that for the global length scale L.

In this study, we want to report another kind of the anomalous behavior of $G(r,t;L)$, which arises from the regime of the global length scale. We study $G(r,t;L)$ of the self-flattening (SF) surface growths [7], which is believed to be physically related to the recently developed other surface growth models with the global constraints [8–10]. The self-flattening model is the same as the restricted solid-on-solid (RSOS) model [11,12] except for one variation to incorporate the global suppression: only when deposition (evaporation) is attempted at the globally highest (lowest) site, is the attempt accepted with probability u and rejected with probability $1-u$. At $u=1$, the ordinary RSOS model is recovered. In equilibrium when the deposition attempt probability p is the same as the evaporation attempt probability q, SF growth models produce an ensemble of RSOS surfaces with an exponentially decreasing weight for increasing surface width. The equilibrium SF model in a one-dimensional (1D) substrate has $\alpha=1/3$ and $z=3/2$ [7], whereas the equilibrium ordinary RSOS model belongs to Edwards-Wilkinson universality class with $\alpha=1/2$ and $z=2$ [13].

Our measurement of $G(r,t;L)$ for the 1D equilibrium SF model shows an anomalous behavior. For the early-time regime ($t\ll L^z$), $G(r,t;L)$ is found to satisfy Eq. (2) with different scaling exponents $\alpha'=1/2(\neq\alpha)$ and $z'=9/4(\neq z)$. Furthermore we found $G(r,t;L)$ for the saturation regime ($t\gg L^z$) is found to show the scaling behavior $G(r,t;L)=L^{2\alpha}f(r/L^{\delta})$ with $\delta=2/3$. These anomalous behaviors for $G(r,t;L)$ will be shown to be understood from a scaling theory based on the competition between the local random-walk-like behavior and the global constraint.

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The dynamic properties of the equilibrium SF model are studied by Monte Carlo simulations. Since the finite size effects on the height-height correlation functions are strong when periodic boundary conditions are imposed, free boundary conditions are applied. Starting from a flat surface, i.e., $h_k=0$ for all $k=1,\ldots,L$ at $t=0$, where h_k is the height of the kth column, a site is chosen randomly. Since we study the equilibrium SF model only, we try to add or subtract the column height by one with the same probability 1/2, i.e., we set $p=q=1/2$ throughout the simulation. If the height change violates the RSOS condition, the tried attempt is rejected. Otherwise, we check if the height change brings a new extremal height. If it does not, the new configuration is accepted with probability 1. If it does, the acceptance probability of the new configuration is reduced by a factor of $u(<1)$.

For the equilibrium RSOS model without the suppression $(u=1)$, it is well known that the W^2 follows the scaling form of Eq. (1) with $\alpha=1/2$ and $z=2$ [13]. We measure the height-height correlation function $\tilde{G}_k(r,t;L)$ at site k defined by

$$\tilde{G}_k(r,t;L)=\langle[h_k(t)-h_{k+r}(t)]^2\rangle. \tag{5}$$

If one uses the periodic boundary condition for the simulation, $\tilde{G}_k(r,t;L)$ is expected to be independent of k, but the finite size effects [or the dependence of $\tilde{G}_k(r,t;L)$ on the system size L] are rather strong for most of r (say, $r>L/10$) for the feasible system sizes. Therefore, we use the free boundary condition and measure $\tilde{G}_k(r,t;L)$ for different k values. Unless k or $k+r$ is very close to the boundary sites, $\tilde{G}_k(r,t;L)$ for systems with the free boundary condition, is also expected to be independent of k. We numerically confirm that this is the case—no noticeable differences in $\tilde{G}_k(r,t;L)$ are observed for different values of k unless k or $k+r$ is very close to or at the boundary sites. We measure $\tilde{G}_k(r,t;L)$ for $r<L/2$ over $L/4$ different values of $k=(L/8)+1,(L/8)+2,\ldots,(3L/8)$ and then average them to obtain $G(r,t;L)$:

$$G(r,t;L)=\frac{4}{L}\sum_{k=(L/8)+1}^{3L/8}\tilde{h}_k(r,t;L). \tag{6}$$

Figure 1 shows the saturated height-height correlation function $G_s(r;L)[=G(r,t\gg L^z)]$ for $u=1$ with the systems sizes $L=32$, 64, 128, and 256. All the data collapse to a single curve and $G_s(r;L)$ increases linearly with r indicating $\alpha'=1/2$. In equilibrium, the model with $u=1$ corresponds to a random walk and therefore $G_s(r;L)\sim r$ is expected [11,12]. The solid line given by $G_s(r;L)=\frac{2}{3}r$ fits the data perfectly. There is no finite size effect due to the free boundary condition.

We now apply the same analysis for the SF model with $u=1/2$. By simulations using free boundary condition and the system sizes $L=16$, 32, 64, 128, and 256, we first confirmed that the surface width $W(t,L)$ satisfies the generic scaling form very well, $W(t;L)/L^\alpha$, with $\alpha=1/3$ and $z=3/2$. Although the exponents are are different from the conventional values of $\alpha=1/2$ and $z=2$ for the random-walk-like surfaces [13], they satisfy the generic scaling form of Eq. (1) very well.

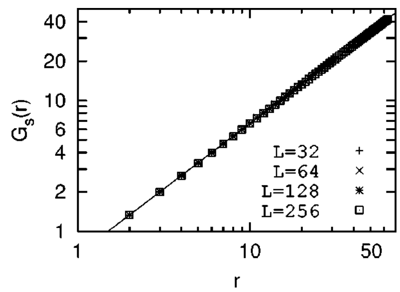

FIG. 1. The height-height correlation function for the ordinary equilibrium RSOS model with $u=1$ [11–13] in the saturated regime $(t\gg L^z)$. The solid line given by $G_s(r;L)=\frac{2}{3}r$ fits the data perfectly, indicating the wandering exponent $\alpha'=1/2$.

However, the height-height correlation functions $G_s(r;L)$ in the saturated regime $(t\gg L^{z'})$ for $u=1/2$ shows quite different behaviors from the conventional cases. According to the generic scaling form of Eq. (2), $G_s(r;L)$ is expected to be the form of

$$G_s(r;L)\sim r^{2\alpha'} \tag{7}$$

in free boundary condition, since $g_G(x)$ in Eq. (2) becomes a constant for large $x(t\gg r^{z'})$ for the conventional cases. Figure 2 shows that this is not the case for our model with $u=1/2$. The saturated height-height correlation functions $G_s(r;L)$ are shown in (a) for the system sizes $L=32$, 64, 128, 256, and 512. We note that $G_s(r;L)$ *does* depend on the

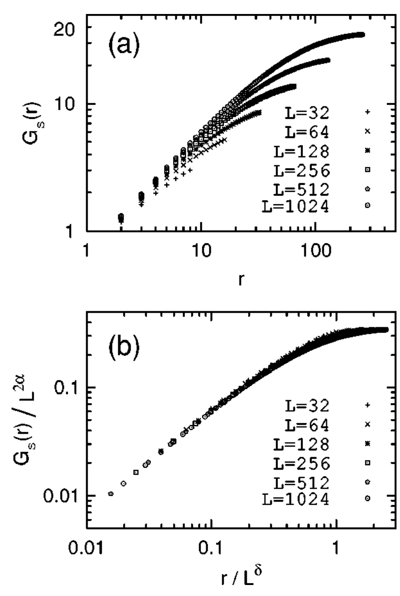

FIG. 2. The height-height correlation functions for $u=1/2$ in the saturated regime $(t\gg L^{z'})$. (a) $G_s(r;L)$ for the systems of sizes $L=32$, 64, 128, 256, and 512 are shown in logarithmic scale. (b) The rescaled height-height correlation function $G_s(r;L)/L^{2\alpha}$ against the rescaled distance r/L^δ with $\alpha=1/3$ and $\delta=2/3$.

system size L being different from the case of $u=1$. Furthermore, $G_s(r;L)$ is not linear in a log-log scale plot, implying that it does not follow the scaling behavior of Eq. (7). There seems to be another length scale over which the scaling ansatz of Eq. (7) should be modified. We rescale the correlation functions $G_s(r;L)$ by $L^{2\alpha}$ and r by L^{δ} with $\delta=2/3$, and find a nice scaling plot. In Fig. 2(b), $G_s(r;L)/L^{2\alpha}$ are plotted against r/L^{δ}. All data collapse to a single curve, implying a new scaling law for the saturated height-height correlation function:

$$G_s(r;L)=L^{2\alpha}f(r/L^{\delta}). \tag{8}$$

Note $f(x)\simeq x^{2\alpha'}(\alpha'=1/2)$ for $x\ll 1$ and $f(x)\simeq$const for $x\gg 1$. In other words, $G_s(r;L)$ increases as $r^{2\alpha'}$ for $r<L^{\delta}$, and reaches a constant value for $r>L^{\delta}$ with $\delta=2/3$. This implies that the saturated correlation length $\xi_s(L)=\xi(t\gg L^z)$ is not the system size L but only L^{δ}. There is a window of the new length scale L^{δ}. Roughly speaking, the surface shows a random-walk-like behavior up to the window size, and then feels the global constraints of the suppression over the window size.

Our analysis on the early time ($t\ll L^z$) behavior of the height-height correlation function also supports the conjecture for $\xi_s(L)\sim L^{\delta}$. Figure 3(a) shows $G(r,t;L)$ for an $L=1024$ system at 16 different times: $t=1,2,2^2,\ldots,2^{15}=32768$. When the height-height correlation functions at small $t[\xi(t)\ll\xi_s(L)]$ are rescaled according to the scaling form of Eq. (2) with $\alpha'=1/2$ and $z'=9/4$ ($\beta=\alpha'/z'=2/9$), they collapse to a single curve representing the early time scaling,

$$G(r,t;L)=t^{2\beta}f_t(r/t^{1/z'}), \tag{9}$$

as shown in (b). The correlation length grows as $t^{1/z'}$ at the beginning and saturates to L^{δ} when it hits the window size $((t_s)^{1/z'}\sim L^{\delta})$, where the saturation time t_s is estimated by $t_s\sim(L^{\delta})^{z'}\sim L^{\delta z'}$. This should be the same as the saturation time for the surface width which is given by L^z according to Eq. (1). Therefore, we have

$$z'=z/\delta. \tag{10}$$

For $r\gg t^{1/z'}$, since $G(r,t;L)$ is independent of r, we expect $G(r,t;L)\sim t^{2\alpha'/z'}$. It should be proportional to $W^2(t)$, so that

$$\alpha'/z'=\beta. \tag{11}$$

From the relation $\alpha'/z'=\beta=\alpha/z$ we get

$$\alpha'=\alpha/\delta. \tag{12}$$

However, as shown in the inset of Fig. 3(b), the same rescaled height correlation functions $G/t^{2\beta}$ do not collapse to a single curve for large t. We can see the clear discrepancy between the rescaled plots as t approaches to L^z. (The data for $t\leq L^z$ are shown here. The deviation becomes even larger for $t>L^z$.)

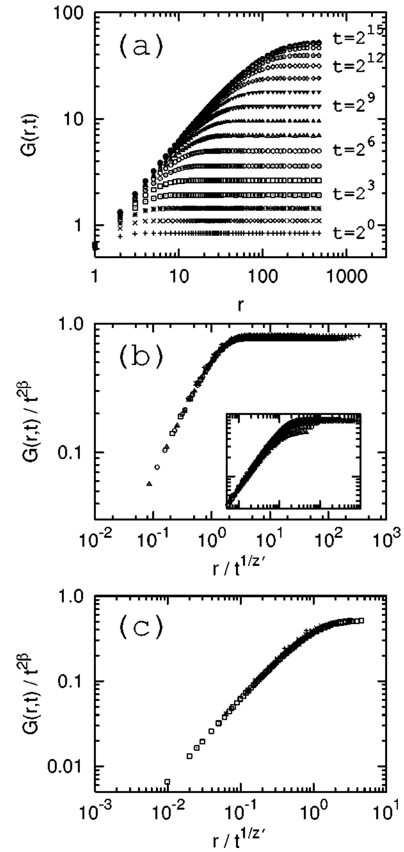

FIG. 3. (a) Height-height correlation functions $G(r,t;L)$ for the $L=1024$ system for $t=1,2,2^2,\ldots,2^{15}$. (b) The rescaled height-height correlation function $G(r,t;L)/t^{2\beta}$ plots against the rescaled distance $r/t^{1/z'}$ with $\beta=2/9$ and $z'=9/4$ ($\alpha'=1/2$) for small $t<2^{10}\ll L^z$. The height-height correlation functions at different times collapse to a single curve, indicating the validity of Eq. (2) for small t. The inset shows the same rescaled height correlation function vs the rescaled distance for all 16 different values of $t\leq 2^{15}=L^z$. The deviations from the small t curve is clear for large t. (c) $G(r,t;L)/t^{2\beta}$ vs $r/t^{1/z'}$ with $L/t^{2/3}=2$ at $t=2^3$, 2^6, 2^9, 2^{12}, and 2^{15}. All data collapse to a single curve indicating the validity of Eq. (15).

We investigate the dynamic scaling behavior of the height-height correlation functions for general t with a scaling ansatz for $G(r,t;L)$. Since our model has a global constraint of extremal height suppression, we introduce a general scaling ansatz where the system size can scale differently from the local length,

$$G(r,t;L)=b^{2\alpha'}G(b^{-1}r,b^{-z'}t,b^{-1/\delta}L), \tag{13}$$

where $b^{-2\alpha'}$, $b^{-z'}$, and $b^{-1/\delta}$ are the scaling factors for the height correlation function, time, and system size, respectively.

We first show that the choice of the above scaling factors reproduce the scaling behaviors of Eqs. (1) and (8). If we choose, $b=L^{\delta}$, Eq. (13) becomes

$$G(r,t;L)=L^{2\delta\alpha'}G(L^{-\delta}r,L^{-\delta z'}t,1)$$
$$=L^{2\alpha}g_1\left(\frac{r}{L^{\delta}},\frac{t}{L^{z}}\right). \qquad (14)$$

Now, the scaling of Eq. (1) is obtained by considering the spatial average of $G(r,t;L)$ since $W^2(t;L)$ is proportional to the average. Figure 2 can be also understood from Eq. (14) by considering a $t\rightarrow\infty$ limit. For a give system size, the height correlation function is expected to be saturated eventually and, therefore, the $y\rightarrow\infty$ limit of the scaling function $g_1(x,y)$ must be exist. This limit is $f(x)$ in Eq. (8).

The dynamic behavior of the height-height correlation function is obtained by considering $b=t^{1/z'}$ case of Eq. (13). Then we have

$$G(r,t;L)=t^{2\alpha'/z'}G\left(\frac{r}{t^{1/z'}},1,\frac{L}{t^{1/\delta z'}}\right)$$
$$=t^{2\beta}g_2\left(\frac{r}{t^{1/z'}},\frac{L}{t^{1/z}}\right). \qquad (15)$$

To check this dynamic scaling behavior, we consider the height-height correlation functions with fixed $L/t^{1/z}$. Then the rescaled correlation $G(r,t;L)/t^{2\beta}$ should be a function of the the rescaled distance $r/t^{1/z'}$ only. We measure $G(r,t;L)$ for five different system sizes with $L/t^{1/z}=2$, $L=2^3$ at $t=2^3$, $L=2^5$ at $t=2^6$, $L=2^7$ at $t=2^9$, $L=2^9$ at $t=2^{12}$, and $L=2^{11}$ at $t=2^{15}$ and plot $G(r,t;L)/t^{2\beta}$ against $r/t^{1/z'}$. As shown in Fig. 3(c), all the scaled data collapse to a single curve supporting the scaling behavior of Eq. (15). Note that Fig. 3(b) implies that $g_2(x,y)$ becomes independent of y for large y or $t\ll L^z$. This $g_2(x,y)$ for large y corresponds to $f_t(x)$ in Eq. (9). However, it depends on y in general, as shown in the inset of Fig. 3(b).

One of the easy ways to understand the anomalous scaling behavior of $G(r,t;L)$ is the scaling theory based on the length scale $\Xi(L,t)$, beyond which one cannot see any local correlation or global constraints. $\Xi(L,t)$ should satisfy

$$\Xi(L,t)\sim\begin{cases} t^{1/z} & \text{for} \quad t\ll L^z \\ L & \text{for} \quad t\gg L^z. \end{cases} \qquad (16)$$

Now assume $G(r,t;L)=G(r,\Xi(L,t))$. Then $G(r,t;L)$ rescales as

$$G(r,t;L)=G[r,\Xi(L,t)]=b^{2\alpha'}G[b^{-1}r,b^{-1/\delta}\Xi(L,t)]. \qquad (17)$$

By taking $b=L^{\delta}$ and $\Xi=L$ for $t\gg L^z$ we can reproduce the scaling relation $G(r,t;L)=L^{2\alpha}f(r/L^{\delta})$ [Eq. (8)] for the saturation regime from Eq. (17). We can also reproduce the scaling relation $G(r,t;L)=t^{2\beta}f_t(t/r^{1/z'})$ [Eq. (9)] for the early-time regime from Eq. (17) by taking $b=t^{1/z'}$ and $\Xi=t^{1/z}$ for $t\ll L^z$. From this result, we can understand that the anomalous behavior of $G(r,t;L)$ comes from the competition between local the random-walk behavior and the global suppression. A similar behavior has also been found in the sort of the local structure function of even visiting random walks [8].

In summary, we measured the wondering exponent $\alpha'\approx 1/2$ and its dynamic exponent $z'\approx 9/4$ from the height correlation function for our model. These values are different from the roughness exponent $\alpha=1/3$ and its dynamic exponent $z=3/2$ of the surface widths. This anomalous scaling behavior can be understood by assuming that the saturated correlation length is given by L^{δ} with $\delta=2/3$. The correlation length grows as $t^{1/z'}$, and approaches a window size L^{δ} asymptotically. We show that the usual way of determining the universality class of surface model by the scaling behavior of the surface width only can miss many important scaling behaviors of the model.

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