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Peak effect in a driven lattice gas model

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We study the peak effect (PE), i.e., a sharp peak observed in the critical current as a function of the particle density, discovered in transport properties of a driven lattice gas model. We show that the PE corresponds to a first-order phase transition found in the undriven system at equilibrium, which in turn gives rise to an "anomalous" second peak in magnetic hysteresis loops. We also explain the "history" dependent phenomena observed in the PE region by investigating the system characteristic time scales, which diverge at low T and have a broad maximum as a function of the external field around the PE. The model we consider can be related to a coarse grained description of vortex lines in superconductors and we discuss the relations of the PE described here and the one experimentally observed in these systems.

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I. INTRODUCTION

Driven lattice gases have been an important area of research for nonequilibrium phenomena in recent years [1,2]. Here we consider a model of driven repulsive particles on a lattice subject of a Monte Carlo dynamics and discuss, the presence of an important phenomenon, known as the peak effect (PE), found in the system. This is a sharp peak observed in the "critical current" when plotted as a function of the particle density. We investigate the physical mechanisms underlying the PE and show that in the present model, the PE is the manifestation in the *driven* system of a first-order phase transition found at *equilibrium* in the absence of drive and associated to a "second peak" in magnetization measures. The origin of the strong "memory effects" observed in the region of the PE traces back, instead, to the nontrivial behavior of the system relaxation times as a function of the particle density and temperature.

Interestingly, our model was originally derived as a coarse grained representation of a system of straight parallel vortex lines described by Ginzburg-Landau equations in the London approximation [3]. In vortex physics of type-II superconductors, the peak effect, i.e., a sharp peak observed in the critical current (or a dip in resistivity) as a function of the applied field [4–7] is known to be an ubiquitous phenomenon. In this research area, in recent years, relevant theoretical [8–10] and experimental [5–7,11] activities have been devoted to such an issue. However, important questions are still open, as those concerning the origin of the strong dynamical anomalies discovered in the PE region, including "memory" effects, history dependence, and apparent metastability phenomena [5,7,12,13].

The present model was already shown to describe a broad range of phenomena occurring in vortex matter including creep dynamics of bean profiles, hysteresis of magnetization, and memory effects in I-V characteristics [3]. In this respect, beyond the specific issues related to driven lattice gas theory, the simple physical mechanisms included in our driven lattice model seem to catch some important aspects of magnetic and transport phenomena of vortex physics and, in particular, can clarify the properties of the PE.

II. THE MODEL

We consider a square lattice model characterized by a field n_i representing the total particles charge present on site i; n_i is an integer number bounded by a given value, N_{c2}: $n_i \in \{-N_{c2}, \ldots, N_{c2}\}$. The Hamiltonian defining the model (called a restricted occupancy model) is Ref. [3]: $\mathcal{H} = \frac{1}{2}\sum_{ij} n_i A_{ij} n_j - \frac{1}{2}\sum_i A_{ii}|n_i| - \sum_i A_i^p |n_i|$. The first two terms describe the interaction between the particles and their self-energy, and the last the interaction with a random pinning background (see Ref. [3] for details). For sake of simplicity, we consider here the simplest version of $\mathcal{H}$: we choose $A_{ii} = A_0 = 1$; $A_{ij} = A_1 < A_0$ if i and j are nearest neighbors; $A_{ij} = 0$ otherwise; the random pinning is δ distributed $P(A_i^p) = (1-p)\,\delta(A_i^p) + p\,\delta(A_i^p - A_0)$ [15]. Such a model can be considered as a schematic representation of a system of straight parallel vortex lines in type-II superconductors coarse grained on a scale l_0 of the order of the London penetration length λ in the plane orthogonal to the field [3]. The charge of vortices on the ith coarse grained cell is mapped into the lattice field n_i and the upper critical field is $B_{c2} = N_{c2}\phi_0/l_0^2$, where $\phi_0 = hc/2e$ is the flux quantum unit.

In analogy with computer investigations of dynamical processes in fluids [14], the time evolution of the model is simulated by a charge conserving Monte Carlo–Kawasaki dynamics on a square lattice of size L at a temperature T [15]. The system is periodic in the y-direction. The two edges parallel to the y axis are in contact with a charge reservoir, i.e., an external "magnetic" field, of given charge density N_{ext}. Particles can enter and leave the system only through the reservoir, which is considered to have an $\mathcal{H}$ of the same form of the system, but with $A^p = 0$. Vortex-antivortex pair creation is not allowed and pairs on the same or nearest neighbor (nn) sites annihilate.

As in standard driven lattice gases [2], the effect of an external drive I is introduced by a bias in the Metropolis coupling of the system to the thermal bath: a particle can jump to a neighboring site with a probability $\min\{1, \exp[-(\Delta\mathcal{H} - \epsilon I)/T]\}$. Here, $\Delta\mathcal{H}$ is the change in $\mathcal{H}$ after the jump and $\epsilon = +1, -1, 0$ ($\epsilon = -1, +1, 0$) for a positive (negative) particle trying to hop along, opposite or or-

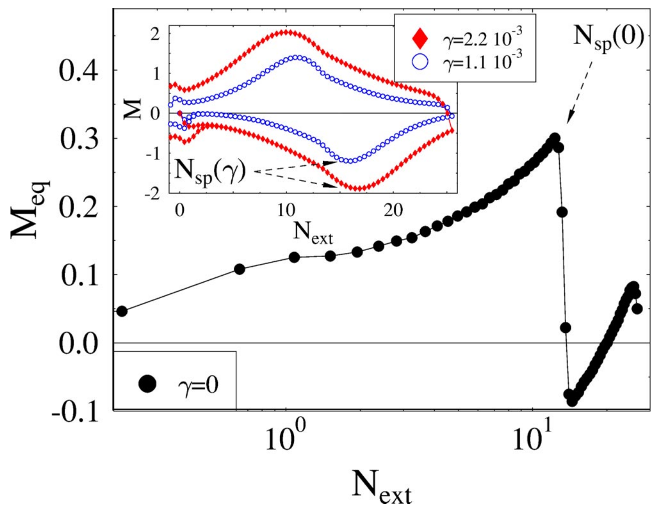

FIG. 1. Inset: In the absence of drive $I=0$, the magnetization M is plotted as a function of the applied field density N_{ext}, which is cycled from and back to zero at $T=0.3$ with the shown sweep rate $\gamma=dN_{ext}/dt$. Notice the appearance of a "second magnetization peak," whose location is γ dependent $N_{sp}(\gamma)$. Main frame: The equilibrium value of M (i.e., the one for $\gamma\to 0$) shows an apparent jump associated with the second magnetization peak $N_{sp}(0)$ signaling a first-order phase transition. The logarithmic horizontal scale is used in order to outline the small N_{ext} region.

thogonal to the direction of the drive. Vortices in superconductors carry a magnetic field and thus they are coupled to an applied electrical current I by the Lorentz force. Under such a drive, vortices start moving at a velocity v, which is in turn proportional to the voltage drop V, generated at the sample boundaries (see Ref. [16]). Thus, in presence of a drive I, a voltage V is generated. As discussed in Ref. [16], V is defined by $V(t)=\langle\overline{v(t)}\rangle$. Here, $\overline{v(t)}$ is an average vortex "velocity" in a small interval around the time t [this is to improve the statistics on $V(t)$] and $v(t)=\Sigma_i v_i(t)/L$ is the "instantaneous velocity" [$v_i(t)=\pm 1$,0, as above, if the vortex i at time t moves along, opposite or orthogonal to I].

III. RESULTS

We first analyze the model properties in absence of an external drive, i.e., by now we set $I=0$. The system, originally empty, is prepared at a given T by increasing the external field N_{ext} from zero at a constant rate $\gamma=dN_{ext}/dt$. While ramping N_{ext}, we record the magnetization defined as $M(t)=N_{in}(t)-N_{ext}(t)$. Here, $N_{in}=\Sigma_i n_i/L^d$ is the charge density inside the sample and the Monte Carlo time t is measured in units of complete Monte Carlo lattice sweeps. During the whole process, the drive is zero, $I=0$.

At low temperatures, pronounced hysteresis magnetization loops are seen when M is parametrically plotted as a function of N_{ext} (which is here cycled back to zero, see inset Fig. 1). Furthermore, when the parameter $\kappa^*=A_1/A_0$ of $\mathcal{H}$ is above a critical threshold, $\kappa_c\simeq 1/4$ [3,17], as in the present case, a definite *second peak* (the so-called "fish-tail" structure of experiments on vortex matter, see Ref. [7], and references therein) appears in M. The shape of loops depends, in particular, on the sweep rate of the external field, $\gamma=dN_{ext}/dt$, as shown in Fig. 1. As soon as the inverse of the sweep rate is smaller than the system characteristic relaxation time τ_V (to be defined in detail below), strong hysteresis effects are, in fact, present. Although the second-peak position N_{sp} does depend on γ [i.e., $N_{sp}=N_{sp}(\gamma)$], it is related to a new first-order equilibrium phase transition: in the $\gamma\to 0$ limit (i.e., when the external field is ramped quasistatically), its location $N_{sp}(\gamma=0)=13.5$ is associated with a sharp jump in the equilibrium magnetization, $M_{eq}(N_{ext})\equiv\lim_{\gamma\to 0}M(\gamma,N_{ext})$, as shown in Fig. 1. For $\kappa^*>\kappa_c$, particles with the same charge strongly repel each other and tend to be disposed in such a way to have no nearest neighbors on the lattice. However, since N_{c2} is finite, such a "staggered" configuration is no longer possible above a given value of the external density N_{ext}. This corresponds to the transition in M.

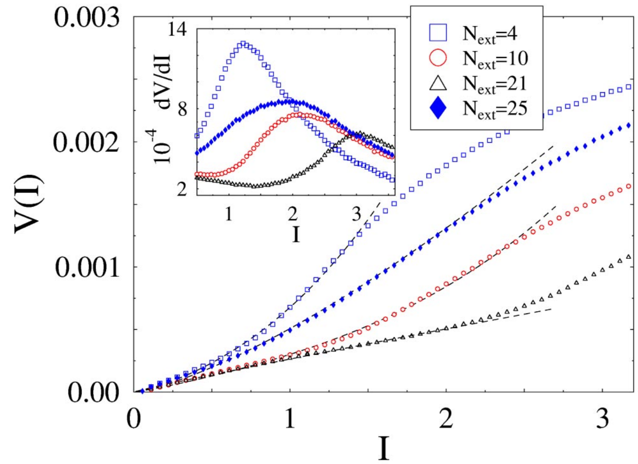

FIG. 2. The I-V characteristic is recorded by ramping I with rate $\gamma_I=dI/dt=5\times 10^{-3}$ at $T=0.1$ for the shown N_{ext}. The dashed lines are the power law fit described in the text. Inset: the differential resistivity $\rho=dV/dI$ for the same data of the main panel.

In the same region where we found the second magnetization peak, we now record the I-V characteristics. The I-V characteristics are here measured as in real experiments: after fixing the working conditions (i.e., temperature T and external field N_{ext}), the function $V(I)$ is recorded by ramping I from zero upwards at a given rate $\gamma_I=dI/dt$. Again, when $\gamma_I>\tau_V^{-1}$, $V(I)$ depends on the system history simply because the system has not been able to follow the applied drive [3]. In fact, it is now experimentally well established that I-V characteristics in superconductors show this kind of memory phenomena (see, for instance, Refs. [7,12,13], and references therein). We show in Fig. 2 the appearance of the I-V characteristics $V(I)$ and differential resistivity $\rho(I)=dV/dI$, recorded for applied fields spanning a decade around the second peak value $N_{sp}(0)$.

At low T, the I-V characteristic appears to have the typical S-shaped form, experimentally found in vortex matter and, as a matter of fact, to be definitely dependent on the ramp rate γ_I. For finite γ_I, the I-V characteristic is non-Ohmic with a power law behavior at low I (the dashed lines in Fig. 2):

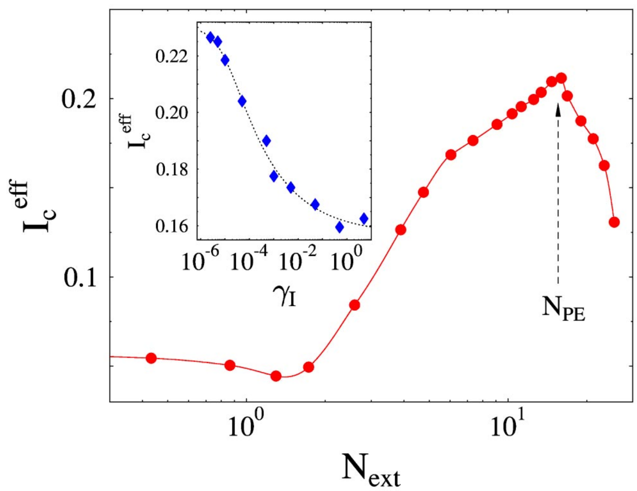

FIG. 3. The critical current I_c^{eff} is plotted as a function of the applied field N_{ext} (in the main panel, at $T=0.1$ and $\gamma_I=5\times10^{-4}$) and current ramp rate $\gamma_I=dI/dt$ (in the inset, at $T=0.1$ and $N_{ext}=10$; here the superimposed dotted curve is the fit discussed in the text). The so-called *peak effect* (PE) in I_c^{eff} as a function of N_{ext} is clearly observable. It corresponds to the second magnetization peak N_{sp} found in hysteresis loops. Also notice the nonequilibrium effects observed at low T, as shown by the γ_I dependence of I_c^{eff} plotted in the inset.

$$V\simeq\rho_0 I^{\alpha}, \tag{1}$$

where the exponent α is, for instance, about 1.3 for $\gamma_I=5\times10^{-3}$, $N_{ext}=10$, and $T=0.1$. In general, α is field and temperature dependent [19]. At larger values of I, an Ohmic behavior is observed [18,19]. As a matter of fact, the S shape tends to disappear when $\gamma_I\to 0$, i.e., when the drive is ramped without altering the equilibrium conditions of the system, and the I-V characteristics become linear. We notice that since the system equilibration time at low T diverges exponentially (see below), it may be very difficult to reach the purely linear regime found indeed at stationarity.

From the I-V characteristics the critical current is derived, which we now discuss in more detailed focusing, in particular, on its dependences on applied field N_{ext} and current ramp rate γ_I. As much as in experiments, we define the *effective* critical current I_c^{eff} by a so-called "voltage criterion": I_c^{eff} is the drive value where V gets larger than a given threshold (here, $V_{thr}=5\times10^{-5}$). We call I_c^{eff} an effective critical current since we show below that generally it does not represent an "intrinsic" material parameter. In fact for a given T and N_{ext}, I_c^{eff} usually depends on γ_I and, more generally, on the sample history. This is apparent from the inset of Fig. 3, where the function $I_c^{eff}(\gamma_I)$ is shown. In particular, in our model we find that, in the present field and temperature range, I_c^{eff} slowly decreases by increasing γ_I. The function $I_c^{eff}(\gamma_I)$ is approximately fitted by a power law with a small exponent, the dotted line in the inset of Fig. 3 (or by an inverse logarithmic function):

$$I_c^{eff}(\gamma_I)\simeq\frac{I_0}{(1+\gamma_I/\gamma_I^0)^{\Delta_I}}, \tag{2}$$

where for $T=0.1$ and $N_{ext}=10$, $I_0\simeq0.7$, $\gamma_I^0\simeq10^{-5}$, and the exponent $\Delta_I\simeq0.2$.

The above effects of γ_I on the critical current of the driven system are the analog of the effects of γ on the magnetization M, discussed above in the undriven system. Interestingly, time dependent critical currents with properties similar to those of the present model are indeed observed in experiments on vortex matter (see Ref. [12], and references therein).

It is important to consider these history dependent effects when analyzing the PE. The typical dependence of I_c^{eff} on the applied field N_{ext} in the low T region is shown in the main panel of Fig. 3 for a given γ_I value. Different from standard models, $I_c^{eff}(N_{ext})$ is not a monotonic function [20]. At low fields, I_c^{eff} slowly decreases with N_{ext}, but above a γ_I dependent turn point, a drastic change is observed in I_c^{eff} which has a broad peak, analogous to the peak effect experimentally found in vortex matter [4–7,12,13]. In fact, a slow dependence of $I_c^{eff}(N_{ext})$ on the current ramp rate γ_I is found as explained above. The location of the maximum in the PE, N_{PE}, is dependent on γ_I, but most important is that it is very close to the values of the second magnetization peak found in magnetic loops, N_{sp}; for $\gamma_I\to0$, N_{PE} is numerically equal to $N_{sp}(\gamma=0)$. Finally, in the present model we find that the second peak in M and the corresponding peak effect in I_c^{eff} are also present when the pinning potential is turned to zero, $A^p=0$. We considered here the more general situation $A^p\geqslant0$ because this is the typical case in real superconducting samples, but, as a consequence of the above remark, we can predict that the peak effect should be present in very clean samples too.

In the above discussion we have stressed the importance of history effects on the PE structure. In order to rationalize these phenomena, we now discuss the system characteristic equilibration time scale.

At low T, upon applying to the system a drive I, its voltage response V slowly relaxes in time towards a stationary value [3]. In particular, as much as experiments in vortex matter (see Ref. [12], and references therein), one finds that for long times, the slow relaxation of $V(t)$ can be well fitted by stretched exponentials [3]: $V(t)\propto\exp(-t/\tau_V)^{\beta}$. The above long time fit defines the characteristic asymptotic scale of relaxation, τ_V, which we discuss now. As a function of the temperature, τ_V has a steep increase with T [3,19], and an approximate Vogel-Tamman-Fulcher behavior: $\tau_V=\tau_0\exp[E_0/(T-T_c)^{\nu}]$, where $\tau_0\simeq7\times10^2$, $\nu\simeq0.9$, and $T_c\simeq0.01$, when $N_{ext}=10$. In the low T regime, a power law fits as well the data, but in all the cases, the best value fit for T_c is practically indistinguishable from zero. This fact confirms that in the present two-dimensional version of the model, the glass transition is pushed at $T=0$ [3]. More generally, we notice that the low-T divergence of τ_V shows that strong off-equilibrium phenomena are to be found when studying the system at small temperatures, a fact in correspondence with experiments on vortex matter [5,7,12,13].

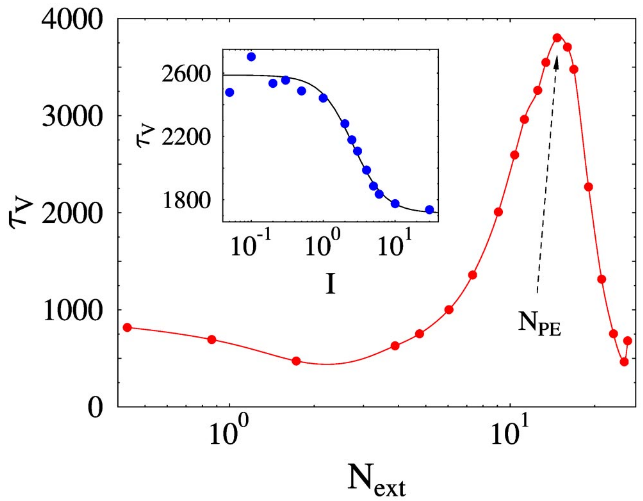

FIG. 4. Main panel: The characteristic time scale of voltage relaxation, τ_V, as a function of N_{ext} is nonmonotonic. It has a maximum corresponding to the location of the PE and second magnetization peak (here $I=1$ and $T=1$). Inset: $\tau_V(I)$, as a function of the applied drive I for $T=1$ and $N_{ext}=10$. The superimposed curve is a power law fit (see Refs. [3,19]).

Since here we are mainly interested with the PE as a function of the applied field, we now discuss in detail the the dependence of τ_V with N_{ext}, shown in the main panel of Fig. 4. Interestingly, τ_V is nonmonotonous with N_{ext} showing an apparent maximum around the value corresponding to the PE observed in the critical current N_{PE}. The fact that for a given T, $\tau_V(N_{ext})$ has a broad maximum around the PE and the second-peak location in M explains why around the peak ''slushy'' regions have been often observed (see, for instance, Refs. [5,7,12,13]): there τ_V is very large and off-equilibrium ''glassy'' features appear whenever the system is observed on time scales too short compared to it.

Finally, it is important to notice that the characteristic time τ_V is strongly affected by the value of the drive I itself as shown in the inset of Fig. 4; $\tau_V(I)$ increases by decreasing I and approaches a finite plateau for $I<I^*$, with $I^*\simeq O(1)$. The higher the drive I the faster is the approach to stationarity and, in this sense, an increase in I has an effect similar to an increase in T. Notice that such a result is in agreement with experimental findings in vortex matter [12] and with analogous features of observed in other driven lattice gas [2].

IV. CONCLUSIONS

Summarizing, in the framework of our driven lattice gas model (a coarse grained description of a system of straight parallel vortex lines [3]), we observe an important phenomenon in its transport properties which can be called the peak effect, i.e., a maximum of I_c^{eff} as a function of N_{ext}. The PE corresponds to a first-order phase transition found in the undriven system at equilibrium, which in turn is manifested as a second peak in magnetic hysteresis loops. The important history dependent phenomena observed in the PE region originate from the properties of the system characteristic time scales, which diverge at low T and have a broad maximum as a function of the external field around the peak of the PE.

In the present scenario, the nature of the analogous PE phenomenon observed in vortex matter can be clarified (for instance, the debated relations between its ''equilibrium'' and ''dynamical'' properties) and a broad range of experimental findings in magnetic and transport properties of superconductors reproduced [3]. The application of driven lattice gases to the study of nonequilibrium properties of vortex physics opens new intriguing perspectives [2].

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[15] Here $A_1=0.28A_0$, $A_0^p=0.3A_0$, $N_{c2}=27$, $p=1/2$, and $L=32$. Our results, averaged over up to 2048 realizations, are checked up to $L=128$ and robust to parameter changes. The system size, L, does not affect the general properties of the model, but it changes the characteristic relaxation time: the larger L, the longer the relaxation time. This is discussed in detail in Refs. [3,19].

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