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Coherent dynamics of an asymmetric particle in a vertically vibrating bed

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Coherent motion is found to emerge out of fluctuations in a vibrated asymmetric particle. Depending on the parameters, amplitude, and frequency of the box, the motion of the particle is classified into several phases. The transition between fluctuating motion and unidirectional motion occurs with constant acceleration in the low-frequency regime and constant amplitude in the high-frequency regime. We show through dimensional analysis that *this behavior does not depend* on the detailed geometry of the particle.

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Consider a particle set on a horizontal plate. When one applies vertical vibrations to the system, a coherent horizontal translational motion emerges. The same type of interesting phenomena can be seen in different systems from toys to micromachines and granular systems. How does this seemingly paradoxical dynamics appear from fluctuations that are orthogonal to the direction of the coherent motion? Since the system has a translational symmetry in a horizontal direction, no horizontal acceleration is possible owing to the momentum conservation as long as the particle is forced only vertically by an external vibration. Even if the particle does not possess spherical symmetry, the same elementary law of physics can be applied to the motion of the centroid of the particle, resulting in no horizontal motion. In this paper, we present experimental results that the coherent transport of a particle emerges out of fluctuations. We observe distinct transitions from Brownian-like motion to coherent motion, on which a simple dimensional analysis is performed to show the generality of the finding.

The emergence of coherent transport phenomena has been studied in various situations. Directional motion or deterministic diffusion out of a chaotic system has been studied in dynamical systems [1–3]. Directional motion out of Brownian systems has been extensively studied in relation to molecular motors [4,5]. Mechanical fluctuation is also expected to be an energy source of the micromachines, since the conventional mode of energy supply into a microscale instrument is not practical. Coherent and incoherent behaviors are also demonstrated on granular systems in a vibrating bed, which have attracted considerable attention [6–9]. It has been clarified individually for specialized systems, however, the generic mechanism of the emergence of coherent motion out of a "noisy" situation has not been well understood. Here we derive conditions for the emergence of coherent motion in an oscillating environment without employing a detailed geometry of the material. We also discuss that dissipation and asymmetry of the system play essential roles in these phenomena.

We use an asymmetric particle sandwiched between two parallel plates. The asymmetric particle is a pipe, made of brass with edges of different sizes as shown in the inset of Fig. 2. The small internal diameter r_1, large external diameter r_2, height h, and thickness of the pipe are 2.0 mm, 3.3 mm, 2.6 mm, and 0.1 mm, respectively. The mass is 30 mg. The asymmetric particle is the simplest one which exhibits nearly one-dimensional deterministic motion in our system. Static electricity is not significant for the motion of the particle. The particle is set in a vibrated thin box as shown in the inset in Fig. 2. The box is made of a plexiglas spacer hollowed cylindrically with a diameter of 100 mm and height of 3.6 mm which is sandwiched between two glass plates. A gap l between the particle and the box is 0.36 mm. The box is mounted on a vibration device (VG-100C:vibration test system) controlled by a function generator (8165A:HP) and is vibrated vertically with a sinusoidal oscillation with an amplitude A and a frequency f. The amplitude is measured by an accelerometer (PV85:RION) and an amplitude calculator (VM80:RION) with an accuracy of one micrometer.

As shown in the inset in Fig. 2, we defined the angle $\theta(t)$ as that between the rotational symmetry axis of the asymmetric particle and the horizontal axis, and the positive direction (forward motion) as the direction of the arrow. The motions of the particle are recorded by a charge coupled device camera with a time resolution of 1/30 sec. The recorded images are digitized with a resolution of 580×430 pixels, 0.117 mm/pixel, and 256 gray scale levels. We measure trajectories of the particle by determining the position of the centroid as a function of time. The x axis is defined by the vector that links the initial point to the final point of the trajectory. The velocity vector is measured as a displacement of the particle per frame divided by 1/30 sec. $V_x(t)$ is defined as a component of the velocity vector projected to the x axis. Since the component $V_y(t)$ is sufficiently smaller than the component $V_x(t)$ [10], we neglect the component $V_y(t)$ and hereafter discuss only the component $V_x(t)$. The alignment of x axis and the axis of symmetry of the particle is always as good as $(\vec{e}_x\cdot\vec{V})/|\vec{V}|\sim0.95$. A dimensionless acceleration amplitude is defined as $\Gamma=A(2\pi f)^2/g$, where g is the gravitational acceleration.

Depending on the parameters, we find that the particle displays several characteristic motions. Figure 1 shows the evolution of the component $V_x(t)$ in two phases. One phase

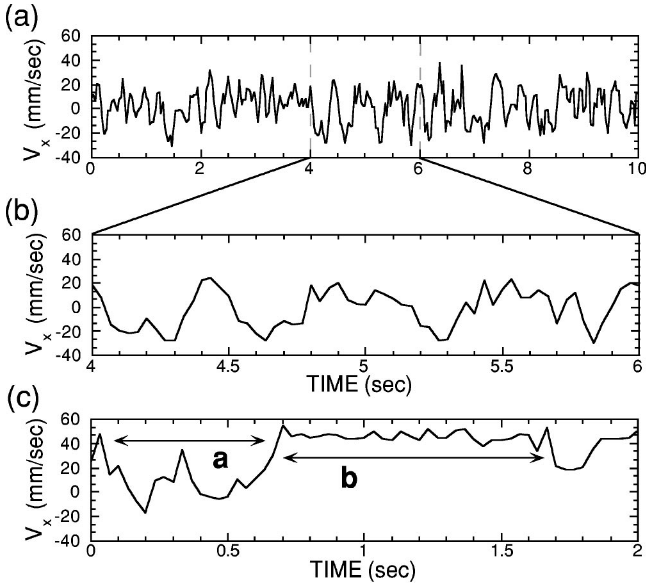

FIG. 1. (a) The evolution of the velocity component $V_x(t)$ of unlocked motion (disordered phase): $A=0.109$ mm, $f=60$ Hz. (b) An expansion of a part of (a). (c) The evolution of the velocity, where qualitatively different motions coexist; an unlocked motion (symbol **a**), and a locked motion (symbol **b**): $A=0.245$ mm, $f=60$ Hz.

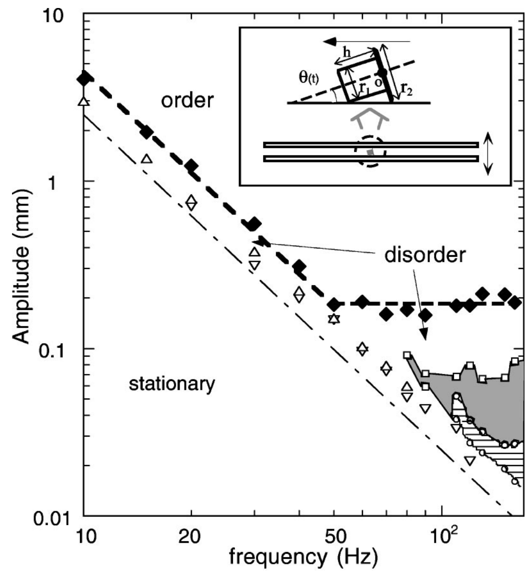

FIG. 2. Phase diagram of the parameters, the driving amplitude and the frequency. ($\triangle$) denotes the onset of the disordered phase from the the stationary phase. (∇) denotes the transition from the disordered phase to the stationary phase. Hysteresis is observed in the transition. Coherent unidirectional motion is observed above the parameters marked by closed diamonds. The dot-dashed line corresponds to $\Gamma=1$. The dashed lines are the best fit to the transition points; $\Gamma_c=1.8$ in the high-frequency regime, and $A=0.18$ mm in the low-frequency regime. Inset: schematic view of our system. The two long rectangles are glass plates.

(disordered phase) is characterized by the motion of rapidly and randomly varying direction as shown in Fig. 1(a), where the component $V_x(t)$ changes its sign with a time scale of the order of 100 msec. This "irregular motion" reminds us of Brownian motion. In the other phase (ordered phase), a qualitatively different motion is observed, where two motions coexist, namely, unlocked motion (the motion in disordered phase) and locked motion, as shown in Fig. 1(c). In the locked motion, as represented by the symbol **b**, the particle coherently moves in the forward direction with almost uniform velocity. Compared with Fig. 1(b), the locked motion continues for a much longer duration than the unlocked motion. The coexistence of unlocked motion (a burst state) and locked motion (a laminar state) reminds us of the intermittency chaos [11]. We also analyze the behavior of the angle $\theta(t)$. $\theta(t)$ represents swinging dynamics of the particle in the vertically vibrating box. In the disordered phase, $\theta(t)$ varies irregularly, while in the ordered phase, $\theta(t)$ varies periodically with the period entrained by the external vibration (data not shown). The periodic evolution leads to a locked motion, as shown by symbol **b** in Fig. 1(c).

Transition between the two phases (disordered phase and ordered phase) are found to occur at a certain driving amplitude and frequency. We characterize these phases using criteria that will be explained later. Figure 2 shows the phase diagram. If the dimensionless amplitude Γ is less than unity ($\Gamma<1$), no motion is observed due to the gravity. When Γ reaches unity, the particle starts moving vertically but without any horizontal motion, and it continues up to $\Gamma\approx1.5$. The particle starts to move horizontally if Γ is beyond 1.5 (onset of the disordered phase). As we increase the parameter further, the ordered phase appears above the transition line, $\Gamma\approx2$, and it continues up to $\Gamma\approx20$ (upper limit for observation in the system). For higher frequency regime ($\geqslant$50 Hz), we find a different transition line that is characterized as a fixed amplitude as shown in Fig. 2. Although we do not discuss them in detail in this paper, there are some other phases of interest in the system. The phases are observed in the high-frequency region as shown in the dotted and hatched areas in Fig. 2. In these phases, $\theta(t)$ does not fluctuate considerably. The particle, however, oscillates between forward and backward motions on the order of 1 s in the dotted area. In the hatched area, it exhibits mostly backward motion.

The criteria for the characterization of the phases are determined by the analysis of the statistical property of the system, i.e., the velocity distribution function (Fig. 3) and the distribution function of the intervals of the consecutive forward motion (Fig. 4). We also consider the evolution of $\theta(t)$. First, by comparing the velocity distribution function in Fig. 3(a) [graph (3)] with that in Fig. 3(a) [graph (4)], it is observed that one of the peaks is shifted from minus to plus. In this region, 0.179 mm$<A<$0.190 mm, the backward motion in the disordered phase disappears gradually and the locked motion emerges as the amplitude is increased. By a further increase of the amplitude, the locked motion becomes dominant in the system. Therefore, the mean velocity as a function of the amplitude A abruptly changes at the amplitude $A=A_c$ as shown in Fig. 3(b) (closed circles). The discontinuity of the mean velocity on the amplitude exists in order-disorder transition in the wide region of the parameters; A

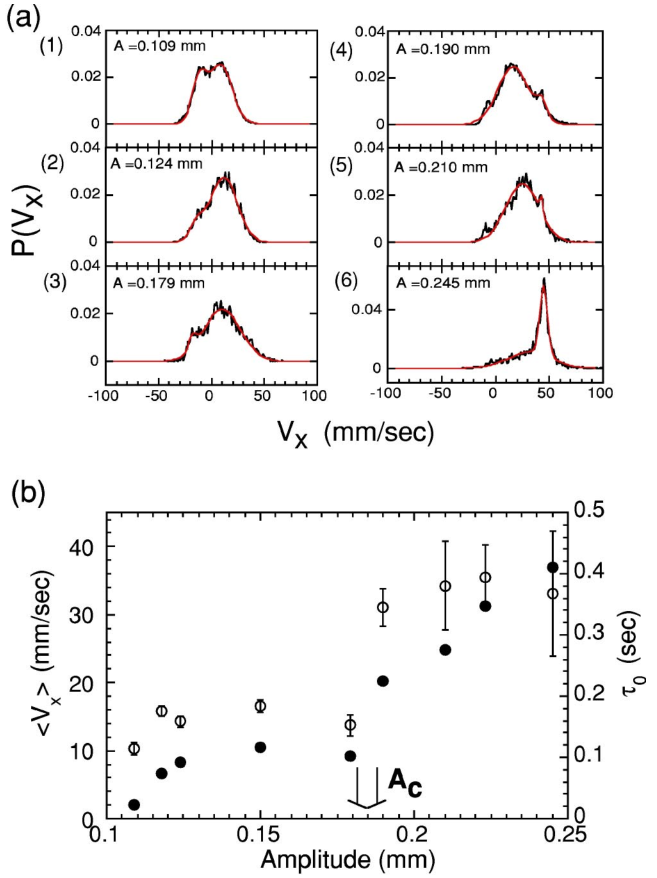

FIG. 3. (a) The velocity distribution function in the disordered phase [graphs (1)–(3)] and ordered phase [graphs (4)–(6)], observed by varying amplitude A with fixed frequency $f=60$ Hz. The distribution functions can be well fitted by double-peaked Gaussian functions $B\exp(-cx^2)+\acute{B}\exp(-\acute{c}x^2)$. (b) The amplitude dependency of the mean velocity $\langle V_x\rangle$ (closed circles) and characteristic decay time τ_0 of the consecutive motion (open circles). Error bars of the mean velocity are smaller than 5.6% of the mean value. The amplitude A_c is the critical value at which the locked motion emerges.

and f. Therefore, one of the criteria is whether the discontinuity appears or not. Second, the probability distribution function of the intervals of the consecutive forward motion suggests that, in both disordered and ordered phases, the exchange process between forward and backward motion is "random" in the sense that the function can be fitted properly by an exponential function, $\exp(-t/\tau_0)$ (see Fig. 4). The decay time of the consecutive forward motion, τ_0, depends on the amplitude of external excitation [12]. Figure 3(b) (open circles) shows the decay time as a function of the amplitude. It also has a discontinuity at $A=A_c$ coinciding with that of the mean velocity. We suspect that at the transition point, a type of resonance mechanism plays a role in the sudden change of dynamics into locked motion. Therefore, the other criterion is whether the discontinuity appears or not in the decay time. The last criterion is whether $\theta(t)$ exhibits periodic motion locked by the external vibration or not. According to these criteria, we characterize the ordered phase.

Close observation of the phase diagram (see Fig. 2), particularly the transition between disordered phase and ordered phase, we find the following characteristics in the phase diagram.

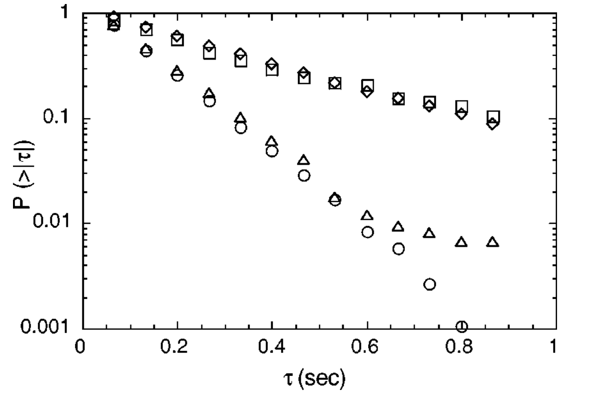

FIG. 4. The distribution function of the intervals of consecutive forward motion. The circles, triangles, squares, and diamonds denote the distributions at the driving amplitude 0.109 mm, 0.179 mm, 0.190 mm, and 0.273 mm, respectively, where the frequency is fixed at $f=60$ Hz.

(1) The transition occurs at $\Gamma_c\approx 1.8$ in a low-frequency region.

(2) The transition occurs at $A_c\approx 0.18$ mm in a high-frequency region.

We adopt dimensional analysis to understand these peculiar characteristics. There are three time scales in the system; (1) a time scale for parabolic motion caused mainly by the gravity, $\tau_g=\sqrt{2H/g}$, where H is the length that the particle can travel freely between the plates, (2) the period of external oscillation, $\tau_f=1/f$, and (3) reorientation time, $\tau_I=\sqrt{(I/m\alpha L)}$, where I is the inertial moment of the particle, α is the acceleration imposed on the particle, m is the mass of the particle, and L is the characteristic length of the particle [13].

To estimate the transition in the low-frequency regime, we choose $H=2A$, since it is the maximum length that the particle can travel vertically for a half period of the external oscillation, and furthermore $l\ll A$ is satisfied in the low-frequency regime. Then the resonance condition between parabolic motion of the particle and external oscillation can be given by $2\tau_g=\tau_f$, where τ_g corresponds to one-half cycle of the external oscillation. It yields $A_c=(g/16f^2)$. Substituting this relation into the the relation $\Gamma=[(2\pi f)^2A/g]$, we obtain $\Gamma_c=2.46$ for the transition. Although the choice of numerical factors in equations needs more elaborate calculation based on the detailed dynamics of the system, the simplest choice results in a reasonable agreement with the experimental result ($\Gamma_c\approx 1.8$). The transition occurring with constant A in the high-frequency regime is explained as follows. When the period of external excitation is much shorter than the acceleration time of the particle, i.e., $\tau_f\ll\tau_g$, the particle hardly moves in the gap of the cell due to inertia. In fact, this is confirmed in the experiment by analyzing the vertical motion of the particle in the box from the video. In such a situation, the particle is hit alternately from top and bottom plates for every one-half cycle of the external oscillation, while the particle remains at the same height. This condition, $2A=l$, yields, $A_c=l/2$. The above estimates for the transitions agree well with the experimental observation ($A_c=1.8$ mm).

Next we consider the crossover of the two regimes in the order-disorder transition at a frequency of around $f=f_c$ ($f_c \approx 50$ Hz). In the low-frequency regime, we assumed that $l \ll A$, and $H=2A$, while in the high-frequency regime, A becomes smaller than l. In that case, one can assume $H=l$, because this is the minimum and necessary length for the particle to travel until it hits the plate. Thus, $\tau_g^{min}=\sqrt{2l/g}$ is obtained. The condition for the resonance between $2\tau_g^{min}$ and τ_f, with $l=0.36$ mm, yields $f_c=58$ Hz.

The dimensional analysis in the above might be an oversimplification if we consider complicated dynamics of the particle in reality. For instance, the resonance condition gave only the phase boundaries, however, the coherent motion continues up to $\Gamma\sim 20$. Elucidating the mechanism working at high Γ is an open problem. We suffice to note the experimental fact that the particle's head always hit the plates, the oscillation of the angle θ is locked to the external frequency at low frequency ordered regime, and stationary in high frequency ordered regime even at large A. We experimentally verified that the analysis also predicted a transition in the case of a bolt whose shape, mass, and size are different from those of the asymmetric particle. An application to micromotors may also be interesting. Honda *et al.* found that the sign of the mean velocity of a micromotor in a narrow pipe quickly varies from minus to plus at a certain frequency by fixing the amplitude [14].

Finally, we mention open problems; how does the coherent motion arise out of orthogonal external perturbations, and what determines the direction of the motion? We suppose that if the frictional force in the forward motion differs from that of the backward motion due to the asymmetry of the particle, the time average of the horizontal force may not be always vanishing. The most efficient acceleration by repeated collisions might be a resonance between swinging motion and collision, otherwise horizontal forces are canceled out due to the chaotic motion of the particle. This mechanism of the emergence of the coherent motion out of fluctuations may be general in different systems. In fact, in thermal ratchet systems, the highest efficiency is attained when the resonance condition is satisfied [15]. Furthermore, in our deterministic system, thermal noise is negligible and resonance is playing more important roles; the exponential distribution of interval may not be just a Poisson distribution that commonly appears in stochastic systems. The exponential distribution of regular intervals interspersed by irregular intermissions can arise through a global bifurcation created by a homoclinic orbit connecting a destabilized saddle point [16]. The existence of chaos behind the motion is indeed indicated by the variance of the velocity distribution [see Fig. 3(a)] and the exponential decay of the temporal correlation function for the velocity component $V_x(t)$ (data not shown). Thus, the understanding of the partial resonance mechanism in deterministic systems may serve as an interesting challenge.

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