Giant clusters in random *ad hoc* networks

G. Németh* and G. Vattay
Communication Networks Laboratory, Eötvös University, Budapest, Hungary
(Received 18 November 2002; published 18 March 2003)

The present paper introduces *ad hoc* communication networks as examples of large scale real networks that can be prospected by statistical means. A description of giant cluster formation based on a single parameter of node neighbor numbers is given along with the discussion of some asymptotic aspects of giant cluster sizes.

DOI: 10.1103/PhysRevE.67.036110 PACS number(s): 89.75.Hc, 05.40.−a, 64.60.Cn

I. INTRODUCTION

Nowadays, natural and designed networks are in the focus of research on different scientific disciplines. Using computers the amount of available empirical data on real world networks has been increased during the past few years. Examples of real networks include the World Wide Web [1,2], the Internet [3–7], collaboration networks of movie actors and scientists [8–10], power grids [11,12], and the metabolic network of living organisms [13–16].

Random graphs are natural candidates for the description of the topology of such large systems of similar units. In Refs. [17–19] the authors have developed a model—which assumes each pair of the graph's vertices to be connected with equal and independent probabilities—that treats a network as an assembly of *equivalent units*.

This model, introduced by the mathematicians Erdös and Rényi, has been much investigated in the mathematical literature [20,21]. However, the increasing availability of large maps of real-life networks has indicated that the latter structures are fundamentally correlated systems, and in many respects their topologies deviate from the uncorrelated random graph model.

Two classes of models, commonly called the *small-world graphs* [11,12,22] and the *scale-free networks* [23,24], have been developed to capture the clustering and the power law degree distribution present in real networks [1,3,8–13,23–29].

Here we present *ad hoc networks* [30] as examples of real structures that can be investigated similarly to the above networks. *Ad hoc* networks arise in the next generations of communication systems and thereby we try to summarize the principal characteristics of such systems. In the *ad hoc* scheme users communicate by means of short range radio devices, which means that every device can connect to those devices that are positioned no farther than a finite maximum geometrical range. We call this range the given device's *transmission range* and the exact value of this range may depend on the transmitter's power and various other physical parameters. See Fig. 1 for an example of *ad hoc* network topology. Neighbor nodes talk the way ordinary radios, such as CBs, do; however, communication between non-neighboring users is also possible. The latter case is accomplished by sending the information from the source user to the destination hop by hop, through intermediate nodes. If the density of users in the area is high compared to their transmission ranges, it is highly possible that more than one alternative route exists between two users. This last feature can be exploited in the case if the shortest route is overloaded or broken, or if the system allows splitting the information flow into separate parallel flows. Moreover, the users are free to move randomly and organize themselves arbitrarily; thus, the network's topology may change rapidly and unpredictably. Such a network may operate in a stand-alone fashion, or may be connected to the Internet.

Giant clusters in *ad hoc* networks are made interesting because a communication network provides a meaningful service only if it integrates as many users as possible within the covered area (e.g., 99% may be considered a good coverage). In this paper we introduce a fractal model that duplicates the giant component formation in the *ad hoc* networks in an area inlaid with obstacles, partially screening radio transmission. Our main result is that in such networks the giant component size can be described by a single parameter—the average number of neighbors a node has. The rest of this paper is structured as follows. Section II gives a detailed description of our random *ad hoc* network model. In Secs. III and IV we delve into the topology differences between random graphs and graphs built using our model. Section V shows the numerical simulation results supporting these analyses.

II. THE RANDOM *AD HOC* NETWORK MODEL

A wireless *ad hoc* network consists of a number of radio devices, also referred to as "nodes" in the following. Every

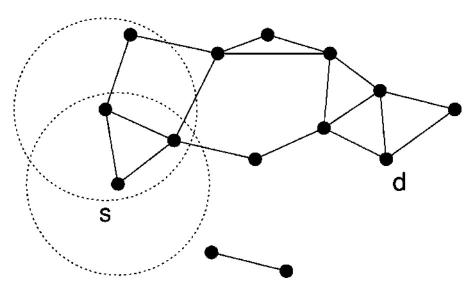

FIG. 1. Nodes and connections of an example *ad hoc* network. The transmission range is the same for all nodes—denoted by the dotted circle for two of the nodes. The shortest path between the *s* source and the *d* destination users touches three intermediate nodes, and there is an alternative route of six hops, which has no common intermediate nodes with the first.

*Electronic address: gab@cnl.elte.hu

node may be connected to one or more other nodes in her vicinity; the actual set of connections depends on the distance of the nodes. In a static environment these connections define the topology of the system; if the nodes are allowed to move then the topology may change; however, at any given point of time there is still a well-defined topology available.

To be precise we define a *random ad hoc network* as a set of uniformly distributed nodes on the arena of the unit Euclidean square $[0;1]\times[0;1]$ with the connections between the pairs of them. The connections are two way in the sense that if node A can communicate to node B, then node B is also able to communicate to node A.

Two nodes are connected if the geometrical distance of the two is less than a certain value r_t, that is, the nodes can communicate up to their "transmission range." We represent a realization of such a system using an undirected graph $G(V,E)$, where the vertices and the edges denote the nodes and the two-way connections, respectively. Sometimes a graph resulting this way is referred to as a *geometric random graph (GRG)*. Note that there are no loops and no multiple edges in G: (1) a node should not communicate to itself; and (2) if two nodes are neighbors, then technically there is no sense to open a second communication channel between them.

Furthermore, all the length parameters in the system are made dimensionless as follows. Length is measured as the multiples of the *unit radius* r_0, which is, in turn, defined by the share of the whole area for each node:

$$r_0 := \sqrt{\frac{A}{N\pi}}, \tag{1}$$

where A denotes the size of the arena and N is the number of nodes. The ratio of the transmission range and the unit radius is called the *normalized transmission range* and is denoted by

$$r_{\mathrm{n}} := \frac{r_t}{r_0}. \tag{2}$$

As mentioned in the Introduction, a communication network may deliver meaningful service only if the network is connected, or at least has a vast subset that is connected. Our work is focused on examining the criteria for giant cluster formation and, in particular, in the networks with fractal connectivity properties.

In the following we give a short overview of the networks on random graphs and afterwards we turn to our model of fractal *ad hoc* connectivity.

III. CONNECTIVITY IN RANDOM NETWORKS

After distributing and connecting the nodes as described previously, the largest connected component of G can be determined. Let S be this components' size fraction:

$$S := \frac{(\text{nodes in the largest component})}{N},$$

which quantity is obtained by counting. This quantity is of particular importance because the network gets fully connected if S diverges and for this end we are to investigate its relationship with other network parameters.

In Ref. [31] the authors present the theory of *random graphs* [18] of arbitrary degree distribution. Among others, an exact result for the giant component size is given, which we shall briefly cite here. Their theory is based on the generating function formalism: given a unipartite undirected graph G and p_k being the probability that a vertex on G has degree k, the generating function for the vertex degree distribution is defined as

$$G_0(x) = \sum_{k=0}^{\infty} p_k x^k$$

and if the distribution p_k is correctly normalized then $G_0(1) = 1$ will hold. Also another function of importance is the one generating the distribution of degree of the vertices pointed to by a randomly chosen edge. Following such an edge one arrives at a vertex with probability proportional to the degree of that vertex, thus the degree distribution is proportional to kp_k, and the normalized distribution is generated by

$$\frac{\sum_k p_k x^k}{\sum_k p_k} = x\frac{G_0'(x)}{G_0'(1)}.$$

Considering the distribution of the remaining edges (i.e., all edges except the one we arrived on), this distribution is the same as above, less by one power of x, making it generated by

$$G_1(x) = \frac{G_0'(x)}{G_0'(1)}.$$

It was shown in Ref. [31] that using $G_0(x)$ and $G_1(x)$, and if there is a giant component in the graph then this component's size can be calculated as

$$S = 1 - G_0(u), \tag{3}$$

where u is the smallest non-negative real value satisfying

$$u = G_1(u). \tag{4}$$

According to these equations we are now able to obtain a closed form expression for S in our GRGs. For more reference on the derivation of the results cited above please see Secs. II A, II C, and II D of Ref. [31].

Let us now use the actual degree distribution of our *ad hoc* networks: it is easily seen that the probability distribution of the number of nodes contained in any disc with radius r_n is the Poisson distribution with expectation value r_n^2. It means that

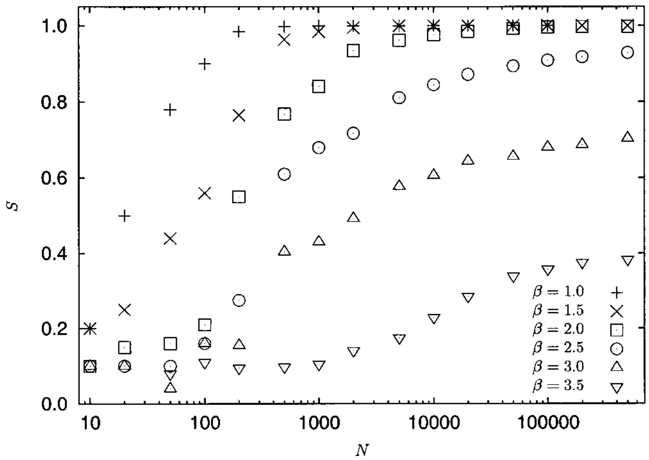

FIG. 2. Giant component sizes for various values of N and β ($a=0.2$ for all cases). Note how $S(N)$ reaches 1 for $\beta \leqslant 2$, yet for $\beta > 2$, $S(N)$ tends to a value strictly <1.

$$p_k = \frac{(r_n^2)^k}{k!} e^{-r_n^2} \tag{5}$$

is the probability that a vertex will have $k-1$ neighbors (-1 is because the node itself does not count for a neighbor). The generating functions are

$$G_0(x) = e^{-r_n^2} \sum_{k=0}^{\infty} \frac{(r_n^2)^k}{k!} x^k = e^{(x-1)r_n^2}$$

and

$$G_1(x) = \frac{r_n^2 G_0(x)}{r_n^2} \equiv G_0(x),$$

which speciality of the Poisson distribution makes

$$u = 1 - S,$$

a solution of Eq. (4), whence Eq. (3) turns into

$$S = 1 - e^{-S r_n^2},$$

and after rearranging, the relation of the size of the giant component and the transmission range finally becomes

$$r_n^2 = \frac{\log(1-S)}{-S}. \tag{6}$$

Applying this relation, one is able to calculate the minimum transmission range needed to achieve a given connectivity ratio in random networks of large node numbers, as illustrated in Fig. 3 (see also Sec. V).

It will be noted here that while Eq. (6) holds for random networks and—as is shown in Sec. V—for fractal *ad hoc* networks, the r_n–S relationship is different for the finite range *ad hoc* case; however, the latter is to be discussed in a separate paper.

IV. THE FRACTAL *AD HOC* NEIGHBORSHIP ALGORITHM

The results of the preceding section apply for scenarios where the arena is ''flat,'' that is, the only limit to build a connection between two nodes is their geometrical distance. In the present section we introduce the idea of generalized obstacles that can screen nodes from each other even if they are positioned within the transmission range. This change produces graphs with extended spatial structure, which is why we call the algorithm fractal.

The obstacles are adopted by changing the algorithm for edge generation. Now two nodes within the transmission range will be connected with a probability, which is given as the function of their geometrical distance. For every two nodes $u,v \in V$, let $p(\mathrm{dist}(u,v))$ be the probability that an edge $e_{uv} \in E$ connecting them is set up.

To choose the actual form of $p(r)$, consider the following. First, the possibility of connections will drop with increasing geometrical distances, which makes $p(r)$ to be in inverse proportion to r. Second, for the description of the obstacles one may think of a hilly landscape. On the one hand, a node may be covered from the view of nearby nodes by an adjacent hill; the falloff (the measure of ''hilliness'') is controlled by the value of parameter β—hills get more dense with increasing β. On the other hand, at any point on the arena there can be directions at which the communication from the given point is not screened for a larger-than-average distance (e.g., sitting in linked valleys or residing on hilltops); this makes long range connections still possible, even though connections are mostly short range. Finally, the singularity caused by the $1/r$ term is shifted to the left to make $p(r)$ finite for all $r \geqslant 0$, and a normalization parameter is introduced, a, which enables to regulate the amplitude of $p(0)$. As a result, $p(r)$ takes the form

$$p(r) = \frac{a}{\left(1 + \frac{r}{r_0 \beta}\right)^{\beta}}, \tag{7}$$

with feasible parameter values $a > 0$ and $\beta > 0$.

Performing computer simulations of networks connected according to Eq. (7), one obtains different results, as β changes. In Fig. 2 we compared the resulting giant cluster sizes for different β values. At lower parameter values $S(N)$ saturates to $S=1$; all nodes become elements of the giant cluster above a certain finite node number. For $\beta = 2.5$ and above, S still converges to a finite value; however, the limit now is strictly less than 1. It means that networks with such parameter values will not become fully connected even at large node numbers. Moreover, the proportion of the largest connected subgraphs drops with β worse than linearly. In the rest of this section we try to interpret this dual behavior of $S(\beta)$.

It is easy to imagine that the more connections the nodes have in average, the larger the giant cluster grows. More accurately we state that the *average vertex degree* $\langle C \rangle$ determines the cardinality of the largest connected subgraph in G. Clearly, if $\langle C \rangle = 0$, then every connected component contains a single node, and in the $N \rightarrow \infty$ limit S becomes 0.

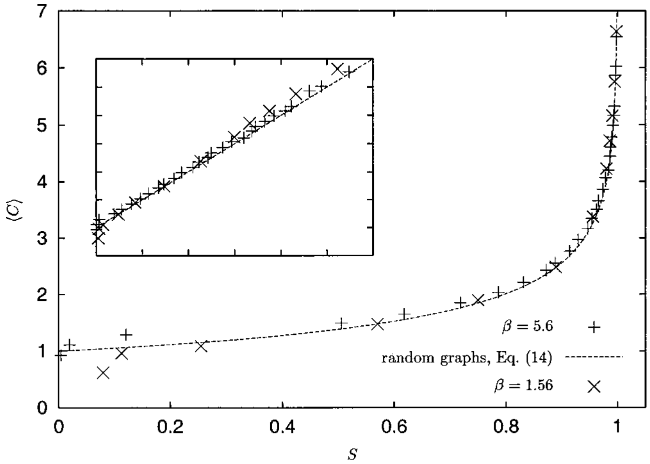

FIG. 3. Simulations of *ad hoc* networks by using the fractal neighborship algorithm with parameter values both $\beta<2$ and $\beta>2$ yield the same giant cluster size vs average vertex degree as random graphs do. Inset displays same plots with $-\log(1-S)/S$ as the abscissa.

Also, if $\langle C\rangle$ diverges or even if only a single node is connected to all the others, the graph obviously gets fully connected. Based on these considerations we are to examine $\langle C\rangle$ in detail.

Vertex degree in G can be calculated by fixing a single node and totaling the $\langle C_r\rangle$ expectation value of the number of neighbors that reside exactly at the distance r away from the fixed one. Assuming that the density of nodes is constant (N/A), $\langle C_r\rangle$ can be expressed by multiplying the average number of nodes at distance r and the probability (7):

$$\langle C_r\rangle=\frac{2r\pi}{A}Np(r).$$

Now if $\overline{\rho}=N/A$, the average vertex degree is

$$\langle C\rangle=\int_{\mathfrak{A}}\langle C_r\rangle dr=\int_{\mathfrak{A}}p(r)2\pi\overline{\rho}r\,dr, \tag{8}$$

where $\mathfrak{A}$ represents the physical boundaries of the arena. As there are no nodes outside this region, the integral will be 0 outside $\mathfrak{A}$.

In general, solving Eq. (8) yields

$$\langle C\rangle=a2\pi\overline{\rho}\frac{r_0\beta}{1-\beta}\left[r\left(1+\frac{r}{r_0\beta}\right)^{1-\beta}-\frac{r_0\beta}{2-\beta}\left(1+\frac{r}{r_0\beta}\right)^{2-\beta}\right]_{\mathfrak{A}}. \tag{9}$$

However, the expectation value of $\langle C\rangle$ is dependent on the value of β. Accordingly, our discussion is separated into several cases.

(1) $\beta>2$. In this case Eq. (9) can be evaluated for $\mathfrak{A}$ being the interval $r\in[0;\infty)$ in the limit where $r_0\to 0$:

$$\langle C\rangle=\frac{a2\pi\overline{\rho}r_0^2\beta^2}{(1-\beta)(2-\beta)}\equiv\frac{2a\beta^2}{(\beta-1)(\beta-2)}. \tag{10}$$

Furthermore, knowing that

$$\lim_{\alpha\to\infty}\frac{1}{(1+x)^{\alpha}}=e^{-\alpha x}$$

in the $\beta\to\infty$ limit, Eq. (10) becomes

$$\langle C\rangle=a2\pi\overline{\rho}r_0^2\equiv 2a.$$

(2) $\beta=1$ or $\beta=2$. Equation (9) diverges logarithmically in r, thus C does not have an expectation value.

(3) $\beta<2$ and $\beta\neq 1$. Here $\langle C\rangle$ will diverge as $N\to\infty$; however, unlike the previous case we try to determine the $\langle C(N)\rangle$ relation. First let us rewrite Eq. (9) as

$$\langle C\rangle=\frac{a2\pi\overline{\rho}r_0\beta}{(1-\beta)(2-\beta)}\left[\frac{r(1-\beta)-r_0\beta}{\left(1+\frac{r}{r_0\beta}\right)^{\beta-1}}\right]_{\mathfrak{A}}. \tag{11}$$

Concerning the r dependence in $[\cdots]_{\mathfrak{A}}$ we can assume that there is a maximal transmission range $r_{\max}$ such that for transmission ranges $r>r_{\max}$ the contribution of the integrand in Eq. (8) is negligible. In this way $[\cdots]_{\mathfrak{A}}$ part of Eq. (11) can be estimated as

$$[\cdots]_{\mathfrak{A}}\simeq-\frac{r_0\beta}{\left(1+\frac{r}{r_0\beta}\right)^{\beta-1}}+\frac{r_{\max}(1-\beta)}{\left(1+\frac{r_{\max}}{r_0\beta}\right)^{\beta-1}}. \tag{12}$$

Now if $r_0\to 0$ (which happens to be the case at sufficiently large node numbers), the first term in Eq. (12) vanishes and $+1$ becomes negligible in the denominator of the second term. After substituting this second term and simplifying the expression, Eq. (9) finally becomes

$$\langle C\rangle\simeq\frac{a2\pi\overline{\rho}}{2-\beta}\left(\frac{r_0\beta}{r_{\max}}\right)^{\beta}r_{\max}^2.$$

The N dependence of $\langle C\rangle$ can be derived from here by substituting definition (1), $\overline{\rho}=N/A$, and the fact that $r_{\max}^2\propto A$. By these means the above expression yields

$$\langle C\rangle\propto N^{1-\beta/2}. \tag{13}$$

To summarize, if $\beta>2$, then a finite neighbor count is expected, and thus such networks are not going to be fully connected (see again Fig. 2). On the other hand, if $\beta<2$, then $\langle C\rangle$ diverges exponentially with increasing node numbers, which in theory leads to fully connected networks at large N, and this means that the more nodes are in the system, the larger the fraction of connected nodes is to become.

V. SIMULATION RESULTS

We carried out computer simulations to illustrate our findings, especially Eqs. (10) and (13). During a simulation run we first pick the random coordinates for the N nodes. Second, the probability p is calculated according to Eq. (7), us-

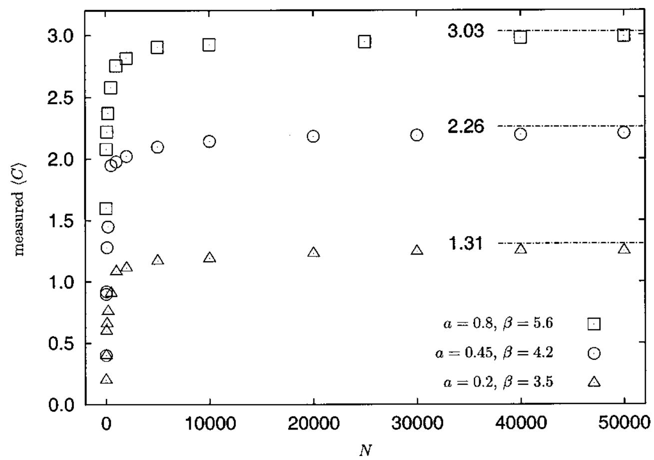

FIG. 4. Average vertex degrees of *ad hoc* graphs for $\beta > 2$. Data points were acquired using the indicated parameter sets. Dashed lines yield the appropriate analytical results, which shall hold in the $N \to \infty$ limit as by Eq. (10).

ing the input parameters a, β, and r. Then for every two nodes a uniform random number $\xi \in [0;1]$ is generated and compared to p: for cases $\xi < p$ an edge connecting those two nodes is recorded. Finally, we count the component sizes and take the largest of these. The output of the simulation run is the average vertex degree $\langle C \rangle$ and the largest components' size S.

As the first test we recorded the giant cluster size vs transmission range relationship. Data points were obtained by repeated runs, changing only the amplitude parameter a of Eq. (7) over an appropriate interval (e.g., $a \in [0.1;1.5]$ for the $\beta = 5.6$ case). The collected output data are shown in Fig. 3. We also shall note the analogy with random graphs: using Eq. (5), the average vertex degree is $\langle C_{\mathrm{rnd}} \rangle = \Sigma_k k p_k = r_n^2$ and, therefore, Eq. (6) can be expressed as

$$\langle C_{\mathrm{rnd}} \rangle = \frac{\log(1-S)}{-S}. \qquad (14)$$

Figure 3 illustrates well that in a network connected using the fractal neighborship algorithm, the observable $S - \langle C \rangle$ relationship matches the equivalent analytical result for random graphs for both relevant cases (1) and (2) in Sec. IV.

On the other hand, the behavior of $\langle C \rangle$ turns out to be sensible to the value of β, as expected. Let us start with the $\beta > 2$ case. Figure 4 presents the simulation results for networks connected as by Eq. (7), using different parameter sets. For example, according to Eq. (10), the average vertex degree for the $a = 0.8, \beta = 5.6$ case is expected to be

$$\langle C \rangle = \frac{2 \times 0.8 \times 5.6^2}{4.6 \times 3.6} \simeq 3.03,$$

which seems to fulfill in Fig. 4: increasing N, the simulation output converges to the analytical result.

Now let us turn to the $0 < \beta < 2$ case. In Fig. 5 the data obtained for $a = 0.1$ and $\beta = 1.56$ are shown along with a numeric function fit according to Eq. (13):

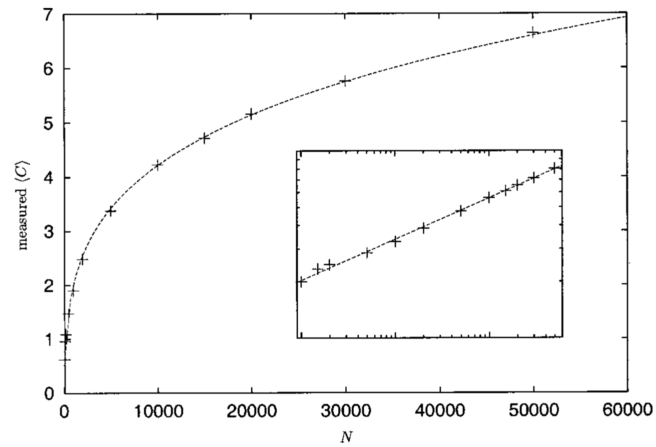

FIG. 5. Divergence of average vertex degrees with N for $\beta < 2$. Crosses, data points for $a = 0.1$, $\beta = 1.56$; the dashed line corresponds to $N^{1-\beta/2} = N^{0.22}$. The inset displays the same plot with both axes logarithmic.

$$\langle C(N) \rangle = c_0 N^{1-(1.56/2)} + c_1$$

(the parameters turn out to be $c_0 = 0.74$ and $c_1 = -1.38$). The simulations agree with the $N^{1-\beta/2}$ divergence well, as calculated in Sec. IV.

Figures 4 and 5 now illustrate the differing S behavior presented in Fig. 2. Data sets for $\beta = 5.6$, 4.2, and 3.5 do not approach full connectivity: with increasing N they converge to $S \simeq 0.93$, 0.84, and 0.33, respectively; on the contrary, the $\beta = 1.56$ case clearly reaches $S = 1$ for node numbers in the magnitude of several thousands.

VI. CONCLUSIONS

In the present paper we have investigated the connected components that are produced in random *ad hoc* networks. Based on the results, the number of nodes needed for a given connectivity ratio can be estimated. Thus, our results may hint about the usefulness of random fractal *ad hoc* networks.

We modified the conventional connection function and made long range connections possible. This way the producing networks become extended in their spatial structure, as thought the network is situated in an area with obstacles screening some of the transmissions. We have found that a single parameter—the average neighbor count $\langle C \rangle$—can characterize the proportion of the largest connected subnetwork. We have also seen that depending on the connection function parameters, this proportion can be either bounded or unbounded as the system size N is increased. For both cases $\langle C(N) \rangle$ was derived analytically and confirmed by the simulations.

ACKNOWLEDGMENT

The authors thank the Hungarian National Science Foundation (OTKA T37903 and T32437) for support.

[1] R. Albert, H. Jeong, and A.-L. Barabási, Nature (London) **401**, 130 (1999).

[2] A. Broder, R. Kumar, F. Maghoul, P. Raghavan, S. Rajalopagan, R. Stata, A. Tomkins, and J. Wiener, in Proceedings of the 9th International World-Wide Web Conference, 2000 (unpublished), http://www.almaden.ibm.com/cs/k53/www9.final.

[3] M. Faloutsos, P. Faloutsos, and C. Faloutsos, Comput. Commun. Rev. **29**, 251 (1999).

[4] A. Medina, I. Matta, and J. Byers, Comput. Commun. Rev. **30**, 18 (2000).

[5] R. Cohen, K. Erez, D. ben Avraham, and S. Havlin, Phys. Rev. Lett. **85**, 4626 (2000).

[6] R. Cohen, K. Erez, D. ben Avraham, and S. Havlin, Phys. Rev. Lett. **86**, 3682 (2001).

[7] Y. Shavitt, X. Sun, A. Wool, and B. Yener, Lucent Technologies Report No. 10009674-000214-01TM, 2000 (unpublished).

[8] A. L. Barabási, H. Jeong, E. Ravasz, Z. Néda, T. Vicsek, and A. Schubert, Physica A **311**, 590 (2002).

[9] M.E.J. Newman, Proc. Natl. Acad. Sci. U.S.A. **98**, 404 (2001).

[10] M.E.J. Newman, e-print cond-mat/0011144.

[11] D.J. Watts and S.H. Strogatz, Nature (London) **393**, 440 (1998).

[12] D.J. Watts, *Small Worlds: The Dynamics of Networks between Order and Randomness (Princeton Reviews in Complexity)* (Princeton University Press, Princeton, NJ, 1999).

[13] H. Jeong, B. Tombor, R. Albert, Z.N. Oltvai, and A.-L. Barabási, Nature (London) **407**, 651 (2000).

[14] P.M. Gleiss, P.F. Stadler, A. Wagner, and D.A. Fell, Santa Fe Institute Report No. 00-10-058, 2000 (unpublished).

[15] J.M. Montoya and R.V. Solé, Santa Fe Institute Report No. 00-10-059, 2000 (unpublished).

[16] R.V. Solé and J.M. Montoya, Santa Fe Institute Report No. 00-10-060, 2000 (unpublished).

[17] P. Erdös and A. Rényi, Publ. Math. (Debrecen) **6**, 290 (1959).

[18] P. Erdös and A. Rényi, Publ. of the Math. Inst. of the Hung. Acad. of Sci. **5**, 17 (1960).

[19] P. Erdös and A. Rényi, Acta Math. Acad. Sci. Hung. **12**, 261 (1961).

[20] B. Bollobás, *Random Graphs* (Academic, London, 1985).

[21] B. Bollobás and A. G. Thomason, *The Annals of Discrete Mathematics* (North-Holland, Amsterdam, 1985), Vol. 28, pp. 47–97.

[22] M. Barthélémy and L.A.N. Amaral, Phys. Rev. Lett. **82**, 3180 (1999).

[23] A.-L. Barabási and R. Albert, Science **286**, 509 (1999).

[24] A.-L. Barabási, R. Albert, and H. Jeong, Physica A **272**, 173 (1999).

[25] S. Redner, Eur. Phys. J. B **4**, 131 (1998).

[26] L.A. Adamic and B.A. Huberman, Nature (London) **401**, 131 (1999).

[27] J. Kleinberg, S. Kumar, P. Raghavan, S. Rajagopalan, and A. Tomkins (unpublished).

[28] L.A.N. Amaral, A. Scala, M. Barthélémy, and H.E. Stanley, e-print cond-mat/0001458.

[29] I.J. Farkas, I. Derényi, A.-L. Barabási, and T. Vicsek, Phys. Rev. E **64**, 026704 (2001).

[30] IETF Mobile Ad-hoc Networks Working Group, http://www.ietf.org/html.charters/manet-charter.html.

[31] M.E.J. Newman, S.H. Strogatz, and D.J. Watts, Phys. Rev. E **64**, 026118 (2001).