Universality of electromagnetic-field correlations within homogeneous and isotropic sources

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We investigate the structure of second-order correlations in electromagnetic fields produced by statistically stationary, homogeneous, and isotropic current distributions. We show that the coherence properties of such fields within a low-loss or nondissipative medium do not depend on the source characteristics, but are solely determined by the propagation properties, and that the degree of coherence of the field is given by the sinc law. Our analysis reproduces the known results for blackbody fields, but it applies to a wider class of sources, not necessarily in thermal equilibrium. We discuss the physics behind the universal behavior of the correlations by comparing the results with those obtained by an electromagnetic plane-wave model.

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I. INTRODUCTION

It has recently been shown that the correlations in scalar wave fields generated by statistically stationary, homogeneous, and isotropic sources fluctuating within a medium of vanishingly small absorption, exhibit spatially universal structures [1,2]. More precisely, the spectral degree of coherence of the field is proportional to the imaginary part of the Green function of the system, indicating that the field correlations are determined by propagation properties only, and not by the source characteristics. This is true for the fields in three and two dimensions [2]. In three-dimensional space, the degree of coherence varies in space as the sinc function, and a superposition of isotropically distributed and angularly uncorrelated plane waves has been shown to produce this functional form for the coherence function [3]. Furthermore, the same coherence function is found for the low-frequency part of statistically homogeneous planar Lambertian sources [4], and for the field within a large δ-correlated primary spherical source [5].

Albeit various investigations on the subject matter have been performed using scalar theory, less attention has been paid to the coherence properties of electromagnetic fields. An exception to this is blackbody radiation, for which the cross-spectral density tensors are known [6]. In particular, the electric cross-spectral density tensor of the blackbody field is proportional to the imaginary part of the Green tensor of the system [6,7], and thus, the normalized trace of the tensor, or the field's degree of coherence, obtains the form of a sinc function. In this work, we show that when the losses in the medium are negligible, this universal character of the field correlations is shared by all electromagnetic fields generated by statistically homogeneous and isotropic current distributions, not only by thermal sources in equilibrium.

The paper is organized as follows. In Sec. II, we derive an expression for the electric cross-spectral density tensor of the field in terms of the corresponding source tensor when the source is statistically homogeneous. The approach is a full electromagnetic analog to the method employed in Ref. [2]. In Sec. III, the formula is applied to sources which are not only homogeneous, but also isotropic. Finally, in Sec. IV, we provide a physical explanation of the results, and summarize the main conclusions of the work. Details of the mathematical calculations are relegated to Appendixes A–D.

II. ELECTRIC CROSS-SPECTRAL DENSITY TENSOR OF THE FIELD GENERATED BY A STATISTICALLY HOMOGENEOUS CURRENT DISTRIBUTION

The second-order spatial correlation properties of a stationary current density distribution and of the electromagnetic field that it generates are described in the space-frequency domain in terms of the cross-spectral density tensors (Ref. [8], Sec. 6.5)

$$\overleftrightarrow{W}_{jj}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\langle \mathbf{j}^*(\mathbf{r}_1,\omega)\mathbf{j}(\mathbf{r}_2,\omega)\rangle, \tag{1}$$

$$\overleftrightarrow{W}_{ee}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\langle \mathbf{E}^*(\mathbf{r}_1,\omega)\mathbf{E}(\mathbf{r}_2,\omega)\rangle. \tag{2}$$

The vectors $\mathbf{j}(\mathbf{r},\omega)$ and $\mathbf{E}(\mathbf{r},\omega)$ represent members of the statistical ensembles of monochromatic current and electric-field realizations at the frequency ω. The angle brackets and the asterisk (*) denote ensemble averaging and complex conjugation, respectively, and $\mathbf{r}_{1,2}$ refer to two points in space.

In a homogeneous, isotropic, and linear medium, the monochromatic realizations obey the inhomogeneous vector wave equation, which (in SI units) reads as

$$\nabla\times\nabla\times\mathbf{E}(\mathbf{r},\omega)-\kappa^2(\omega)\mathbf{E}(\mathbf{r},\omega)=i\omega\mu(\omega)\mathbf{j}(\mathbf{r},\omega). \tag{3}$$

Here $\kappa(\omega)=k_0 n(\omega)$, with k_0 being the free-space wave number, and $n(\omega)$ is the complex refractive index of the medium, which is expressed as $n^2(\omega)=\epsilon_r(\omega)\mu_r(\omega)$ in terms of the relative permittivity $\epsilon_r(\omega)=\epsilon(\omega)/\epsilon_0$ and permeability $\mu_r(\omega)=\mu(\omega)/\mu_0$, given as ratios of the corresponding value in the medium to that in vacuum. In order to

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simplify the notation we shall, from now on, drop the frequency dependence of the material parameters. In addition, the real and imaginary parts of the parameters will be indicated by primed and double-primed symbols, respectively.

Since the source fluctuations are assumed to be statistically homogeneous, it is advantageous to transfer into the Fourier space by introducing the spatial Fourier transforms of the current density and of the field as

$$\tilde{\mathbf{j}}(\mathbf{k},\omega)=\int d^3R\,\mathbf{j}(\mathbf{R},\omega)e^{-i\mathbf{k}\cdot\mathbf{R}}, \tag{4}$$

$$\tilde{\mathbf{E}}(\mathbf{k},\omega)=\int d^3r\,\mathbf{E}(\mathbf{r},\omega)e^{-i\mathbf{k}\cdot\mathbf{r}}. \tag{5}$$

For later convenience, we have here adopted a notation in which a capitalized spatial vector refers to a source point. From the wave equation, Eq. (3), we find that the Fourier transforms satisfy the equation

$$\mathbf{k}\times[\mathbf{k}\times\tilde{\mathbf{E}}(\mathbf{k},\omega)]+\kappa^2\tilde{\mathbf{E}}(\mathbf{k},\omega)=-i\omega\mu\tilde{\mathbf{j}}(\mathbf{k},\omega), \tag{6}$$

or equivalently

$$[\mathbf{kk}-(k^2-\kappa^2)\overleftrightarrow{U}]\cdot\tilde{\mathbf{E}}(\mathbf{k},\omega)=-i\omega\mu\tilde{\mathbf{j}}(\mathbf{k},\omega), \tag{7}$$

where $k=|\mathbf{k}|$ and $\overleftrightarrow{U}$ is the unit tensor. Since Eq. (7) is linear, it can straightforwardly be solved for $\tilde{\mathbf{E}}(\mathbf{k},\omega)$ and the result is

$$\tilde{\mathbf{E}}(\mathbf{k},\omega)=-\frac{i\eta_0}{k_0\epsilon_r(k^2-\kappa^2)}(\mathbf{kk}-\kappa^2\overleftrightarrow{U})\cdot\tilde{\mathbf{j}}(\mathbf{k},\omega), \tag{8}$$

where $\eta_0=\sqrt{\mu_0/\epsilon_0}$ is the impedance of vacuum. Equation (8) is central in our analysis, as it expresses the Fourier components of the field in terms of the corresponding components of the source distribution that produces the field.

For a statistically homogeneous source, the cross-spectral density tensor is of the form

$$\overleftrightarrow{W}_{jj}(\mathbf{R}_1,\mathbf{R}_2,\omega)\equiv\overleftrightarrow{W}_{jj}(\mathbf{R},\omega), \tag{9}$$

where $\mathbf{R}=\mathbf{R}_1-\mathbf{R}_2$. We can then directly write in the Fourier space

$$\langle\tilde{\mathbf{j}}^*(\mathbf{k}_1,\omega)\tilde{\mathbf{j}}(\mathbf{k}_2,\omega)\rangle=\int d^3R_1\int d^3R_2\langle\mathbf{j}^*(\mathbf{R}_1,\omega)\mathbf{j}(\mathbf{R}_2,\omega)\rangle \times e^{i\mathbf{k}_1\cdot\mathbf{R}_1-i\mathbf{k}_2\cdot\mathbf{R}_2} \tag{10}$$

$$=(2\pi)^3\delta(\mathbf{k}_1-\mathbf{k}_2)\tilde{\overleftrightarrow{W}}_{jj}(-\mathbf{k}_1,\omega), \tag{11}$$

where

$$\tilde{\overleftrightarrow{W}}_{jj}(\mathbf{k},\omega)=\int d^3R\,\overleftrightarrow{W}_{jj}(\mathbf{R},\omega)e^{-i\mathbf{k}\cdot\mathbf{R}}. \tag{12}$$

Equation (11) states that the different Fourier components of an infinite homogeneous source are δ correlated.

We are now in a position to express the cross-spectral density tensor of the field in terms of the corresponding source tensor. On inserting the inverse transform of Eq. (5) into Eq. (2), we obtain

$$\overleftrightarrow{W}_{ee}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}\langle\tilde{\mathbf{E}}^*(\mathbf{k}_1,\omega)\tilde{\mathbf{E}}(\mathbf{k}_2,\omega)\rangle \times e^{-i\mathbf{k}_1\cdot\mathbf{r}_1+i\mathbf{k}_2\cdot\mathbf{r}_2}. \tag{13}$$

Furthermore, using Eq. (8), we get

$$\langle\tilde{\mathbf{E}}^*(\mathbf{k}_1,\omega)\tilde{\mathbf{E}}(\mathbf{k}_2,\omega)\rangle=\frac{\eta_0^2}{k_0^2|\epsilon_r|^2(k_1^2-\kappa^2)^*(k_2^2-\kappa^2)} \times[\mathbf{k}_1\mathbf{k}_1-(\kappa^2)^*\overleftrightarrow{U}]\cdot\langle\tilde{\mathbf{j}}^*(\mathbf{k}_1,\omega) \times\tilde{\mathbf{j}}(\mathbf{k}_2,\omega)\rangle\cdot[\mathbf{k}_2\mathbf{k}_2-(\kappa^2)\overleftrightarrow{U}], \tag{14}$$

which, when substituted together with Eqs. (11) and (12) into Eq. (13), yields

$$\overleftrightarrow{W}_{ee}(\mathbf{r},\omega) =\frac{\eta_0^2}{k_0^2|\epsilon_r|^2}\int d^3R\int\frac{d^3k}{(2\pi)^3}\frac{1}{|k^2-\kappa^2|^2} \times[\mathbf{kk}-(\kappa^2)^*\overleftrightarrow{U}]\cdot\overleftrightarrow{W}_{jj}(\mathbf{R},\omega)\cdot[\mathbf{kk}-\kappa^2\overleftrightarrow{U}]e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{r})}, \tag{15}$$

where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$. Thus, we see that the field generated by a homogeneous source distribution is also homogeneous, as expected. Equation (15) can be developed further by noting that

$$\mathbf{k}e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{r})}=i\nabla_{\mathbf{r}}e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{r})}, \tag{16}$$

where $\nabla_{\mathbf{r}}$ operates on the vector $\mathbf{r}$. We use this to rewrite Eq. (15) in the form

$$\overleftrightarrow{W}_{ee}(\mathbf{r},\omega)=\eta_0^2k_0^2\mu_r^2\int d^3R\left[\overleftrightarrow{U}+\frac{1}{(\kappa^2)^*}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right]\cdot\overleftrightarrow{W}_{jj}(\mathbf{R},\omega) \cdot\left[\overleftrightarrow{U}+\frac{1}{\kappa^2}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right]\int\frac{d^3k}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{r})}}{|k^2-\kappa^2|^2}. \tag{17}$$

In this formula we have assumed, as we shall also do later on, that the orders of integration and differentiation can be interchanged. The integration over k in Eq. (17) can be carried out analytically by applying the residue theorem as is shown in Appendix A. Making use of Eq. (A3), we have

$$\int\frac{d^3k}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{r})}}{|k^2-\kappa^2|^2}=\frac{1}{(\kappa^2)''}\mathrm{Im}[G(\mathbf{R}-\mathbf{r},\omega)], \tag{18}$$

where Im denotes the imaginary part, and $G(\mathbf{R}-\mathbf{r},\omega)$ is the (scalar) Green function of the system given by

$$G(\mathbf{R}-\mathbf{r},\omega)=\frac{e^{i\kappa|\mathbf{R}-\mathbf{r}|}}{4\pi|\mathbf{R}-\mathbf{r}|}. \tag{19}$$

Applying Eq. (18), the cross-spectral density tensor in Eq. (17) takes on the form

$$\overleftrightarrow{W}_{ee}(\mathbf{r},\omega)=\frac{\eta_0^2\mu_r}{\epsilon_r''}\left[\overleftrightarrow{U}+\frac{1}{(\kappa^2)^*}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right]\cdot\int d^3R\,\overleftrightarrow{W}_{jj}(\mathbf{R},\omega)\cdot\left[\overleftrightarrow{U}+\frac{1}{\kappa^2}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right]\mathrm{Im}[G(\mathbf{R}-\mathbf{r},\omega)]. \tag{20}$$

We note that the integral in Eq. (20) contains a term that resembles the imaginary part of the Green tensor (see Appendix B), but strictly is not since κ, here, is a complex quantity. In the limit of small absorption, however, κ will become (almost) purely real, and to good approximation, the tensor in the brackets in the integrand of Eq. (20) becomes equal to the imaginary part of the Green tensor, i.e.,

$$\overleftrightarrow{W}_{ee}(\mathbf{r},\omega)=\frac{\eta_0^2\mu_r}{\epsilon_r''}\left(\overleftrightarrow{U}+\frac{1}{\kappa^2}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right)\cdot\int d^3R\,\overleftrightarrow{W}_{jj}(\mathbf{R},\omega)\cdot\mathrm{Im}[\overleftrightarrow{G}(\mathbf{R}-\mathbf{r},\omega)]. \tag{21}$$

We stress that in the limit of vanishing losses the imaginary part ϵ_r'' approaches zero, and thus the elements of the cross-spectral density tensor diverge. This divergence, however, disappears for normalized quantities such as the degree of coherence (or if the current source is restricted to a large but finite volume). Since we are also interested in the explicit functional form of the cross-spectral density tensor, we do not, at this stage, perform any normalization, but keep these facts in mind.

III. ELECTRIC CROSS-SPECTRAL DENSITY TENSOR OF THE FIELD GENERATED BY A STATISTICALLY HOMOGENEOUS AND ISOTROPIC CURRENT DISTRIBUTION

We next apply Eq. (21) to sources which are not only homogeneous, but also statistically isotropic. The general form for the cross-spectral density tensor of such a source is explicitly given by [9,10]

$$\overleftrightarrow{W}_{jj}(\mathbf{R},\omega)=A(R,\omega)\overleftrightarrow{U}+B(R,\omega)\hat{\mathbf{R}}\hat{\mathbf{R}}, \tag{22}$$

where $A(R,\omega)$ and $B(R,\omega)$ are scalar functions, and $\hat{\mathbf{R}}=\mathbf{R}/R$ with $R=|\mathbf{R}|$. In fact, the functions $A(R,\omega)$ and $B(R,\omega)$ are not entirely independent, but are connected by a continuity equation. Furthermore, Eq. (22) is symmetric and its form is invariant under rotation of the coordinate system. For convenience, we set

$$\overleftrightarrow{W}^A_{jj}(\mathbf{R},\omega)=A(R,\omega)\overleftrightarrow{U}, \tag{23}$$

$$\overleftrightarrow{W}^B_{jj}(\mathbf{R},\omega)=B(R,\omega)\hat{\mathbf{R}}\hat{\mathbf{R}}, \tag{24}$$

and treat the tensors $\overleftrightarrow{W}^A_{jj}(\mathbf{R},\omega)$ and $\overleftrightarrow{W}^B_{jj}(\mathbf{R},\omega)$ separately.

A. Field correlations generated by the tensor $\overleftrightarrow{W}^A_{jj}(\mathbf{R},\omega)$

On substituting Eq. (23) into Eq. (21) we obtain

$$\overleftrightarrow{W}^A_{ee}(\mathbf{r},\omega)=\frac{\eta_0^2\mu_r}{\epsilon_r''}\left(\overleftrightarrow{U}+\frac{1}{\kappa^2}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right)\cdot\int d^3R\,A(R,\omega)\times\mathrm{Im}[\overleftrightarrow{G}(\mathbf{R}-\mathbf{r},\omega)], \tag{25}$$

which can also be expressed as

$$\overleftrightarrow{W}^A_{ee}(\mathbf{r},\omega)=\frac{\eta_0^2\mu_r}{\epsilon_r''}\left(\overleftrightarrow{U}+\frac{1}{\kappa^2}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right)\cdot\left(\overleftrightarrow{U}+\frac{1}{\kappa^2}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right)\times\int d^3R\,A(R,\omega)\mathrm{Im}[G(\mathbf{R}-\mathbf{r},\omega)]. \tag{26}$$

This equation can be simplified further by performing the angular integrations as outlined in Appendix C. Making use of Eq. (C3) in the low-loss limit, we obtain

$$\int d^3R\,A(R,\omega)\mathrm{Im}[G(\mathbf{R}-\mathbf{r},\omega)]=\kappa C_A(\omega)j_0(\kappa r)=4\pi C_A(\omega)\mathrm{Im}[G(\mathbf{r},\omega)], \tag{27}$$

where

$$C_A(\omega)=\int_0^\infty dR\,R^2A(R,\omega)j_0(\kappa R), \tag{28}$$

and consequently Eq. (26) simplifies to

$$\overleftrightarrow{W}^A_{ee}(\mathbf{r},\omega)=\frac{4\pi\eta_0^2\mu_r}{\epsilon_r''}C_A(\omega)\left(\overleftrightarrow{U}+\frac{1}{\kappa^2}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right)\cdot\mathrm{Im}[\overleftrightarrow{G}(\mathbf{r},\omega)]. \tag{29}$$

As is demonstrated in Appendix B, in the low-loss limit we have

$$\nabla_{\mathbf{r}}\cdot\mathrm{Im}[\overleftrightarrow{G}(\mathbf{r},\omega)]=\mathbf{0}. \tag{30}$$

Making use of this relation, we end up with the expression

$$\overleftrightarrow{W}^{A}_{ee}(\mathbf{r},\omega)=\frac{4\pi\eta_0^2\mu_r}{\epsilon_r''}C_A(\omega)\mathrm{Im}[\overleftrightarrow{G}(\mathbf{r},\omega)] \quad (31)$$

for the cross-spectral density tensor of the field, when the source correlations are of the form $\overleftrightarrow{W}^{A}_{jj}(\mathbf{R},\omega)=A(R,\omega)\overleftrightarrow{U}$. We see that $\overleftrightarrow{W}^{A}_{ee}(\mathbf{r},\omega)$ is proportional to the imaginary part of the Green tensor with the proportionality factor depending on the source characteristics and on the medium.

B. Field correlations generated by the tensor $\overleftrightarrow{W}^{B}_{jj}(\mathbf{R},\omega)$

The tensor $\overleftrightarrow{W}^{B}_{jj}(\mathbf{R},\omega)$ defined in Eq. (24), when inserted into Eq. (21), yields

$$\overleftrightarrow{W}^{B}_{ee}(\mathbf{r},\omega)=\frac{\eta_0^2\mu_r}{\epsilon_r''}\left(\overleftrightarrow{U}+\frac{1}{\kappa^2}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right)\cdot\int d^3R B(R,\omega)\hat{\mathbf{R}}\hat{\mathbf{R}}\cdot\mathrm{Im}[\overleftrightarrow{G}(\mathbf{R}-\mathbf{r},\omega)]. \quad (32)$$

Making use of the fact that $\hat{\mathbf{R}}\hat{\mathbf{R}}\cdot\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}=(\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\cdot\hat{\mathbf{R}}\hat{\mathbf{R}})^T$, where the superscript T denotes the transpose, Eq. (32) can be expressed as

$$\overleftrightarrow{W}^{B}_{ee}(\mathbf{r},\omega)=\frac{\eta_0^2\mu_r}{\epsilon_r''}\left(\overleftrightarrow{U}+\frac{1}{\kappa^2}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right)\cdot\left\{\left(\overleftrightarrow{U}+\frac{1}{\kappa^2}\nabla_{\mathbf{r}}\nabla_{\mathbf{r}}\right)\cdot\int d^3R B(R,\omega)\hat{\mathbf{R}}\hat{\mathbf{R}}\,\mathrm{Im}[G(\mathbf{R}-\mathbf{r},\omega)]\right\}^T. \quad (33)$$

The angular integrations can again be performed analytically as outlined in Appendix D. In the low-loss limit, Eq. (D10) implies that

$$\begin{aligned}\int d^3R B(R,\omega)\hat{\mathbf{R}}\hat{\mathbf{R}}\,\mathrm{Im}[G(\mathbf{R}-\mathbf{r},\omega)]&=\frac{\kappa}{3}[C_{B1}(\omega)j_0(\kappa r)\\&\quad-C_{B2}(\omega)j_2(\kappa r)]\overleftrightarrow{U}+\kappa C_{B2}(\omega)j_2(\kappa r)\hat{\mathbf{r}}\hat{\mathbf{r}}\\&=4\pi C_{B2}(\omega)\mathrm{Im}[\overleftrightarrow{G}(\mathbf{r},\omega)]+\frac{4\pi}{3}[C_{B1}(\omega)\\&\quad-2C_{B2}(\omega)]\mathrm{Im}[G(\mathbf{r},\omega)]\overleftrightarrow{U},\end{aligned} \quad (34)$$

where

$$C_{B1}(\omega)=\int_0^\infty dR R^2 B(R,\omega)j_0(\kappa R), \quad (35)$$

$$C_{B2}(\omega)=\int_0^\infty dR R^2 B(R,\omega)j_2(\kappa R). \quad (36)$$

The last expression in Eq. (34) is obtained with the help of the explicit form for the imaginary part of the Green tensor, Eq. (B5). Substituting Eq. (34) into Eq. (33), and making use of Eq. (30) and the symmetry of the Green tensor, i.e., the fact that $\overleftrightarrow{G}(\mathbf{r},\omega)=\overleftrightarrow{G}(\mathbf{r},\omega)^T$, the cross-spectral density tensor of the field reduces to

$$\overleftrightarrow{W}^{B}_{ee}(\mathbf{r},\omega)=\frac{4\pi\eta_0^2\mu_r}{3\epsilon_r''}[C_{B1}(\omega)+C_{B2}(\omega)]\mathrm{Im}[\overleftrightarrow{G}(\mathbf{r},\omega)]. \quad (37)$$

This formula can further be simplified by combining the spectral coefficients $C_{B1}(\omega)$ and $C_{B2}(\omega)$ in terms of the relation $j_0(\kappa R)+j_2(\kappa R)=3j_1(\kappa R)/\kappa R$ for the spherical Bessel functions, and we find that

$$\overleftrightarrow{W}^{B}_{ee}(\mathbf{r},\omega)=\frac{4\pi\eta_0^2\mu_r}{\epsilon_r''}C_B(\omega)\mathrm{Im}[\overleftrightarrow{G}(\mathbf{r},\omega)], \quad (38)$$

where

$$C_B(\omega)=C_{B1}(\omega)+C_{B2}(\omega)=\int_0^\infty dR R^2 B(R,\omega)\frac{j_1(\kappa R)}{\kappa R}. \quad (39)$$

Thus, also for the source tensor $\overleftrightarrow{W}^{B}_{jj}(\mathbf{R},\omega)$, the spatial correlation properties of the field are described by the imaginary part of the Green tensor.

C. Degree of coherence

When combining Eqs. (31) and (38), we find that

$$\overleftrightarrow{W}_{ee}(\mathbf{r},\omega)=\frac{4\pi\eta_0^2\mu_r}{\epsilon_r''}[C_A(\omega)+C_B(\omega)]\mathrm{Im}[\overleftrightarrow{G}(\mathbf{r},\omega)]. \quad (40)$$

Hence the spatial correlation properties of a fluctuating field, generated by any statistically homogeneous and isotropic current distribution within an infinite low-loss or nondissipative medium, are determined by the imaginary part of the Green tensor of the system. In particular, the normalized trace of the electric cross-spectral density tensor, commonly regarded as the electromagnetic field's degree of spatial coherence [4], acquires the universal form

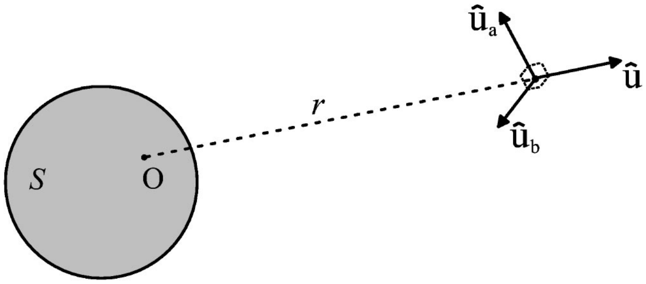

FIG. 1. Illustration of notations for analyzing the far-field polarization. The field emitted by a spherical source occupying the volume S is studied at the distance r from the origin O in the direction specified by the unit vector $\hat{u}$. The vectors $\hat{u}_a$, $\hat{u}_b$, and $\hat{u}$ constitute an orthonormal set of vectors.

$$\mu_{ee}(\mathbf{r},\omega)=\frac{\mathrm{tr}\overleftrightarrow{W}_{ee}(\mathbf{r},\omega)}{\mathrm{tr}\overleftrightarrow{W}_{ee}(\mathbf{0},\omega)}=\frac{\sin\kappa r}{\kappa r}=\frac{4\pi}{\kappa}\mathrm{Im}[G(\mathbf{r},\omega)], \qquad (41)$$

where tr denotes the trace operation. This equation shows that when the conditions assumed for the random source currents and the medium hold, the degree of coherence of the electromagnetic field does not depend on the source characteristics, but only on the propagation properties of the medium. We emphasize that owing to the small but nonzero absorbtion by the medium, the result is valid even for fully coherent current distributions.

D. Polarization of the far field radiated by finite, statistically homogeneous, and isotropic spherical source

We next show that the far field radiated by a finite, statistically homogeneous, and isotropic spherical source is fully unpolarized in all directions. This result will be useful in the following section where we discuss the universal behavior of the correlations. Consider the field generated by a source occupying a spherical volume denoted by the symbol S (see Fig. 1). The far-field realization of the electric field at distance r in the direction specified by unit vector $\hat{u}$ is given by (Ref. [11], Sec. 2.8; see also Ref. [10])

$$\mathbf{E}(r\hat{u},\omega)\sim\frac{i\kappa\eta_0\mu_r e^{i\kappa r}}{4\pi r}(\overleftrightarrow{U}-\hat{u}\hat{u})\cdot\tilde{\mathbf{j}}(\boldsymbol{\kappa},\omega), \qquad (42)$$

where $\boldsymbol{\kappa}=\kappa\hat{u}$. Using Eq. (42), the 3×3 coherence tensor $\overleftrightarrow{\Phi}_3(r\hat{u},\omega)$, which contains all information about the polarization state of the field (at the point $r\hat{u}$), takes the form

$$\overleftrightarrow{\Phi}_3(r\hat{u},\omega)=\overleftrightarrow{W}_{ee}(r\hat{u},r\hat{u},\omega)=\left(\frac{\kappa\eta_0\mu_r}{4\pi r}\right)^2 \times(\overleftrightarrow{U}-\hat{u}\hat{u})\cdot\langle\tilde{\mathbf{j}}^*(\boldsymbol{\kappa},\omega)\tilde{\mathbf{j}}(\boldsymbol{\kappa},\omega)\rangle\cdot(\overleftrightarrow{U}-\hat{u}\hat{u}). \qquad (43)$$

Making use of Eq. (10), we obtain

$$\langle\tilde{\mathbf{j}}^*(\boldsymbol{\kappa},\omega)\tilde{\mathbf{j}}(\boldsymbol{\kappa},\omega)\rangle=\int d^3R_1\int d^3R_2\overleftrightarrow{W}_{jj}(\mathbf{R},\omega)e^{i\boldsymbol{\kappa}\cdot\mathbf{R}}, \qquad (44)$$

where, as before, $\mathbf{R}=\mathbf{R}_1-\mathbf{R}_2$. Since the source is confined to a finite volume, its cross-spectral density tensor is of the form

$$\overleftrightarrow{W}_{jj}(\mathbf{R},\omega)=[A(R,\omega)\overleftrightarrow{U}+B(R,\omega)\hat{\mathbf{R}}\hat{\mathbf{R}}]\mathcal{B}(\mathbf{R}_1)\mathcal{B}(\mathbf{R}_2), \qquad (45)$$

where $\mathcal{B}(\mathbf{R}_i)$ is a blocking function defined such that $\mathcal{B}(\mathbf{R}_i)=1$ if $\mathbf{R}_i\in S$, and $\mathcal{B}(\mathbf{R}_i)=0$ otherwise ($i=1,2$). By setting $B'(R,\omega)=B(R,\omega)/R^2$, transforming into the variables $\mathbf{R}=\mathbf{R}_1-\mathbf{R}_2$ and $\mathbf{R}_c=(\mathbf{R}_1+\mathbf{R}_2)/2$, and making use of the fact that $\mathbf{R}e^{i\boldsymbol{\kappa}\cdot\mathbf{R}}=-i\nabla_{\boldsymbol{\kappa}}e^{i\boldsymbol{\kappa}\cdot\mathbf{R}}$, Eq. (44) assumes the form

$$\langle\tilde{\mathbf{j}}^*(\boldsymbol{\kappa},\omega)\tilde{\mathbf{j}}(\boldsymbol{\kappa},\omega)\rangle=\tilde{A}_{\mathcal{B}}(\kappa,\omega)\overleftrightarrow{U}-\nabla_{\boldsymbol{\kappa}}\nabla_{\boldsymbol{\kappa}}\tilde{B}'_{\mathcal{B}}(\kappa,\omega). \qquad (46)$$

In this formula

$$\tilde{A}_{\mathcal{B}}(\kappa,\omega)=\int d^3RA(R,\omega)I_{\mathcal{B}}(R)e^{-i\boldsymbol{\kappa}\cdot\mathbf{R}}, \qquad (47)$$

$$\tilde{B}'_{\mathcal{B}}(\kappa,\omega)=\int d^3RB'(R,\omega)I_{\mathcal{B}}(R)e^{-i\boldsymbol{\kappa}\cdot\mathbf{R}} \qquad (48)$$

are the Fourier transforms of $A(R,\omega)I_{\mathcal{B}}(R)$ and $B'(R,\omega)I_{\mathcal{B}}(R)$, respectively, and

$$I_{\mathcal{B}}(R)=\int d^3R_c\mathcal{B}(\mathbf{R}_c+\mathbf{R}/2)\mathcal{B}(\mathbf{R}_c-\mathbf{R}/2), \qquad (49)$$

which depends only on the magnitude R for a spherical source region. In Eqs. (47) and (48) we also have made use of the spherical symmetry of the integrands in order to get, in accordance with Eq. (12), a negative sign in the exponent. By performing the derivations in Eq. (46), we find that

$$\langle\tilde{\mathbf{j}}^*(\boldsymbol{\kappa},\omega)\tilde{\mathbf{j}}(\boldsymbol{\kappa},\omega)\rangle=\left[\tilde{A}_{\mathcal{B}}(\kappa,\omega)-\frac{1}{\kappa}\frac{d\tilde{B}'_{\mathcal{B}}(\kappa,\omega)}{d\kappa}\right]\overleftrightarrow{U}+\left[\frac{1}{\kappa}\frac{d\tilde{B}'_{\mathcal{B}}(\kappa,\omega)}{d\kappa}-\frac{d^2\tilde{B}'_{\mathcal{B}}(\kappa,\omega)}{d\kappa^2}\right]\hat{u}\hat{u}, \qquad (50)$$

which, when substituted into Eq. (43) gives

$$\overleftrightarrow{\Phi}_3(r\hat{u},\omega)=\left(\frac{\kappa\eta_0\mu_r}{4\pi r}\right)^2\left[\tilde{A}_{\mathcal{B}}(\kappa,\omega)-\frac{1}{\kappa}\frac{d\tilde{B}'_{\mathcal{B}}(\kappa,\omega)}{d\kappa}\right](\overleftrightarrow{U}-\hat{u}\hat{u}). \qquad (51)$$

We see that $\hat{u}\cdot\overleftrightarrow{\Phi}_3(r\hat{u},\omega)=\overleftrightarrow{\Phi}_3(r\hat{u},\omega)\cdot\hat{u}=0$, i.e., the field in the far zone is transverse with respect to the direction $\hat{u}$. This, of course, is as expected since the far field in the direction of $\hat{u}$ behaves locally as a plane wave propagating in that direction. Hence, we may describe the polarization properties of the far field locally in terms of the 2×2 coherence tensor associated with the two orthogonal transverse components $\hat{u}_a$ and $\hat{u}_b$ (see Fig. 1). The elements of this 2×2 coherence tensor, denoted by $\overleftrightarrow{\Phi}_2(r\hat{u},\omega)$, are explicitly given by

$$\Phi_{2,ij}(r\hat{u},\omega)=\hat{u}_i\cdot\overleftrightarrow{\Phi}_3(r\hat{u},\omega)\cdot\hat{u}_j=\left(\frac{\kappa\eta_0\mu_r}{4\pi r}\right)^2 \times\left[\tilde{A}_{\mathcal{B}}(\kappa,\omega)-\frac{1}{\kappa}\frac{d\tilde{B}'_{\mathcal{B}}(\kappa,\omega)}{d\kappa}\right]\delta_{ij}, \quad (52)$$

where $(i,j)=(a,b)$. Hence the diagonal elements $\Phi_{2,aa}$ and $\Phi_{2,bb}$ are equal, i.e., the spectral densities of the transverse components are the same. Furthermore, the transverse components are mutually uncorrelated since $\Phi_{2,ab}=\Phi_{2,ba}=0$. These two facts indicate that the far field radiated by a finite statistically homogeneous and isotropic spherical source (of any radius) is fully unpolarized in every direction. Mathematically, the degree of polarization of plane waves, defined by (Ref. [8], Sec. 6.3)

$$P(r\hat{u},\omega)=\sqrt{1-\frac{4\det\overleftrightarrow{\Phi}_2(r\hat{u},\omega)}{\mathrm{tr}^2\overleftrightarrow{\Phi}_2(r\hat{u},\omega)}}, \quad (53)$$

where tr and det stand for the trace and the determinant, respectively, assumes the value $P(r\hat{u})=0$ for all $\hat{u}$.

If the source is finite but nonspherical, the function $I_{\mathcal{B}}(R)$ characterizing the source domain in Eq. (49) will, in general, depend not only on the magnitude but also on the direction of the vector $\mathbf{R}$. In such a case the Fourier transforms $\tilde{A}_{\mathcal{B}}(\kappa,\omega)$ and $\tilde{B}'_{\mathcal{B}}(\kappa,\omega)$ in Eqs. (47) and (48) are not necessarily spherically symmetric in $\boldsymbol{\kappa}$ space, and consequently the 2×2 coherence tensor $\overleftrightarrow{\Phi}_2(r\hat{u},\omega)$ need not be proportional to a unit tensor, i.e., the field in the direction $\hat{u}$ is not unpolarized. However, if the source domain is sufficiently large (in relation to important values of R), the quantity $I_{\mathcal{B}}(R)$ is approximately equal to the volume of the source and the field is unpolarized. Hence, for large statistically homogeneous and isotropic current distributions (spherical or otherwise), the far field is fully unpolarized in every direction.

IV. DISCUSSION AND CONCLUSIONS

The universal behavior of correlations in electromagnetic fields that we found above can physically be justified by arguments which are parallel to those presented in Ref. [3] for the scalar case, but which go somewhat beyond it. In analogy with the field that consists of angularly uncorrelated and isotropically distributed scalar plane waves, i.e., a scalar field whose spatial correlations obey the sinc law [3], we may construct a full vectorial counterpart in terms of electromagnetic-plane waves within a nonabsorbing medium [12]. In addition to directional isotropy and angular noncorrelation, the vectorial plane waves in the electromagnetic ensemble are taken to be fully unpolarized. For this particular (free) field, the electric cross-spectral density tensor is proportional to the imaginary part of the Green tensor [12], and thus the field shares its coherence properties with that produced by a statistically homogeneous and isotropic current distribution in a low-loss or nondissipative medium.

Since the medium has a small but nonzero absorbtion, in the neighborhood of a given source point the field correlations extend effectively over a finite region. Therefore, we may think of the whole infinite source as being divided into finite, uniformly distributed, and mutually uncorrelated domains whose dimensions depend on the correlation length. As in Ref. [3], we may refer to these domains as source correlation regions. Each source correlation region produces an electromagnetic field, which at large distances behaves approximately as a plane wave. Thus, in any observation region, the contributions from the (very) distant parts of the source can be viewed as consisting of a superposition of isotropically distributed and angularly uncorrelated plane waves. For source regions containing statistically homogeneous and isotropic current distributions, these plane waves are also fully unpolarized as was shown in Sec. III D.

Hence, the model and calculations show that the field correlations in any observation region, given by the imaginary part of the infinite-domain Green tensor in Eq. (40), are determined by the distant contributions. Although the local currents at every point also generate a near field, with the associated correlation tensor having both real and imaginary parts (see, for example, Ref. [13]), this contribution from a statistically homogeneous and isotropic current in an infinite low-loss or nonabsorbing medium is negligible as compared to the propagating far-zone contributions. Thus, despite local current sources, the field at any point behaves effectively as a free electromagnetic field. We note that these remarks are consistent with the earlier result that a homogeneous free field can be expressed in terms of angularly uncorrelated plane waves [14]. Besides unbounded current sources, our analysis and arguments should, to a good approximation, hold also for an electromagnetic field well inside a finite, but large, source region.

When the losses are significant, the correlations do not show universal behavior as noted in Ref. [2]. This can be physically explained using the plane-wave model discussed above. In the presence of losses, the contribution to the field from the distant source correlation regions weakens in relation with that from the nearby regions. Consequently, the plane-wave model no longer describes the physical situation, and no universality is found. Finally, we also note that when the losses in an infinite source region decrease, the intensity of the field at a certain observation point increases, since more waves can reach that point, i.e., waves from distant source regions start to dominate. This explains physically the divergent behavior of the cross-spectral density tensor of Eq. (20), in the limit $\epsilon''_r\rightarrow0$.

To summarize, we have investigated the structure of spatial (spectral) correlations in the electromagnetic fields produced by stationary, statistically homogeneous and isotropic current distributions. We showed that for any such field, within a medium of vanishingly small losses, the coherence properties are determined by the imaginary part of the Green tensor of the system, i.e., solely by the propagation properties of waves in the medium. This demonstrates that the field's correlation properties are independent of the source characteristics, and that the degree of coherence is of a universal form. Our analysis covers the known results of blackbody radiation fields, but it applies to a wider class of sources, without requiring thermal equilibrium. We also discussed the physics behind the universal behavior of the correlations by comparing the results with those obtained by an electromagnetic plane-wave model. Our results generalize the scalar analysis of Refs. [1,2] to homogeneous and isotropic electromagnetic fields.

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APPENDIX A: CALCULATION OF THE INTEGRAL IN EQ. (17)

Consider the integral

$$I(\mathbf{x})=\int \frac{d^3k}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{|k^2-\kappa^2|^2}. \tag{A1}$$

By performing the angular integrations we obtain (as in the Appendix of Ref. [2])

$$\begin{aligned} I(\mathbf{x})&=\frac{1}{2\pi^2|\mathbf{x}|}\int_0^\infty dk\,\frac{k\sin(k|\mathbf{x}|)}{|k^2-\kappa^2|^2}\\ &=\frac{1}{(2\pi)^2|\mathbf{x}|i}\int_{-\infty}^\infty dk\,\frac{ke^{ik|\mathbf{x}|}}{|k^2-\kappa^2|^2}. \end{aligned} \tag{A2}$$

Since the integrand in Eq. (A2) is an analytic function everywhere in the upper half of the complex k plane, except for the two poles at $k=k_0(\pm n'+in'')$, and it decays more rapidly than $1/k^2$ for $|k|\to\infty$ (with $0\leqslant \arg k\leqslant\pi$), we may apply the residue theorem to evaluate it. Since the medium is assumed to be lossy, the poles are not on the real axis, and we may choose the contour of integration to be a semicircle in the upper half of the complex k plane. By performing the integration find that

$$I(\mathbf{x})=\frac{1}{(\kappa^2)''}\mathrm{Im}[G(\mathbf{x},\omega)], \tag{A3}$$

where $(\kappa^2)''$ is the imaginary part of κ^2, and

$$G(\mathbf{x},\omega)=\frac{e^{i\kappa|\mathbf{x}|}}{4\pi|\mathbf{x}|}. \tag{A4}$$

Thus, we see that $I(\mathbf{x})$ is proportional to the imaginary part of the diverging spherical wave, or the scalar Green function of the system.

APPENDIX B: SOME PROPERTIES OF THE GREEN TENSOR

The Green tensor, denoted by $\overleftrightarrow{G}(\mathbf{x},\omega)$, is explicitly written as (Ref. [11], Sec. 4)

$$\overleftrightarrow{G}(\mathbf{x},\omega)=\left(\overleftrightarrow{U}+\frac{1}{\kappa^2}\boldsymbol{\nabla}\boldsymbol{\nabla}\right)G(\mathbf{x},\omega), \tag{B1}$$

where $G(\mathbf{x},\omega)$ is the scalar Green function presented in Eq. (A4). The Green tensor satisfies the wave equation

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\overleftrightarrow{G}(\mathbf{x},\omega)-\kappa^2\overleftrightarrow{G}(\mathbf{x},\omega)=\overleftrightarrow{U}\delta(\mathbf{x}), \tag{B2}$$

where $\delta(\mathbf{x})$ is the Dirac delta function. By taking the divergence of Eq. (B2), we obtain

$$\boldsymbol{\nabla}\cdot\overleftrightarrow{G}(\mathbf{x},\omega)=-\frac{1}{\kappa^2}\boldsymbol{\nabla}[\delta(\mathbf{x})]. \tag{B3}$$

Thus, when the losses are negligible, we have

$$\boldsymbol{\nabla}\cdot\mathrm{Im}[\overleftrightarrow{G}(\mathbf{x},\omega)]=\mathbf{0}. \tag{B4}$$

Furthermore, in this low-loss case, the imaginary part of the Green tensor is explicitly written as

$$\mathrm{Im}[\overleftrightarrow{G}(\mathbf{x},\omega)]=\frac{\kappa}{4\pi}\left\{\left[j_0(\kappa x)-\frac{j_1(\kappa x)}{\kappa x}\right]\overleftrightarrow{U}+j_2(\kappa x)\hat{\mathbf{x}}\hat{\mathbf{x}}\right\}, \tag{B5}$$

where $\hat{\mathbf{x}}=\mathbf{x}/x$. It is straightforward to verify that this form satisfies Eq. (B4).

APPENDIX C: ANGULAR INTEGRATIONS IN EQ. (26)

The angular integrations in the expression

$$\begin{aligned} I_A(\mathbf{r})&=\int d^3R A(R,\omega)\mathrm{Im}[G(\mathbf{R}-\mathbf{r},\omega)]\\ &=\int d^3R A(R,\omega)\mathrm{Im}\left(\frac{e^{i\kappa|\mathbf{R}-\mathbf{r}|}}{4\pi|\mathbf{R}-\mathbf{r}|}\right), \end{aligned} \tag{C1}$$

where κ may, in general, be complex, can be performed by making use of the expansion [cf. Eq. 8.533(1) of Ref. [15]]

$$\begin{aligned} \frac{e^{i\kappa|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}&=i\kappa\sum_{l=0}^{\infty} j_l(\kappa r_<)h_l^{(1)}(\kappa r_>)\sum_{m=-l}^{l}\\ &\quad\times Y_l^{m*}(\theta',\varphi')Y_l^m(\theta,\varphi), \end{aligned} \tag{C2}$$

where $r_<=\min\{|\mathbf{r}|,|\mathbf{r}'|\}$, $r_>=\max\{|\mathbf{r}|,|\mathbf{r}'|\}$, $j_l(x)$ and $h_l^{(1)}(x)$ are spherical Bessel and spherical Hankel functions of the first kind and of order l, and $Y_l^m(\theta,\varphi)$ are spherical harmonics. The angular integration in Eq. (C2) leads to the Kronecker delta $\delta_{l,0}$, and consequently

$$I_A(\mathbf{r})=\int_0^\infty dRR^2A(R,\omega)\mathrm{Im}[i\kappa j_0(\kappa r_<)h_0^{(1)}(\kappa r_>)]. \tag{C3}$$

In the low-loss limit, the imaginary part in the integrand of Eq. (C3) is obtained directly by noting that $h_l^{(1)}(x)=j_l(x)+in_l(x)$, where $n_l(x)$ is spherical Neumann function of order l.

APPENDIX D: ANGULAR INTEGRATIONS IN EQ. (33)

The integration over the angular coordinates in the integral

$$\overleftrightarrow{I}_B(\mathbf{r})=\int d^3RB(R,\omega)\hat{\mathbf{R}}\hat{\mathbf{R}}\,\mathrm{Im}[G(\mathbf{R}-\mathbf{r},\omega)]$$
$$=\int d^3RB(R,\omega)\hat{\mathbf{R}}\hat{\mathbf{R}}\,\mathrm{Im}\left(\frac{e^{i\kappa|\mathbf{R}-\mathbf{r}|}}{4\pi|\mathbf{R}-\mathbf{r}|}\right) \tag{D1}$$

can be carried out analogously to that in Appendix C. Making use of Eq. (C2), and expressing the components of $\hat{\mathbf{R}}$ in a spherical polar coordinate system, we obtain

$$\overleftrightarrow{I}_B(\mathbf{r})=\int_0^\infty dRR^2B(R,\omega)\overleftrightarrow{\mathcal{M}}(R,\mathbf{r}), \tag{D2}$$

where

$$\overleftrightarrow{\mathcal{M}}(R,\mathbf{r})=\mathrm{Im}\Bigg[i\kappa\sum_{l=0}^{\infty}j_l(\kappa r_<)h_l^{(1)}(\kappa r_>)\sum_{m=-l}^{l}Y_l^{m*}(\theta',\varphi')\times(-1)^m\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}}\;\overleftrightarrow{\mathcal{N}}\Bigg], \tag{D3}$$

with

$$\overleftrightarrow{\mathcal{N}}=\int_0^{2\pi}d\varphi\int_0^{\pi}d\theta e^{im\varphi}P_l^m(\cos\theta)\overleftrightarrow{\Omega}(\theta,\varphi)\sin\theta. \tag{D4}$$

In Eq. (D4), $P_l^m(\cos\theta)$ are associated Legendre functions, and the angular tensor $\overleftrightarrow{\Omega}(\theta,\varphi)$ has the form

$$\overleftrightarrow{\Omega}(\theta,\varphi)=\begin{pmatrix}\sin^2\theta\cos^2\varphi & \sin^2\theta\sin\varphi\cos\varphi & \sin\theta\cos\theta\cos\varphi\\ \sin^2\theta\sin\varphi\cos\varphi & \sin^2\theta\sin^2\varphi & \sin\theta\cos\theta\sin\varphi\\ \sin\theta\cos\theta\cos\varphi & \sin\theta\cos\theta\sin\varphi & \cos^2\theta.\end{pmatrix}. \tag{D5}$$

The angular integrations in Eq. (D4) are performed by making use of the orthogonality properties of Legendre functions, and we find that the elements of $\overleftrightarrow{\mathcal{N}}$ are of the form

$$\mathcal{N}_{xx}=\frac{4\pi}{3}\delta_{m,0}\delta_{l,0}-\frac{4\pi}{15}\delta_{m,0}\delta_{l,2}+\frac{8\pi}{5}\delta_{m,2}\delta_{l,2}+\frac{\pi}{15}\delta_{m,-2}\delta_{l,2},$$

$$\mathcal{N}_{yy}=\frac{4\pi}{3}\delta_{m,0}\delta_{l,0}-\frac{4\pi}{15}\delta_{m,0}\delta_{l,2}-\frac{8\pi}{5}\delta_{m,2}\delta_{l,2}-\frac{\pi}{15}\delta_{m,-2}\delta_{l,2},$$

$$\mathcal{N}_{zz}=\frac{4\pi}{3}\delta_{m,0}\delta_{l,0}+\frac{8\pi}{15}\delta_{m,0}\delta_{l,2}, \tag{D6}$$

$$\mathcal{N}_{xy}=\mathcal{N}_{yx}=\frac{8\pi i}{5}\delta_{m,2}\delta_{l,2}-\frac{\pi i}{15}\delta_{m,-2}\delta_{l,2},$$

$$\mathcal{N}_{yz}=\mathcal{N}_{zy}=\frac{4\pi i}{5}\delta_{m,1}\delta_{l,2}+\frac{2\pi i}{15}\delta_{m,-1}\delta_{l,2},$$

$$\mathcal{N}_{xz}=\mathcal{N}_{zx}=\frac{4\pi}{5}\delta_{m,1}\delta_{l,2}-\frac{2\pi}{15}\delta_{m,-1}\delta_{l,2}.$$

Substituting these into Eq. (D3), we get

$$\mathcal{M}_{pp}(R,\mathbf{r})=\mathrm{Im}\left\{\frac{i\kappa}{3}[j_0(\kappa r_<)h_0^{(1)}(\kappa r_>)-j_2(\kappa r_<)h_2^{(1)}(\kappa r_>)\times(1-3\hat{r}_p\hat{r}_p)]\right\}, \tag{D7}$$

$$\mathcal{M}_{pq}(R,\mathbf{r})=\mathrm{Im}\{i\kappa j_2(\kappa r_<)h_2^{(1)}(\kappa r_>)\hat{r}_p\hat{r}_q\},\quad \text{when}\quad p\neq q. \tag{D8}$$

Here, $\hat{r}_p$ $(p=x,y,z)$ are the components of the unit vector $\hat{\mathbf{r}}=\mathbf{r}/r$. The elements in Eqs. (D7) and (D8) can be expressed compactly in a tensorial form

$$\overleftrightarrow{\mathcal{M}}(R,\mathbf{r})=\mathrm{Im}\left\{\frac{i\kappa}{3}[j_0(\kappa r_<)h_0^{(1)}(\kappa r_>)-j_2(\kappa r_<)h_2^{(1)}(\kappa r_>)]\overleftrightarrow{U}+i\kappa j_2(\kappa r_<)h_2^{(1)}(\kappa r_>)\hat{\mathbf{r}}\hat{\mathbf{r}}\right\}. \tag{D9}$$

Substituting this into Eq. (D2), we obtain

$$\overleftrightarrow{I}_B(\mathbf{r})=\int_0^\infty dRR^2B(R,\omega)\mathrm{Im}\left\{\frac{i\kappa}{3}[j_0(\kappa r_<)h_0^{(1)}(\kappa r_>)-j_2(\kappa r_<)h_2^{(1)}(\kappa r_>)]\overleftrightarrow{U}+i\kappa j_2(\kappa r_<)h_2^{(1)}(\kappa r_>)\hat{\mathbf{r}}\hat{\mathbf{r}}\right\}. \quad \text{(D10)}$$

[1] H.M. Nussenzveig, J.T. Foley, K. Kim, and E. Wolf, Phys. Rev. Lett. **58**, 218 (1987).

[2] S.A. Ponomarenko and E. Wolf, Phys. Rev. E **65**, 016602 (2001).

[3] F. Gori, D. Ambrosini, and V. Bagini, Opt. Commun. **107**, 331 (1994).

[4] W.H. Carter and E. Wolf, J. Opt. Soc. Am. **65**, 1067 (1975).

[5] J.T. Foley, W.H. Carter, and E. Wolf, J. Opt. Soc. Am. A **3**, 1090 (1986).

[6] C.L. Mehta and E. Wolf, Phys. Rev. **161**, 1328 (1967).

[7] G.S. Agarwal, Phys. Rev. A **11**, 230 (1975).

[8] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, UK, 1995).

[9] G.K. Batchelor, *The Theory of Homogeneous Turbulence* (Cambridge University Press, Cambridge, UK, 1970), Secs. 3.3 and 3.4.

[10] G. Gbur, D. James, and E. Wolf, Phys. Rev. E **59**, 4594 (1999).

[11] C.-T. Tai, *Dyadic Green's Functions in Electromagnetic Theory* (Intext, Scranton, PA, 1971).

[12] T. Setälä, M. Kaivola, and A. T. Friberg (unpublished).

[13] T. Setälä, M. Kaivola, and A.T. Friberg, Phys. Rev. E **59**, 1200 (1999).

[14] E. Wolf, Phys. Rev. D **13**, 869 (1976).

[15] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products*, 5th ed. (Academic Press, New York, 1980).