Energy loss of ions in a magnetized plasma: Conformity between linear response and binary collision treatments

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The energy loss of a heavy ion moving in a magnetized electron plasma is considered within the linear response (LR) and binary collision (BC) treatments with the purpose to look for a connection between these two models. These two complementary approaches yield close results if no magnetic field is present, but there develop discrepancies with growing magnetic field at ion velocities that are lower than, or comparable with, the thermal velocity of the electrons. We show that this is a peculiarity of the Coulomb interaction which requires cutoff procedures to account for its singularity at the origin and its infinite range. The cutoff procedures in the LR and BC treatments are different as the order of integrations in velocity and in ordinary (Fourier) spaces is reversed in both treatments. While BC involves a velocity average of Coulomb logarithms, there appear in LR Coulomb logarithms of velocity averaged cutoffs. The discrepancies between LR and BC vanish, except for small contributions of collective modes, for smoothened potentials that require no cutoffs. This is shown explicitly with the help of an improved BC in which the velocity transfer is treated up to second order in the interaction in Fourier space.

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I. INTRODUCTION

The energy loss of ion beams and the related processes in magnetized plasmas are important in many areas of physics such as transport, heating, magnetic confinement of thermonuclear plasmas, and astrophysics. Recent applications are the cooling of heavy-ion beams by electrons [1–3] and the energy transfer for heavy-ion inertial confinement fusion (see Ref. [4] for an overview).

For a theoretical description of the energy loss of ions in a plasma, there exist two standard approaches. The dielectric linear response (LR) treatment considers the ion as a perturbation of the target plasma and the stopping is caused by the polarization of the surrounding medium. It is only valid if the ion couples weakly to the target. Alternatively, the stopping is calculated as the result of the energy transfers in successive binary collisions (BCs) between the ion and the electrons. Here it is essential to consider appropriate approximations for the shielding of the Coulomb potential by the plasma.

Since the early 1960s, a number of theoretical calculations of the stopping power within the LR treatment in a magnetized plasma have been presented (see, e.g., Refs. [5–14], and references therein). Recently, new theoretical investigations [8–14] have been stimulated by a number of experiments, e.g., the electron cooling of heavy-ion beams in the presence of a magnetic field or in magnetized target fusion research.

The problem of two charged particles in an external magnetic field cannot be solved in closed form as the relative motion and the motion of the center of mass are coupled to each other. Therefore no closed solution exists for BCs, which is uniformly valid for any strength of the magnetic field and the Coulomb force between the particles. Numerical calculations have been performed for BCs between magnetized electrons [15,16] and for collisions between magnetized electrons and ions [17–21]. As an ion is much heavier than an electron, its uniform motion is only weakly perturbed by collisions with the electrons. In this paper, we consider the Coulomb interaction with the ion as a perturbation to the helical motion of the magnetized electrons, while the ion motion remains unchanged. This has been done previously in first order in the ion charge Z and for an ion at rest [19]. However, it has been shown that a second-order treatment is both necessary and sufficient for the conservation of a generalized energy [20].

Both treatments, LR and BC, can be regarded as complementary to each other and both of them are of physical interest. Within the LR treatment (dielectric theory), the stopping power can receive a dynamic contribution from collective plasma excitations. It requires a cutoff at small distances, where hard collisions between the ion and electrons cannot be treated any more as a weak perturbation. Within the BC picture, the interaction between the plasma electrons is only treated approximately by an effective interaction or an upper cutoff for the impact parameters, to account for screening. In this case, the stopping power of an ion is the result of the energy transfer in successive binary collisions. In the limit of a noninteracting electron gas ($V \to 0$, where V is the interaction potential between the electrons), the LR and BC treatments should therefore provide the same result for the stopping power. But even in the absence of a magnetic field, both approaches give slightly different results. Exactly the same results can be achieved if physically reasonable cutoffs are used in the Coulomb logarithms [1,2]. In the presence of a magnetic field, the situation is dramatically changed. Here the agreement between the LR and BC treatments breaks down for intermediate and low ion

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velocities as discussed in Ref. [20]. The disagreement is larger for an ion motion along the magnetic field than transverse to it. In addition, the BC treatment also predicts an energy gain (negative stopping power) for very low ion velocities.

In this paper, we consider BCs between ion and electrons in the presence of an arbitrary magnetic field and demonstrate that full agreement between BC and LR treatments in the limit of a noninteracting electron plasma is guaranteed for a smoothened interaction which is both of finite range and less singular than the Coulomb interaction at the origin. The paper is organized as follows: We start in Sec. II with a brief discussion of the basic results of the LR treatment for the stopping power in a magnetized plasma. We derive an analytical expression for the stopping power in the case of noninteracting electron plasma. In Sec. III, we consider the LR and BC treatments without and with an infinitely strong magnetic field. In Sec. IV, we discuss the velocity and energy transfer during BCs of magnetized electrons with ions for arbitrary magnetic fields and strengths of the electron-ion interaction potential. We assume that the ion mass M is much larger than the electron mass m. The equations of motion are solved in a perturbative manner up to the second order in Z starting from the unperturbed helical motion of the electrons in the magnetic field. Then in Sec. V we turn to the energy loss of ions in a magnetized electron plasma. We show that the stopping power obtained within the BC treatment completely coincides with the LR result. The results are summarized and discussed in Sec. VI.

II. LINEAR RESPONSE FORMULATION

As a basis for further considerations, we recall briefly the main aspects of the LR theory for the ion-plasma interaction in the presence of an external magnetic field. Within the LR, the electron plasma is described as a continuous, polarizable fluid (medium), which is represented by the phase-space density of the electrons $f(\mathbf{r},\mathbf{v},t)$. Usually only a mean-field interaction between the electrons is considered and hard collisions are neglected. The evolution of the distribution function $f(\mathbf{r},\mathbf{v},t)$ is determined by the Vlasov-Poisson equation. This is valid for weakly coupled plasmas where the number of electrons in the Debye sphere $N_{\mathrm{D}}=4\pi n_0\lambda_{\mathrm{D}}^3\gg 1$ is very large. Here, n_0 is the electron density and $\lambda_{\mathrm{D}}=(\epsilon_0 k_{\mathrm{B}}T/n_0e^2)^{1/2}$ is the Debye length.

We consider a nonrelativistic projectile ion with charge Ze and with a velocity $\mathbf{v}_i$, which moves in a magnetized plasma at an angle α with respect to the magnetic field $\mathbf{B}_0$. We shall consider here the limit of heavy ions and neglect recoil effects. The strength of the coupling between the moving ion and the electron plasma is given by the coupling parameter

$$\mathcal{Z}=\frac{g}{[1+v_i^2/v_{\mathrm{th}}^2]^{3/2}}. \tag{1}$$

Here $v_{\mathrm{th}}=(k_{\mathrm{B}}T/m)^{1/2}$ is the thermal velocity of an electron, $g=Z/N_{\mathrm{D}}$. The derivation of Eq. (1) is discussed in detail in Refs. [21,22]. The parameter $\mathcal{Z}$ characterizes the ion-target coupling, where $\mathcal{Z}\ll 1$ corresponds to weak, almost linear coupling and $\mathcal{Z}\gtrsim 1$ to strong, nonlinear coupling.

For a sufficiently small perturbation ($\mathcal{Z}\ll 1$), the linearized Vlasov equation of the plasma may be written as

$$\frac{\partial f_1}{\partial t}+\mathbf{v}\cdot\frac{\partial f_1}{\partial \mathbf{r}}-\Omega[\mathbf{v}\times\mathbf{b}]\cdot\frac{\partial f_1}{\partial \mathbf{v}}=-\frac{e}{m}\frac{\partial \phi}{\partial \mathbf{r}}\cdot\frac{\partial f_0}{\partial \mathbf{v}}, \tag{2}$$

where $f=f_0+f_1$ and $\phi=\phi_{\mathrm{ie}}+\phi_{\mathrm{sc}}$ is the electrostatic potential. It consists of the perturbing ion potential $\phi_{\mathrm{ie}}=Ze/4\pi\epsilon_0|\mathbf{r}-\mathbf{v}_i t|$ and the self-consistent polarization contribution ϕ_{sc} which is determined by the Poisson equation

$$\nabla^2\phi_{\mathrm{sc}}=\frac{e}{\epsilon_0}\int d\mathbf{v} f_1(\mathbf{r},\mathbf{v},t). \tag{3}$$

Further, $\mathbf{b}$ is the unit vector parallel to $\mathbf{B}_0$, $-e$ and $\Omega=eB_0/m$ are the charge and cyclotron frequency of the plasma electrons, respectively, and f_0 is the unperturbed distribution function of plasma electrons.

By solving Eqs. (2) and (3) in space-time Fourier components, we obtain the electrostatic potential

$$\phi(\mathbf{r},t)=\frac{Ze}{(2\pi)^3\varepsilon_0}\int d\mathbf{k}\frac{\exp[i\mathbf{k}\cdot(\mathbf{r}-\mathbf{v}_i t)]}{k^2\varepsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v}_i)} \tag{4}$$

which provides the dynamic response of the plasma to the motion of the projectile ion in the presence of the external magnetic field. The dielectric function $\varepsilon(\mathbf{k},\omega)$ of a homogeneous plasma is given by $\varepsilon(\mathbf{k},\omega)=1+V(\mathbf{k})\chi^{(0)}(\mathbf{k},\omega)$, where $(e^2/4\pi\epsilon_0)V(\mathbf{k})$ is the Fourier transformed two-body interaction potential; in case of the repulsive Coulomb potential for electron systems $V(\mathbf{k})=1/2\pi^2k^2$. The susceptibility of the magnetized electrons is (see, e.g., Ref. [23]) the causality of

$$\chi^{(0)}(\mathbf{k},\omega)=-\frac{(2\pi)^3\omega_p^2}{2}\sum_{n=-\infty}^{+\infty}\int_0^\infty v_\perp dv_\perp \times\int_{-\infty}^{+\infty}dv_\parallel\left(\frac{n\Omega}{v_\perp}\frac{\partial f_0}{\partial v_\perp}+k_\parallel\frac{\partial f_0}{\partial v_\parallel}\right)\frac{J_n^2(\beta)}{k_\parallel v_\parallel-\alpha_n-i0}. \tag{5}$$

Here $\omega_p=(n_0e^2/m\varepsilon_0)^{1/2}$ is the plasma frequency, J_n is the Bessel function of the nth order and $\alpha_n=\omega-n\Omega$, and $\beta=k_\perp v_\perp/\Omega$. The symbols $\parallel$ and $\perp$ denote the components of the vectors $\mathbf{k}$ and $\mathbf{v}$ parallel or perpendicular to the external magnetic field, respectively. The positive infinitesimal $+i0$ in Eq. (5) guarantees the vanishing of the response.

The stopping power S_{LR} of an ion is now defined as the energy loss of the ion per unit length due to the plasma polarization, that is, the electric field $\mathbf{E}=-\nabla\phi$, from Eq. (4)

$$S_{\mathrm{LR}}=-\frac{dE_i}{d\ell}=\frac{Z^2e^2}{\epsilon_0 v_i(2\pi)^3}\int d\mathbf{k}\frac{\mathbf{k}\cdot\mathbf{v}_i}{k^2}\mathrm{Im}\frac{-1}{\varepsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v}_i)}. \tag{6}$$

The collective excitations (i.e., magnetized plasma modes) contributing to the stopping power are contained in $\varepsilon(\mathbf{k},\mathbf{k}\cdot\mathbf{v}_i)$. But in general, Eq. (6) cannot be evaluated in closed form except for the limiting cases $B_0 \to 0$ and $B_0 \to \infty$. In intermediate situations we assume weakly interacting electrons, $e^2 n_0^{1/3} \to 0$ (or $\omega_p \to 0$), with

$$\mathrm{Im}\frac{-1}{\varepsilon(\mathbf{k},\omega)} = \frac{\mathrm{Im}\varepsilon(\mathbf{k},\omega)}{|\varepsilon(\mathbf{k},\omega)|^2} \simeq V(\mathbf{k})\mathrm{Im}\chi^{(0)}(\mathbf{k},\omega). \tag{7}$$

In this approximation the stopping power thus reads

$$S'_{\mathrm{LR}} = \frac{Z^2 e^2}{4\pi\epsilon_0 v_i}\int d\mathbf{k}|U(\mathbf{k})|^2(\mathbf{k}\cdot\mathbf{v}_i)\mathrm{Im}\chi^{(0)}(\mathbf{k},\mathbf{k}\cdot\mathbf{v}_i), \tag{8}$$

where we have introduced the Fourier transformed ion-electron interaction potential, $-(Ze^2/4\pi\epsilon_0)U(\mathbf{k})$. Now the stopping power does not receive any contribution from the dynamic collective plasma modes, but static collective contributions (i.e., screening) can be easily reintroduced by replacing $U(\mathbf{k})$ with a shielded ion-electron interaction potential. Then, Eq. (8) amounts to neglecting the electron-electron interaction in the target except for a static shielding of the ion.

In the case of a bare Coulomb interaction between the projectile ion and plasma electrons, i.e., $U(\mathbf{k}) = V(\mathbf{k})$, the cutoff parameters $k_{\mathrm{min}} = 1/r_{\mathrm{max}}$ and $k_{\mathrm{max}} = 1/r_{\mathrm{min}}$ (where r_{min} is the effective minimum impact parameter) must be introduced in Eq. (8) to avoid the logarithmic divergence at small and large k. The divergence at large k corresponds to the incapability of the linearized Vlasov theory to treat close encounters between the projectile ion and the plasma electrons properly, while k_{min} accounts for screening. For r_{min} we use the effective minimum impact parameter excluding hard Coulomb collisions with a scattering angle larger than 90^0,

$$r_{\mathrm{min}} = \frac{Ze^2}{4\pi\epsilon_0 m v_r^2}, \quad r_{\mathrm{max}} = \frac{v_r}{\omega_p}. \tag{9}$$

The cutoff r_{max} describes the dynamic screening at high relative electron-ion velocities v_r (see, e.g., Ref. [22] for more detail).

In the LR treatment, cutoffs and impact parameters are used which are averaged with respect to the electron velocity distribution function. With the averaged relative velocity $\langle v_r \rangle \simeq (v_i^2 + v_{\mathrm{th}}^2)^{1/2}$, they read

$$\langle r_{\mathrm{min}} \rangle = \frac{1}{\langle k_{\mathrm{max}} \rangle} = \frac{Ze^2}{4\pi\epsilon_0 m(v_i^2 + v_{\mathrm{th}}^2)},$$

$$\langle r_{\mathrm{max}} \rangle = \frac{1}{\langle k_{\mathrm{min}} \rangle} = \frac{(v_i^2 + v_{\mathrm{th}}^2)^{1/2}}{\omega_p}. \tag{10}$$

The cutoff parameters (9) and (10) are well known (see, e.g., Refs. [1–3,18–22]) for stopping power calculations without magnetic field. In particular, the minimum impact parameter, r_{min}, is provided by the Rutherford scattering formula [24]. However, in the presence of a magnetic field, the cutoff r_{min} must be deduced by a comparison of the LR and the full nonperturbative BC treatments.

III. LR AND BC TREATMENTS WITHOUT AND WITH STRONG MAGNETIC FIELD

To illustrate the problem and to motivate the forthcoming considerations on the interrelation of the LR and BC treatments, we consider the cases without and with an infinitely strong magnetic field. We show that even in the absence of a magnetic field both approaches yield slightly different results when using the standard averaging procedure. The discrepancy grows with the strength of the magnetic field.

A. LR and BC treatments without magnetic field

Without magnetic field, the imaginary part of the susceptibility of an isotropic Maxwellian plasma is given by (see, e.g., Ref. [23])

$$\mathrm{Im}\chi^{(0)}(\mathbf{k},\omega) = \frac{(2\pi)^{5/2}}{4\lambda_{\mathrm{D}}^2}\frac{\omega}{k v_{\mathrm{th}}}\exp\left(-\frac{\omega^2}{2k^2 v_{\mathrm{th}}^2}\right). \tag{11}$$

The expression (8) for the simplified LR stopping power in the case of the Coulomb interaction, $U(\mathbf{k}) = V(\mathbf{k})$, yields

$$S'_{\mathrm{LR}} = S_0 \frac{g^2}{\lambda^2}\Lambda_{\mathrm{LR}}(\lambda)\left[\mathrm{erf}\left(\frac{\lambda}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}}\lambda \exp\left(-\frac{\lambda^2}{2}\right)\right], \tag{12}$$

where erf(x) is the error function, $\lambda = v_i/v_{\mathrm{th}}$, $S_0 = 4\pi\epsilon_0(k_{\mathrm{B}}T/e)^2$, and

$$\Lambda_{\mathrm{LR}}(\lambda) = \ln\frac{\langle k_{\mathrm{max}}\rangle}{\langle k_{\mathrm{min}}\rangle} = \ln\frac{(1+\lambda^2)^{3/2}}{g} = \ln\frac{1}{\mathcal{Z}(\lambda)}. \tag{13}$$

Here $\mathcal{Z}$ is defined by Eq. (1). In the Coulomb logarithm Λ_{LR}, the averaged lower and upper cutoffs [see Eq. (10)] have been used.

Within the perturbative BC treatment (see Ref. [20] for details), we need to consider only the second-order energy transfer during an electron-ion collision, as the first-order energy transfer is proportional to the impact parameter **s** and vanishes after averaging over **s**. The angular averaged second-order energy transfer reads

$$\langle \Delta E_i^{(2)} \rangle = \left(\frac{Ze^2}{4\pi\epsilon_0 s}\right)^2 \frac{2\mathbf{v}_i\cdot\mathbf{v}_r}{m v_r^4}, \tag{14}$$

where $\mathbf{v}_r = \mathbf{v}_{e0} - \mathbf{v}_i$ is the relative velocity of the colliding particles. The energy loss of the ion in a homogeneous electron plasma is obtained by integrating Eq. (14) over an area element $d^2\mathbf{s}$ perpendicular to the relative current density $n_0\mathbf{v}_r$ and averaging over the unperturbed electron distribution function f_0,

$$S_{\mathrm{BC}} = -\left(\frac{dE_i}{d\ell}\right)_{\mathrm{BC}} = -\frac{2\pi n_0}{v_i}\int d\mathbf{v}_{e0} f_0(\mathbf{v}_{e0}) v_r \times \int_{r_{\mathrm{min}}}^{r_{\mathrm{max}}} ds s \langle \Delta E_i^{(2)} \rangle. \tag{15}$$

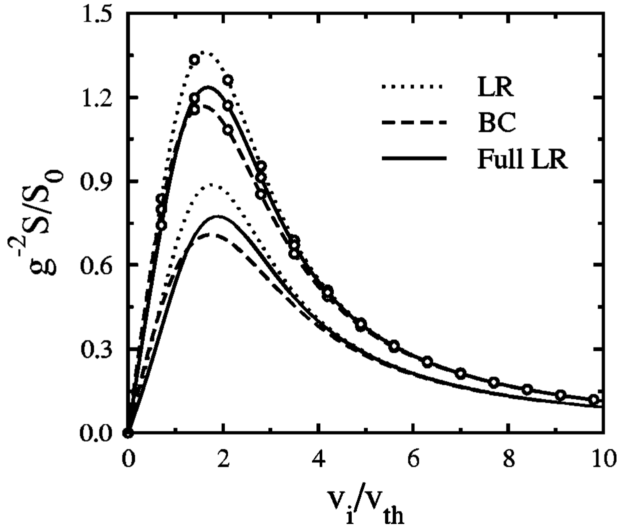

FIG. 1. Stopping powers (in units of g^2S_0) within the simplified LR, Eq. (8) (dotted lines); full LR, Eq. (6) (solid lines); and BC (dashed lines) treatments as a function of the ion velocity v_i (in units of $v_{\rm th}$) in a plasma without magnetic field for $g=0.1$ (lines without circles) and $g=0.01$ (lines with circles).

Here the upper cutoff $r_{\max}$ accounts for screening, while $r_{\min}=Ze^2/(4\pi\varepsilon_0 m v_r^2)$ is the cutoff below which the perturbative treatment of the Coulomb interaction fails [see Eq. (9)]. However, it is well known that for Rutherford scattering hard collisions are taken into account by regularizing the s integral in Eq. (15) according to

$$\int_{r_{\min}}^{r_{\max}}\frac{ds}{s}=\ln\frac{r_{\max}}{r_{\min}}\rightarrow\int_0^{r_{\max}}\frac{s\,ds}{s^2+r_{\min}^2}=\frac{1}{2}\ln\left(1+\frac{r_{\max}^2}{r_{\min}^2}\right), \tag{16}$$

which yields the exact result [24].

For an isotropic Maxwellian distribution, we finally obtain from Eqs. (14)–(16),

$$S_{\rm BC}=S_0\frac{g^2}{2\lambda^2\sqrt{2\pi}}\int_0^\infty\frac{dx}{x^2}\ln\left(1+\frac{x^6}{g^2}\right)[(\lambda x-1)e^{-(x-\lambda)^2/2}+(\lambda x+1)e^{-(x+\lambda)^2/2}], \tag{17}$$

where $x=v_r/v_{\rm th}$.

In Fig. 1, the normalized stopping powers within the simplified LR (dotted lines) and BC (dashed lines) treatments, Eqs. (12) and (17), are plotted versus ion velocity. The full LR results, $S_{\rm LR}$, including the electron-electron interaction are also plotted for comparison (solid lines). All these approaches yield close results except for some deviations in the intermediate velocity range.

To make a contact between LR and BC results, Eqs. (12) and (17), we note that the integral in Eq. (17) divided by the factor $\sqrt{2\pi}$ is identical with the expression in square brackets in Eq. (12) if the logarithmic factor in Eq. (17) is taken out of the x integral. Taking thus out the logarithmic function with some average value $\langle x\rangle=\langle v_r\rangle/v_{\rm th}$, the stopping power in the BC treatment can be rewritten as

$$\overline{S}_{\rm BC}=\frac{1}{2\Lambda_{\rm LR}(\lambda)}\ln\left(1+\frac{\langle x^6\rangle}{g^2}\right)S'_{\rm LR}. \tag{18}$$

This demonstrates that both approaches are equivalent if comparable cutoff procedures, resulting in equal Coulomb logarithms, are used. Equation (18) shows that for the conformity between both approaches one must choose $\langle x^6\rangle=(1+\lambda^2)^3-g^2$. In the limit $g\ll 1$, this condition becomes $\langle x^2\rangle\simeq 1+\lambda^2$.

B. LR and BC treatments with strong magnetic fields

In the presence of a strong magnetic field, the imaginary part of $\chi^{(0)}(\mathbf{k},\omega)$ reads (see, e.g., Ref. [23])

$$\mathrm{Im}\chi^{(0)}(\mathbf{k},\omega)=\frac{(2\pi)^{5/2}}{4\lambda_{\rm D}^2}\frac{\omega}{|k_\parallel|v_{\rm th}}\exp\left(-\frac{\omega^2}{2k_\parallel^2v_{\rm th}^2}\right), \tag{19}$$

where $k_\parallel$ is the component of $\mathbf{k}$ along the magnetic field. Substituting Eq. (19) into Eq. (8), we obtain

$$S'_{\rm LR}=S_0\frac{g^2}{2\sqrt{2\pi}}\lambda\Lambda_{\rm LR}(\lambda)\sin^2\alpha\int_{-\infty}^{\infty}\frac{x^2e^{-x^2/2}dx}{q^{3/2}(x,\lambda)}. \tag{20}$$

Here α is the angle between the magnetic field and the ion velocity $\mathbf{v}_i$, the Coulomb logarithm $\Lambda_{\rm LR}$ is the same as in Eq. (13) and $q(x,\lambda)=x^2-2\lambda x\cos\alpha+\lambda^2$, where $x=v_{e\parallel}/v_{\rm th}$.

The second-order energy transfer for an electron-ion collision in the presence of a strong magnetic field is [20]

$$\langle\Delta E_i^{(2)}\rangle=\left(\frac{Ze^2}{4\pi\epsilon_0 s}\right)^2\frac{v_{i\perp}^2}{mv_r^6}(v_{e\parallel}^2-v_i^2), \tag{21}$$

where $\mathbf{v}_r=v_{e\parallel}\mathbf{b}-\mathbf{v}_i$, and $v_{e\parallel}$ is the electron velocity along the magnetic field. The last result has already been given in Ref. [1] for the case $v_{e\parallel}=0$. In the general case, this term also leads to an energy gain for $v_i^2<v_{e\parallel}^2$.

The integration of Eq. (21) over the impact parameter s is similar to that used in the preceding section, Eq. (16), with $r_{\min}$ and $r_{\max}$ from Eq. (9). However, now v_r is replaced by the relative velocity of the guiding center $[(v_{e\parallel}-v_{i\parallel})^2+v_{i\perp}^2]^{1/2}$. Averaging expression (21) over an isotropic Maxwell distribution function f_0, we arrive at

$$S_{\rm BC}=S_0\frac{g^2}{4\sqrt{2\pi}}\lambda\sin^2\alpha\int_{-\infty}^{\infty}\frac{dx(\lambda^2-x^2)e^{-x^2/2}}{q^{5/2}(x,\lambda)}\times\ln\left[1+\frac{q^3(x,\lambda)}{g^2}\right]. \tag{22}$$

When the logarithmic factor in Eq. (22) is now taken out with some average value $\langle q^3(x,\lambda)\rangle$, the x integral does not necessarily coincide with the x integral in Eq. (20) as it has been the case in the absence of a magnetic field, cf. Eq. (18). Moreover, at low ion velocity ($\lambda\rightarrow 0$), the stopping power $\overline{S}_{\rm BC}$ behaves as $1/\lambda$ and tends to infinity. The LR stopping

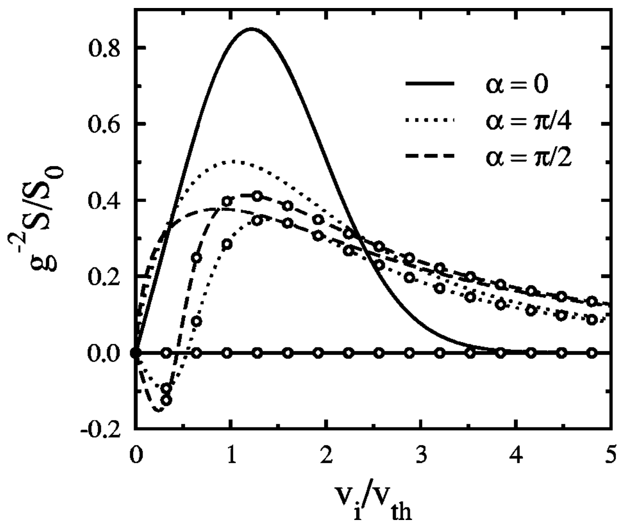

FIG. 2. Stopping powers (in units of g^2S_0) within the simplified LR (lines without circles) and the BC (lines with circles) treatment as a function of the ion velocity v_i (in units of v_{th}) in a plasma with a strong magnetic field for $g=0.1$, $\alpha=0$ (solid lines), $\alpha=\pi/4$ (dotted lines), and $\alpha=\pi/2$ (dashed lines).

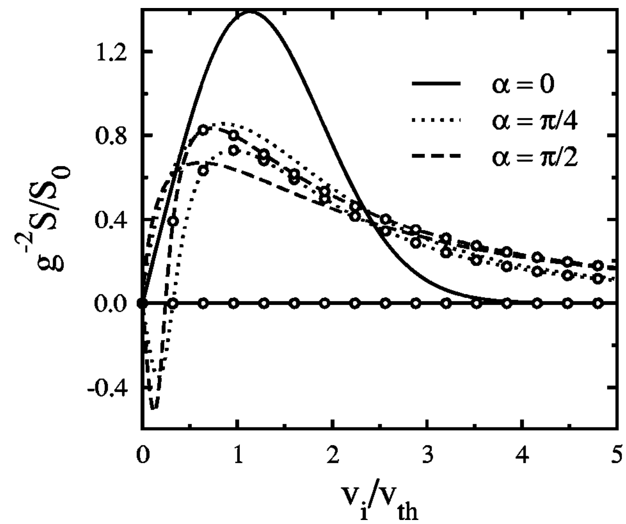

FIG. 3. As Fig. 2, but here $g=0.01$.

power Eq. (20), on the other hand, leads at low ion velocities to a term which behaves $\propto\lambda\ln(1/\lambda)$ [10], for both the full and the simplified LR treatments. This is a quite unexpected behavior compared to the well-known linear velocity dependence without magnetic field [22,25,26].

The stopping powers (20) and (22) depend on the angle α. For small $\alpha\to 0$, Eq. (20) yields

$$S'_{\mathrm{LR}}(\alpha\to 0)=S_0\frac{g^2}{\sqrt{2\pi}}\Lambda_{\mathrm{LR}}(\lambda)\lambda e^{-\lambda^2/2}, \tag{23}$$

whereas for BC, the stopping power vanishes as

$$S_{\mathrm{BC}}(\alpha\to 0)=S_0\frac{g^2}{4\sqrt{2\pi}}\lambda\sin^2\alpha\int_{-\infty}^{\infty}\frac{dx(\lambda^2-x^2)e^{-x^2/2}}{|x-\lambda|^5}\times\ln\left[1+\frac{(x-\lambda)^6}{g^2}\right]. \tag{24}$$

This result coincides with the exact behavior (no perturbation treatment) of the BC stopping power for small α [17,18,21]. In the presence of a strong magnetic field, the electrons move parallel to the magnetic field. For reasons of symmetry, no velocity can be transfered to positively charged ions that also move parallel to the field. The energy transfer and hence the stopping power within the BC treatment must therefore vanish.

In Figs. 2 and 3, the stopping powers within the LR (the curves without circles) and BC (the curves with circles) treatments are plotted for plasmas in a strong magnetic field for three values of α: $\alpha=0$ (solid lines), $\alpha=\pi/4$ (dotted lines), $\alpha=\pi/2$ (dashed lines). The difference between the two treatments is noteworthy especially in the low and intermediate ion velocity limits. It is related to the different cutoff procedures, i.e., $\langle k_{\mathrm{min}}\rangle$, $\langle k_{\mathrm{max}}\rangle$ in the LR and r_{min}, r_{max} in the BC. In particular, the large stopping power predicted by the simplified LR for $\alpha=0$ is unrealistic, since it vanishes within an exact (nonlinear) BC treatment, as discussed above. This is not healed by including collective effects, see Fig. 4, where we compare the simplified [Eq. (8)] and full [Eq. (6)] LR stopping powers for an infinitely strong magnetic field. The role of collective excitations is not as important here as in the limiting case considered in Refs. [1,8]. These conclusions are supported by a numerical solution of the nonlinearized Vlasov-Poisson equations [13].

In the low-velocity limit when $\lambda\ll 1$, the BC stopping power is linear in λ and negative,

$$S_{\mathrm{BC}}(\lambda\to 0)=-S_0\frac{g}{8\sqrt{2\pi}}\lambda\sin^2\alpha\int_0^{\infty}\frac{dx}{x^2}e^{-gx}\ln(1+8gx^3). \tag{25}$$

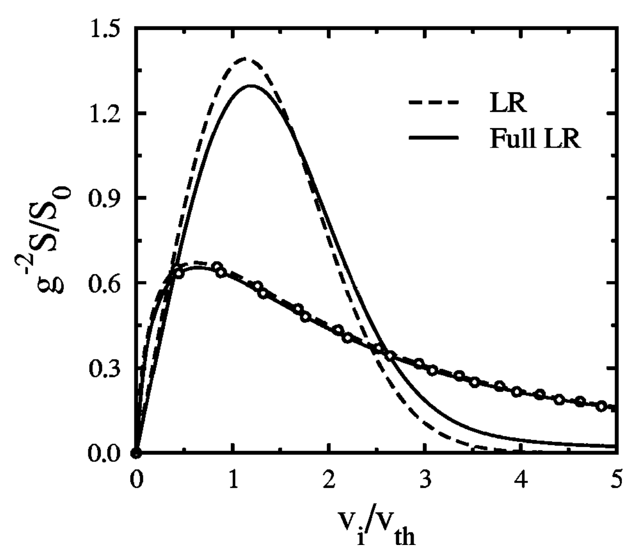

FIG. 4. Stopping powers (in units of g^2S_0) within the simplified (dashed lines) and the full (solid lines) LR treatment as a function of the ion velocity v_i (in units of v_{th}) in a plasma with strong magnetic field for $g=0.01$, $\alpha=0$ (lines without circles), and $\alpha=\pi/2$ (lines with circles).

This corresponds to an energy gain of an ion at slow perpendicular motion. For a finite magnetic field strength and anisotropic (nonequilibrium) velocity distributions typical for electron cooling, such energy gains have indeed been observed in nonperturbative numerical simulations of binary collisions [18] as well as in a numerical solution of the nonlinearized Vlasov-Poisson equation [13]. The perturbing ion drives the equilibration between the longitudinal and transverse electron temperatures. There results, in the average, a shrinking of the cyclotron radii of the electrons and an energy gain of the ion caused by the released transverse electron energy. For an infinitely strong magnetic field and an effective one-dimensional electron motion, however, this mechanism does not work. Both the unexpected $\propto\lambda \ln(1/\lambda)$ behavior in the LR and the energy gain in (second-order) BC therefore indicate a breakdown of a perturbation treatment for $B\to\infty$ and small ion velocities. This is also supported by considering again the case where the ion moves along the magnetic field. As discussed above, the energy transfer in binary collisions is zero for positively charged ions due to symmetry. But for negatively charged ions, the electron can either pass over the potential well, which gives again no energy transfer, or it is reflected with a momentum transfer of two times its initial momentum. Thus all scattering events contributing to the stopping power are nonperturbative in this case. It is evident that this situation cannot be treated by either the LR or the second-order BC, which both depend on the square of the ion charge. However, the cutoff procedures employed in the (perturbative) BC lead to stopping powers which are much closer to numerical simulations [13,18] than the LR predictions [20] for positively charged ions and finite magnetic fields.

IV. BINARY COLLISION FORMULATION

The results obtained so far strongly suggest that the large discrepancies between the LR and the BC seen at strong magnetic fields are peculiar to the Coulomb interaction that requires cutoffs. It is the main concern of this paper to show that these discrepancies are in fact a consequence of the different cutoff procedures in the LR and BC and that the standard cutoff recipes for the nonmagnetized case are not guaranteed to work in the presence of an external magnetic field. To this end we replace the Coulomb interaction by an effective, smoothened interaction potential, which decays faster than r^{-1} at large distances and increases slower than r^{-1} at small ones. The introduction of such a smoothened potential can be viewed as an alternative implementation of cutoffs. It is justified by the same line of arguments: At large distances, the bare Coulomb interaction is shielded by the polarization of the electrons. At small distances, a perturbative treatment of the Coulomb interaction leads to divergencies. One thus attempts to approximate the finite cross section of a nonperturbative treatment by either a cutoff or a smoothening of the interaction.

Below we first discuss the general equations of motion for two charged particles moving in a homogeneous magnetic field and the remaining conservation laws. From the velocity transfer we then proceed to the energy loss of particles during binary collision process. In contrast to previous work [20], the present treatment becomes more transparent in Fourier space.

A. Relative motion and conservation laws

We consider two point charges with masses m, M and charges $-e$, Ze, respectively, moving in a homogeneous magnetic field $\mathbf{B}_0=B_0\mathbf{b}$. We assume that the particles interact with the potential $-(Ze^2/4\pi\epsilon_0)U(\mathbf{r})$, where $\mathbf{r}$ is the relative coordinate of colliding particles. For charged particles the function $U(\mathbf{r})$ can be expressed, for instance, by the Coulomb potential, $U_{\mathrm{C}}(\mathbf{r})=1/r$ or, more realistically, for application in plasmas, by the Debye screened potential, $U_{\mathrm{D}}(\mathbf{r})=\exp(-r/\lambda_{\mathrm{D}})/r$. In the presence of an external magnetic field, the Lagrangian and the corresponding equations of particles' motion cannot, in general, be separated into parts describing the relative motion and the motion of the center of mass with velocities $\mathbf{v}$, $\mathbf{V}_{\mathrm{cm}}$ and coordinates $\mathbf{r}$, $\mathbf{R}_{\mathrm{cm}}$, respectively (see, e.g., Refs. [15,18,20]). Introducing the reduced mass $1/\mu=1/m+1/M$ the equations of motion are

$$\dot{\mathbf{v}}(t)+\Omega_4[\mathbf{v}(t)\times\mathbf{b}]=-\Omega_3[\mathbf{V}_{\mathrm{cm}}(t)\times\mathbf{b}]-\frac{Ze^2}{4\pi\epsilon_0\mu}\mathbf{F}(\mathbf{r}(t)), \tag{26}$$

$$\dot{\mathbf{V}}_{\mathrm{cm}}(t)-\Omega_1[\mathbf{V}_{\mathrm{cm}}(t)\times\mathbf{b}]=-\Omega_2[\mathbf{v}(t)\times\mathbf{b}], \tag{27}$$

where $-(Ze^2/4\pi\epsilon_0)\mathbf{F}(\mathbf{r}(t))$ ($\mathbf{F}=-\partial U/\partial\mathbf{r}$) is the force acting on each particle. The frequencies Ω_1, Ω_2, Ω_3, and Ω_4 are expressed in terms of the electron cyclotron frequency $\Omega=eB_0/m$,

$$\Omega_1=\frac{m(Z-1)}{M+m}\Omega,\quad \Omega_2=\frac{m(M+Zm)}{(M+m)^2}\Omega, \tag{28}$$

$$\Omega_3=\left(1+\frac{Zm}{M}\right)\Omega,\quad \Omega_4=\left(1-\frac{Zm^2}{M^2}\right)\frac{\mu}{m}\Omega. \tag{29}$$

From Eqs. (26) and (27) follows the conservation of total energy

$$W=\frac{(M+m)V_{\mathrm{cm}}^2}{2}+\frac{\mu v^2}{2}-\frac{Ze^2}{4\pi\epsilon_0}U(\mathbf{r})=\mathrm{const}, \tag{30}$$

but the relative and center of mass energies are not conserved separately.

The coupled, nonlinear differential equations (26) and (27) completely describe the motion of the particles. They have to be integrated numerically for a complete set of the initial conditions for solving the scattering problem. In the case of heavy ions, i.e., $M\gg m$, the equations of motion can be further simplified, since $\mu\to m$, $\Omega_1,\Omega_2\to 0$, and $\Omega_3,\Omega_4\to\Omega$ [see Eqs. (28) and (29)]. Equation (27) leads to $\mathbf{V}_{\mathrm{cm}}\to\mathbf{v}_i=\mathrm{const}$, where $\mathbf{v}_i$ is the heavy-ion velocity, and Eq. (26) turns into

$$\dot{\mathbf{v}}(t)+\Omega[\mathbf{v}(t)\times\mathbf{b}]=-\Omega[\mathbf{v}_i\times\mathbf{b}]-\frac{Ze^2}{4\pi\epsilon_0 m}\mathbf{F}(\mathbf{r}(t)). \tag{31}$$

With the help of the equation of motion (31) it can be easily proven that the quantity

$$K=\frac{mv^2}{2}-\frac{Ze^2}{4\pi\epsilon_0}U(\mathbf{r})+m\Omega\mathbf{r}[\mathbf{v}_i\times\mathbf{b}] \tag{32}$$

is a constant of motion. In contrast to the unmagnetized case, it thus follows that the relative energy transfer during ion-electron collision is proportional to $\delta r_\perp v_{i\perp}$, where $\delta r_\perp$ and $v_{i\perp}$ are the perpendicular components of the change of relative position and the ion velocity.

B. Trajectory correction

It is now useful to introduce the velocity correction through relations $\delta\mathbf{v}(t)=\mathbf{v}_e(t)-\mathbf{v}_{e0}(t)=\mathbf{v}(t)-\mathbf{v}_0(t)$, where $\mathbf{v}_{e0}(t)$ and $\mathbf{v}_0(t)$ are the unperturbed electron and relative velocities, respectively, with

$$\dot{\mathbf{v}}_0(t)+\Omega[\mathbf{v}_0(t)\times\mathbf{b}]=-\Omega[\mathbf{v}_i\times\mathbf{b}]. \tag{33}$$

Note that $\delta\mathbf{v}(t)\to 0$ at $t\to-\infty$. The equation of motion for $\delta\mathbf{v}(t)$ then follows from Eq. (31) as

$$\delta\dot{\mathbf{v}}(t)+\Omega[\delta\mathbf{v}(t)\times\mathbf{b}]=-\frac{Ze^2}{4\pi\epsilon_0 m}\mathbf{F}(\mathbf{r}(t)), \tag{34}$$

where $\mathbf{r}(t)=\mathbf{r}_e(t)-\mathbf{v}_i t$ is the ion-electron relative coordinate.

We seek an approximate solution of Eq. (34) in which the interaction force between the ion and electrons is considered as a perturbation. Thus we have to look for the solution of Eq. (34) for the variables $\mathbf{r}$ and $\mathbf{v}$ in a perturbative manner

$$\mathbf{r}(t)=\mathbf{r}_0(t)+\mathbf{r}_1(t)+\mathbf{r}_2(t)+\cdots, \tag{35}$$

$$\mathbf{v}(t)=\mathbf{v}_0(t)+\mathbf{v}_1(t)+\mathbf{v}_2(t)+\cdots, \tag{36}$$

where $\mathbf{r}_0(t),\mathbf{v}_0(t)$ are the unperturbed ion-electron relative coordinate and velocity, respectively, $\mathbf{r}_n(t),\mathbf{v}_n(t)\propto Z^n\mathbf{F}_{n-1}$ $(n=1,2,\ldots)$ are the nth order perturbations of $\mathbf{r}(t)$ and $\mathbf{v}(t)$, which are proportional to Z^n. $\mathbf{F}_n(t)$ is the nth order correction to the ion-electron interaction force. Using the expansion (35) for the nth order corrections $\mathbf{F}_n$, we obtain

$$\mathbf{F}(\mathbf{r}(t))=\mathbf{F}_0(\mathbf{r}_0(t))+\mathbf{F}_1(\mathbf{r}_0(t),\mathbf{r}_1(t))+\cdots, \tag{37}$$

where

$$\mathbf{F}_0(\mathbf{r}_0(t))=\mathbf{F}(\mathbf{r}_0(t))=-i\int d\mathbf{k}U(\mathbf{k})\mathbf{k}\exp[i\mathbf{k}\cdot\mathbf{r}_0(t)], \tag{38}$$

$$\mathbf{F}_1(\mathbf{r}_0(t),\mathbf{r}_1(t))=\left(\mathbf{r}_1(t)\cdot\frac{\partial}{\partial\mathbf{r}}\right)\mathbf{F}(\mathbf{r})|_{\mathbf{r}=\mathbf{r}_0(t)} = \int d\mathbf{k}U(\mathbf{k})\mathbf{k}[\mathbf{k}\cdot\mathbf{r}_1(t)]\exp[i\mathbf{k}\cdot\mathbf{r}_0(t)]. \tag{39}$$

In Eqs. (38) and (39), we have introduced the ion-electron interaction potential $U(\mathbf{r})$ through $\mathbf{F}(\mathbf{r})=-\partial U(\mathbf{r})/\partial\mathbf{r}$ and the force corrections have been written using a Fourier transformation in space.

We start with the zero-order unperturbed helical motion of the electrons. From Eq. (33), we obtain

$$\mathbf{v}_0(t)=\mathbf{v}_r+v_{0\perp}\{\mathbf{u}\cos(\Omega t)+[\mathbf{b}\times\mathbf{u}]\sin(\Omega t)\}, \tag{40}$$

$$\mathbf{r}_0(t)=\mathbf{R}_0+\mathbf{v}_r t+a\{\mathbf{u}\sin(\Omega t)-[\mathbf{b}\times\mathbf{u}]\cos(\Omega t)\}, \tag{41}$$

where $\mathbf{u}=(\cos\varphi,\sin\varphi)$ is the unit vector perpendicular to the magnetic field, $v_{0\parallel}$ and $v_{0\perp}$ (with $v_{0\perp}\geqslant 0$) are the electron unperturbed velocity components parallel and perpendicular to $\mathbf{b}$, respectively, $\mathbf{v}_r=v_{0\parallel}\mathbf{b}-\mathbf{v}_i$ is the relative velocity of the electron guiding center, and $a=v_{0\perp}/\Omega$ is the cyclotron radius. It should be noted that in Eqs. (40) and (41), the variables $\mathbf{u}$ and $\mathbf{R}_0$ are independent and are defined by the initial conditions.

The equation for the first-order velocity correction is given by

$$\dot{\mathbf{v}}_1(t)+\Omega[\mathbf{v}_1(t)\times\mathbf{b}]=-\frac{Ze^2}{4\pi\epsilon_0 m}\mathbf{F}_0(\mathbf{r}_0(t)) \tag{42}$$

with the solutions

$$\mathbf{v}_1(t)=\frac{Ze^2}{4\pi\epsilon_0 m}\{-\mathbf{b}V_\parallel(t)+\mathrm{Re}[\mathbf{b}(\mathbf{b}\cdot\mathbf{V}_\perp(t))-\mathbf{V}_\perp(t) + i[\mathbf{b}\times\mathbf{V}_\perp(t)]]\}, \tag{43}$$

$$\mathbf{r}_1(t)=\frac{Ze^2}{4\pi\epsilon_0 m}\{-\mathbf{b}P_\parallel(t)+\mathrm{Re}[\mathbf{b}(\mathbf{b}\cdot\mathbf{P}_\perp(t))-\mathbf{P}_\perp(t) + i[\mathbf{b}\times\mathbf{P}_\perp(t)]]\}, \tag{44}$$

where we have introduced the following abbreviations

$$V_\parallel(t)=\int_{-\infty}^{t}d\tau\mathbf{b}\cdot\mathbf{F}_0(\mathbf{r}_0(\tau)), \qquad \mathbf{V}_\perp(t)=e^{i\Omega t}\int_{-\infty}^{t}d\tau e^{-i\Omega\tau}\mathbf{F}_0(\mathbf{r}_0(\tau)), \tag{45}$$

$$P_\parallel(t)=\int_{-\infty}^{t}d\tau V_\parallel(\tau), \qquad \mathbf{P}_\perp(t)=\int_{-\infty}^{t}d\tau\mathbf{V}_\perp(\tau), \tag{46}$$

and have assumed that all corrections vanish at $t\to-\infty$. For instance, in the unscreened Coulomb case, the interaction force $\mathbf{F}_0$ must behave as $\mathbf{F}_0(\mathbf{r}_0(t))\to 1/t^2$ for $|t|\to\infty$. Thus from Eq. (45) at $t\to\infty$ we obtain $V_\parallel(t)\to V_{0\parallel}=\text{const}$ and $\mathbf{V}_\perp(t)\to e^{i\Omega t}\mathbf{V}_{0\perp}$, where $\mathbf{V}_{0\perp}=\text{const}$. The quantities $V_{0\parallel}$ and $\mathbf{V}_{0\perp}$ give the first-order velocity correction in Eq. (43) after an electron-ion collision. In this limit, we find for the first-order trajectory correction from Eqs. (45) and (46), $P_\parallel(t)=V_{0\parallel}t+P_{0\parallel}$, $\mathbf{P}_\perp(t)=-i(\mathbf{V}_{0\perp}/\Omega)e^{i\Omega t}+\tilde{\mathbf{P}}_{0\perp}$, where

$$P_{0\parallel}=\nu-\int_{-\infty}^{\infty}d\tau\tau\mathbf{b}\cdot\mathbf{F}_0(\mathbf{r}_0(\tau)),$$
$$\tilde{\mathbf{P}}_{0\perp}=\frac{i}{\Omega}\int_{-\infty}^{\infty}d\tau\mathbf{F}_0(\mathbf{r}_0(\tau)). \tag{47}$$

For the Coulomb interaction $\nu=-tV_\parallel(t)|_{t\to-\infty}=\mathbf{b}\mathbf{v}_r/v_r^3$ and $\nu=0$ for any screened interaction potential. Note that for the Coulomb interaction, the second term in Eq. (47) tends to infinity (see, e.g., Ref. [20]). However, the contribution of this term to the ion energy change vanishes after averaging over impact parameters.

Substituting Eqs. (38) and (41) into Eqs. (45) and (46), and using the expression [27]

$$\exp(iz\sin\theta)=\sum_{n=-\infty}^{\infty}J_n(z)e^{in\theta}, \tag{48}$$

where J_n is the Bessel function of the nth order, we obtain for an arbitrary interaction potential

$$P_\parallel(t)=i\int d\mathbf{k}U(\mathbf{k})(\mathbf{k}\cdot\mathbf{b})e^{i\mathbf{k}\cdot\mathbf{R}_0}\times\sum_{n=-\infty}^{+\infty}e^{in\psi}J_n(k_\perp a)\frac{e^{i\zeta_n(\mathbf{k})t}}{[\zeta_n(\mathbf{k})-i0]^2}, \tag{49}$$

$$\mathbf{P}_\perp(t)=i\int d\mathbf{k}U(\mathbf{k})\mathbf{k}e^{i\mathbf{k}\cdot\mathbf{R}_0}\sum_{n=-\infty}^{+\infty}e^{in\psi}J_n(k_\perp a)\times\frac{e^{i\zeta_n(\mathbf{k})t}}{[\zeta_n(\mathbf{k})-i0][\zeta_{n-1}(\mathbf{k})-i0]}. \tag{50}$$

Here $\zeta_n(\mathbf{k})=n\Omega+\mathbf{k}\cdot\mathbf{v}_r$, $\psi=\varphi-\theta$, and $\text{tg}\theta=k_y/k_x$. The quantities $V_\parallel(t)$ and $\mathbf{V}_\perp(t)$ are obtained directly from Eqs. (49) and (50) through the relations $V_\parallel(t)=\dot{P}_\parallel(t)$ and $\mathbf{V}_\perp(t)=\dot{\mathbf{P}}_\perp(t)$.

V. FROM THE BC ENERGY TRANSFER TO THE STOPPING POWER

A. General formulation

Previously (see, e.g., Ref. [20], and references therein), the energy gain ΔE_e of the electron in terms of the velocity transfer $\delta\mathbf{v}$ was considered. But this equals the energy loss of the ion, $\Delta E_i=-\Delta E_e$.

The total energy change of the ion during an ion-electron collision is given by

$$\Delta E_i=\frac{Ze^2}{4\pi\epsilon_0}\int_{-\infty}^{\infty}dt\mathbf{v}_i\cdot\mathbf{F}(\mathbf{r}(t)). \tag{51}$$

Insertion of Eq. (37) into the general expression (51) yields

$$\Delta E_i=\Delta E_i^{(1)}+\Delta E_i^{(2)}+\cdots, \tag{52}$$

where

$$\Delta E_i^{(1)}=\frac{Ze^2}{4\pi\epsilon_0}\int_{-\infty}^{\infty}dt\mathbf{v}_i\cdot\mathbf{F}_0(\mathbf{r}_0(t)),$$
$$\Delta E_i^{(2)}=\frac{Ze^2}{4\pi\epsilon_0}\int_{-\infty}^{\infty}dt\mathbf{v}_i\cdot\mathbf{F}_1(\mathbf{r}_0(t),\mathbf{r}_1(t)) \tag{53}$$

are the first- and second-order energy transfer, respectively.

B. First-order energy transfer

The first-order energy transfer can be obtained by substituting Eqs. (38) and (41) into the first one of Eqs. (53). This yields

$$\Delta E_i^{(1)}=-i\frac{Ze^2}{2\epsilon_0}\int d\mathbf{k}U(\mathbf{k})(\mathbf{k}\cdot\mathbf{v}_i)e^{i\mathbf{k}\cdot\mathbf{R}_0}\times\sum_{n=-\infty}^{+\infty}e^{in\psi}J_n(k_\perp a)\delta(\zeta_n(\mathbf{k})). \tag{54}$$

We now introduce the variable $\mathbf{s}=\mathbf{R}_{0\perp}^{(r)}$ which is the component of $\mathbf{R}_0$ perpendicular to the relative velocity vector $\mathbf{v}_r$. From Eqs. (40) and (41) we can see that $\mathbf{s}$ is the distance of closest approach for the guiding center of the electron helical motion. The stopping power is now given by the average of ΔE_i with respect to the initial phase of the electrons φ and the azimuthal angle of $\mathbf{s}$. For spherically symmetric interaction potentials $[U(\mathbf{r})=U(r)$ and $U(\mathbf{k})=U(k)]$, the first-order energy transfer gives no contribution due to symmetry and the ion energy change receives a contribution only from higher orders. In fact, Eq. (54) for the averaged first-order energy change gives

$$\langle\Delta E_i^{(1)}\rangle=-i\frac{Ze^2}{2\epsilon_0}\int d\mathbf{k}U(k)(\mathbf{k}\cdot\mathbf{v}_i)J_0(\kappa s)J_0(k_\perp a)\delta(\mathbf{k}\cdot\mathbf{v}_r), \tag{55}$$

where $\kappa=\sqrt{k^2-(\mathbf{k}\cdot\mathbf{n}_r)^2}$ and $\mathbf{n}_r=\mathbf{v}_r/v_r$. As the integrand is an odd function of $\mathbf{k}$ we have $\langle\Delta E_i^{(1)}\rangle=0$.

C. Second-order energy transfer

Inserting Eqs. (39), (41), (44), (49), and (50) into the second equation of Eqs. (53) one obtains

$$\Delta E_i^{(2)}(\mathbf{R}_0,\varphi)=\frac{\pi i Z^2 e^4}{(4\pi\epsilon_0)^2 m}\int d\mathbf{k}d\mathbf{k}'U(\mathbf{k})U(\mathbf{k}')\times(\mathbf{k}\cdot\mathbf{v}_i)e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{R}_0}\times\sum_{n;m=-\infty}^{+\infty}e^{in\psi+im\psi'}J_n(k_\perp a)J_m(k'_\perp a)\times\delta(\zeta_n(\mathbf{k})+\zeta_m(\mathbf{k}'))G_m(\mathbf{k},\mathbf{k}'), \quad (56)$$

where $\psi'=\varphi-\theta'$, and

$$G_m(\mathbf{k},\mathbf{k}')=\frac{-2(\mathbf{k}\cdot\mathbf{b})(\mathbf{k}'\cdot\mathbf{b})}{[\zeta_m(\mathbf{k}')-i0]^2}+\frac{(\mathbf{k}\cdot\mathbf{b})(\mathbf{k}'\cdot\mathbf{b})-\mathbf{k}\cdot\mathbf{k}'+i\mathbf{k}\cdot[\mathbf{b}\times\mathbf{k}']}{[\zeta_m(\mathbf{k}')-i0][\zeta_{m-1}(\mathbf{k}')-i0]}+\frac{(\mathbf{k}\cdot\mathbf{b})(\mathbf{k}'\cdot\mathbf{b})-\mathbf{k}\cdot\mathbf{k}'-i\mathbf{k}\cdot[\mathbf{b}\times\mathbf{k}']}{[\zeta_m(\mathbf{k}')-i0][\zeta_{m+1}(\mathbf{k}')-i0]}. \quad (57)$$

Next, $\Delta E_i^{(2)}$ is averaged with respect to the initial phase of electrons φ (see the Appendix for details). The φ-averaged ion energy change, $\langle\Delta E_i^{(2)}\rangle_\varphi$, is then integrated over the impact parameters $\mathbf{s}$ in the full two-dimensional (2D) space.

Thus we can introduce an effective transport cross section [17,21] through the relation (see the appendix)

$$\sigma_{\rm tr}(\mathbf{v}_r,\mathbf{v}_i)=-\frac{2}{mv_r^2}\int d^2\mathbf{s}\langle\Delta E_i^{(2)}\rangle_\varphi =\frac{\pi^2 Z^2 e^4}{\epsilon_0^2 m^2 v_r^3}\int d\mathbf{k}|U(\mathbf{k})|^2(\mathbf{k}\cdot\mathbf{v}_i)\sum_{n=-\infty}^{+\infty}J_n^2(k_\perp a)\times\left\{k_\parallel^2\delta'(\zeta_n(\mathbf{k}))+\frac{k_\perp^2}{2\Omega}[\delta(\zeta_{n+1}(\mathbf{k}))-\delta(\zeta_{n-1}(\mathbf{k}))]\right\}, \quad (58)$$

where $\delta'(x)$ defines the derivative of the δ function with respect to the argument.

For the Coulomb interaction $U(k)=V(k)$, the full 2D integration over the $\mathbf{s}$ space results in a logarithmic divergence of the $\mathbf{k}$ integration in Eq. (58). To cure this, we introduce cutoff parameters $\langle k_{\rm min}\rangle$ and $\langle k_{\rm max}\rangle$ as it was done in the linear response formulation [see Eq. (10)].

For applications to the energy loss of ions moving in a magnetized homogeneous plasma, we average the ion energy change during binary collision over the distribution function of the electrons f_0. The standard procedure for averaging over distribution function yields

$$S_{\rm BC}=-\frac{dE}{d\ell}=\int d\mathbf{v}_0 f_0(\mathbf{v}_0)\frac{n_0 v_r}{v_i}\frac{mv_r^2}{2}\sigma_{\rm tr}(\mathbf{v}_r,\mathbf{v}_i), \quad (59)$$

where n_0 is the density of plasma electrons. Substituting Eq. (58) for the effective transport cross section $\sigma_{\rm tr}$ into the last expression, we obtain Eq. (8) derived in the simplified LR treatment (see the Appendix for details). This shows the complete conformity between both approaches.

In the presence of a strong magnetic field, the simplified LR expression (8) for the stopping power with an arbitrary spherically symmetric interaction potential $U(r)$ and the imaginary part of the susceptibility (19) yields

$$S'_{\rm LR}(\lambda)=S_0\frac{g^2\mathcal{U}_0}{2\sqrt{2\pi}}\lambda\sin^2\alpha\int_{-\infty}^{\infty}\frac{x^2e^{-x^2/2}dx}{q^{3/2}(x,\lambda)}=S_{\rm BC}(\lambda). \quad (60)$$

Here,

$$\mathcal{U}_0=\frac{(2\pi)^4}{4}\int_0^\infty k^3U^2(k)dk \quad (61)$$

and $U(k)$ is the Fourier transformed interaction potential. Now there appears in Eq. (60) the numerical factor $\mathcal{U}_0$ instead of the Coulomb logarithm (13) in Eq. (20). Equation (61) also gives a criterion in Fourier space for the smoothened potential, which is equivalent to the conditions considered above. Indeed, it must behave like $U(k)\simeq k^{-2-\sigma}$ at $k\to 0$ and $k\to\infty$ with negative or positive values, $\sigma<0$ and $\sigma>0$, respectively. Note that for the Coulomb potential $\sigma=0$ in both limits and the lower and upper cutoffs must be introduced in Eq. (61).

VI. SUMMARY AND CONCLUSIONS

In this paper, we have presented a detailed theoretical investigation of the stopping power of ions moving in a magnetized electron plasma within two complementary approaches: the dielectric linear response (LR) and the binary collision (BC) treatments. The full LR including the dynamic collective response of the electrons can only be evaluated in closed form in the limiting cases of a vanishing and an infinitely strong magnetic field, respectively. A simplified LR which includes only static screening is used for intermediate cases. The BC treatment developed here is valid for arbitrary strengths of the magnetic field and arbitrary shapes of the interaction potential up to second order in the interaction strength. The purpose of this work was to investigate the connection between the complementary BC and LR approaches.

We compare both treatments for a vanishing and very strong magnetic fields. The results obtained within both approaches differ slightly at intermediate ion velocities in the field-free case (see Fig. 1) but significantly at low and intermediate ion velocities for strong magnetic fields (see Fig. 2 and 3). In particular, if the ion moves parallel to a strong magnetic field, the stopping power becomes unrealistic within the simplified LR treatment and this is not healed in the full LR treatment (see Fig. 4). The discrepancies can be traced to the different cutoff procedures employed in both treatments. The cutoffs are required by the infinite range of the Coulomb interaction and its singularity at the origin. We showed the complete conformity between both treatments for smoothened potentials, which need no cutoffs.

In order to look for a connection between the two models, we start within the BC approach from Eqs. (56) and (57), which represent the energy transfer to the ion for arbitrary magnetic fields and shapes of smooth electron-ion interaction potentials. Following the standard procedure, Eq. (56) must be averaged with respect to the impact parameters. For a smoothened potential, Eq. (56) can be integrated over all possible impact parameters in 2D space. However, for a Coulomb potential the averaging requires a cutoff parameter in the $\mathbf{k}$ integration. It is found that for a velocity averaged cutoff parameter $\langle k_{\max}\rangle$ [see Eq. (10)], the energy loss within the BC [Eq. (59)] coincides with the simplified LR result [Eq. (8)].

We note that previous BC and LR treatments (see, e.g., Refs. [1–3,20–22]) for the Coulomb interaction differ somewhat in their approaches. In the BC model, the modified Coulomb logarithm (16) is considered under the integral with respect to the velocity distribution of the electrons. For small relative velocities v_r, Eqs. (9) and (16) show that the modified Coulomb logarithm is proportional to $r_{\max}^2/r_{\min}^2 \propto v_r^6$. Therefore, this approach is self-cutting for small relative velocities v_r. In the LR model the integral with respect to the velocity distribution of the electrons enters in the susceptibility, Eq. (5) and hence in the dielectric function. Here an average Coulomb logarithm $\Lambda_{\rm LR}$ arises due to the $\mathbf{k}$ integration in Eq. (6) or (8) with averaged cutoffs $\langle k_{\min}\rangle$ and $\langle k_{\max}\rangle$ [see Eq. (10)]. This leads to a large energy loss at low ion velocities, which behaves as $S_{\rm LR}\propto v_i \ln(v_{\rm th}/v_i)$ for low ion velocities [10], as shown in Fig. 2–4.

Finally, we would like to mention that our current results still leave some questions open. Usually, the dependence of cutoff parameters on the magnetic field is ignored in both approaches. Besides, the regularization procedure given by Eq. (16) has been performed on the basis of the exact Rutherford formula in the field-free case. However, it is known that for a strong magnetic field the Rutherford scattering formula breaks down and the transport coefficients and the Coulomb logarithm are strongly modified by the magnetic field (see, e.g., Refs. [19,23] and references therein). These topics are presently under investigation by the authors.

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APPENDIX: CALCULATION OF EFFECTIVE TRANSPORT CROSS SECTION AND STOPPING POWER

We now give a more detailed derivation of Eqs. (56)–(58) and show that Eq. (8) follows from Eqs. (58) and (59). We start with the angular averaging of the ion energy transfer. From Eq. (56), we have

$$\langle \Delta E_i^{(2)}\rangle_\varphi = \frac{\pi i Z^2 e^4}{(4\pi\epsilon_0)^2 m}\int d\mathbf{k} d\mathbf{k}' U(\mathbf{k})U^*(\mathbf{k}') \times (\mathbf{k}\cdot\mathbf{v}_i) e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_0} \times \sum_{n=-\infty}^{+\infty} e^{in(\theta'-\theta)} J_n(k_\perp a) J_n(k'_\perp a) \times \delta((\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}_r) G_{-n}(\mathbf{k},-\mathbf{k}'), \qquad (A1)$$

where the function $G_n(\mathbf{k},\mathbf{k}')$ is given by Eq. (57). Here U^* is the complex conjugate of U and we used the relation $J_{-n}=(-1)^n J_n$ for the Bessel functions [27].

For calculation of the $\mathbf{s}$ integral in Eq. (58) we split all variables $\mathbf{A}=A_\parallel^{(r)}\mathbf{n}_r+\mathbf{A}_\perp^{(r)}$ in Eq. (A1) into components parallel ($A_\parallel^{(r)}$) and perpendicular ($\mathbf{A}_\perp^{(r)}$) to the relative velocity $\mathbf{v}_r$, where $\mathbf{n}_r=\mathbf{v}_r/v_r$ is the unit vector along $\mathbf{v}_r$. Note that the component of $\mathbf{k}$ perpendicular to the magnetic field, $\mathbf{k}_\perp$, is now a function of $k_\parallel^{(r)}$ and $\mathbf{k}_\perp^{(r)}$. Performing the $\mathbf{s}$ integration in Eq. (58) we obtain

$$\sigma_{\rm tr}(\mathbf{v}_r,\mathbf{v}_i) = \frac{(2\pi)^4 Z^2 e^4}{(4\pi\epsilon_0)^2 m^2 v_r^3}\int d\mathbf{k} |U(\mathbf{k})|^2 (\mathbf{k}\cdot\mathbf{v}_i) \times \sum_{n=-\infty}^{+\infty} J_n^2(k_\perp a) g_n(\mathbf{k}), \qquad (A2)$$

where $g_n(\mathbf{k})=(-i/2\pi)G_{-n}(\mathbf{k},-\mathbf{k})$ is given by

$$g_n(\mathbf{k}) = \frac{1}{\pi i}\left\{ \frac{k_\parallel^2}{[\zeta_n(\mathbf{k})+i0]^2} + \frac{k_\perp^2}{2\Omega}\left[\frac{1}{\zeta_{n-1}(\mathbf{k})+i0} - \frac{1}{\zeta_{n+1}(\mathbf{k})+i0}\right]\right\}. \qquad (A3)$$

Here we used the following relations between the functions $\zeta_n(\mathbf{k})$:

$$\zeta_{n+1}(\mathbf{k})+\zeta_{n-1}(\mathbf{k}) = 2\zeta_n(\mathbf{k}), \quad \zeta_{n+1}(\mathbf{k})-\zeta_{n-1}(\mathbf{k}) = 2\Omega. \qquad (A4)$$

The second-order singularity in Eq. (A3) (the first term) must be understood as

$$\frac{1}{(\zeta_n+i0)^2} \to \frac{1}{\zeta_n(\zeta_n+i0)} = \frac{1}{\zeta_n^2} - \frac{\pi i}{\zeta_n}\delta(\zeta_n). \qquad (A5)$$

It is easy to see that the contribution of the imaginary part of $g_n(\mathbf{k})$ to the effective cross section, Eq. (A2), vanishes. The contribution of the real part together with Eq. (59) leads to

$$S_{\rm BC} = \frac{Z^2 e^2}{4\pi\epsilon_0 v_i}\int d\mathbf{k} |U(\mathbf{k})|^2 (\mathbf{k}\cdot\mathbf{v}_i)\, \Xi(\mathbf{k},\mathbf{k}\cdot\mathbf{v}_i), \qquad (A6)$$

where

$$\Xi(\mathbf{k},\omega)=2\pi^2\omega_p^2\sum_{n=-\infty}^{+\infty}\int d\mathbf{v}_0 f_0(\mathbf{v}_0)J_n^2(k_\perp a)\left\{k_\parallel^2\delta'(\xi_n)+\frac{k_\perp^2}{2\Omega}[\delta(\xi_{n+1})-\delta(\xi_{n-1})]\right\}, \tag{A7}$$

and $\xi_n=\mathbf{k}\cdot\mathbf{v}_0-\alpha_n$ with $\alpha_n=\omega-n\Omega$.

With the relations

$$J_{n-1}^2(z)-J_{n+1}^2(z)=\frac{4n}{z}J_n(z)J_n'(z), \tag{A8}$$

for the Bessel functions (see, e.g., Ref. [27]), the function $\Xi(\mathbf{k},\omega)$ becomes

$$\Xi(\mathbf{k},\omega)=2\pi^2\omega_p^2\sum_{n=-\infty}^{+\infty}\int d\mathbf{v}_0 f_0(\mathbf{v}_0)\left\{k_\parallel J_n^2(k_\perp a)\frac{\partial}{\partial v_{0\parallel}}\delta(\xi_n)+\delta(\xi_n)\frac{n\Omega}{v_{0\perp}}\frac{\partial}{\partial v_{0\perp}}J_n^2(k_\perp a)\right\}. \tag{A9}$$

After a partial integration of Eq. (A9) we arrive at $\Xi(\mathbf{k},\omega)=\mathrm{Im}\chi^{(0)}(\mathbf{k},\omega)$, where $\chi^{(0)}(\mathbf{k},\omega)$ is the susceptibility of magnetized electrons given by Eq. (5). The comparison of Eqs. (A6) and (8) then yields $S_{\mathrm{BC}}=S'_{\mathrm{LR}}$.

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