PHYSICAL REVIEW E **67**, 025602(R) (2003)

Electromagnetic waves focused by a negative-index planar lens

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(Received 3 October 2002; published 26 February 2003)

We demonstrate that negative refraction occurs for both cw and pulsed electromagnetic waves when traversing from a "right-handed" (index >0) to a "left-handed" (index <0) material (LHM) which has causal dispersive intrinsic properties. We also demonstrate that a divergent line source spaced a distance H in front of a planar LHM slab and excited by either an impulse cw or a Gaussian frequency pulse is imaged at a distance H away, inside the LHM, and at H to the other side of the slab. The image size is $\sim\lambda$ consistent with limitations dictated by wave optics. We find no evidence of evanescent mode amplification. The studies were performed using numerical experiments with finite difference time domain solutions and incorporating a causal Lorentzian form for the frequency-dependent material properties.

DOI: 10.1103/PhysRevE.67.025602 PACS number(s): 41.20.Jb, 42.70.Qs, 78.20.Ci

I. INTRODUCTION

Veselago [1] was the first author to postulate the unusual behavior for a hypothetical negative index, left-handed medium (LHM). He predicted that the Poynting vector $\vec{S}$ would always form a right-handed triad with $\vec{E}$ and $\vec{H}$ (in a normal material or a LHM). However, he predicted that the propagation vector $\vec{k}$ would form a left-handed triad with $\vec{E}$ and $\vec{H}$ inside a LHM. Hence the phase velocity in the LHM would appear to move in the negative direction opposite to the positive moving energy flow, $\vec{S}$. Through imposition of boundary conditions he also concluded that a negative refraction would occur inside the LHM. That is, a ray would be directed to the opposite side of the normal to the surface from that expected in a conventional $(n>0)$ material.

The existence of negative refraction has recently been the subject of considerable discussion and some disagreements [2–11]. For example, Valanju, Walser, and Valanju [2] conclude that others [3–7] have erroneously deduced that an electromagnetic pulse diverging from a point source could be focused by a plane-parallel slab of LHM. On the other hand, Pendry and co-workers [6,8] have taken the focusing argument to another level by insisting that a LHM can amplify evanescent modes allowing a complete reconstruction of the source to a perfect point image, with none of the conventional optical limitations.

In light of the continuing dialog on these topics we have elected to perform careful numerical simulations to examine the refractive and focusing properties of left-handed materials. Much of the prior work has been done assuming unrealistic material properties (i.e., no frequency dispersion and zero loss). We have used a finite difference time domain (FDTD) solution, which includes causal frequency dependent permittivity and permeability. Our results clearly show negative refraction for either a cw or pulsed wave. They also demonstrate focusing of a line source at the center and to the right of a LHM slab. However, the focus is far from perfect. Its size is $\sim\lambda$, consistent with limitations imposed by wave optics.

II. SIMULATIONS

In order to investigate electromagnetic propagation in a medium of any type it is important that the constitutive parameters be causal. To obtain negative index properties, both the intrinsic permeability $\mu(f)$ and permittivity $\epsilon(f)$ must be strongly dispersive and must be negative over a limited frequency range. To meet this requirement we have selected a Lorentzian frequency form for both μ and ϵ. Without loss of generality we take these to have identical functional forms

$$F(f)=1+\frac{K-1}{1-i\left(\frac{fG}{f_0^2}\right)-\left(\frac{f}{f_0}\right)^2}, \tag{1}$$

where $F(f)=\mu(f)$ or $\epsilon(f)$ and $K=\mu_{DC}$ or ϵ_{DC}. The FDTD code [12] we use to perform the numerical experiments has been validated extensively with laboratory experiments. The code uses perfectly matched layer [13] absorbing boundaries in the dimensions of finite extent and periodic boundary conditions in the dimensions of infinite extent. The volume of our two-dimensional FDTD simulation space for this study was 700 hundred cells in the x direction and 600 cells in the z direction (direction of propagation). Each spatial cell was 0.05 cm on a side and the time step interval was $\Delta t=\Delta z/\sqrt{3}c=1ps$ (Courant condition). The simulation space was gridded even more finely for our one-dimensional simulations.

The complex index of refraction is given by $n(f)=\pm\sqrt{\epsilon(f)\mu(f)}$. Analysis of Maxwell's equations leads to selection of the negative sign for the case when both ϵ and μ are negative. To minimize reflection at interfaces and to reduce the effect of losses we have selected parameters in Eq. (1) of $G=0.04$ GHz, $\epsilon_{DC}=\mu_{DC}=4.0$, and $f_0=6.3$ GHz. The selected frequency for our cw and the centroid of our

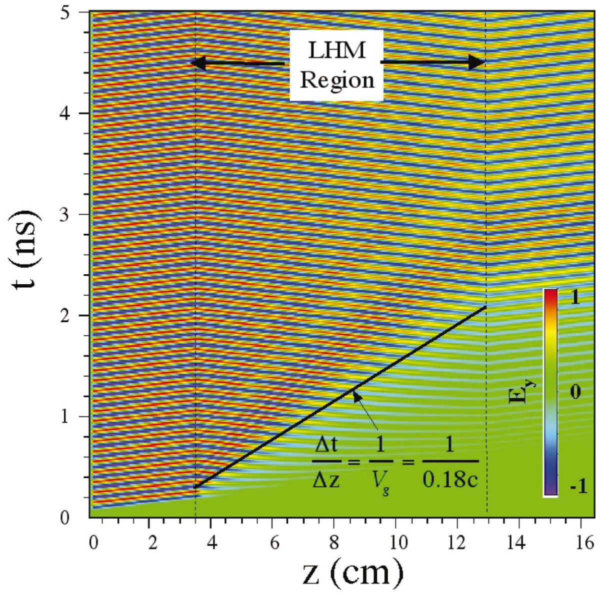


FIG. 1. (Color) Electric field amplitude versus time for an impulse-generated cw plane wave traveling in the z direction. A left-handed material with index $n=-1.001+i0.013$ at the particular 10 GHz chosen is situated between the two dashed lines. The inverse of the slopes of the crests gives the phase velocity. On either side of the slab the phase velocity is the speed of light, c. Inside the LHM the phase velocity is also approximately the speed of light, but is negative. The slope of the early time arriving impulse corresponds to the group velocity of $0.18c$ and is positive.

pulse is 10 GHz. At this frequency and using the selected parameters we obtain an index of $n=-1.001+i0.013$. Further reductions in the loss component of n were used to extrapolate the results, but it is not possible to take the loss to zero and maintain causality.

A. One-dimensional cw plane wave

We first examine the direction of phase velocity within a LHM having causal properties given by Eq. (1). A 10 GHz cw plane wave is propagated in the z direction, normal to a semi-infinite, plane-parallel LHM slab of thickness $2H=9.5$ cm. Figure 1 displays our computer generated phase fronts for this plane wave to the left, within, and to the right of the slab. The left edge of the slab is at 3.5 cm. Notice that the slopes of the phase fronts give the phase velocity outside and inside the material. In both cases the amplitude of the phase velocity is equal or approximately equal to the speed of light c but it is negative within the slab. Thus we demonstrate the relatively noncontroversial point that the phase velocity of a monochromatic wave is negative within a LHM. The leading edge of the impulse used to generate the cw wave is shown by the solid line. Using Eq. (1), the group velocity $V_g=2\pi(df/dk)$ is equal to $0.18c$, very close to that deduced from the solid line and it is positive.

B. Two-dimensional divergent cw wave: Infinite line source

Much of the discussion and controversy about refraction, propagation, and focusing in LHM has dealt with divergent point sources and planar slabs of LHM. To produce a point source we must move to a two-dimensional FDTD simulation. We use an infinitely long line source oriented along the y axis. This divergent source is spaced a distance $H=4.7$ cm in front of the LHM slab which has a thickness of 9.5 cm in the z direction, a height of 30 cm in the x direction, and is also infinite in the y direction.

The line source is first excited by an impulse cw signal with the same carrier frequency, 10 GHz, as before and the same amplitude. It takes ≈ 5 ns for a steady state to develop within the spatial extent of our 30 cm long FDTD simulation. Time animations then show the phase fronts moving backwards within the LHM but forward outside of the slab. The Figs. 2(a) and 2(b) show snapshots at 5 and 21 ns after the pulse was excited.

There is a clear indication of focusing at the midpoint of the slab and again to the right of the slab. In contrast with Ziolkowski [9] we find that the positions and intensities of the focused areas are stable. We note that at early times, before the steady state develops, we too see the unstable focus [9]. The focused image is not a perfect line. Its size is $\sim\lambda$, consistent with conventional optical limitations. The focus appears to be distorted primarily by aberration effects including material losses, compounded by the finite width of the material. The focus is not broadened by chromatic aberrations since this is a single frequency wave. Separate cw calculations at 20 GHz, with the same negative index scaled up to this higher frequency, show similar results.

C. Two-dimensional divergent pulsed wave: Infinite line source

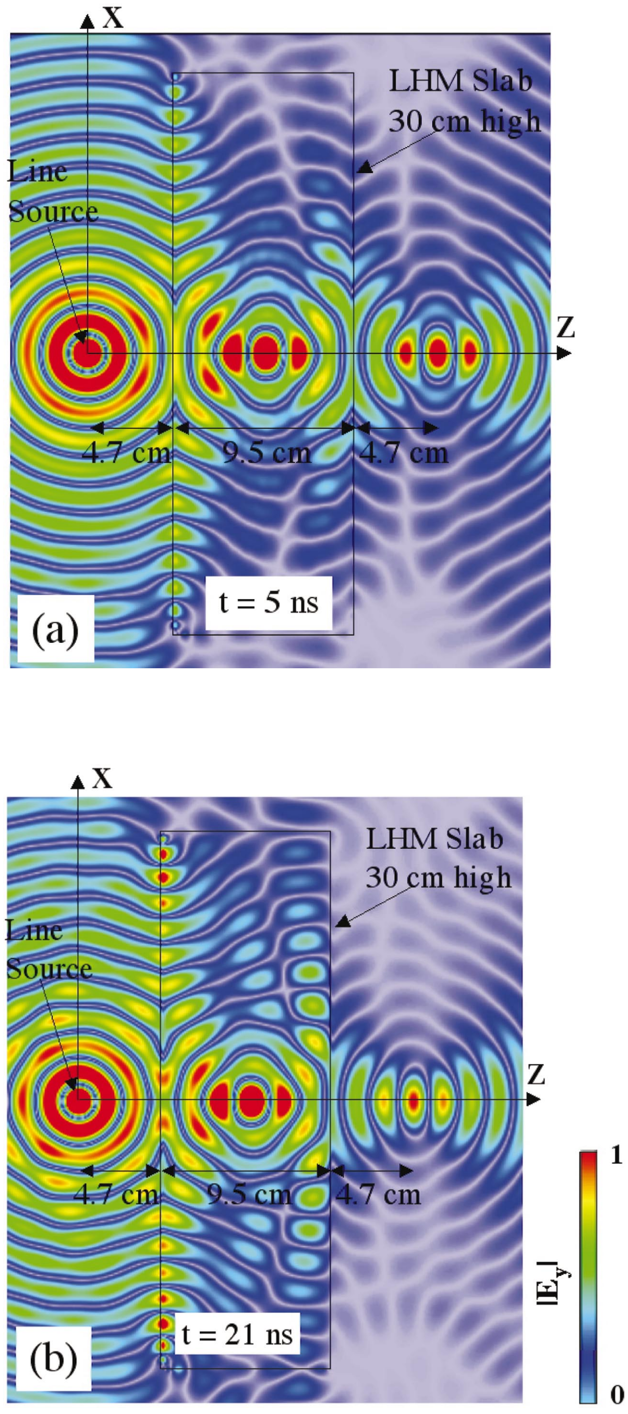
Still more controversy revolves about phase velocity, group velocity, and negative refraction for finite duration pulses passing between right- and left-handed materials. For this simulation we again use the line source from above but excite it with a Gaussian sine wave modulation [see Eq. (2)], again at a center frequency of 10 GHz:

$$\frac{E(t')}{E_{max}} = e^{-1/2(t'/\tau_B)^2} \sin\left(\frac{2\pi t'}{\tau_P}\right). \tag{2}$$

The modulation parameter is $\tau_B=1.67$ ns, the period is $\tau_P=0.10$ ns, and $t'=t-t_0$. The pulse peaks at a time $t_0=6.8$ ns. A Fourier transform of Eq. (2) yields frequency components having a range of ± 0.21 GHz (measured at 10% of the peak amplitude) on either side of 10 GHz. The index of refraction is closely equal to -1 over this range of frequency components.

Figure 3 displays snapshots of the electric-field amplitude produced by the Gaussian pulse. The first snapshot is at $t=6.6$ ns [Fig. 3(a)] as the pulse excitation nears its peak. Figure 3(b) is a snapshot at $t=9.4$ ns, well after the pulse has passed. Distinct focal areas are present both at the center of the slab and equidistant to the right of the slab. As in Fig. 2, the size of the focus is $\sim\lambda$, consistent with the limitations imposed by wave optics.

In Fig. 4 we present the time evolution of the E_y field generated by the same Gaussian-pulse-excited line source



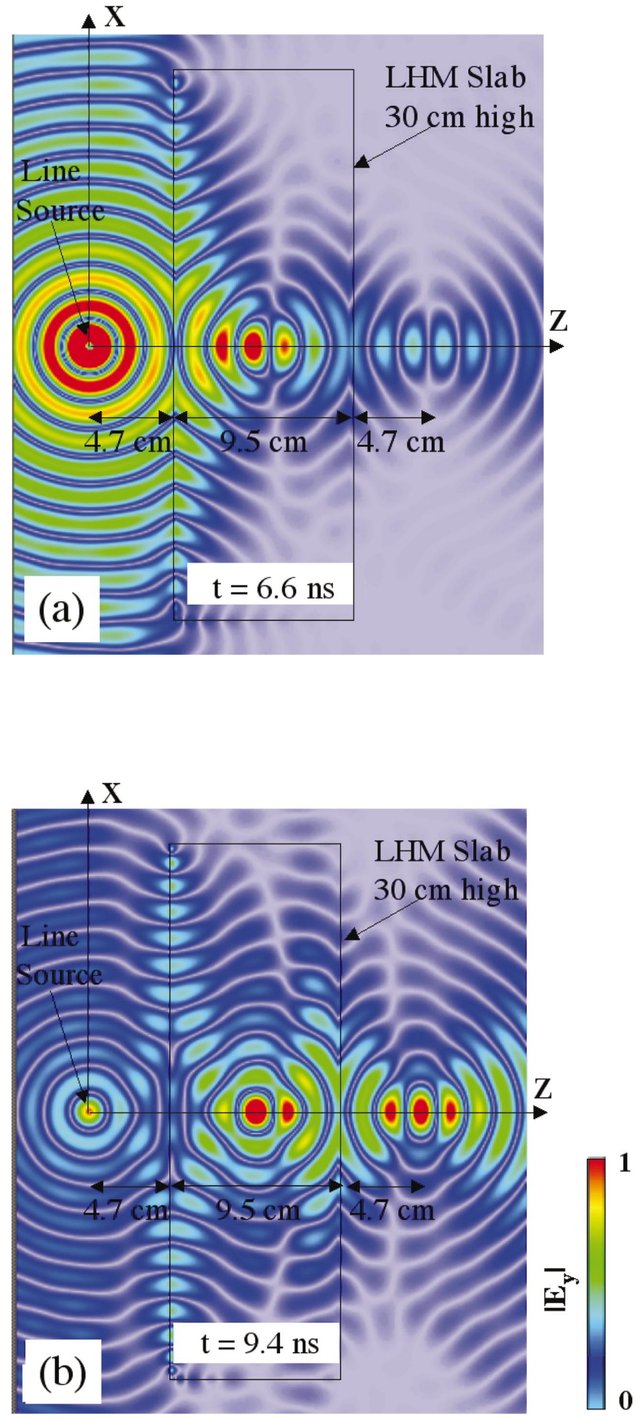


FIG. 2. (Color) Snapshots of the electric field amplitude (absolute magnitude) produced by an infinite line source (y axis) excited by a *cw impulse* at 10 GHz. A left-handed material occupies the rectangular region denoted in the x-z plane and extends to infinity in the y direction. At 10 GHz the index of the LHM is $-1.001+i0.013$. (a) The snapshot at 5 ns after generation. (b) The same scene at 21 ns. Distinct focal areas are present at the center of the slab and equidistant to the right of the slab. Separate simulations for taller slabs indicate that edge diffraction plays a minimal role in the results.

FIG. 3. (Color) Snapshots of the electric field amplitude (absolute magnitude) produced by an infinite line source (y axis) excited by a *Gaussian frequency pulse* [Eq. (2)] with a 10 GHz center frequency. A "left-handed" material occupies the region which is denoted in the x-z plane and extends to infinity in the y direction. At the centroid frequency, 10 GHz, the index of the LHM is $n=-1.001+i0.013$. (a) The snapshot taken at $t=6.6$ ns after generation. (b) The same scene at $t=9.4$ ns. The group velocity is positive but the wave components undergo negative refraction in the LHM.

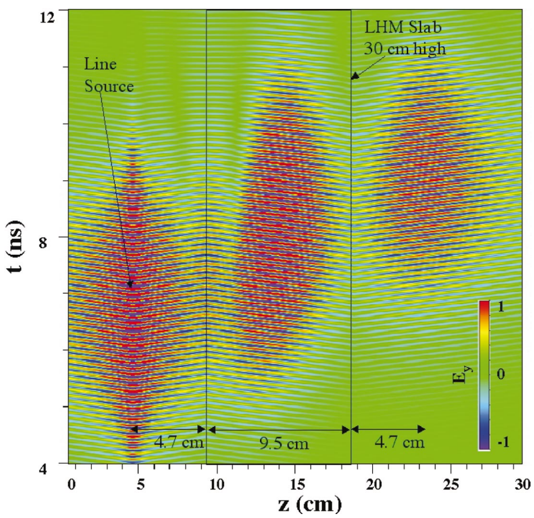


FIG. 4. (Color) E_y component amplitude as a function of z and time for a Gaussian pulse propagating in the z direction through a 2D left-handed material slab. The pulse shape shown in Fig. 3 has a narrow bandwidth and peaks at $t\approx 7$ ns at the line source location. At a later time the amplitude peaks at a focal region at the slab center. Still later it peaks at a second focal region, 4.7 cm to the right of the slab. The focal regions are basically the same as for the cw case in Fig. 2, indicating a minor effect due to chromatic aberration. Inspection of the phase fronts clearly shows the negative phase velocity noted earlier for the cw case.

with respect to position along the z axis. At the position of the source, the wave packet amplitude builds with time, reaches a peak at $t\approx 7$ ns, and then decays. The field spreads equally in the $+z$ and $-z$ directions with time and decays in amplitude as it does so. Later in time the wave packet evolves to a focus area in the center of the LHM slab. Dispersion and loss have led to a reshaping and reduction of amplitude. The wave packet is again focused in the free-space region to the right of the slab at $t\approx 9$ ns. The focal regions are basically the same as for the cw case in Fig. 2 indicating a minor effect due to chromatic aberration. The lack of a ‘‘perfect’’ focus is consistent with limitations imposed by conventional wave optics. In separate calculations the loss was progressively reduced but had little effect on the focal region. Inspection of the phase fronts in Fig. 4 clearly shows the negative phase velocity within the LHM noted earlier for the cw case. A slight forward tilt of the focused wave packet is also evident inside the LHM. This is due to dispersion that leads to the focusing of lower frequency components, that travel more slowly than the pulse centroid, slightly beyond the LHM slab center and with the reverse effect for higher frequency components. Attenuation of the field eliminates any noticeable tilt to the wave packet to the right of the slab. Additional calculations were done with pulses having frequency components well outside the negative index region. These pulses travel through the LHM slab with no evidence of negative refraction. This work will be presented later in a more detailed paper.

III. CONCLUSION

We have demonstrated that negative refraction occurs for either continuous waves or pulsed Gaussian wave packets traversing the boundary from a normal $(n>0)$ material into a left-handed material $(n<0)$ each with causal properties. We show that the phase velocity is negative within the LHM and that the wave or group velocity, while positive, is refracted to the opposite side of the normal incidence plane. In contrast with others [2], our results clearly show that pulsed electromagnetic waves do refract negatively in dispersive left-handed materials. Our simulations show a distinct, Albeit imperfect focusing of divergent waves proceeding from a line source through a plane parallel LHM slab. There is no evidence of the amplification of evanescent waves. Both monochromatic cw and Gaussian pulsed waves are focused in a very similar manner. For our chosen geometry there is a focused region within and to the right of the slab as predicted by earlier work [1,6]. However, the image we observe is broadened significantly by optical aberrations and losses in the slab. Through simulations using successive reductions in the LHM loss we are led to the conclusion that our results do not support a perfect ‘‘superlens’’ as postulated by Pendry [6].

ACKNOWLEDGMENT

This work was supported by the DARPA Metamaterials Program.

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