Instabilities in slowly driven granular packing

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In this work, a digital imaging technique is used to study the superficial fluctuations observed when a granular packing is slowly driven to the threshold of instability. The experimental results show the presence of three types of events. Small superficial rearrangements of grains are observed during all the experiments. They present a power-law behavior although the system is not in a critical state as predicted by self-organized criticality models. In thick granular piles, large rearrangements are detected at regular angular intervals. They are related to the threshold of instability of the contact network that relaxes to stable configurations producing internal rearrangements of the grains. Finally, an avalanche is triggered when the superficial beads that are set in motion acquire enough momentum to destabilize grains from layers below.

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I. INTRODUCTION

Avalanche studies have been (and are) of great interest because of their implications in geophysical problems (talus slopes, sand dune formations, and rock falls) or technological processes connected with granular material processing (loading and unloading operations).

Some years ago, Bak *et al.* [1] introduced the notion of self-organized criticality (SOC) to describe the behavior of driven dynamical systems in a steady state. However, several experimental [2–6], theoretical, and numerical [7–9] studies have demonstrated that piles of grains, in most of cases, cannot be described by SOC. In fact, their behavior often presents characteristics of a system in equilibrium close to a first-order transition phase (with hysteresis) and not a second-order transition phase (a power-law behavior) as suggested by SOC.

Recently, Daerr *et al.* [10] analyzed the growth and the propagation front of avalanches generated by externally perturbing the free surface of a metastable layer on a rough bed. The perturbation propagates downhill or uphill depending upon the thickness of the layer below and the inclination angle of the bed. A similar approach in a two-dimensional (2D) rotating drum was taken by Rajchenbach [11] in order to study the dynamics of avalanche growth. He characterized the avalanche process by two velocities of both up and down fronts independent of the dissipative properties of the flowing grains.

In the present work, we study the preavalanche dynamics on a 3D granular packing constituted by glass beads contained in a rectangular box. During the experiment, the pile is inclined up to the threshold of instability, where a large slide of granular material is produced. When this ‘‘catastrophic event,’’ which we shall call the avalanche, is triggered, the slope of the free surface of the pile defines a critical angle, currently called the maximum angle of stability, θ_M. During the slide, a decrease of the slope of the free surface is observed until the avalanche stops. At this point, the free surface defines a second critical angle: the angle of repose, θ_R. While the packing is being tilted, one can observe many superficial rearrangements of beads for angles just below θ_M.

In previous studies [12–14], we have extensively studied the event called the avalanche. The critical angles and the avalanche size were measured for systems with different sample lengths, packing volume fractions, and numbers of layers. The main results show that the surface flow and the avalanches depend strongly upon the height of the packing (or the number of grain layers). There is a given height above which the avalanche characteristics are independent of the pile height. When an avalanche occurs, its dynamics develop according to two regimes. For thin packings, the flow can be characterized as a bouncing one; for thick packings, the regime is governed by a creeping flow. The avalanche mass flowing out the system is strongly correlated with the characteristic angle θ_M.

In this work, we focus on the very rich behavior of the events that precede an avalanche. For this purpose, we developed an image processing technique that allows us to determine their size and the angle of inclination where they occur. We analyze the influence of the structure (ordered or disordered) and the roughness of the free surface on the characteristic events occuring before an avalanche is produced.

In the following section, we describe the experimental setup and the image processing technique. In Sec. III, we present experimental observations that give a qualitative insight into the relevant physical mechanisms involved. The experimental results are presented and analyzed in Sec. IV. We finally discuss all these results and conclude in Sec. V.

II. EXPERIMENTAL SETUP

In order to study these phenomena thoroughly, one must construct an experimental device where parameters such as the size of the pile, the packing fraction, the roughness of the

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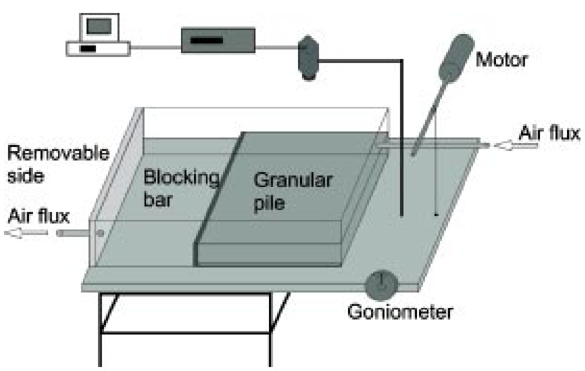

FIG. 1. Scheme of the experimental device used for the experiments.

free surface, and the air humidity can be controlled. The experimental study of superficial rearrangements of beads requires, most importantly, to get a precise measurement of their size and the time when they occurred. For this reason, the development of an accurate image processing technique has been a central aspect of the design of the experimental setup. The experimental system we have constructed is described in detail in Ref. [12]. Briefly, the granular packing is constituted of glass beads contained in a rectangular box. The mean bead diameter is $d=2.2\pm0.2$ mm and their density $\rho=2.5$ g/cm^3. We have decided to work with this granular material because it is easy to identify the movements of beads. The box is made of transparent glass that allows us to visualize the pile inside. The bottom of the box was prepared by gluing down the same glass beads that constitute the piles over a flat piece of glass in order to give a rough surface bed and to avoid sliding. The experiments are carried out under a controlled humidity atmosphere of 50% which ensures that spurious effects due to capillary or electrostatic forces are negligible [12,15]. Finally, the box is secured on a heavy plane able to be inclined at different rates by means of a pulley system linked to a stepping motor. We have verified that this apparatus does not produce vibrations which can trigger avalanches or any other motion in the free surface of the pile. Fig. 1 shows a scheme of the experimental device.

A. Preparation of the granular piles

The piles are prepared by pouring glass beads into a rectangular box. The length of the box, $L=320$ mm, and its width, $W=260$ mm, are fixed. The height of the outlet wall of the box is adjusted such that it is always equal to the packing height. Therefore, we study the process in a filled box of granular material. We state that the system increases its height in this box in *one* layer when 230 g of beads are added to it. This is the amount of beads necessary to cover the surface of the box with a superficial packing fraction equal to 0.7. The height of the system, then, will be noted by the number of layers, N, that it contains. In order to study how the structure of the packing affects the evolution of the rearrangements, we performed experiments with two sorts of piles: disordered piles and partially ordered piles. These piles have different structures and packing fractions depending on how they were prepared.

Disordered piles (D). All the beads are poured into the box at once. They are then spread with a spoon until a flat free surface is obtained. The measured volume packing fraction of these piles is $C_D=0.62\pm0.01$.

Ordered piles (O). In this case, the beads are poured layer by layer and they are spread and pressed down hard with a piece of glass that covers the whole surface of the box. This procedure is repeated for every layer added to the pile. In these systems, the volume packing fraction is $C_O=0.67\pm0.01$, which represents an increase of 8% compared to C_D.

We have also studied the influence of the roughness of the free surface on the development of the rearrangements. This parameter was not measured quantitatively but it was clearly and easily modified by pressing a piece of glass over the free surface. In this way, two other sorts of piles are obtained.

Smooth piles (S). When these piles are prepared, the free surface is first flattened by spreading the beads with a spoon and then pressing a piece of glass over the whole surface.

Rough piles (R). In this case, the free surface is only flattened with the spoon.

In summary, we study four different systems, which for simplicity are noted as follows: The number that precedes the letter N identifies the number of layers. Then, D (disordered) or O (ordered) note the packing structure. Finally, R (rough) or S (smooth) label the free surface roughness. As an example, a 20 layer disordered pile with a rough free surface is noted as *20NDR.*

B. Image processing technique

The aim of the image processing technique is to determine the size of the rearrangements that are observed on the free surface and the angle of inclination or time when they occur. During an experiment, the free surface is repeatedly scanned by means of a charge-coupled device image sensor (CCD camera) interfaced, via a ScionCorp LG-3 card, to a computer equipped with an image processing software. The resolution of the camera is 768×512 pixels and 256 gray levels. The camera is firmly fixed to the plane 100 cm above the free surface with its optical axis perpendicular to it. Twin fluorescent lamps located beside the camera provide diffuse lighting and a black cloth isolates the experimental device from ambient light.

The images are systematically acquired and processed by a subroutine developed for the ScionCorp PCIMAGE software. The steps performed by the subroutine can be summarized as follows. First, two successive images are digitally subtracted. If between those images, a displacement of beads is produced, the subtracted image displays a figure with different gray levels due to the change of brightness of the grains that moved. Three filters are then applied: (a) The image is digitized, applying a gray level threshold. This cutoff threshold is chosen to accurately detect the grains that have moved and, at the same time, to erase the low intensity pixels of the grains that did not move. (b) An eroding algorithm is applied to erase any remaining noise. (c) A dilating algorithm is applied so that the rearrangement recovers its original size. Finally, the number of black pixels of the image and the time are stored. The conversion between pixels and number of beads and between time and angle of inclination are performed by means of a calibration previously established. Under our experimental conditions, one bead diameter corresponds to 9 pixels. This calibration was obtained from an image of a single bead and by calculating the corresponding number of pixels. The program requires 0.7 s to process two successive images.

From a practical point of view, an experiment is carried out by performing the following steps. First, we prepare a pile with a desired number of layers, structure, and superficial roughness as described above. Second, the humidity inside the box is controlled until it reaches 50%. This step takes about 2 h. The plane then starts to be tilted. The rotation speed, w, may vary from 0.3°/min to 6°/min. The results reported in the present work were obtained at $w=0.3$°/min. The rotation is halted when a slide that displaces out of the box a mass of granular material of the order of the whole system mass is detected. Once the avalanche stops, the critical angles θ_M and θ_R are measured by means of a goniometer fixed to the plane. Finally, an electronic balance measures the mass of granular material displaced by the avalanche. The uncertainty in these measurements for θ_M, θ_R, and the displaced mass are $\Delta\theta_M=0.2°$, $\Delta\theta_R=0.5°$, and $\Delta M=0.1$ g, respectively. During the whole experiment, the camera scans the surface of the pile and the image processing program determines the size of the superficial rearrangements s and the angle of inclination where they occur, θ.

In the following section we describe some relevant experimental observations.

III. EXPERIMENTAL OBSERVATIONS

In this section, we describe qualitatively the events observed when an experiment is performed. From very small angles of inclination to angles close to θ_M, several beads are set in motion during a very short period of time (<1 s), that is, they quickly come to rest. These mobile beads are mainly confined to the free surface and do not change its slope. We shall call these events *internal avalanches*. Within internal avalanches, we distinguish two different scales: *small rearrangements*, where a few number of beads are set in motion, and large rearrangements or *precursors*, where the number of beads that move is of the order of the number of grains of the free surface. When the pile reaches θ_M, an *avalanche* is triggered. In this case, beads belonging to several layers are set in motion and some of them flow out of the box. Let us discuss in detail these different scales.

Small rearrangements. Figure 2 shows two small rearrangements of 8 and 57 beads respectively. These events form tiny clusters of beads located somewhere on the free surface of the pile. When a small rearrangement starts, some beads move to neighboring sites and sometimes these relocations destabilize other beads so that the event grows in two or three successive steps. These events occur during all the experiments and anywhere on the free surface. In some cases, two or three clusters are observed simultaneously at different places of the surface.

Precursors. These are events that involve from 7% to almost all the beads of the free surface. Figure 3 shows two

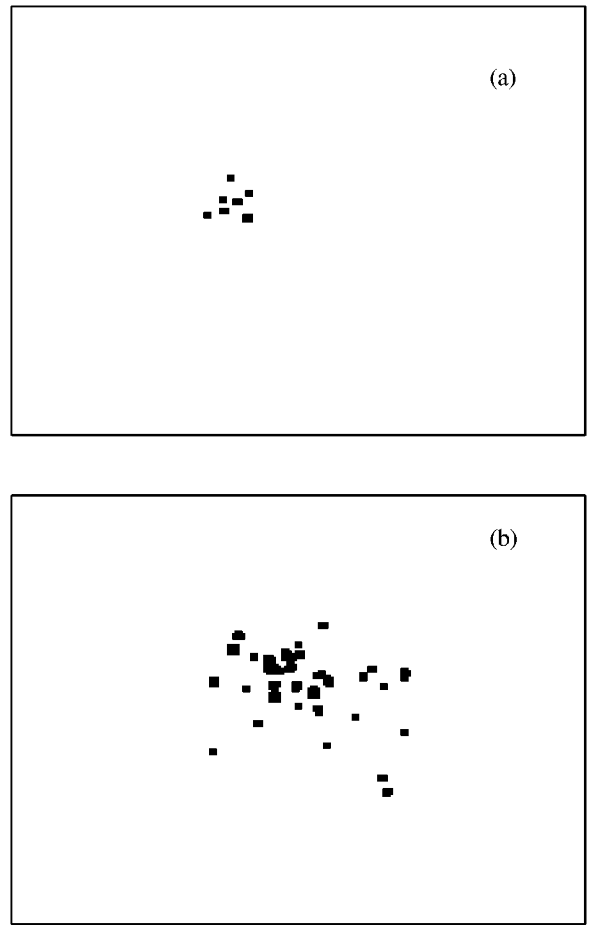

FIG. 2. Small rearrangements of eight beads (a) and 57 beads (b). These events involve less than 7% of the beads of the free surface and form tiny clusters located somewhere on the surface.

precursors of 3043 and 11 086 beads, respectively. One can observe that the beads involved are uniformly distributed on the free surface. That is, precursors are not localized events. In this case, all the beads move simultaneously, mainly in one step. Precursors are only observed in piles with a number of layers $N>10$ [13].

Avalanches. These are events that start as a small perturbation located somewhere on the free surface which rapidly grows and propagates to several layers below. Remarkably, the avalanche perturbs only beads belonging to a certain number of layers from the free surface. This means that independent of the number of layers of the pile, a constant number of superficial layers contribute to the avalanche process. In our experimental conditions, this critical number of layers was found to be close to 13. In this case, the avalanches develop as a creeping flow. A detailed study of these phenomena can be found in Refs. [12,14].

IV. EXPERIMENTAL RESULTS

As an example of the results obtained with the image processing technique, in Fig. 4, the size of the small rearrangements, the precursors, and the avalanche obtained in a $20NDR$ pile, are plotted as a function of the angle of

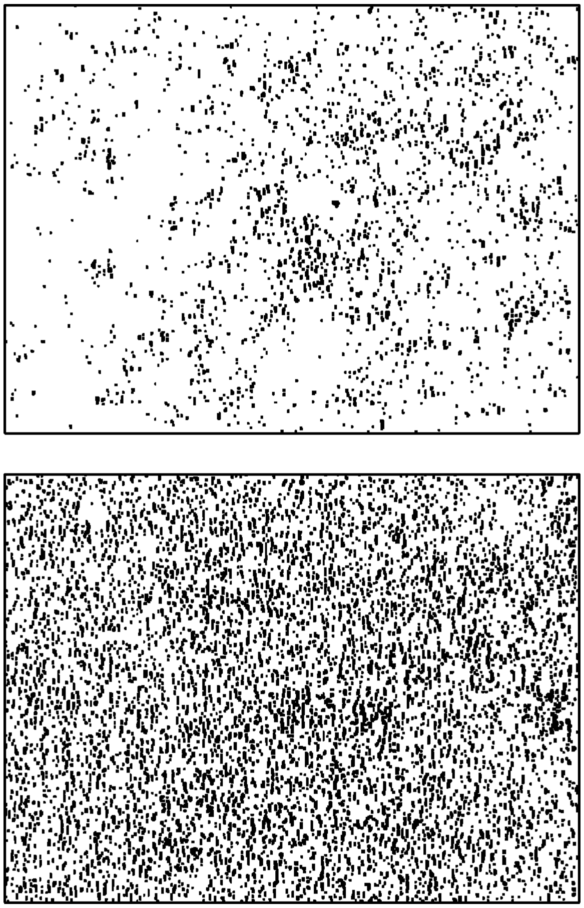

FIG. 3. Precursors of 3034 beads (top) and 11 086 beads (bottom). These are events only observed in piles with more than ten layers. They affect more than 7% to almost all the beads of the free surface. The beads involved are uniformly distributed on the surface.

inclination in a semilog scale. The size of the avalanche in number of beads involved was computed from the measurement of its mass, as indicated below. We can remark how different the characteristics of small rearrangements and precursors are as can be appreciated in Fig. 4. The small rearrangements occur during all the experiments (independent of the inclination angle) and displays a noisy structure. On the contrary, precursors are only observed for angles of inclination larger than $\sim 17°$ and occur at approximately regular angular intervals, $\Delta\theta$. In the example of Fig. 4: $\Delta\theta = 1.8°$ $\pm 0.4°$. If we analyze the size of the precursors, an exponential growth with the angle of inclination is observed as shown by the dotted line in Fig. 4. Finally, an avalanche started at $\theta_M = 27.3° \pm 0.2°$. This is the only event where a considerable mass fraction of the pile (46%) is displaced out of the system decreasing the slope of the free surface down to $\theta_R = 21.1° \pm 0.5°$. Note that the same angular interval $\Delta\theta$ separates the avalanche from the last precursor. This result suggests that, in this case, the avalanche was triggered by a precursor. In Sec. IV D, we will show that precursors are not produced by external mechanical perturbations.

Next, we present three different ways of analyzing these experimental results. First, we study their statistical proper-

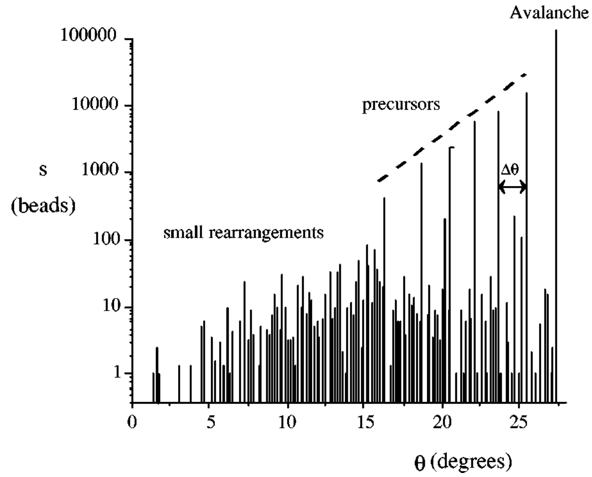

FIG. 4. Size of the events as a function of the angle of inclination obtained with a $20NDR$ pile. Small rearrangements occur during all the experiment and present a noisy structure. On the contrary, precursors are only observed for $\theta > 17°$ and occur at almost regular angular intervals, $\Delta\theta = 1.8° \pm 0.4°$. The size of the precursors grows exponentially with θ as the dotted line shows in the figure. Finally, an avalanche started at $\theta_M = 27.3° \pm 0.2°$.

ties by computing the size distribution function. Then, we analyze how the number of events and the cumulated activity vary with θ.

A. Size distribution function

In order to study the statistical properties of the events observed during an experiment, we analyze the distribution function computed from the size of the events. Since the sizes range over five decades and the total number of events in an experiment is less than 500, we have decided to determine the size distribution function $D(s)$ by computing a histogram whose bins grow exponentially. In addition, in order to increase the size of the sample and, therefore, the statistical properties of this analysis, $D(s)$ was calculated from the results of several experiments performed under identical conditions.

In Fig. 5, we have plotted the size distribution obtained from five experiments performed in $20NDR$ piles. The three scales mentioned above can also be identified in this plot. For small rearrangements, $D(s)$ presents a clear power-law behavior that lasts over two decades and it is characterized by an exponent $\tau = 1.98 \pm 0.08$. Similar results were obtained in experiments carried out with smooth piles: $\tau_{20NDS} = 1.8$ ± 0.2, ordered piles: $\tau_{20NOR} = 2.0 \pm 0.1$, and when varying the number of layers: $\tau_{2NDR} = 1.9 \pm 0.1$. For the precursors, however, the function deviates from a power-law behavior. It should be noted finally that no events are seen with sizes between that of the precursor and the avalanche.

The three scales observed experimentally present different statistical properties. Small rearrangements display a power-law behavior with an exponent $\tau \approx 2$, which is independent of the way the packing was initially prepared. Similar results were found by Held *et al.* [5] in conical piles built with few grains (see also Rosendhal *et al.* [6]).

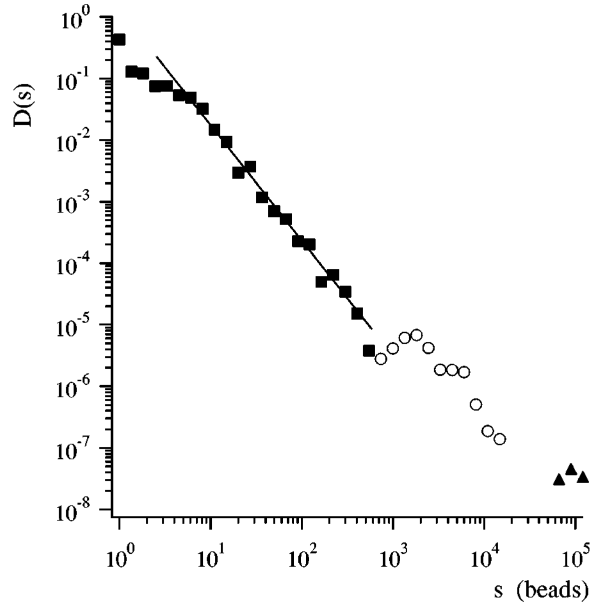

FIG. 5. Size distribution function computed from the results of five experiments performed in identical conditions with $20NDR$ piles. Three scales can be identified. The small rearrangements (■) present a power-law behavior that lasts over two decades and it is characterized by an exponent $\tau = 1.98 \pm 0.08$. Precursors (○) present a larger probability than that predicted by the power law and the distribution has a discontinuity between precursors and avalanches (▲).

Precursors present a larger probability than that predicted by the power law and the distribution has a discontinuity between precursors and avalanches. This means that the underlying physical mechanisms of these events are probably different from that of the small rearrangements.

Let us point two important features concerning the dynamics:

(1) Small rearrangements are dominated by local mechanisms (on the surface). This is not true for the avalanche. Let us remember that during a small rearrangement, the beads move from one site to a neighbor one, they do not travel long distances and no inertia effects are observed. On the contrary, for the avalanche, a considerable inertia effect is observed that leads to a nonlocal process. During this event, the mobile beads travel long distances and most of them flow out of the system.

(2) A power-law behavior is observed when clearly the system is not in a critical state. The slope of the free surface increases continuously while small rearrangements occur.

In summary, our results confirm that the absence of inertia effects leads to the development of small events where local mechanisms dominate their dynamics. Even if the system is not in a critical state, these events present a power-law behavior. This behavior is robust with respect to macroscopic parameters of the system such as roughness, internal structure, or number of layers. This interpretation of the results allows us to understand which are the underlying physical mechanisms. For small rearrangements, the process can be understood in terms of the loss of equilibrium of the superficial beads as long as the plane is being tilted. In a simplified scenario, a bead falls to a neighboring site when it loses its equilibrium out of its "sustentation polygon" and finds a new one in a neighboring site. As long as the plane is inclined, the slope of some beads exceeds the equilibrium value and they fall to a neighboring site. In some cases, this process modifies the equilibrium of other neighboring beads, and the rearrangement grows in successive steps. We shall see that this interpretation of the results let us easily understand the results presented in the following section.

On the other hand, a considerable inertia effect is observed when the system reaches θ_M. At this point, the superficial beads that are set in motion, by a small rearrangement in thin piles or a precursor in thick ones, acquire enough momentum to destabilize beads from layers below and the avalanche process is able to grow and propagate to the bulk. When the slope of the free surface reaches the angle of repose θ_R, the momentum gained by the beads is not enough to carry on the process and the avalanche is halted. Let us remark that this hypothesis has been successfully tested on a cellular automata model [8,9] that takes into account the transfer of momentum to layers below. This model is able to reproduce the small rearrangements and the avalanche just by considering that the critical slope which the bead has to overcome to fall is a decreasing function of the momentum of the bead.

B. Number of events

Another way of analyzing the experimental results is to study the variation of the number of events n, with the inclination angle θ; where n represents how many events, of any type and size, have already occurred when the plane is tilted in θ degrees. In Fig. 6, the curves obtained for two experiments performed in identical conditions with $20NDR$ and $2NDR$ piles are plotted. It can be observed that after a certain transient ($\theta < 10°$), the curves grow approximately linearly. In piles with few layers, where no precursors occur, the range of linear growth (RLG) extends up to the maximum angle of stability. On the contrary, in piles with more than ten layers, the RLG is reduced to $10° < \theta < 17°$. For larger inclination angles, the number of events decreases from the linear trend due to the presence of precursors. This decrease can be easily understood considering that the precursors involve a large number of beads that move to stable sites. Therefore, after each precursor, the number of beads that remain in unstable sites falls remarkably. In piles with few layers, where no precursors occur, this decrease is obviously not observed. In the RLG, the frequency of events $n_\theta = \partial n / \partial \theta$ remains almost constant. This quantity characterizes the dynamics of the small rearrangements (in this range, only small rearrangements take place). $n_\theta = \partial n / \partial \theta$ gives us a tool to study how the roughness, order, and number of layers of the pile affect these events. In Table I, the value of n_θ computed for systems with different roughness surface, order, and number of layers are presented. Each value is obtained by averaging over five identical experiments. A clear correlation between n_θ and the roughness of the free surface can be observed. This parameter falls remarkably when the system is smooth. No correlation can be found with the number of layers or the

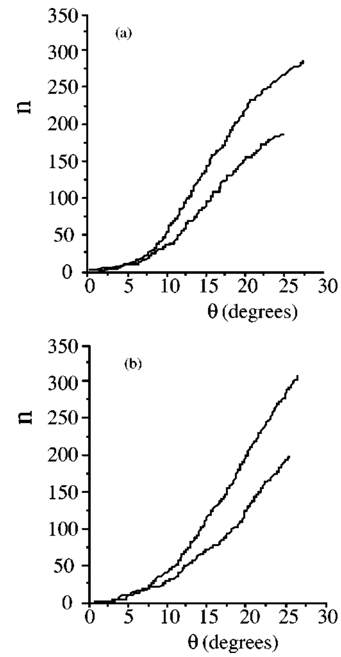

FIG. 6. Number of events as a function of the angle of inclination for experiments performed with 20*NDR* (a) and 2*NDR* (b) piles. From $\theta > 10°$, the curves grow approximately linearly. For the 2*NDR* pile, the RLG extends up to θ_M. On the contrary, in the 20*NDR* pile, the RLG is reduced to $\theta < 17°$. For larger angles of inclination, the number of events decreases from the linear trend due to the presence of precursors.

structure of the packing. This result corroborates that the small rearrangements are only superficial events. Its frequency strongly depends on the initial roughness of the free surface. In the smooth free-surface packings, the number of small rearrangements observed is smaller than in the rough ones. In the experiments performed on a conical pile by Rosendhal *et al.* [6], the evolution of the number of events is different. During the buildup period, they observed many small events and the relaxation of the system also occurs through big avalanches. In this case, however, the frequency of events grows linearly with time. What is not clearly understood is whether this difference with our results is due to the different geometry of the piles or due to the different driving mechanisms.

TABLE I. n_θ values for different packings: *N* is the number of layers, *D* and *O* indicate disordered and ordered piles, respectively (volume packing fraction characteristic), and *S* and *R* indicate smooth and rough piles, respectively (free-surface characteristic).

N	Packing structure	Surface roughness	n_θ (number of events/min) ($\pm 25\%$)
40	*D*	*R*	8.8
30	*D*	*R*	12.7
20	*D*	*R*	15.3
10	*D*	*R*	11.6
5	*D*	*R*	21.2
2	*D*	*R*	17.5
20	*O*	*R*	11
20	*D*	*S*	3.25
20	*O*	*S*	3.95
5	*D*	*S*	5.1

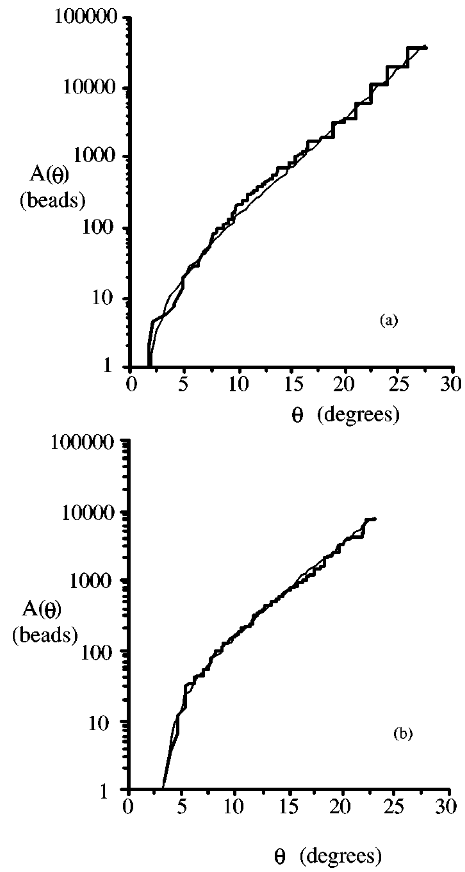

FIG. 7. Cumulated activity as a function of the angle of inclination for two experiments performed with (a) 20*NDR* and (b) 2*NDR* piles. The cumulated activity is defined as the sum of the size of all events occurred up to an angle θ. The functions $\tilde{A}(\theta)$, defined by Eq. (5), obtained by fitting the values of $A(\theta)$ are also plotted.

C. Cumulated activity

The cumulated activity $A(\theta_k)$ is here defined as the sum of the size of all events occurring up to a certain angle θ_k:

$$A(\theta_k) = \sum_{j=0}^{k} s(\theta_j), \tag{1}$$

where θ_j is the angle of inclination where the event number j occurs and $s(\theta_j)$ is its size. In other words, $A(\theta_k)$ is the total number of beads that have moved between $\theta = 0$ and $\theta = \theta_k$.

In Fig. 7, the cumulated activity curves obtained with 2*NDR* and 20*NDR* piles are plotted in a semilog scale. It can be observed that in both experiments $A(\theta)$ increases in a similar way and it apparently fluctuates around a unique function that grows monotonously. In order to determine this function, which we shall note $\tilde{A}(\theta)$, we first compute analytically $A(\theta)$ by only considering the fluctuations introduced by the precursors. Let us remember that the size of the precursors grows exponentially with the inclination angle

$$s_p = s_o e^{(\theta/\alpha)}, \tag{2}$$

where s_o and α are characteristic parameters. Then, as the precursors occur at regular intervals $\Delta\theta$, one can evaluate the inclination angle in terms of the number of precursors, i

$$\theta = \theta_{po} + i\Delta\theta, \tag{3}$$

where θ_{po} is the inclination angle, at which the first precursor takes place $(i=0)$. Replacing Eqs. (2) and (3) in Eq. (1), a function that determines the cumulated activity by k precursors is obtained:

$$A_p(\theta_k) = \sum_{i=0}^{k} s_o e^{[(\theta_{po}+i\Delta\theta)/\alpha]}. \tag{4}$$

In order to get a continuum description of $A(\theta)$, one can rewrite Eq. (4) by assuming that the precursors do not occur at intervals $\Delta\theta$ but at $\Delta\theta/m$, where $m>0$. We need to redefine, therefore, the amplitude s_o by s_o/m, to maintain the unchanged, cumulated activity given by Eq. (4). Finally, to obtain $\tilde{A}(\theta)$, we compute the limit when m goes to infinity and extend the dominion of the function considering that θ_{po} is now the angle θ_o, where the first small rearrangement occurs. The final expression for $\tilde{A}(\theta)$ is

$$\tilde{A}(\theta) = \frac{s_o\alpha}{\Delta\theta}\left(e^{(\theta/\alpha)} - e^{(\theta_o/\alpha)}\right). \tag{5}$$

It should be noted that $\tilde{A}(\theta)$ is only a function of three parameters: α, $s_o/\Delta\theta$, and θ_o. In Fig. 7, the functions $\tilde{A}(\theta)$ obtained by fitting the values of $A(\theta)$ are also plotted. First, it can be pointed out that in the piles with many layers such as a 20NDR pile, the fluctuations due to both small rearrangements and precursors oscillate around the same function $\tilde{A}(\theta)$. Up to 17°, the fluctuations are small because the activity is only due to small rearrangements. For $\theta>17°$, the fluctuations are larger due to the presence of precursors. On the other hand, although $\tilde{A}(\theta)$ was derived from the activity cumulated by the precursors, it fits in a very good agreement with the results of an experiment performed with few layers, where there are no precursors. Such is the case for the 2NDR pile in Fig. 7. Considering these results, we conjecture that the small rearrangements and the precursors are not independent events. This means that the underlying physical mechanisms are nearly the same. In order to understand the connection between small rearrangements and precursors, let us analyze in detail the curves in Fig. 7. In the range $17°<\theta<24°$, every time a precursor occurs, $A(\theta)$ grows abruptly and in the following interval $\Delta\theta$, it remains almost constant. Also in this range, the frequency of events decreases remarkably (see preceding section). In summary, we can state that all the activity observed in an interval $\Delta\theta$ essentially originates from the precursor's activity. In other words, the increase of the cumulated activity observed in an interval $\Delta\theta$ is equal to the size of the precursor that occurs in this interval. Now, let us suppose that the precursors would not have occurred. In that case, $A(\theta)$ would have grown around the same function $\tilde{A}(\theta)$ but the activity cumulated in the same interval $\Delta\theta$ would have been due to several small rearrangements, such as occurs in the range $10°<\theta<17°$ or observed in the 2NDR pile. The connection between these two scenarios gives us the answer we are looking for. The size of the precursors that occur at intervals $\Delta\theta$ is equal to the sum of the sizes of the small rearrangements that would have occurred in the same interval if the precursor had not existed. This result leads us to the following conclusion. The precursors are several small rearrangements that occur simultaneously. Then, a question arises: Why do they occur simultaneously? Possibly, because the origin of the precursors is a perturbation that simultaneously destabilizes the free surface at different locations. We shall study this hypothesis in the following section.

D. Precursors

An important question that arises from the preceding section is that whether the origin of precursors is due to an external mechanical perturbation, for example, provided by the experimental device, or due to a physical mechanism of the packing. We study the first hypothesis analyzing how a 20NDR pile responds to external perturbations. The following set of experiments was performed: the plane is raised at a constant velocity and smoothly stopped every 5° before the angle of repose ($\theta_R \approx 21°$) is reached, and every 1° beyond and up to the maximum angle of stability ($\theta_M \approx 27°$). Each time, the system is exposed to both weak and strong perturbations provided by blocks of two different weights (50 g and 500 g) thrown from a constant height (300 mm) on the plane that held the granular pile. During the experiment, the CCD camera scans the free surface and the image processing program determines the size of the superficial rearrangements that each perturbation triggers. Up to 15°, the experiment develops normally and the perturbation does not produce additional events. From $15°<\theta<22°$ for the strong perturbation and $20°<\theta<24°$ for the weak one, small rearrangements are observed following the external perturbations. From 22° or 24°, the perturbations trigger large superficial rearrangements with similar sizes as the precursors. Finally, at $\theta=25°$, an avalanche is initiated by the perturbation. In summary, while the precursors occur spontaneously from $\theta\sim17°$, the external perturbations only produce similar events when the angle of inclination is larger than 22°. Thus, as the experimental device does not provide perturbations stronger than those used in the experiments described above, it is very unlikely to induce the precursors observed during a regular experiment. We can then conclude that precursors do not originate from external perturbations. It is necessary, therefore, to know if the underlying physical mechanism is affected by parameters such as the order of the packing, the roughness of the free surface, and the number of layers.

We now present the experimental results concerning the variation of the angular interval between successive precur-

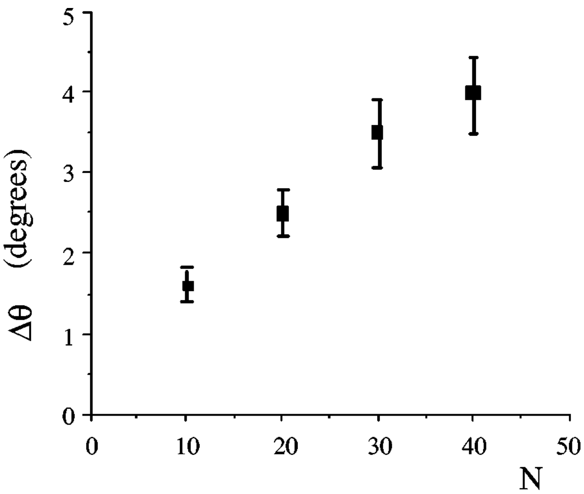

FIG. 8. Variation of the angular interval between successive precursors as a function of the number of layers of the pile. Each value is obtained by averaging over five identical experiments.

sors that seem to characterize this mechanism when the control parameters of the experiments are changed.

In Fig. 8, we have plotted the variation of $\Delta\theta$ with the number of layers for DR (*disordered and rough*) piles. Each value was obtained by averaging over five identical experiments. It can be appreciated that $\Delta\theta$ increases with N. This result suggests that the mechanism that triggers precursors is affected by collective effects, which involve all the layers of the pile. In other words, precursors are not only superficial events. Let us add that the correlation between $\Delta\theta$ and N is even observed on systems with $N>13$, which is the maximum number of layers the avalanche perturbs as was observed in Ref. [12]. Let us see now, in Table II, the results obtained for $\Delta\theta$ in 20-layer piles, while varying superficial roughness and pile structure. Each value was also obtained by averaging over five identical experiments. Although there is no significant difference between the values of $\Delta\theta$, a small increase can be appreciated for ordered systems. On the contrary, the variation of the roughness does not seem to modify $\Delta\theta$. In both, ordered and disordered piles, the average coordinance number is 12 and the geometry of the contact network is not modified by the structure of the pile. In our case, $\Delta\theta$ does not depend on the type of order of our piles. What is clear in our experiments, however, is that the number of layers dramatically modifies the stability of the pile. If a small perturbation is induced in the bulk of the pile, it will propagate to the whole contact network producing internal rearrangement of the grains until it reaches the free surface [16]. At this point, the perturbation destabilizes beads at the free surface at different places, where several small rearrangements are triggered simultaneously. After each internal rearrangement, the contact network relaxes to a new stable configuration until the inclination angle is increased by an interval $\Delta\theta$. When the pile has many layers, the perturbation grows faster and produces a much larger internal rearrangement than in a pile with a smaller number of bead layers. In this situation, the pile reaches a more stable state. The angle of inclination must then be increased by a larger interval to reach an unstable configuration.

TABLE II. Angular interval $\Delta\theta$ between successive precursors for piles of $N=20$ layers with different structure and surface roughness.

N	Structure	Roughness	$\langle\Delta\theta\rangle$ (in degrees) ($\pm 12\%$)
20	D	R	2.3
20	D	S	2.6
20	O	R	3.45
20	O	S	3

In order to get more information about the latter mechanism, we have carried out a qualitative experiment. We prepared a 20NDR pile placing several black beads, three layers below the free surface and performed a regular experiment. Although the shape of the black beads is not very well defined when recording images of the free surface, the contrast between black and transparent beads allows us to trace the position of the black ones. Initially, we could not detect any displacement of any black bead between the images taken before and after a precursor occurred. After around five precursors, however, we observed that the black beads had moved a distance of the order of one-bead diameter. In addition, we noticed that no displacement of the beads placed in the lateral side of the pile (in contact with the lateral walls) had been observed. Precursors were never observed in experiments performed with a 20NDR pile whose width was five bead diameter ($\sim$10 mm). The observation reported above confirm that the internal rearrangement of beads occurs mainly in the bulk of the pile and not only on the free surface.

V. CONCLUSIONS

In this work, we designed an experimental technique addressed to study the superficial fluctuations produced when a granular packing is slowly driven.

The techniques, developed to analyze the experimental results, allowed us to identify the underlying physical mechanisms of the events observed during an experiment.

In the first place, the size distribution function and the analysis of the number of events confirm the presence of superficial small events which exhibit a power-law behavior. This behavior is observed in a system that is not in a critical state but where the inertia effect is negligible. The existence of these events can be straightforwardly understood in terms of the loss of equilibrium of the superficial beads when the pile is tilted. We have shown the existence of precursors before the avalanche occurs. These events are observed in 3D piles with more than ten layers and occur at regular angular intervals.

The analysis of the cumulated activity and the interval between successive precursors indicate that these events are produced by the propagation of an internal rearrangement of beads. We predict that when a thick 3D pile is slowly inclined, the contact network will oscillate between stable and unstable configurations at regular angular intervals. Every time the pile reaches an unstable state, the system relaxes producing an internal rearrangement of grains.

The analysis of the avalanche process lets us argue that the onset of the avalanche is produced when some superficial beads that are set in motion by an instability, for example, a small rearrangement or a precursor, acquire enough momentum to destabilize grains from layers below. Finally, we can assure that in piles with more than ten layers, there will always be some precursors to the avalanche. This is precisely the origin of the name we have chosen for these events.

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