Generation of single-frequency coherent transition radiation by a prebunched electron beam traversing a vacuum beam tunnel in a periodic medium

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A classical Maxwell equation longitudinal boundary value problem analysis of a prebunched ac electron beam traversing a vacuum tunnel in a periodic layered dielectric medium is used to calculate the single-frequency coherent transition radiation power generated per unit length. For low voltage electron beams in the kilovolt range, only transition radiation is produced, the Cerenkov effect being below threshold. A numerical example indicates that power levels of the order of milliwatts per centimeter can be produced in the 35 GHz range with 3–10 keV beams. An interesting aspect of transition versus Cerenkov radiation is that the transition "cone" of radiation is in the backward direction of the charge beam.

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I. INTRODUCTION

Cerenkov and transition radiation (CR, TR) have a long history [1,2]. The first experimental studies of CR were by Mallet [3] in 1926–1929 and Cerenkov [4] in 1934–1938. In 1937, Frank and Tamm [5] were the first to publish a classical Maxwell treatment of the problem.

Most of the hundreds of papers written on CR have been the use of the effect for the detection of high energy particles. There was a proposal by Ginzburg [6] in 1947 to use CR for the production of microwaves but no serious experimental work seems to have been done until 1960 when Coleman [7] and his students [8,9] focused on producing single-frequency coherent CR at the watt level in isotropic dielectrics, ferrites, and anisotropic plasmas.

The physical basis for this large increase in power level over previous work was (1) the use of a prebunched relativistic electron beam rich in harmonics of the base bunching frequency, with a small velocity spread to retain its harmonic content over a long interaction distance, (2) using a vacuum tunnel for the beam in the material to avoid scattering, ionization, and destruction of the bunching.

The power radiated varies as the square of the charge. If N charges are in a bunch whose dimensions are small compared to the wavelength, spacial coherence will be obtained and the power will vary as the square of N. A bunched periodic beam achieves this spacial coherence plus permits the generation of harmonic fields.

The first treatment of TR was by Frank [10] in 1946 and by 1990, over 300 papers had appeared in the literature [2]. Most of these papers have considered a charge or single-charge bunch passing from vacuum into a dielectric or through a thin metal sheet. This radiation is incoherent with a broad spectrum. Also it is difficult to separate the CR from the TR.

The interest of using TR to produce x-rays has existed for over 20 years. This has been achieved with MeV and GeV electron beams passing through thin single or multiple metal films. Two examples are the 1980 paper by Chu *et al.* [11] and the 2000 paper by Lastdrager *et al.* [12]. In-phase addition of radiation from M multiple films to obtain spectral intensities varying as M^2 was a key point in the Chu paper. Also, foils separated by a vacuum yield two orders of magnitude greater intensity than striated media.

One objective of this paper is to apply the prebunched electron beam-vacuum beam tunnel technique used in the CR study and experiment to the TR problem. Similar to the CR problem, the TR can also be realized by passing the charges near as well as through the dielectric structure. The vacuum tunnel will again provide for a long interaction length and a low voltage keV beam to obtain detectable power.

There will be similarities and differences in the CR and TR problems driven by bunched beams. For the CR case, coherent radiation will be produced at all harmonic frequencies where the beam velocity exceeds the phase velocity of the medium. A periodic dielectric medium will also have pass bands. Hence for the TR case only harmonic frequencies in the pass bands can be produced. Also the periodic dielectric medium will produce an infinite set of coupled TM mode fields all with different phase velocities. In the CR case, only a single TM mode field is produced. Needless to say the TR problem, with its many new features, is more challenging to analyze than the CR problem.

As will be seen directly, the TR problem is more complicated to solve than the CR problem. The periodic medium will result in an infinite number of coupled TM mode fields to be produced precluding a compact closed form solution like those obtained for the CR problem. Also to represent a layer dielectric structure requires an infinite Fourier series. To minimize the math but still display the basic physics of the TR problem, only three terms will be used in both the Fourier series and TM fields. This will mean that these approximations will underestimate the power that can be produced.

The problem of computing the power expected from an infinite set of coupled TM mode fields of the same frequency is tedious. If one were to choose to consider a finite number of fields as few as three for example, it would not change the basic physics of the problem, only the accuracy of the results. From a practical point of view, if three terms in the

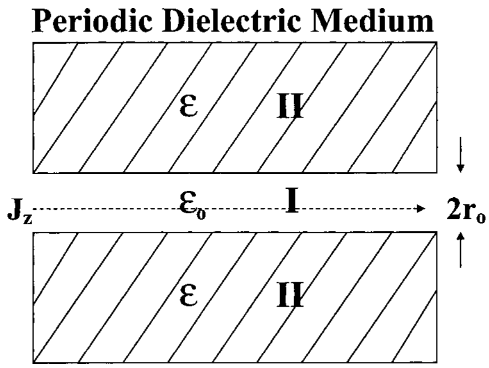

FIG. 1. Prebunched electron beam injected into a vacuum beam tunnel in a periodic dielectric medium.

series would yield negligible power, it is unlikely that adding more terms would save the problem since the series must converge.

In this paper, only two and three field models have been analyzed. In the numerical calculations frequencies in the 35 GHz range were arbitrarily chosen in order to perform possible experiments to verify the theoretical calculation.

II. COUPLED FIELD ANALYSIS

The structure to be analyzed is shown in Fig. 1. It consists of a periodic dielectric containing a beam tunnel of radius r_o into which a prebunched ac beam is injected. It is assumed that the beam fills the beam tunnel so that the problem to be analyzed is a two-region Maxwell boundary value problem. End effects are neglected at the beginning and end of the tunnel.

The dielectric and beam current density will be described by the equations

$$\varepsilon=\xi_o+2\varepsilon_1\cos(kz)=\xi_o+\varepsilon_1 e^{ikz}+\varepsilon_{-1}e^{-ikz}, \tag{1}$$

$$J_z=\rho u=J_o+\sum J_n e^{in\omega(t-z/u)}, \tag{2}$$

where ρ is the charge density and u is the beam velocity.

The working equation for the TR problem is obtained by separating the electric field E in the classical Maxwell equations [13],

$$\vec{\nabla}\times\vec{H}=i\omega\varepsilon\vec{E}+\vec{J},$$

$$\vec{\nabla}\cdot\overline{H}=0,$$

$$\vec{\nabla}\times\vec{E}=-i\omega\mu\vec{H}, \tag{3}$$

$$\vec{\nabla}\cdot(\varepsilon\vec{E})=\rho,$$

where a time variation $e^{-i\omega t}$ has been assumed for the fields,

$$\nabla^2\vec{E}+\omega^2\mu\varepsilon\vec{E}=i\omega\mu\vec{J}+\vec{\nabla}\left(\frac{\rho}{\varepsilon}\right)-\vec{\nabla}[\vec{E}\cdot\vec{\nabla}\ln\varepsilon]. \tag{4}$$

The analysis will apply equally to each beam harmonic, hence without loss of generality only the fundamental beam current will be considered.

The z component of the electric field obtained from Eq. (4) in cylindrical coordinates is given by the expression

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right)+\frac{\partial^2 E_z}{\partial z^2}+\left(\frac{\omega}{c}\right)^2 E_z=-i\omega\mu\left[\left(\frac{c}{u}\right)^2-1\right]J_1 \tag{5}$$

with solution for region I seen to be

$$E_{zc1}=\left[A_1 I_o(\alpha_{1c}r)+\frac{iJ_1}{\omega\varepsilon_o}\right]e^{-i(\omega/u)z}, \tag{6}$$

where $I_o(\alpha r)$ is the modified Bessel function [14] of the first kind of order zero.

The separation constant α_{1c}^2 is given by the relation

$$\alpha_{1c}^2=\left(\frac{\omega}{c}\right)^2\left[\left(\frac{c}{u}\right)^2-1\right]\geqslant 0. \tag{7}$$

In region II, the equation for E_z is from Eq. (4),

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right)+\frac{\partial^2 E_z}{\partial z^2}\right]+\omega^2\mu[\xi_o+\varepsilon_1 e^{ikz}+\varepsilon_{-1}e^{-ikz}]E_z$$
$$=-\frac{\partial}{\partial z}\left[E_z\frac{\partial}{\partial z}\ln\varepsilon\right]. \tag{8}$$

A part of the physics of the periodic dielectric medium can be seen immediately from this equation. If a field E_z varying as $e^{-i(w/u)z}$ were substituted into Eq. (8), fields varying as $e^{-i(w/u-k)z}$ and $e^{-i(w/u+k)z}$ would appear. If these fields were substituted back into the equation, more fields would appear. Thus the periodic dielectric medium would produce an infinite set of fields, in this case the TM mode fields. At this point only three terms will be used in all the series to limit the mathematical analysis required. This approach will not mask the physics of the problem and any calculations will underestimate the TR that can be produced.

The periodic dielectric medium using a three-field model will require two additional fields in region I of the form

$$E_{ztI}=B_1 I_o(\alpha_{1t}r)e^{-i(\omega/u-k)z} \tag{9}$$

and

$$E_{zsI}=C_1 I_o(\alpha_{1s}r)e^{-i(\omega/u+k)z} \tag{10}$$

having the separation constants

$$\alpha_{1t}^2=\left[\left(\frac{\omega}{u}-k\right)^2-\left(\frac{\omega}{c}\right)^2\right]>0 \tag{11}$$

and

$$\alpha_{1s}^2=\left[\left(\frac{\omega}{u}+k\right)^2-\left(\frac{\omega}{c}\right)^2\right]>0. \tag{12}$$

It is to be noted that α_{1c}^2 and α_{1s}^2 will remain positive for all ranges of ω, u, and k, but not α_{1t}^2. For a specific example, let $\omega = 2.2\times 10^{11}$, $u = 0.1c$, and $k = 2\pi\times 10^3$. Then the ranges for which α_{1t}^2 is negative is seen to be

$$1.71\times 10^{11} < \omega < 2.09\times 10^{11},$$
$$6600 < k < 8066, \tag{13}$$
$$0.1045 < \frac{u}{c} < 0.1332.$$

For these ranges the solutions for E_{ztI} become

$$E_{ztI} = B_1 J_o(\alpha_{1t} r) e^{-i(\omega/u - k)z}, \tag{14}$$

where

$$\alpha_{1t}^2 = \left[\left(\frac{\omega}{c}\right)^2 - \left(\frac{\omega}{u} - k\right)^2\right] > 0 \tag{15}$$

with the associated $H_{\theta tI}$ field given by

$$H_{\theta tI} = \frac{i\omega\varepsilon_o}{\alpha_{1t}} B_1 J_1(\alpha_{1t} r) e^{-i(\omega/u - k)z}. \tag{16}$$

The three E_z fields of Eqs. (9) and (10) will be associated with three H_θ and E_r fields through Maxwell's equations to form a TM set of fields. No TE fields are excited by the prebunched beam and there will be no coupling of the TM and TE modes. This lack of coupling between TM and TE modes is a property of the periodic dielectric medium [15].

$$H_{\theta cI} = \frac{i\omega\varepsilon_o}{\alpha_{1c}} A_1 I_1(\alpha_{1c} r) e^{-i\omega/uz}, \tag{17}$$

$$H_{\theta tI} = \frac{i\omega\varepsilon_o}{\alpha_{1t}} B_1 I_1(\alpha_{1t} r) e^{-i(\omega/u - k)z}, \tag{18}$$

$$H_{\theta sI} = \frac{i\omega\varepsilon_o}{\alpha_{1s}} C_1 I_1(\alpha_{1s} r) e^{-i(\omega/u + k)z}, \tag{19}$$

with

$$E_{rcI} = \frac{i\omega}{u\alpha_{1t}} A_1 I_1(\alpha_{1c} r) e^{-i\omega/uz}, \tag{20}$$

$$E_{rtI} = \frac{i}{\alpha_{1t}}\left(\frac{\omega}{u} - k\right) B_1 I_1(\alpha_{1t} r) e^{-i(\omega/u - k)z}, \tag{21}$$

$$E_{rsI} = \frac{i}{\alpha_{1s}}\left(\frac{\omega}{u} + k\right) C_1 I_1(\alpha_{1s} r) e^{-i(\omega/u + k)z}, \tag{22}$$

where $I_1(\alpha r)$ is the modified Bessel function of the first kind of order 1. The time factor $e^{i\omega t}$ has been deleted from the expressions [16].

Anticipating that the periodic dielectric medium will couple the various TM fields together in region II, a coupled field solution for E_{zII} will be taken in the form

$$\begin{aligned} E_{zII} = {} & [A_2 K_o(\alpha_{2c} r) + aB_2 H_o^{(2)}(\alpha_{2t} r) \\ & + cC_2 K_o(\alpha_{2s} r)] e^{-i\omega/uz} + [B_2 H_o^{(2)}(\alpha_{2t} r) \\ & + bA_2 K_o(\alpha_{2c} r) + dC_2 K_o(\alpha_{2s} r)] e^{-i(\omega/u - k)z} \\ & + [C_2 K_o(\alpha_{2s} r) + eA_2 K_o(\alpha_{2c} r) \\ & + fB_2 H_o^{(2)}(\alpha_{2t} r)] e^{-i(\omega/u + k)z}, \end{aligned} \tag{23}$$

where only the $B_2 H_o^{(2)}(\alpha_{2t} r)$ field is above threshold [17]. The "constants" a, b, c, d, e, and f are the field coupling constants, which of course will vary with the parameters of the problem. For the solutions given for the fields to hold, the separation constants must satisfy the conditions $\alpha_{2c}^2 > 0$, $\alpha_{2t}^2 > 0$, and $\alpha_{2s}^2 > 0$. These conditions will define pass and stop bands for radiation, again a property of a periodic medium [15]. $K_o(\alpha r)$ is the modified Bessel function of the second kind while $H_o^{(2)}(\alpha r) = J_o(\alpha r) - iN_o(\alpha r)$ is the Bessel function of the third kind (Hankel function) of the order zero. For radiation to occur a Hankel function must be present. The expression for E_{zII} given by Eq. (23) is only the first three terms in the infinite set of TM mode fields of the TR problem. Nevertheless, it is a valid partial solution since it will be seen to satisfy Maxwell's equations and the boundary conditions. The solution will yield a lower bound on the power that can be generated.

The field varying as $e^{-i\omega/uz}$ is termed c. It would be the Cerenkov field if the medium were not periodic. The fields varying as $e^{-i(\omega/u - nk)z}$ and $e^{-i(\omega/u + nk)z}$ are the TR fields which have been labeled t and s, respectively. All are at the same frequency.

It is of interest to observe how the bunched electron beam excites the TR fields, only one of which is above threshold. For the bunched beam to interact with a travelling wave electric field over an appreciable distance, the electron velocity and field phase velocity must be nearly matched. This is true only for the Cerenkov field. Hence, the beam excites the Cerenkov field in region I which in turn excites the Cerenkov field in region II. In region II the Cerenkov field couples to the t and s TR fields, one of which is above threshold. The coupling of all the fields will cause radiation from all the three fields through terms like $aB_2H_0^{(2)}$ and $fB_2H_0^{(2)}$.

Next consider the term $\partial/\partial z(\ln\varepsilon)$ in Eq. (8). It is a periodic function of kz, i.e.,

$$\frac{\partial}{\partial z}(\ln\varepsilon) = \frac{1}{\varepsilon}\frac{\partial\varepsilon}{\partial z} = \frac{ik\varepsilon_1 e^{ikz}}{\varepsilon} - \frac{ik\varepsilon_{-1} e^{-ikz}}{\varepsilon} \tag{24}$$

that can be expanded in a Fourier series,

$$\frac{1}{\varepsilon}\frac{\partial\varepsilon}{\partial z} = c_1 e^{ikz} - c_1 e^{-ikz} + \cdots. \tag{25}$$

The first terms in the series are

$$ic_1 = g \approx \frac{-k\varepsilon_1}{[\xi_o^2 - (2\varepsilon_1)^2]^{1/2}}. \tag{26}$$

If an E_z field varying as $e^{-i(\omega/u)z}$ were substituted into the term on the right side of Eq. (8), the following expression would appear

$$-\frac{\partial}{\partial z}[e^{-i(\omega/u)z}(C_1e^{ikz}-C_1e^{-ikz})]$$
$$=g\left(\frac{\omega}{u}-k\right)e^{-i(\omega/u-k)z}-g\left(\frac{\omega}{u}+k\right)e^{-i(\omega/u+k)z}. \quad (27)$$

Thus, the $-(\partial/\partial z)[E_z(\partial/\partial z)(\ln\varepsilon)]$ term generates TM fields in the same manner as the ε term. For CR in a nonperiodic medium a single field in both regions I and II would exist and the radiation problem can be solved conveniently without the need to consider an infinite series of fields or make any approximations.

The separation constants α_{2c}^2, α_{2t}^2, and α_{2s}^2 and the coupling constants are obtained by substituting E_{zII} into Eq. (8) and equating terms in $e^{-i(\omega/u)z}$, $e^{-i(\omega/u-k)z}$, and $e^{-i(\omega/u+k)z}$ to zero.

To illustrate the procedure, consider the $e^{-i(\omega/u)z}$ field and just the $A_2K_o(\alpha_{2c}r)$ term to obtain

$$\frac{1}{r}\frac{d}{dr}[rA_2K_o(\alpha_{2c}r)]+\omega^2\mu\xi_oA_2K_o(\alpha_{2c}r)$$
$$+b\omega^2\mu\varepsilon_{-1}A_2K_o(\alpha_{2c}r)+e\omega^2\mu\varepsilon_1A_2K_o(\alpha_{2c}r)$$
$$-\left(\frac{\omega}{u}\right)^2A_2K_o(\alpha_{2c}r)+\frac{\omega g}{u}bA_2K_o(\alpha_{2c}r)$$
$$-\frac{\omega g}{u}cA_2K_o(\alpha_{2c}r)=0. \quad (28)$$

Collecting terms and using the Bessel relation $(1/r)(d/dr)\times[rK_o(\alpha_{2c}r)]=\alpha_{2c}^2K_o$ the result is

$$\left[\alpha_{2c}^2+\omega^2\mu\xi_o-\left(\frac{\omega}{u}\right)^2\right]+b\left[\omega^2\mu\varepsilon_1+\frac{\omega}{u}g\right]$$
$$+e\left[\omega^2\mu\varepsilon_1-\frac{\omega}{u}g\right]=0. \quad (29)$$

In a similar fashion, one can obtain the remaining eight equations

$$b\left[\alpha_{2c}^2+\omega^2\mu\xi_o-\left(\frac{\omega}{u}-k\right)^2\right]+\omega^2\mu\varepsilon_1-\left(\frac{\omega}{u}-k\right)g=0, \quad (30)$$

$$e\left[\alpha_{2c}^2+\omega^2\mu\xi_o-\left(\frac{\omega}{u}+k\right)^2\right]+\omega^2\mu\varepsilon_1+\left(\frac{\omega}{u}+k\right)g=0, \quad (31)$$

$$d\left[\alpha_{2s}^2+\omega^2\mu\xi_o-\left(\frac{\omega}{u}-k\right)^2\right]+c\left[\omega^2\mu\varepsilon_1-\left(\frac{\omega}{u}-k\right)g\right]=0, \quad (32)$$

$$\left[\alpha_{2t}^2-\omega^2\mu\xi_o+\left(\frac{\omega}{u}-k\right)^2\right]-a\left[\omega^2\mu\varepsilon_1-\left(\frac{\omega}{u}-k\right)g\right]=0, \quad (33)$$

$$\left[\alpha_{2s}^2+\omega^2\mu\xi_o-\left(\frac{\omega}{u}+k\right)^2\right]+c\left[\omega^2\mu\varepsilon_1+\left(\frac{\omega}{u}+k\right)g\right]=0, \quad (34)$$

$$f\left[\alpha_{2t}^2-\omega^2\mu\xi_o+\left(\frac{\omega}{u}+k\right)^2\right]-a\left[\omega^2\mu\varepsilon_1+\left(\frac{\omega}{u}+k\right)g\right]=0, \quad (35)$$

$$a\left[\alpha_{2t}^2-\omega^2\mu\xi_o+\left(\frac{\omega}{u}\right)^2\right]-f\left[\omega^2\mu\varepsilon_1-\frac{\omega}{u}g\right]$$
$$-\left[\omega^2\mu\varepsilon_1+\frac{\omega}{u}g\right]=0, \quad (36)$$

$$c\left[\alpha_{2s}^2+\omega^2\mu\xi_o-\left(\frac{\omega}{u}\right)^2\right]+d\left[\omega^2\mu\varepsilon_1+\frac{\omega}{u}g\right]$$
$$+\left[\omega^2\mu\varepsilon_1-\frac{\omega}{u}g\right]=0. \quad (37)$$

These nine equations are the dispersion equations for the periodic medium problem.

The $H_{\theta II}$ field can be determined from the Maxwell equation

$$\frac{1}{r}\frac{\partial}{\partial r}(rH_o)=i\omega[\xi_o+\varepsilon_1e^{ikz}+\varepsilon_{-1}e^{-ikz}]E_z \quad (38)$$

using the Bessel relations

$$\frac{1}{r}\frac{d}{dr}[rK_1(\alpha r)]=-\alpha K_o(\alpha r) \quad (39)$$

and

$$\frac{1}{r}\frac{d}{dr}[rH_1^{(2)}(\alpha r)]=\alpha H_o^{(2)}(\alpha r), \quad (40)$$

to be the following expression:

$$\frac{H_{\theta\Pi}}{i\omega}=\left\{-\left(\frac{\xi_o+b\varepsilon_1+e\varepsilon_1}{\alpha_{2c}}\right)A_2K_1(\alpha_{2c}r)\right.$$
$$+\left(\frac{a\xi_o+\varepsilon_{-1}+f\varepsilon_1}{\alpha_{2T}}\right)B_2H_1^{(2)}(\alpha_{2t}r)$$
$$\left.-\left(\frac{c\xi_o+d\varepsilon_1+\varepsilon_1}{\alpha_{2s}}\right)C_2K_1(\alpha_{2s}r)\right\}e^{-i(\omega/u)z}$$
$$+\left\{-\left(\frac{b\xi_o+\varepsilon_1}{\alpha_{2c}}\right)A_2K_1(\alpha_{2c}r)+\left(\frac{\xi_o+\varepsilon_1}{\alpha_{2t}}\right)B_2H_1^{(2)}\right.$$
$$\left.\times(\alpha_{2t}r)-\left(\frac{d\xi_o+c\varepsilon_1}{\alpha_{2s}}\right)C_2K_1(\alpha_{2s}r)\right\}e^{-i(\omega/u-k)z}$$

$$+\left\{-\left(\frac{e\xi_o+\varepsilon_1}{\alpha_{2c}}\right)A_2K_1(\alpha_{2c}r)+\left(\frac{f\xi_o+\varepsilon_1}{\alpha_{2t}}\right)B_2H_1^{(2)}\times(\alpha_{2t}r)-\left(\frac{\xi_o+c\varepsilon_1}{\alpha_{2s}}\right)C_2K_1(\alpha_{2s}r)\right\}e^{-i(\omega/u+k)z}. \quad (41)$$

The final Maxwell chore is to satisfy the boundary conditions on E_z and H_θ at $r=r_o$, to determine the field amplitudes A_1, B_1, C_1, A_2, B_2, and C_2. These six equations for the complex amplitudes will take the form

$$-D_1A_1+D_2A_2+D_3B_2+D_4C_2=i\frac{J_1}{\omega\varepsilon o}, \quad (42)$$

$$D_5A_2-D_6B_1+D_7B_2+D_8C_2=0, \quad (43)$$

$$D_9A_2+D_{10}B_2-D_{11}C_1+D_{12}C_2=0, \quad (44)$$

$$D_{13}A_1+D_{14}A_2-D_{15}B_2+D_{16}C_2=0, \quad (45)$$

$$D_{17}A_2+D_{18}B_1-D_{19}B_2+D_{20}C_2=0, \quad (46)$$

$$D_{21}A_1-D_{22}B_2+D_{23}C_1+D_{24}C_2=0, \quad (47)$$

where the dimensionless quantities D are given by the expressions

$$D_1=I_o(\alpha_{1c}r_o), \quad D_5=bK_o(\alpha_{2c}r_o), \quad D_9=eK_o(\alpha_{2c}r_o),$$

$$D_2=K_o(\alpha_{2c}r_o), \quad D_6=\left\{\begin{matrix} I_o(\alpha_{1t}r_o) \\ J_o(\alpha_{1t}r_o)\end{matrix}\right\},$$

$$D_{10}=fH_o^{(2)}(\alpha_{2t}r_o),$$

$$D_3=aH_o^{(2)}(\alpha_{2t}r_o), \quad D_7=H_o^{(2)}(\alpha_{2t}r_o), \quad D_{11}=I_o(\alpha_{1s}r_o),$$

$$D_4=cK_o(\alpha_{2s}r_o), \quad D_8=cK_o(\alpha_{2s}r_o), \quad D_{12}=K_o(\alpha_{2s}r_o),$$

$$D_{13}=I_1(\alpha_{1c}r_o), \quad D_{19}=\frac{\alpha_{1t}}{\alpha_{2t}}\left(\frac{\xi_o+a\varepsilon_1}{\varepsilon_o}\right)H_1^{(2)}(\alpha_{2t}r_o),$$

$$D_{14}=\frac{\alpha_{1c}}{\alpha_{2c}}\left(\frac{\xi_o+b\varepsilon_1+e\varepsilon_1}{\varepsilon_o}\right)K_o(\alpha_{2c}r_o),$$

$$D_{20}=\frac{\alpha_{1t}}{\alpha_{2s}}\left(\frac{\xi_o+c\varepsilon_1}{\varepsilon_o}\right)K_1(\alpha_{2s}r_o),$$

$$D_{15}=\frac{\alpha_{1c}}{\alpha_{2t}}\left(\frac{a\xi_o+\varepsilon_1+f\varepsilon_1}{\varepsilon_o}\right)H_1^{(o)}(\alpha_{2T}r_o),$$

$$D_{21}=\frac{\alpha_{1s}}{\alpha_{2c}}\left(\frac{e\xi_o+\varepsilon_1}{\varepsilon_o}\right)K_1(\alpha_{2c}r_o),$$

$$D_{16}=\frac{\alpha_{1c}}{\alpha_{2s}}\left(\frac{a\xi_o+d\varepsilon_1+\varepsilon_1}{\varepsilon_o}\right)K_1(\alpha_{2s}r_o),$$

$$D_{22}=\frac{\alpha_{1s}}{\alpha_{2t}}\left(\frac{f\xi_o+a\varepsilon_1}{\varepsilon_o}\right)H_1^{(2)}(\alpha_{2t}r_o),$$

$$D_{17}=\frac{\alpha_{1t}}{\alpha_{2c}}\left(\frac{b\xi_o+\varepsilon_1}{\varepsilon_o}\right)K_1(\alpha_{2c}r_o), \quad D_{23}=I_1(\alpha_{1s}r_o),$$

$$D_{18}=\left\{\begin{matrix} I_1(\alpha_{1t}r_o) \\ J_1(\alpha_{1t}r_o)\end{matrix}\right\}, \quad D_{24}=\frac{\alpha_{1s}}{\alpha_{2s}}\left(\frac{\xi_o+c\varepsilon_1}{\varepsilon_o}\right)K_1(\alpha_{2s}r_o).$$

These expressions hold only when the all the squares of the separation constants are positive. When a squared separation constant is negative, the Bessel function changes from a modified to a regular Bessel function and vice versa.

Finally the average power radiated per meter is given by the equation

$$\frac{dP_{av}}{dz}=\frac{1}{2}\mathrm{Re}\int_o^{r_o}E_{zI}J_z^*2\pi r dr. \quad (48)$$

Integration yields the compact result

$$\frac{dP_{av}}{dz}=\frac{\pi r_oJ_1}{\alpha_{1c}}I_1(\alpha_{1c}r_o)\mathrm{Re}(A_1). \quad (49)$$

CR and TR are stated as a result of charges moving with constant velocity through a medium faster than the phase velocity or through a medium having a spacial or time varying property. Equation (48) indicates that if the field is out of phase with the current, it will decelerate the charge changing its velocity. A corner stone of Einstein-Maxwell theory is that the accelerated charges radiate. Thus the assumption of constant velocity is not strictly valid. The physics of this apparent dilemma is that for the CR and TR small decelerations occur. The deceleration radiation is negligible compared to the CR or TR. The assumption of constant velocity is a valid approximation and the power calculated is ‘‘pure’’ TR.

III. NUMERICAL EXAMPLE OF EXPECTED RADIATION

In this section a specific numerical example will be presented with two objectives, (1) a tentative design of a 35 GHz experiment and (2) extract the physics of the problem from the lengthy equations and formulas. Also the energy of the electron beam will be restricted to the 3–10 keV range to demonstrate that readily detectable TR power can be achieved in a ‘‘table top’’ experiment.

There are six parameters (u/c, k, ω, ξ_o, $2\varepsilon_1$, and r_o) in the problem to be chosen. Values of $\xi_o=12\varepsilon_o$ and $\varepsilon_1=2\varepsilon_o$ were chosen with a periodic dielectric of layered semiconductors in mind; r_o is taken as 10^{-3} m. Larger values of r_o would appreciably increase the power output since more charges would be moving over an increased surface area. The beam velocities chosen are well below the Cerenkov threshold.

Figures 2, 3, and 4 display $-(1/J_1^2)(dP_{av}/dz)$ versus u/c, k, and ω over the pass band range where α_{2t}^2 is positive. The $(-)$ sign is associated with power being produced. This differs from the case of a charge passing from a vacuum into

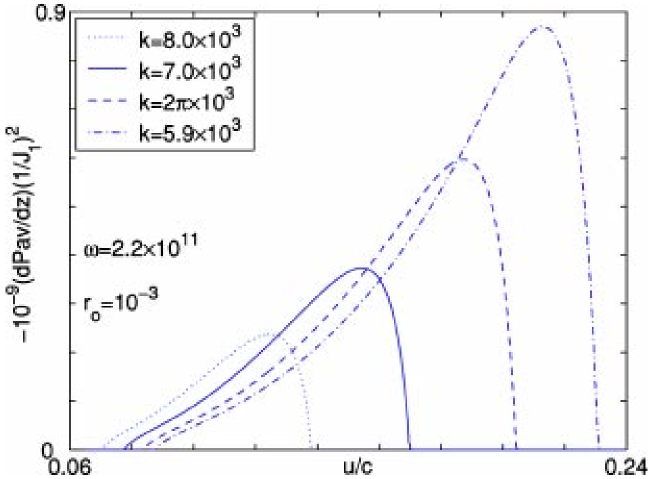

FIG. 2. Power term $-(1/J_1^2)(dP_{av}/dz)$ versus velocity ratio u/c for a range of k values with $\omega=2.2\times 10^{11}$ constant.

a dielectric where TR is produced for all charge velocities.

While there are peaks in the power in all the three curves, this does not imply that resonances exist. Power either increases or decreases monotonically from the bottom or top of the bands falling to zero at the band edges. The quantity ω/ku appears in most of the calculations. Power output increases as ω/ku decreases.

The coupling constants shown in Figs. 5, 6, 7, and 8 also change monotonically across the bands with no resonances. Similarly the values of $|A_2|^2$, $|B_2|^2$, and $|C_2|^2$ change monotonically with $|A_2|^2$ and $|C_2|^2$, being larger at the top of the band while $|B_2|^2$ is larger at the bottom of the band.

Peak values of $-(1/J_1^2)(dP_{av}/dz)$ vary from 4.3×10^{-10} to 9.0×10^{-10}, the larger value being associated with $u/c=0.214$ and $k=5.0\times 10^3$ with $\omega=2.2\times 10^{11}$.

Assuming the beam is well bunched $J_1=2J_o$, the power generated per meter is given by the equation

$$-\frac{dP_{av}}{dz}=3.6\times 10^{-9}\left(\frac{\vec{I}_0}{\pi r_o^2}\right)\text{W/m}, \tag{50}$$

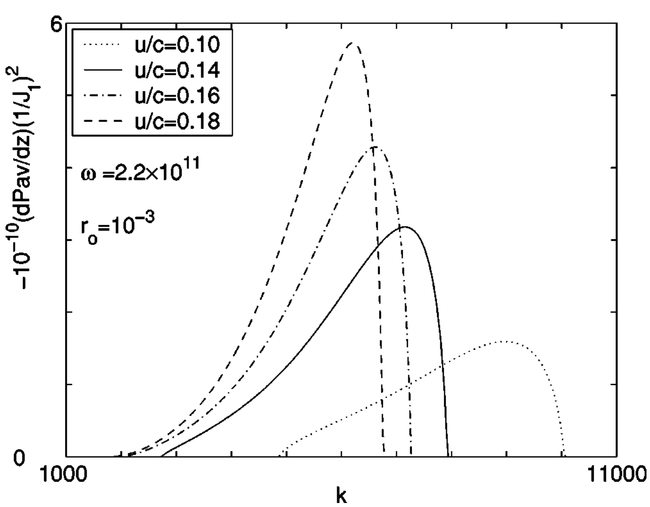

FIG. 3. Power term $-(1/J_1^2)(dP_{av}/dz)$ versus k for a range of u/c values with $\omega=2.2\times 10^{11}$ constant.

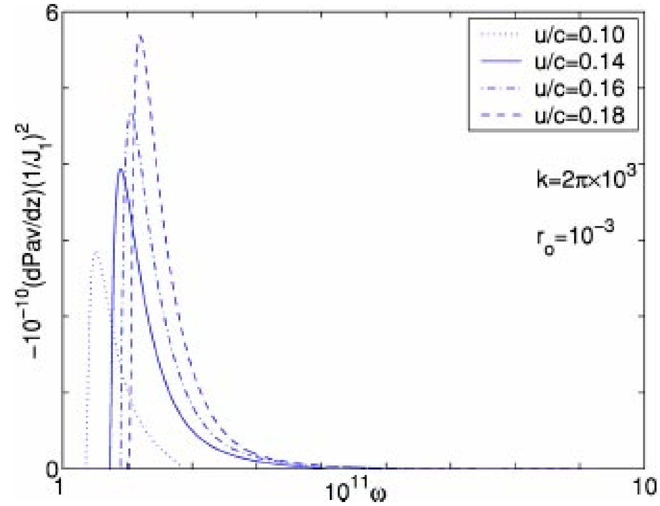

FIG. 4. Power term $-(1/J_1^2)(dP_{av}/dz)$ versus $\omega=2\pi f$ for a range of u/c values with $k=2\pi\times 10^3$ constant.

where $\vec{I}_0$ is the average dc current at 11.23 keV. Assuming a dc beam current of I=0.020 A with the tunnel radius $r_o=10^{-3}$ m and $\omega/2\pi=35.01$ GHz, the calculated coherent power is

$$-\frac{dP_{av}}{dz}=1.46\ \text{mW/cm}. \tag{51}$$

In these calculations the dielectric has been assumed to have no attenuation. Obviously, a lossy dielectric will attenuate the TR radiation. However, for example, GaAs [20] will have an attenuation of less than 0.5 cm^{-1} at 300 K up to a THz, hence attenuation in the periodic medium should not pose a major problem.

Phase and group velocities, Bragg reflection

Consider the E_{zII} field given by Eq. (23) for large r where the $K_o(\alpha r)$ terms can be neglected and

$$H_o^{(2)}(\alpha_{2t}r)\rightarrow\sqrt{\frac{2}{\pi\alpha_{2t}r}}e^{-i\pi/4}e^{-i\alpha_{2t}r}. \tag{52}$$

Then the far field value of E_{zII} becomes

$$\begin{aligned}E_{zII}\frac{e^{i\pi/4}}{B_2}\sqrt{\frac{\pi\alpha_{2t}r}{2}}&=ae^{i[\omega t-\alpha_{2t}r-(\omega/u)z]}\\&+e^{i[\omega t-\alpha_{2t}r-(\omega/u-k)z]}\\&+fe^{i[\omega t-\alpha_{2t}r-(\omega/u+k)z]}.\end{aligned} \tag{53}$$

Each of the three waves has a phase of the form

$$\psi=\omega t-\vec{\beta}\cdot\vec{r}=\omega t-\beta_r r-\beta_z z. \tag{54}$$

Constant phase ψ with time t requires a phase velocity $u_p=\omega/\beta$ with components

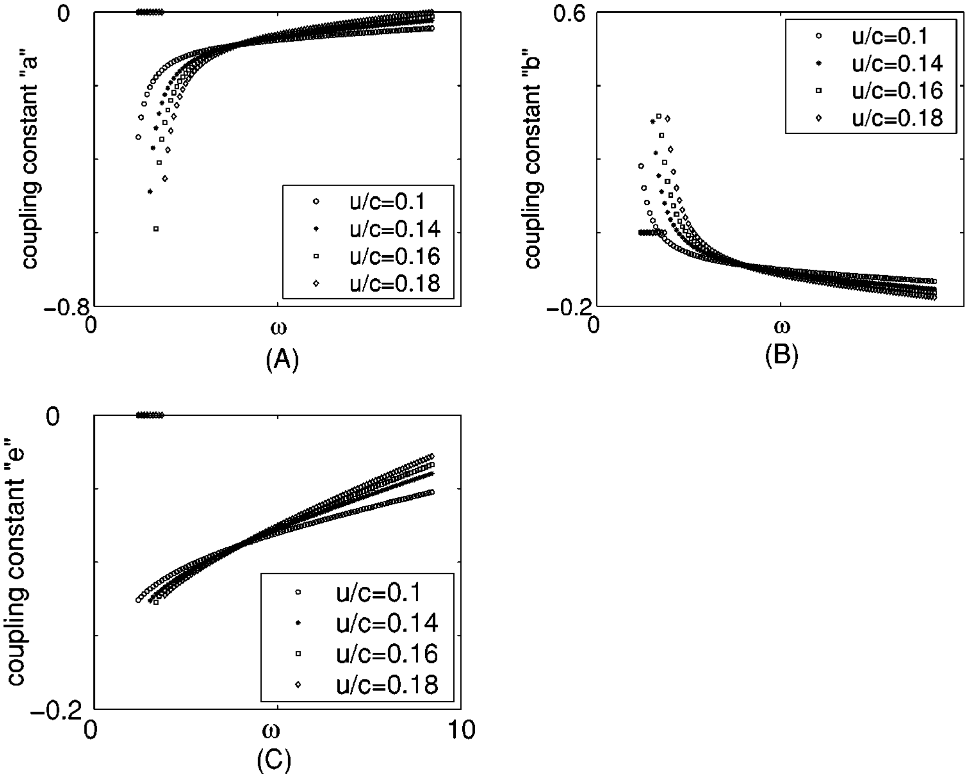

FIG. 5. Coupling constants a, b, and e versus angular frequency $\omega = 2\pi f$ for a range of u/c values with $k = 2\pi\times10^3$ constant.

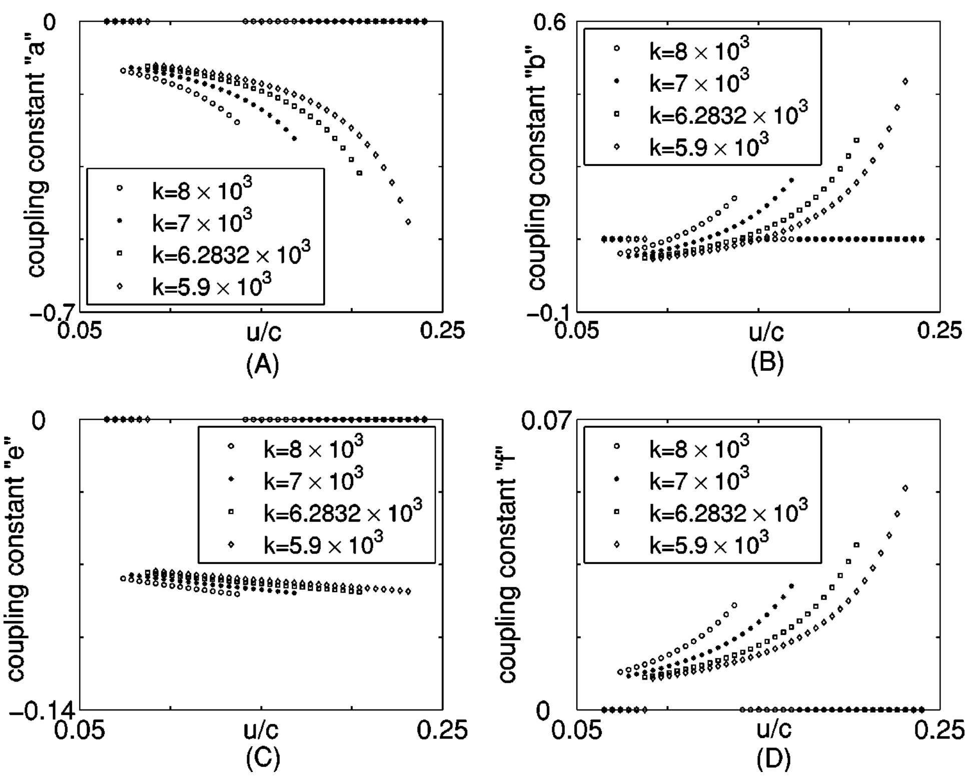

FIG. 6. Coupling constants a, b, e, and f versus beam velocity ratio u/c for a range of k with $\omega = 2.2\times10^{11}$ constant.

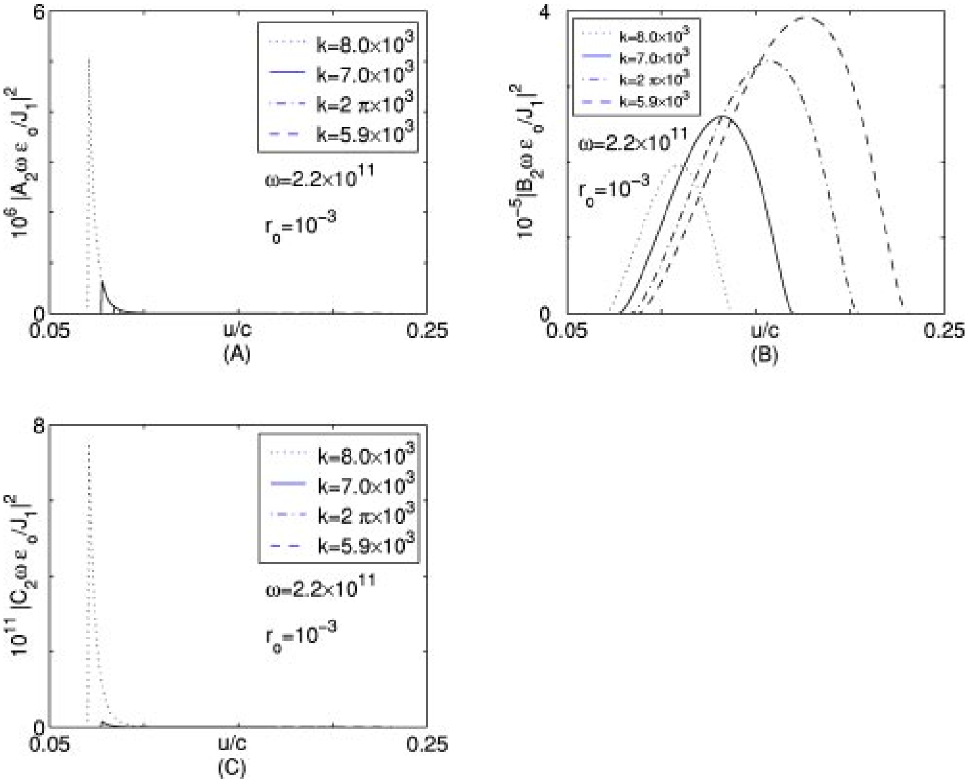

FIG. 7. Absolute squared values of $|A_2|^2$, $|B_2|^2$, and $|C_2|^2$ versus beam velocity ratio u/c for a range of values of k with $\omega = 2.2 \times 10^{11}$ constant.

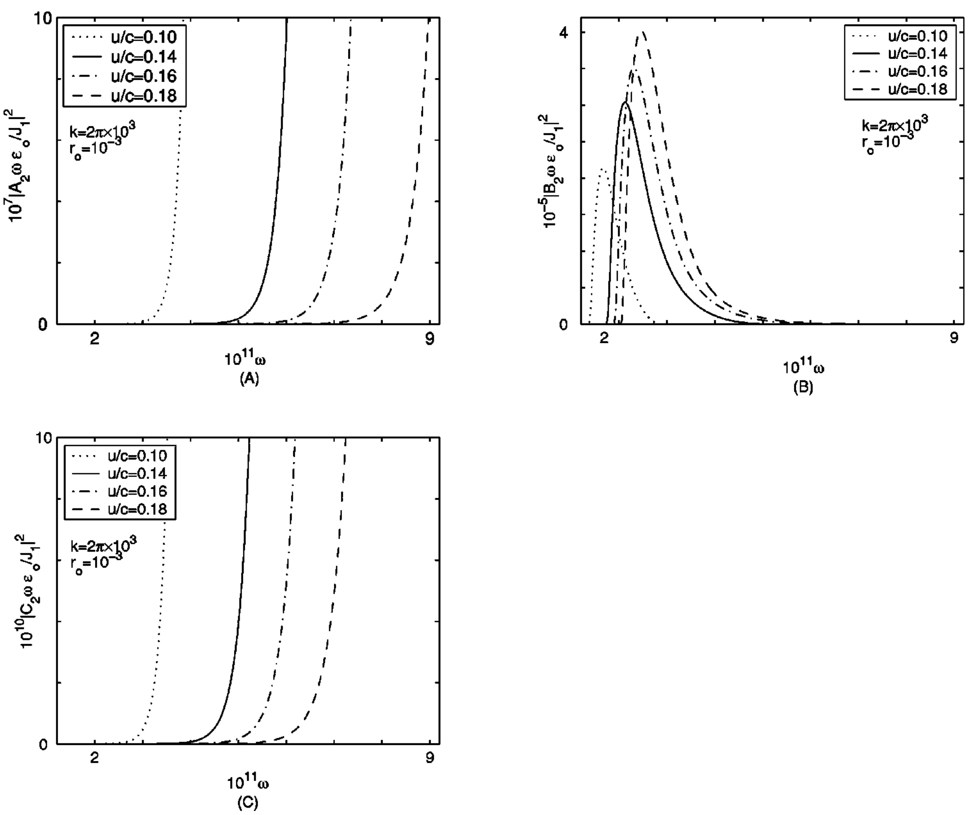

FIG. 8. Absolute squared values of $|A_2|^2$, $|B_2|^2$, and $|C_2|^2$ versus angular frequency $\omega = 2\pi f$ for a range of beam velocity ratios u/c with $k = 2\omega \times 10^3$ constant.

$$(u_p)_r=\frac{\omega\beta r}{\beta^2},\quad (u_p)_z=\frac{\omega\beta z}{\beta^2}, \tag{55}$$

$$(u_{pc})_r=\frac{\omega\alpha_{2t}}{\alpha_{2t}^2+\left(\frac{\omega}{u}\right)^2},\quad (u_{pc})_z=\frac{\omega^2/u}{\alpha_{2t}^2+\left(\frac{\omega}{u}\right)^2}. \tag{56}$$

Hence the phase-velocity components for the three waves are seen to be

$$(u_{pt})_r=\frac{\omega\alpha_{2t}}{\alpha_{2t}^2+\left(\frac{\omega}{u}-k\right)^2},\quad (u_{pt})_z=\frac{\omega\left(\frac{\omega}{u}-k\right)}{\alpha_{2t}^2+\left(\frac{\omega}{u}-k\right)^2}, \tag{57}$$

$$(u_{ps})_r=\frac{\omega\alpha_{2t}}{\alpha_{2t}^2+\left(\frac{\omega}{u}+k\right)^2},\quad (u_{ps})_z=\frac{\omega\left(\frac{\omega}{u}+k\right)}{\alpha_{2t}^2+\left(\frac{\omega}{u}+k\right)^2}. \tag{58}$$

All velocity components are positive (in the z direction) except $(u_{pt})_z$ when $k>\omega/u$.

The angles between the phase-velocity components are

$$\cos\theta_c=\frac{\omega/u}{\left[\alpha_{2t}^2+\left(\frac{\omega}{u}\right)^2\right]^{1/2}},\quad \cos\theta_t=\frac{\omega/u-k}{\left[\alpha_{2t}^2+\left(\frac{\omega}{u}-k\right)^2\right]^{1/2}},$$

$$\cos\theta_s=\frac{\omega/u+k}{\left[\alpha_{2t}^2+\left(\frac{\omega}{u}+k\right)^2\right]^{1/2}}. \tag{59}$$

Given the data $\omega=2.2\times10^{11}$, $u/c=0.18$, $k=2\pi\times10^3$, $\xi_0=12\epsilon_0$ and $\epsilon_1=\epsilon_{-1}=2\epsilon_0$, as an example, the various angles are computed to be $\theta_c=17.1°$, $\theta_t=150.4°$, $\theta_s=6.92°$.

The group velocity $\vec{u}_g$ is given by the equation

$$\vec{u}_g=\frac{\partial\omega}{\partial\beta_r}\hat{r}+\frac{\partial\omega}{\partial\beta_z}\hat{z}. \tag{60}$$

To a good approximation, Eqs. (30), (33), and (36) yield

$$\alpha_{2c}^2\cong\left(\frac{\omega}{u}\right)^2-\omega^2\mu\xi_o=\beta_{zc}^2-\omega^2\mu\xi_o, \tag{61}$$

$$\alpha_{2t}^2\cong\left(-\frac{\omega}{u}-k\right)^2+\omega^2\mu\xi_o=\beta_{rt}^2=\omega^2\mu\xi_o-\beta_{2t}^2, \tag{62}$$

$$\alpha_{2s}^2\cong\left(\frac{\omega}{u}+k\right)^2-\omega^2\mu\xi_o=\beta_{Zs}^2-\omega^2\mu\xi_o. \tag{63}$$

Using these dispersion relationships, one can calculate the various components of the group velocity for each of the three waves to be

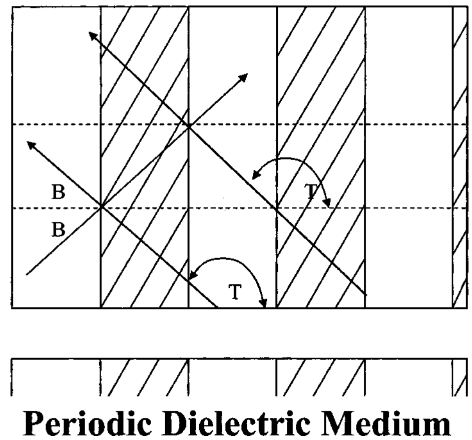

FIG. 9. Angles θ_T and θ_B associated with the Bragg reflection.

$$(u_{gt})_r=\frac{2\omega}{\beta_{rt}}=\frac{\beta_{ct}}{\omega\mu\xi_o}=(u_{gc})_r=(u_{gs})_r, \tag{64}$$

$$(u_{gc})_z=\frac{2\omega}{\beta_{zt}}=\frac{\beta_{zt}}{\omega\mu\xi_o}, \tag{65}$$

$$(u_{gt})_z=\frac{2\omega}{\beta_{zc}}=\frac{\beta_{zc}}{\omega\mu\xi_o}, \tag{66}$$

$$(u_{gs})_z=\frac{2\omega}{\beta_{zs}}=\frac{\beta_{zs}}{\omega\mu\xi_o}. \tag{67}$$

The angles between the group-velocity components are the same as between the phase-velocity components as in Eq. (60).

Since the Poynting vector and group velocity are in the same direction, the angle between the Poynting vector components are the same as given by Eq. (60).

As seen from Eq. (53), the coupling of fields will result in radiation being produced by three fields with three different phase constants even though only the "transition field" $\exp[-i(\omega/u-k)z]$ is above threshold. Two of the fields propagate in the positive z direction with the transition field propagating in the negative z direction for $k>\omega/u$. Each field will have its own cone of radiation as given by Eq. (59).

Consider Fig. 9 and the cosines of the two angles θ_B and θ_T. Using the values

$$\lambda_p=\frac{2\pi v_p}{\omega}=\frac{2\pi}{\omega\sqrt{\mu\xi_o}}\quad\text{and}\quad k=2\pi/l, \tag{68}$$

Eq. (59) can be rearranged into the following Bragg form for a layered medium:

$$2d\cos\theta_b=l\cos\theta_b=\lambda_p\left[1-\frac{\omega}{ku}\right]=\lambda_{eff}. \tag{69}$$

Note that for the Bragg reflection to occur, that $\omega/ku<1$ and $\lambda_{eff}<l$, i.e., the transition field must propagate in the negative z direction.

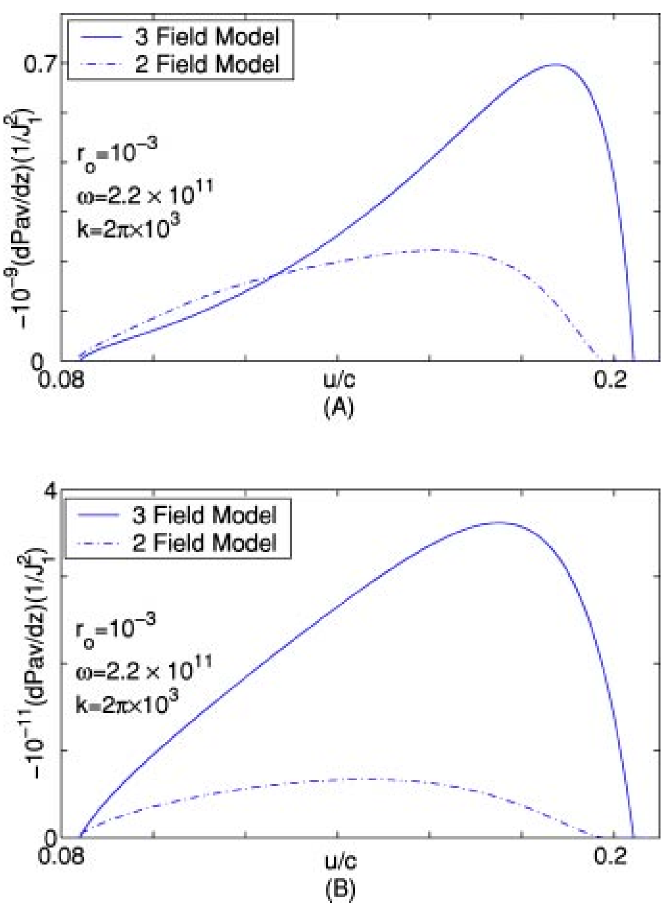

FIG. 10. Power term $-(1/J_1^2)(dP_{av}/dz)$ versus velocity ratio u/c for $\omega=2.2\times10^{11}$, and $k=2\pi\times10^3$ for $r_o=10^{-3}$ m and $r_o=3\times10^{-4}$ m and for the three- and two-field model.

The effect of changing the tunnel radius r_o and deleting the field varying as $\exp[-i(\omega+k/u)z]$ for a two-field model is displayed in Figs. 10(a) and 10(b). The peak power term for $r_o=10^{-3}$ m is 20 times that for $r_o=3\times10^{-4}$ m. Increasing r_o leads to more charges passing over a larger surface area. Including only two fields (Cerenkov and Transition) decreases the peak power term by a factor of 3 from the three-field model as seen from Fig. 10(b). Adding more field terms should further increase the power term making a TR source a practical 1–10 mW/cm device.

IV. CONCLUSIONS

The numerical calculations indicate that the practical values of detectable single-frequency coherent TR can be generated in the millimeter region with a prebunched ac beam in the 3–10 keV range using a layered periodic dielectric medium. Both CR an TR can be generated simultaneously with relativistic beams . Since the CR will be emitted in the forward direction while the TR can be emitted in the backward direction when both effects are above threshold, the radiation can be conveniently separated experimentally.

Some insight on the power output as a function of the number of coupled TM fields included in the calculations can be gained from Fig. 10 for a two- and three-field model. These data are encouraging as they show that a three-field model yields a power increase of over two times that of a two-field model. All the data presented are lower bounds on the total power that can be produced.

The vacuum beam tunnel is of major importance in the TR problem. Without a beam tunnel, efficient excitation of the system coherently with a keV beam would be difficult. The interaction length would be negligible if a KeV beam were to pass through the material. Smith-Purcell [18], Cerenkov [19], and Transition Radiation can all be produced by an electron beam moving near a periodic metallic surface, a uniform or periodic dielectric surface. Palôcz and Oliner view the problem as "leaky" space charge waves. A cylindrical electron beam tunnel effectively couples the beam to the media and simplifies the Maxwell boundary analysis of the problem.

The TR problem of this paper can be viewed as a frequency multiplier. Radiation at frequency ω is used to bunch an electron beam to obtain beam harmonics at $n\omega$. The periodic dielectric structure is then used to couple to the beam harmonics to yield coherent radiation at $n\omega$. The coherence of the system should be comparable to that of any harmonic multiplication system.

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